

## Transmission Performance of $\mu \cong 255$ Quantization in a Local Digital Office

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*Analog-to-digital conversion is required to terminate voice frequency loops and analog facilities in a digital central office. We demonstrate the appropriateness of the  $\mu \cong 255$  quantization law for application in a local digital office. We show that the transmission performance for parameters determined by the choice of quantization law can all theoretically meet reasonable objectives with the  $\mu \cong 255$  law. However, the crosstalk coupling loss required of analog plant preceding the digital switch may need to be improved to retain an acceptably low probability of intelligible crosstalk. This improvement in analog crosstalk loss may be obviated by more stringent control of the quantizer bias, by maintaining a minimum noise on the disturbed channels, or by a combination of both of these.*

### I. INTRODUCTION

The piecewise uniform, logarithmically companded,  $\mu \cong 255$  quantizing characteristic is the logical choice for an analog-to-digital converter in a digital office because of its widespread use in the North American telephone network in digital facility terminals and No. 4 ESS switch applications. This paper describes the theoretical and practical voice transmission performance attainable with this quantization law and discusses the requirements imposed on the quantizer in order for it to meet reasonable transmission objectives for a local digital switch,<sup>1</sup> and the implications of introducing quantization on crosstalk performance.

We address in this paper only the transmission performance parameters that are affected by the choice of quantizer characteristic. These include tracking (net loss and overload), idle channel noise (noise in the absence of signal), crosstalk, and signal-to-distortion ratio. The nonlinear effects introduced into the above transmission parameters by quantization are examined.

The fact that the transmission performance attained is adequate using the  $\mu \cong 255$  quantization law in PCM carrier facilities does not prove that this law is adequate for use in a local digital switch. Because of differences in environment (e.g., less average loss from the decoder to the customer), some transmission objectives (e.g., idle channel noise) for the local office are more severe than they are for a carrier channel.

In the next section, the  $\mu \cong 255$  quantizer is specified, and in the following sections the theoretical and practical transmission performance of each of the above parameters are considered relative to reasonable transmission objectives suggested in Ref. 1.

## II. QUANTIZER DESCRIPTION

The  $\mu \cong 255$  quantizer is defined by specifying its decision and output levels. The relative magnitudes of the decision levels are given in Table I.<sup>2</sup> The corresponding output levels can be found using the equations following the table.

Ideally, the quantizer is biased so that a zero or low-level input signal (one whose amplitude does not exceed the first decision levels) produces steady zero output. Bias at an output level is referred to as mid-tread bias, i.e., bias ( $=0, 1, 2, \dots$ ) at the center of a (stairway) tread midway between two decision levels. For this bias condition, any low power input noise or crosstalk signal is eliminated by the quantizer. In a PCM system such as a D2-type channel bank,<sup>3</sup> the quantizer may be preceded by a multiplexer, which allows the quantizer to be shared by a number of channels. In concept, the quantizer can have an automatic biasing circuit, which maintains the bias for the composite signal from the multiplexer very near the ideal midtread position.

Table I—Encoder decision levels for 15-segment  
 $\mu \cong 255$  coding law

Level Number $\eta$	Level Magnitude* $ X_\eta $	Segment Number
0	0	1
$1 \leq \eta \leq 16$	$\eta - 0.5$	1
$17 \leq \eta \leq 32$	$2\eta - 16.5$	2
$33 \leq \eta \leq 48$	$4\eta - 80.5$	3
$49 \leq \eta \leq 64$	$8\eta - 272.5$	4
$65 \leq \eta \leq 80$	$16\eta - 784.5$	5
$81 \leq \eta \leq 96$	$32\eta - 2064.5$	6
$97 \leq \eta \leq 112$	$64\eta - 5136.5$	7
$113 \leq \eta \leq 128$	$128\eta - 12304.5$	8

$$Y_0 = X_0 = 0 \quad \eta = 0.$$

$$Y_\eta = \frac{X_\eta + X_{\eta+1}}{2} \quad \eta = 1, 2, 3, \dots, 128.$$

\* Normalized to unity for smallest step size.

However, in a practical system an offset can be produced by the gates in the multiplexer which (relative to the input) biases a channel away from ideal midtread bias by the amount of the offset voltage of the gate. When this effect results in midriser bias, i.e., bias at a decision level ( $=0.5, 1.5, \dots$ ), low power input signals to the quantizer are enhanced rather than suppressed (an extremely small input variation will produce a full-step output variation). This phenomenon affects both idle channel noise and crosstalk performance.

The minimum power of idle channel noise and crosstalk coupling loss, when the quantizer is biased at a decision level on the first segment, depends upon the smallest step size. The smallest step size is, in turn, determined by the absolute scale of the  $\mu \cong 255$  quantizer. The next sections discuss the tracking characteristic, overload point, idle channel noise, and crosstalk performance of the  $\mu \cong 255$  quantizer.

### III. TRACKING

The tracking of a system can be displayed by a plot of gain or loss through the system as a function of the input signal power. This gain or loss through the system will depend upon the amplitude density function of the signal. Figure 1 is a plot of loss through the  $\mu \cong 255$  quantizer, in decibels, for a sine wave whose input power is expressed in decibels below overload. The overload power for this figure,  $P_{VOL}$ , is defined as the power of a sine wave whose peak amplitude is equal to a "virtual" decision level,  $X_{128}$ , placed one half step above the largest quantizer output level. This definition is one of at least three that might reasonably be employed. The word "virtual" is used for this decision level since the same highest output level is encoded independent of whether the signal is just below or just above this level. The justification for using this "virtual" overload definition is based on the fact that an error up to one-half step in amplitude, introduced by the quantizer, is considered quantization distortion. It seems reasonable, therefore, to define that portion of the error above the highest quantizer output level, up to a "virtual" decision level (placed where the next decision level would have been, had the last segment of the quantizer been uniformly extended) as quantization error.

An overload defined as the power of a sine wave whose peak is equal to the largest output level magnitude,  $P_{LOL}$ , has also been used. This level is about 0.14 dB below the virtual overload point (see Appendix A).

A third overload point, not easily analytically derived, is the highest input power for which the net loss through the quantizer is exactly zero. In the figure, this point is seen to fall between the above two points.

A reasonable objective for overload is +3 dBm<sub>0</sub>, the level currently

used in D-type channel banks. A sequence of code words, called the digital milliwatt reference signal, is currently used to scale the quantizer levels. With the decoder adjusted so that it produces an analog output signal of 0 dBm0 when decoding the digital milliwatt sequence, the quantizer will have a virtual overload point of +3.17 dBm0. A point in the system that measures 0 dBm with this code sequence (or an equivalent sequence which produces the same power) is defined to be a 0 TLP (Transmission Level Point). The level point is indicated by the 0 following the dBm. The derivation of this overload point is given in Appendix A. This value for overload was verified in measurements of the D2 channel bank as reported in Ref. 3.

A reasonable tracking requirement for the digital central office is the same as the present tracking requirement for the D-type channel bank. The deviation in gain, from the gain at 0 dBm0, should be 0.5 dB maximum and <0.25 dB averaged over all production for any level sine wave between +3 and -37 dBm0. The deviation requirement is increased for powers from -37 to -50 dBm0 to 1.0 dB maximum and <0.5 dB average. A perusal of Fig. 1 shows that these requirements can theoretically be met with substantial margin. Typical D4 codec performance as given in Ref. 9 demonstrates substantially better performance than the requirement.

#### IV. IDLE CHANNEL NOISE

The theoretical output idle channel noise (ICN), or output noise in the absence of signal, produced by a quantizer depends upon the amplitude of the input noise and bias of the quantizer. Ideal or zero bias occurs when the input zero level corresponds to the exact center of the quantizing characteristic of the  $\mu \cong 255$  15-segment quantizer. An offset voltage may be introduced between the input of the system and the quantizer by a device such as a multiplexing gate. If very low-level Gaussian noise is put into the quantizer when biased at midread, no decision levels will be crossed, and the output will be a constant zero amplitude. However, if the quantizer is biased midriser relative to the input (e.g., by a pedestal in the multiplexing gate), no matter how low the input signal is, the output will fluctuate between two levels and produce an idle channel noise. The magnitude of this noise is determined by the size of the smallest step. For a quantizer whose smallest step size is normalized to one unit, it is shown in Appendix A that the peak of the digital milliwatt reference signal is 2831.31 units. The power of a square wave with peak to peak equal to one unit is thus

$$10 \log \frac{|\frac{1}{2}|^2}{(2831.31^2/2)} = -72.05 \text{ dBm0.}$$

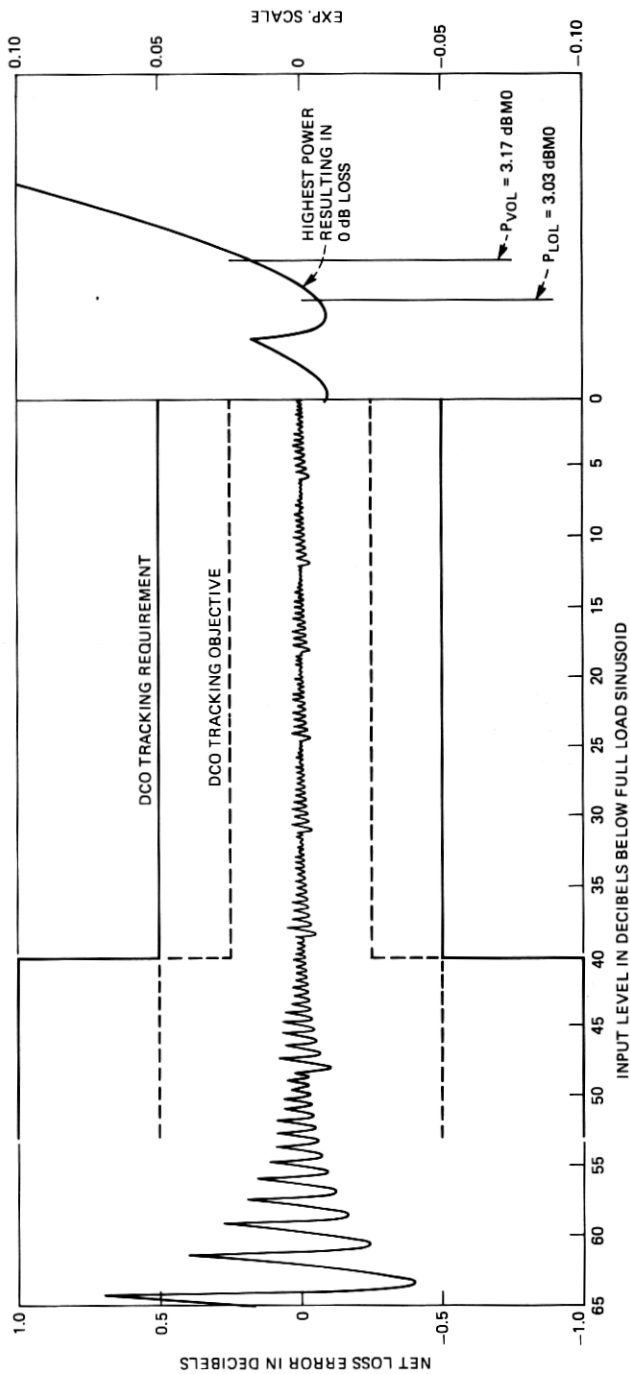


Fig. 1—Tracking. Sine wave density input — bias = 0.

Bennett has shown that the mean square value of the response of an ideal low-pass filter to a train of unit impulses multiplied by the instantaneous samples occurring at double the cutoff frequency is equal to the mean square value of the samples, provided no harmonic of the sampling frequency is equal to twice the frequency of one component or equal to the sum or the difference of two component frequencies of the sampled signal (Ref. 4, Theorem I). Since the mean square of a signal equals the mean square of the samples of the signal in such a case (Ref. 4, Appendix I), the power out of an ideal low-pass filter will equal the power of the square wave out of the quantizer. The  $-72.05$  dBm0 can be converted to a dBrnC0 measure provided the spectrum of the output noise is known. Assuming a low-level, zero-mean, white Gaussian input noise, the samples at the Nyquist interval from an ideal low-pass filter into the quantizer will be uncorrelated and thus independent. The samples out of the quantizer will thus also be independent and hence uncorrelated, and the output spectrum of an ideal low-pass reconstruction filter will also be white. Zero dBm of white noise in a 4-kHz band will measure 87.18 dBrnC.\* Thus for midriser bias, the noise will be  $87.18 - 72.05 = 15.13$  dBrnC. Figure 2 is a plot of output noise vs input noise for bias values from zero to 0.5 step in 0.1 step. Appendix B gives the derivation of the formula used to calculate these curves. Note that, for midtread bias ( $=0$ ), the output noise rapidly decreases as the input noise decreases in level.

The bias encountered in D4 production channel banks is nearly uniformly distributed over the first six steps of the first segment of the quantizer.<sup>5</sup> For a given input noise level, the output noise depends upon the quantizer bias. If we assume a uniform distribution of bias over the first six steps, Fig. 3 shows the resulting cumulative distribution of output noise for input noise levels of 0, 5, 10, and 15 dBrnC0. Also shown are the mean and standard deviations of the output noise. The step-like structure is due to approximating the continuous uniform bias distribution by a discrete distribution with 0.01 step intervals. Observe that, for an input noise level equal to 10 dBrnC0, the mean output noise level is 12.65 dBrnC0. A reasonable objective for the idle channel noise is an average of 16 dBrnC0 and a reasonable requirement is a maximum of 20 dBrnC0.<sup>1</sup> As can be seen from the figure, this noise objective can theoretically be met with substantial margin. Even if the coder bias were maintained at worst case midriser position, the objective can still theoretically be met, as seen by the maximum of the distribution, which corresponds to the value of noise produced by one-

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\* *Transmission Systems for Communications*, Bell Laboratories, revised fourth edition. Page 176 gives  $-4.8$  dBm/kHz  $\rightarrow$  88.4 dBrnC. Thus the power in 4 kHz is  $6.02 - 4.8 = 1.22$  dBm  $\rightarrow$  88.4 dBrnC or 0 dBm  $\rightarrow$  87.18 dBrnC. This calculation neglects any contribution above 4 kHz, since the C-message weighting gives high attenuation to these values.

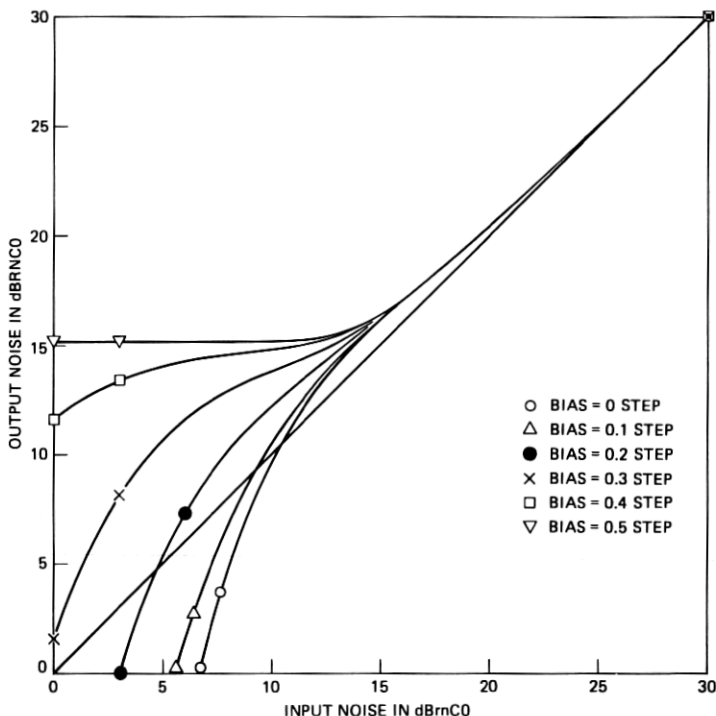


Fig. 2—Output noise vs input noise, Gaussian density input.

step perturbation previously calculated. However, should the bias wander off the first segment, the asymptotic idle channel noise at a decision level on the second segment will increase by 6 dB, thus exceeding the reasonable maximum idle channel noise requirement.

#### 4.1 Measured idle channel noise performance

Measurements were made of idle channel noise on D4 PCM channel bank codecs with  $\mu \cong 255$  quantization used by the operating telephone companies. The average and maximum idle channel noise were 14 dBrnC0 and 19 dBrnC0, respectively.<sup>9</sup> The fact that higher-than-worst-case bias idle channel noise is measured, in some instances, indicates that noise sources other than the quantizer, such as amplifiers, are contributing significant noise. From these data and the foregoing theoretical analysis, one sees not only that the  $\mu \cong 255$  quantization is theoretically capable of meeting the reasonable idle channel noise objective, but that the current D4 PCM channel bank codec does meet this objective. The digital central office will use the same digital milliwatt reference signal and will therefore have the same step sizes, and so a reasonable idle channel noise objective for the digital office is feasible.

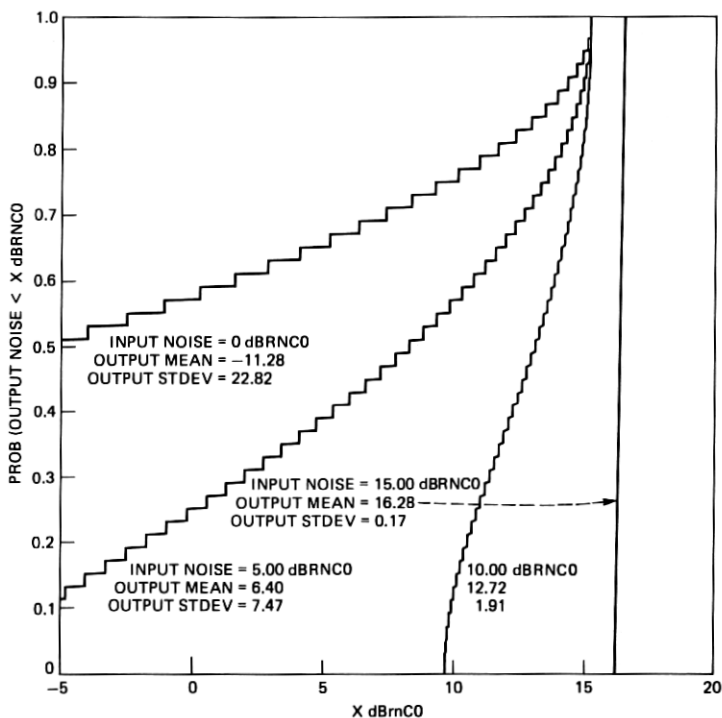


Fig. 3—Cumulative distribution of output noise, Gaussian noise input and uniform bias over six steps.

## V. CROSSTALK

In addition to the normal crosstalk mechanisms found in transmission equipment, a PCM system has an additional nonlinear mechanism caused by the quantizer. If a quantizer is effectively biased so that a zero volt input reaches the quantizer at a decision level (i.e., midriser bias), then a very small input signal will be enhanced as shown on Fig. 4. The enhanced output signal has a peak-to-peak height of one step (no matter how small the input crosstalk signal is), and has zero crossings corresponding to the original input signal (which is essentially one-bit PCM encoding). Speech encoded into one-bit PCM is somewhat intelligible, if its absolute level is high enough. Thus, for very small input levels, the power of the enhanced crosstalk signal out of the quantizer is independent of the power of the input crosstalk signal. On the other hand, for a bias at midtread, corresponding to the zero input level being half-way between two decision levels, the very small signals will be annihilated, and any signal large enough to result in a crosstalk signal out of the quantizer suffers center clipping, which reduces its



power and substantially reduces its intelligibility even further than the one-bit PCM encoding has already done.

Crosstalk is typically measured by inserting a zero dBm0 sine wave on the disturbing circuit and measuring the signal power on the disturbed circuit with a frequency selective meter.<sup>6</sup> Since the minimum square-wave power out of the quantizer is  $-72.05$  dBm0, or  $-73$  dBm0 in the fundamental, one might conclude that 73 dB of crosstalk loss is an upper bound when biased at a decision level. In a realistic situation, however, some Gaussian noise will be present on the disturbed channel, and it is shown in the next section that this 73-dB worst case (midriser bias) maximum coupling loss is not, in fact, an upper bound. It is shown that noise present on the disturbed channel at the input to the quantizer increases the crosstalk coupling loss at the output of the quantizer.

### 5.1 Crosstalk enhancement and mitigation

The results of a paper published in 1962 by R. H. Shennum and J. R. Gray<sup>7</sup> are used in this discussion of enhanced crosstalk. Shennum and Gray show that when a small Gaussian noise signal is mixed with a sine wave crosstalk signal, the enhancement effect is reduced and, in fact, that a sufficient amount of noise will mitigate quantizer enhancement entirely. Equation (14) in the above paper was used to calculate the crosstalk power as a function of input signal power and quantizer

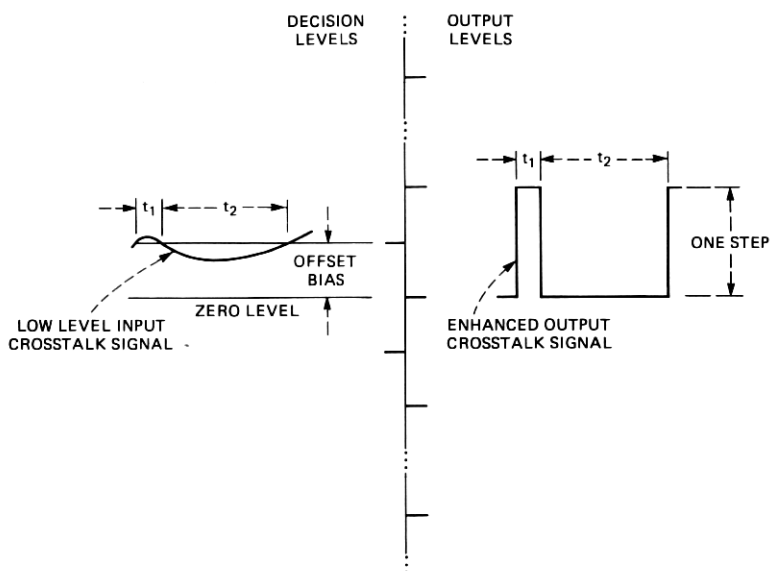


Fig. 4—Crosstalk enhancement.

bias. After correction of a typographical error, this equation is

$$\chi = \frac{1}{2T^2} \left[ E + \frac{2V'}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} J_1 \left( \frac{2\pi n E}{V'} \right) \cos \left( \frac{2\pi n X'_0}{V'} \right) e^{\frac{-2(n\pi)^2 \phi_0}{V'^2}} \right]^2,$$

where  $\chi$  is the crosstalk power,  $T$  is the sampling interval,  $E$  is the peak of the sine wave input to the quantizer,  $V'$  is the quantizer step size at the bias point,  $J_1$  is the first order Bessel function of the first kind,  $X'_0$  is the quantizer bias, and  $\phi_0$  is the input noise power. Figure 5 shows the crosstalk output power as a function of input power for midriser and midread (dashed) bias with input noise as a parameter.

The present performance objective for an analog switch of 75 dB minimum coupling loss (80 dB–95 percent point) for repeated exposures between the same pair of loops is intended to ensure that the probability of a customer hearing intelligible crosstalk is <0.1 percent.<sup>8</sup> Somewhat more lenient requirements for random exposures and unintelligible voice-like crosstalk (babble) may be acceptable and are

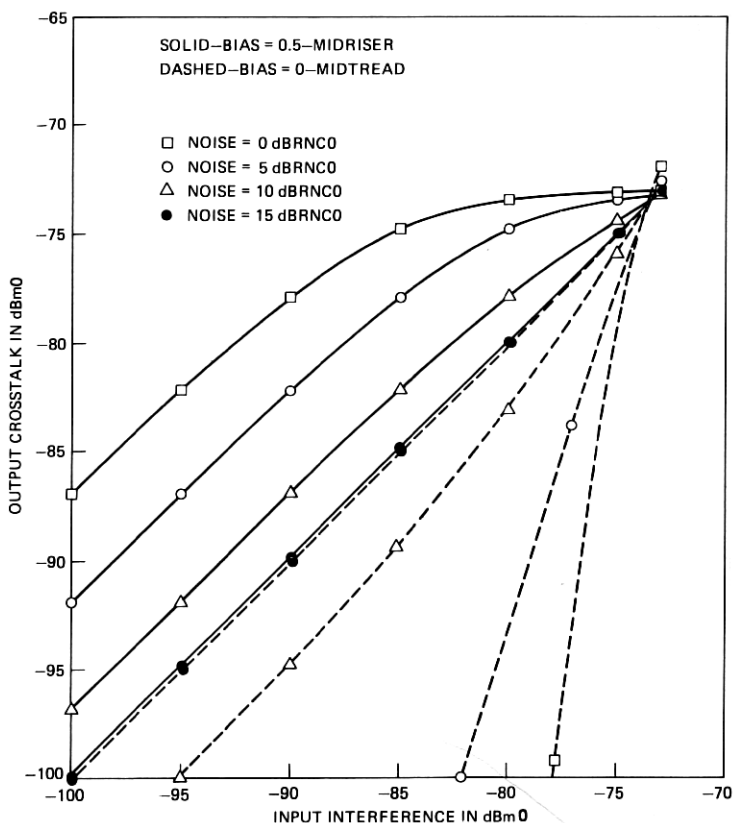


Fig. 5—Crosstalk enhancement for sine wave input.

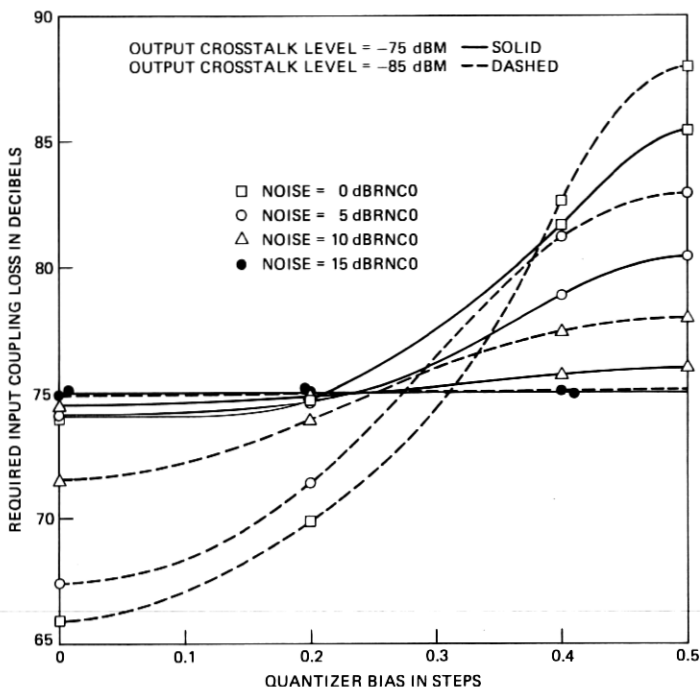


Fig. 6—Required input coupling loss vs bias for 75-dB output coupling loss sine wave.

under study. This 75-dB minimum crosstalk coupling loss represents a 10-dB more stringent requirement than is placed on present PCM channel banks at 0 dBm0 signal level. This increased coupling loss is desired because (i) masking noise is likely to be less on intra-office calls, (ii) there may be a higher likelihood that the disturbing/disturbed customers will be acquainted, and (iii) the exposure may be more likely to be repeated in a central office compared to the channel bank path because the possible number of paths may be much smaller.

Because of enhancement of crosstalk by the quantizer, the computation of the performance requirement for coupling loss is further complicated by its dependence upon quantizer bias, noise on the disturbed circuit, and signal power. For these reasons, the determination of the necessary crosstalk coupling loss required of a digital switch to insure 0.1% probability of intelligible crosstalk on a connection is currently being investigated at Bell Laboratories and is unavailable at this time.

Figure 6 shows the required quantizer input crosstalk coupling loss for a 75-dB output quantizer coupling loss. The loss is given as a function of quantizer bias with input noise as a parameter for two levels of output crosstalk power, -75 and -85 dBm0. The -75-dBm0

value was chosen to reflect the standard method of measuring crosstalk coupling loss of PCM channel banks with a 0-dBm0 test signal. The -85-dBm0 value was chosen because, for this value, the enhancement reaches a maximum for any noise equal to or greater than 0 dBrnC0, as can be seen from Fig. 5. The required input coupling loss for a given output coupling loss depends upon the amount of noise on the affected channel, the bias of the coder, and the signal level. For example, for worst-case bias of 0.5 step (bias at a decision level), if the input noise is only 0 dBrnC0, then the required coupling loss at the quantizer input for a 75-dB quantizer output coupling loss must be about 85.5 dB for -75 dBm0 at the output, but increases to a maximum of 88.6 dB for lower output signal powers, such as are representative of actual speech powers. These losses correspond to enhancements of 10.5 and 13.6 dB. If the noise is 5 dBrnC0, then the required input coupling loss for 75-dB output coupling loss reduces to 80.5 dB (83.7 dB maximum) for a received -75 dBm0, and if the noise is 10 dBrnC0, the required coupling loss must only be about 76 dB (78.7 dB maximum).

Figure 7 shows the required input coupling loss for a 75-dB output coupling loss as a function of noise with bias as a parameter, for a -75-

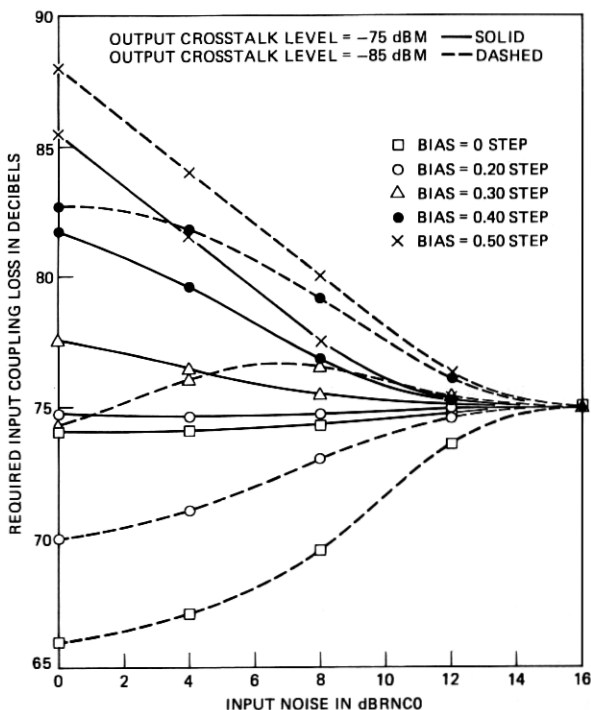


Fig. 7—Required input coupling loss vs noise for 75-dB output coupling loss sine wave.

and  $-85\text{-dBm}_0$  crosstalked signal out of the quantizer. Notice that, as the noise increases, the effect of bias becomes less important, and in fact with noise of about  $15\text{ dBm}_0$ , the quantizer is essentially linearized.

Using these curves, the designer can make tradeoffs among bias control, required analog input crosstalk coupling loss, and input noise insertion.

Measurements of crosstalk on D4 codecs are reported in Ref. 9. A distribution of crosstalk measured on D4 codecs given in the reference shows crosstalk coupling loss ranges from 75 to 90 dB, and is typically  $>80\text{ dB}$ . However, the channels were terminated, and the amount of noise present on the disturbed channel and the bias of the quantizer were not measured and controlled. In addition, these measurements were only made with a  $0\text{-dBm}_0$  test tone and not at lower levels where quantizer enhancement is more pronounced. Typically, the bias is uniformly distributed over the first six steps of the quantizer. Because of the uncertainties in noise levels and bias conditions and the high test signal level, these measurements should not be considered to imply that crosstalk in such a bank could not impose restrictions on the crosstalk loss path preceding the channel bank.

A main point to be observed is that the  $73\text{-dB}$  bound for crosstalk coupling loss inferred from the minimum step size is only applicable for the actually nonexistent theoretical case of midriser bias with no noise present. The next section gives a heuristic explanation of the crosstalk dependence on noise.

### ***5.2 Heuristic explanation of quantizer linearization with noise***

With no noise present at the quantizer input, the output will be a square wave of one step peak-to-peak height for a low level sine wave input biased at a decision level (e.g. bias = 0.5). This will be true for any sine wave whose peak amplitude is less than one step. The period of this square wave will be equal to the period of the input sine wave. The crosstalk power is defined to be the power in a small frequency band centered at the input signals' frequency. A Fourier series expansion of the square wave shows that the power of the fundamental is about 1 dB less than the power of the square wave. Since the power of the square wave is  $-72\text{ dBm}_0$ , its fundamental is  $-73\text{ dBm}_0$ . As noise is added to the input sine wave preceding the quantizer, the zero crossings of the composite signal are perturbed. For low level noise and signal power, the total output power remains constant at  $-72\text{ dBm}_0$ ; however, the power in the band centered at the crosstalk frequency decreases with increasing noise, since the period of the square type output signal is no longer constant at the frequency of the input sine wave frequency (the zero crossings being changed by the noise).

As shown on Fig. 5, the gain for bias = 0.5 (midriser), is constant as the input crosstalk signal decreases in level; i.e., for input signal powers low relative to the input noise power. This implies a limiting crosstalk gain in decibels. This constant gain is approximately realized for input signal power equal to and less than the noise power. Because of this behavior, a crosstalk model can be used which is essentially linear with a constant decibel penalty for a given assumed input noise power.

The linear gain that occurs for signals which are small relative to the noise present can be explained as follows. For a given noise power, say 5 dBrnC0, samples of the combined noise and signal will produce -72.05 dBm0, or 15.13 dBrnC0. Since the input noise power contribution is constant and it dominates the signal, the gain or enhancement is also constant. The sine wave which is mixed with the noise is also amplified by this same amount, regardless of the sine wave power, if it is low relative to the noise.

As the sine wave power increases and exceeds the noise power, the sine wave will determine the control of the zero crossings, causing more output power to be in the band near the sine-wave frequency, and hence the gain decreases (departure of the curves from linear) and becomes 0 when the sine-wave amplitude of one step peak to peak is approached.

For signal levels small relative to the noise, the constant gain is approximately 13 minus the input noise power in dBrnC0, e.g., for 0 dBrnC0 the gain is 13 dB, for 5 dBrnC0 the gain is  $13 - n = 13 - 5 = 8$  dB, etc. The gain is 13 rather than 15.13 dB for 0 dBrnC0 of input noise because the different noise amplitudes are enhanced by different amounts, i.e., the larger ones are enhanced less, and the smaller ones more than 15 dB so that on the average they are enhanced around 13 dB. (In fact, the one sigma point of the Gaussian distribution is 0.34, so 34 percent of the sine wave samples will be amplified by 15 dB or more, and 66 percent will be amplified by less than 15 dB.) The above approximation ( $13 - n$ ) is good for noises up to 10 dBrnC0 where the gain is about 3 dB. For 12 and 14 dBrnC0, the gain is 1.5 and 0.5 dB, respectively. Finally, at 15 dBrnC0 and above, the gain is 0 dB. Also note that, for 15 dBrnC0 noise and above, the bias is immaterial (see Fig. 6). This occurs, since any input signal is dithered over the entire smallest step range at this height of noise.

## VI. SIGNAL-TO-DISTORTION RATIO (SDR)

A reasonable analog-port-to-analog-port SDR requirement for the local digital office is the same as it is for current PCM channel banks. Figure 8 shows the D4 channel bank analog-to-analog and receive or transmit requirement and theoretical performance for a sine-wave test signal for ideal bias (=0), end of the first segment (bias = 15.5 steps),

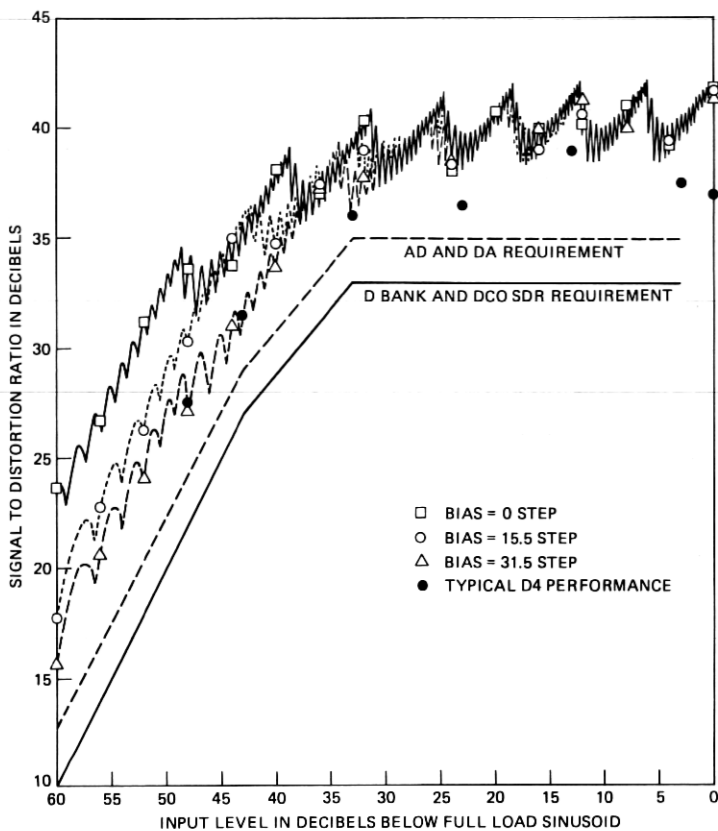


Fig. 8—Signal to distortion ratio vs level for 8-bit PCM.

and middle of the second segment (bias = 31.5 steps). For a bias at the end of the first segment, the minimum margin from theoretical performance to the reasonable requirement is about 1.5 dB near the knee of the curve. Since the performance curves are for ideal analog-to-digital and digital-to-analog converters and the only impairment is bias, these curves represent perfect converters. The margin from the theoretical performance curves to the transmit-and-receive requirement curve is thus the margin available for both the encoder and decoder. The transmit-and-receive requirement curve has been raised above the overall requirement curve to allow coders manufactured by different companies to be used together and still meet the overall analog-to-analog requirement.

Also shown on the figure is the mean performance for D4 channel banks for input levels of 0, 3, 13, 23, 33, 43, and 48 dB below overload.<sup>9</sup> It should be pointed out that this performance is for 7% bit coding,

whereas the curves are for 8-bit coding. This one-sixth bit lower resolution causes a reduction of about 1.8 dB in SDR performance. The method of analysis used to calculate the SDR is described in Appendix B. Using this method, the distortion power is defined to be the difference between the output signal and the least-mean-square error linear predictor of the output. This corresponds to minimizing the distortion relative to gain and offset in the system. In addition to a reasonable signal-to-total-distortion objective, a second and third harmonic distortion objective of at least 43 dB for an input signal of  $-10$  dBm is reasonable, which is the current analog switch objective. W. R. Bennett has shown that the harmonic distortion for a single sine wave oscillates with signal level.<sup>4</sup> It is therefore recommended that harmonic distortion be measured using a set of four sine waves.<sup>6</sup> Measurements of D4 channel banks using this method which have the  $\mu \cong 255$  quantization law reveal that the second harmonic distortion has a mean of about 52 dB.<sup>9</sup> Thus, this objective can be met with current technology.

The jagged signal-to-distortion curve vs level can be smoothed by using a Gaussian test signal rather than a sine wave. Using a Gaussian test signal is a recommended method of the CCITT.<sup>10</sup> Figure 9 shows the SDR performance for a Gaussian input signal for 0 bias, bias at the center and end of the first segment and center, two-thirds point, and end of the second segment. The CCITT requirement is also shown. The performance for high-level signals is little affected by bias offset. However, for lower signal levels one sees that for any bias past the end of the first segment, the margin will be less for low levels than it is for higher levels. It also should be pointed out that bias at a decision level on the second segment will increase the idle channel noise by 6 dB, up to 21 dBm, which does not meet the reasonable maximum requirement. Note that the distortion has been adjusted for 3100-kHz bandwidth, i.e.  $(10 \log(4000/3100)) = 1.11$  dB).

## VII. CONCLUSIONS

Transmission parameters affected by the choice of quantization law have been analyzed for the  $\mu \cong 255$  8-bit quantizer. The theoretical transmission performance characterized by these parameters has been shown to permit the reasonable transmission objectives and requirements to be met. Tradeoffs permitted and transmission degradation resulting from system bias alignment are discussed. It is shown that the enhancement of crosstalk due to the quantizer can: (i) impose more severe restrictions on the analog crosstalk paths preceding the encoder than are necessary in an analog office; (ii) impose close to a midtread bias constraint on the encoder; (iii) impose a minimum required noise on the disturbed channel preceding the encoder; or (iv) impose a combination of these.



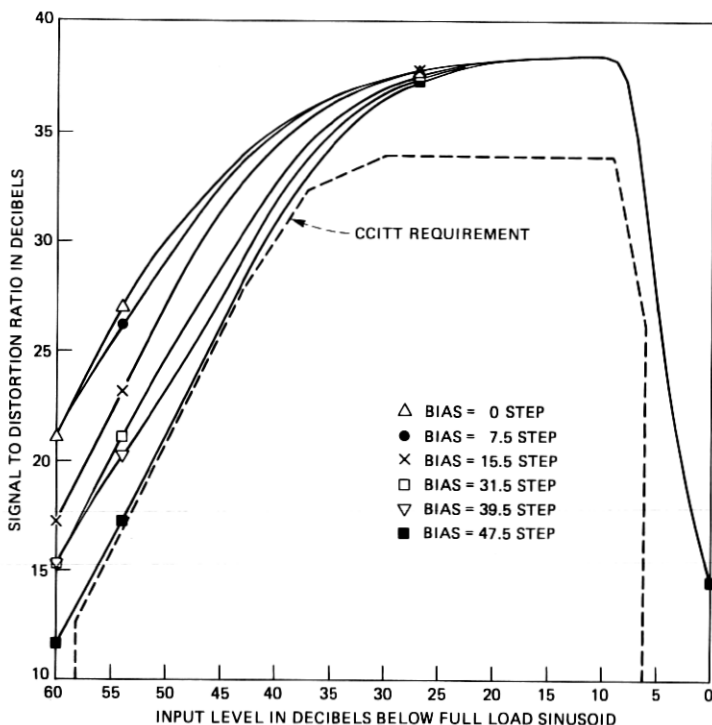


Fig. 9—Signal to distortion ratio vs level, Gaussian density input.

### VIII. ACKNOWLEDGMENTS

H. M. Zydney suggested the appropriateness of this paper and W. L. Ross, I. J. Shapiro, J. E. Abate, and Behram Bharucha are acknowledged for encouragement and helpful discussions concerning this work. I thank R. W. Hatch, F. A. Saal, E. J. Anderson, Martin Brilliant, John Cavanaugh, and John Sullivan for comments on the draft.

### APPENDIX A

#### Quantizer Scale Specification

The scale of the quantizer is specified by using a digital milliwatt reference signal. The signal is generated by repeating a line code word sequence of length eight, comprised of four words,  $A$ ,  $-A$ ,  $B$ , and  $-B$ , where  $A = 10011110$ ,  $-A = 00011110$ ,  $B = 10001011$ , and  $-B = 00001011$ .<sup>3</sup> The sequence is  $\{A, B, B, A, -A, -B, -B, -A\}$ . With a sampling rate of 8 kHz, this sequence has a fundamental frequency of 1 kHz and fundamental peak level,  $V_p = 2831.31$ , relative to a smallest step of unit value. This value is derived below along with the third harmonic magnitude, which is more than 42 dB down from the fundamental. The line code is the ones complement of the sign-magnitude

code, i.e., the levels near zero have the highest ones density. For typical speech signals, which have approximately exponential amplitude density functions, this complementing of the magnitude causes the coded line signal to have a higher ones density, which is advantageous for timing on the digital repeatered line.

The decision levels and output levels of the quantizer can be expressed by simple mathematical expressions of the sign-magnitude code. The uncomplemented magnitude,  $M$ , of  $A$  (0011110) is  $(1100001)_2 = 97$ , and of  $B$  (0001011) is  $(1110100)_2 = 116$ . From the magnitudes 97 and 116, the segment number,  $V$ , and the step number on the segment,  $L$ , are calculated. The values  $V$  and  $L$  must satisfy the equation:

$$M = 16V + L,$$

where  $V$  is an integer from 0 to 7 denoting the segment number, and  $L$  is an integer from 0 to 15 denoting the step number on the segment.  $L$  is thus  $M$  modulo 16. For example, for  $M = 97$  we have

$$97 = 16(6) + 1, \quad V = 6 \quad \text{and} \quad L = 1.$$

For  $M = 116$  we have  $V = 7$  and  $L = 4$ . To find the quantizer output levels for these values of  $V$  and  $L$ , one uses the following equation:

$$OL(V, L) = 2^V(L + 16.5) - 16.5.$$

The decision levels are given by

$$DL(V, L) = 2^V(L + 16) - 16.5.$$

These output and decision levels correspond to decision level assignment as defined by Kaneko.<sup>11</sup> Substituting the values of  $V$  and  $L$  into the above equation for output level, we obtain

$$97 \text{ or } OL(6, 1) = 2^6(1 + 16.5) - 16.5 = 1103.5 \text{ units} = \hat{A}$$

and

$$116 \text{ or } OL(7, 4) = 2^7(4 + 16.5) - 16.5 = 2607.5 \text{ units} = \hat{B}.$$

$\hat{A}$  and  $\hat{B}$  are the output levels for a quantizer with unit smallest step, corresponding to codes  $A$  and  $B$ .

The quantizer is related to an absolute scale, by making the power of the reconstructed analog signal produced by repeating the digital milliwatt sequence equal 0 dBm0. Since the digital milliwatt sequence repeats every eight samples and the sampling rate is 8 kHz, the fundamental frequency for this signal is 1 kHz.

To find the fundamental and harmonic content of the periodic milliwatt sequence of samples, one can expand it into a Fourier series. If one considers the origin to be half-way between  $-\hat{A}$  and  $\hat{A}$ , then the sequence is odd and only sine wave terms will appear in the expansion. In addition, due to the symmetry about the midpoint between  $\hat{B}$  and

$\hat{B}$  (also  $-\hat{B}$  and  $-\hat{B}$ ), only odd harmonic sine waves will be in the expansion. The fundamental and third harmonic coefficients can be found by integrating over one-fourth the period and multiplying by four, because of the symmetry of the sequence. Letting  $T$  be the sampling interval in microseconds, the period is  $8T$  and we have

$$b_1 = \frac{2}{8T} \int_0^{8T} f(t) \sin \frac{2\pi}{8T} t dt$$

$$= \frac{4.2}{8T} \int_0^{2T} [\hat{A} \cdot T\delta(t - 62.5) + \hat{B} \cdot T\delta(t - 187.5)] \sin \frac{2\pi}{8T} t dt^*$$

and

$$b_3 = \frac{4.2}{8T} \int_0^{2T} [\hat{A} \cdot T\delta(t - 62.5) + \hat{B} \cdot T\delta(t - 187.5)] \sin 3 \frac{2\pi}{8T} t dt.$$

Solving these gives  $b_1 = 2831.31$  and  $b_3 = 21.66$ . The fifth and higher harmonics are suppressed by the low-pass reconstruction filter. The fundamental and third harmonic are inband, where the reconstruction filter is within  $\pm 0.15$  dB of the response of an ideal low-pass filter. Hence, the fundamental-to-third harmonic ratio is

$$20 \log \left| \frac{b_1}{b_3} \right| = 42.33 \text{ dB.}$$

Thus, the milliwatt reference tone is essentially a pure 1-kHz sine wave whose peak is 2831.31 units.

#### A.1 Overload calculation

Several definitions of overload are possible in describing a PCM system. One can define it to be the power in a sine wave whose peak equals the virtual decision level,  $P_{VDL}$ , or a sine wave whose peak equals the largest output level,  $P_{LOL}$ .

The overload will now be calculated, assuming the virtual decision level definition and also the overload assuming the peak of the sine wave equal to the largest output level. The relative value of the virtual decision level is

$$DL(V = 7, L = 16) = 2^7(16 + 16) - 16.5 = 4079.5 \text{ units.}$$

The mean power,  $P_{VDL}$ , of a sine wave whose peak is 4079.5 units is

$$P_{VDL} = 20 \cdot \log \left| \frac{4079.5}{2831.31} \right| = +3.17 \text{ dBm.}$$

\* The  $\delta$  is multiplied by the  $T$  so that the energy of the samples will equal the energy of the sampled wave, as the sampling rate,  $f_s \rightarrow \infty$  and the period,  $T \rightarrow 0$ .

The mean power,  $P_{LOL}$ , of a sign wave whose peak is coded to the largest output level (4015.5 units) relative to a sine wave whose peak is 4079.5 units is

$$P_{VDL} - P_{LOL} = 20 \cdot \log \left| \frac{4079.5}{4015.5} \right| = 0.14 \text{ dB.}$$

Thus

$$P_{LOL} = 3.17 - 0.14 = 3.03 \text{ dBm.}$$

If one plots net loss versus input sine wave power, a saw-tooth-like function results, as shown in Fig. 1. The input levels on this plot are referred to the *VDL* definition of overload. The greatest output level definition is also shown. A level at which exactly zero loss occurs is between  $P_{LOL}$  and  $P_{VDL}$ .

Notice the asymmetrical structure of the net loss vs level plot. The maxima are quite sharp, while the minima are more rounded. This behavior is due to the peakedness of the amplitude density function of the samples of a sine wave. The rapid change of net loss at the maxima points occur when the peaks of the sine wave cross a decision level. The minima correspond to points of maximum gain and occur after the peak passes a decision level but before it reaches the level of the corresponding output level.

The value derived using the *VDL* definition is in agreement with the overload point of the D2 bank, as given in Ref. 2. Also described in Ref. 2 is the bank alignment procedure that utilizes the digital milliwatt reference signal.

## APPENDIX B

### Noise and SDR Calculation

The output noise is simply the variance of the output signal with a Gaussian density input. The signal-to-distortion ratio (SDR) is the ratio of the output signal power, i.e., the variance of the output signal, to the mean square error (MSE). Let  $X$  be the input to the quantizer,  $Y$  the output, and  $\hat{Y}$  the least-mean-square-error linear predictor of  $Y$ ,  $\hat{Y} = AX + B$ .<sup>10</sup> Then define the mean square error to be

$$\text{MSE} = E[(Y - \hat{Y})^2] = (1 - \rho^2)\text{var}(Y),$$

where

$$A = \frac{\text{cov}(x,y)}{\text{var}(x)}, B = E(x) - \frac{\text{cov}(x,y)}{\text{var}(x)} E(x), \text{ and } \rho = \frac{\text{cov}(X, Y)}{\sigma_x \sigma_y}.$$

Calculating the MSE by this method adjusts the offset,  $B$ , and linear gain,  $A$ , to minimize the MSE.

The covariance of  $X$  and  $Y$  and the variance of  $Y$  can be found using

the density function of  $X$  and the known functional relationship between  $X$  and  $Y$  given by the quantizer characteristic. Let  $DL_i$  be the  $i$ th decision level, as shown on Fig. 10. For any input signal  $X$  such that  $DL_{i+1} \leq X < OL_i$ , the output  $Y$  will be the  $i$ th output level, termed  $OL_i$ . Letting  $p_X(x)$  be the density function of  $X$ , one has

$$\text{VAR}[Y] = E[Y^2] - E^2[Y]$$

and

$$E[Y^2] = \int_{-\infty}^{\infty} y(x)^2 p_X(x) dx = \int_{v_{\min}}^{DL_{i_{\min}+1}} OL_{i_{\min}}^2 p_X(x) dx$$

$$+ \sum_{i=i_{\min}+1}^{i_{\max}-1} \int_{DL_i}^{DL_{i+1}} OL_i^2 p_X(x) dx + \int_{DL_{i_{\max}}}^{v_{\max}} OL_{i_{\max}}^2 p_X(x) dx,$$

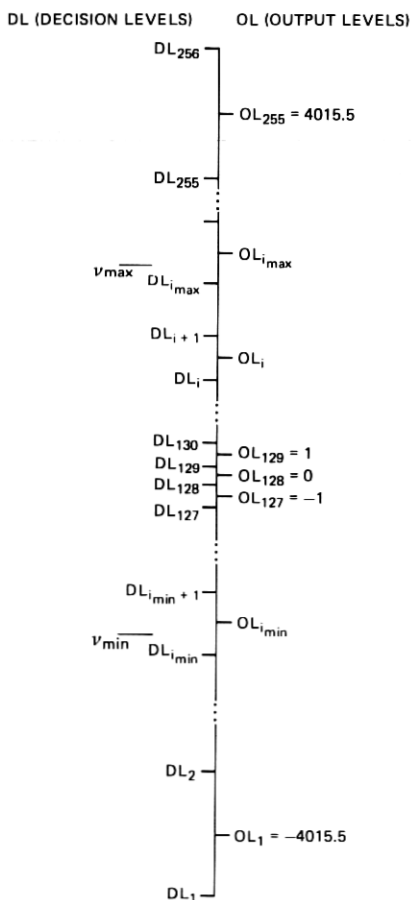


Fig. 10—Assignment level detail.

where  $\nu_{\min}$  and  $\nu_{\max}$  are the minimum and maximum values of the support of  $p_X(x)$ , and  $i_{\min}$  and  $i_{\max}$  are the least and greatest quantizing intervals over which the input signal density has support. The density of the samples of the input is  $p_X(x)$ , where the relative phase of the input wave and sampler is assumed uniform over  $2\pi$  radians. For the sine wave case, we have

$$p_X(x) = \frac{1}{\pi} \frac{1}{\sqrt{V^2 - (x - b)^2}} \quad |x - b| < V,$$

$$p_X(x) = 0 \quad |x - b| > V,$$

where  $V$  is the peak of the sine wave and  $b$  is the bias. The equations for the mean and covariance are the same as for  $E[Y^2]$ , except that the  $OL_i^2$  is replaced by  $OL_i$  and  $x \cdot OL_i$ , respectively. The noise calculation is made by simply using the Gaussian density for  $p_X(x)$  and evaluating  $\text{VAR}[Y]$ .

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