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Impairment Mechanisms for SSB Mobile Communications at UHF with Pilot-Based Doppler/Fading Correction

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Several impairments are associated with the pilot-based correction of a single sideband (SSB) voice signal for the effects of vehicle motion in a Rayleigh multipath field. These include the distortion caused by a limit on the available correction gain, the distortion resulting from frequency selective fading, the degradation of the signal-to-interference (S/I) [noise (S/N)] ratio, and the distortion induced because of interference to the pilot from a cochannel pilot or from noise. We have investigated these impairments for a specific feedforward, phase- and gain-correcting, ideal receiver. For this receiver, the S/I ratio is degraded at baseband because of the time-varying nature of the gain correction signal and the assumption that the correction signal is correlated with the desired voice signal fading envelope but is independent of the interferer fading envelope. A limit on correction gain appears necessary to limit the magnitude of several of the impairments, and has been considered as a parameter in the analysis of each impairment type. The results are here extended to characterize an "equal-gain" space diversity receiver. Diversity combining is applied after phase correction, but before gain correction to maintain proper voice correction while retaining the benefits of the improved envelope statistics. This paper deals only with the SSB idealized channel, not with a system employing SSB modulation; the authors recognize that many complicated systems problems (e.g., antenna combining, channel generation, and cost reduction) must be solved before a working system using the channel analyzed herein can become a reality.

I. INTRODUCTION

Conventional suppressed carrier single sideband (SSB) UHF mobile communication links exhibit severe voice-quality impairments because of the phase and amplitude modulation associated with motion in a multipath field. In view of this, various voice correction methods have been occasionally considered over the years.[1,2] These include the technique of incorporating a separate pilot RF frequency at constant level to provide a means of correcting the voice information at the receiver. This idea is now receiving increased attention, since re-evaluation of SSB for providing land-mobile radio communication.[3] Although pilot-based correction is now often presented as a probable solution to the fading-induced voice quality problem,[4–6] an analysis of the residual impairments and the performance of a pilot-based SSB channel in the presence of cochannel interference has not been given. Such an analysis is the object of this paper. The analysis is based on a specific receiver configuration employing ideal elements. This paper deals only with the SSB idealized channel, not with a system employing SSB modulation; the authors recognize that many complicated systems problems (e.g., antenna combining, channel generation, and cost reduction) must be solved before a working system using the channel analyzed herein can become a reality.

The intuitive expectations for pilot-based operation, that stem from considerations of suppressed-carrier, pilotless, SSB operation, are often incorrect. Such is the case with the plausible and common assumption that the signal-to-interference (S/I) [or signal-to-noise (S/N)] ratio at baseband is the same as the corresponding ratio at RF. In fact, we will show that the S/I (S/N) ratio of a pilot-based system is deteriorated at baseband in the presence of either flat or frequency-selective Rayleigh fading, even when cochannel pilot interference is absent. This can be thought of as the SSB signal-to-interference "disadvantage," analogous to the textbook FM S/N advantage for FM operation in the nonfading environment. The deterioration of the S/I at baseband is caused by the time-varying nature of the gain-correction signal and the assumption that the gain correction signal is correlated with the desired voice-signal fading envelope but is independent of the interferer-fading envelope. Although effects such as this may be established empirically through system trials, an understanding of their basis is desirable for the design of efficient SSB systems and for the most realistic comparison of different, and as yet untested, "paper" systems.

The effect of a limit on the gain available for fast-gain correction is considered in Section II, as in a previous analysis of gain correction for AM mobile radio by M.J. Gans.[2] While some limit is a consequence of practical hardware, a limit is actually required to bound impairments in the presence of frequency-selective fading, as well as to bound the S/I disadvantage. The tradeoff is essentially between residual-correction distortion in the absence of delay spread and the distortion in the presence of significant (3 μs) delay spread. The effect on the S/I disadvantage also influences the choice of a correction-gain limit.

Frequency-flat Rayleigh fading was assumed in the analysis of residual-gain limit distortion and S/I disadvantage. A frequency-selective model[7] was assumed for the analysis of delay-spread effects on the correction process. In Section III, N-branch space diversity was applied to the correcting receiver and the effects on the above impairments were analyzed; the impairments in this latter case were found to be bounded even when no limit is placed on the available correction gain.

II. RECEIVER ANALYSIS

Figure 1 diagrams the idealized pilot-based receiver under consideration. Both gain and phase correction are included because both have been experimentally found to be necessary to provide a natural rendition of voice information. The voice-channel selective filtering is placed before the phase and gain correction stages to avoid introducing inband adjacent channel interference, resulting from the complex multiplication of the incoming RF by a relatively wideband signal which is likely to be uncorrelated with any adjacent channel signals that are present. This filter placement requires the voice filter to be delay equalized across its passband. In any case, the absolute delay of the voice and the pilot paths must be matched prior to correction in phase and amplitude. In practice, a slow agc would be included before the receiver elements in Fig. 1, with the intention of removing the slow, log-normally distributed, shadow fading that is imposed on the rapidly fading signal. The agc would receive a control signal from the pilot envelope detector and adjust the gain to keep the average power in the pilot channel constant. An agc time constant of 0.1 to 1.0 s seems appropriate for this purpose. In the following sections, we assume that this slow shadow fading can be successfully removed, and consequently we will not consider it in the analysis.

Amplitude companding is applicable to a pilot-based SSB system

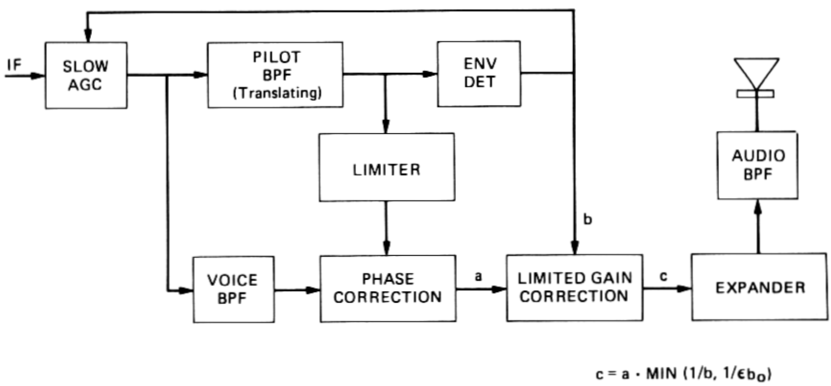

Fig. 1—Pilot-based Doppler/fading correction.

because of the constant-gain nature of the corrected channel. As is customary, amplitude companding is expected to improve the noise and interference (cross talk) performance of the system. The purpose of this paper is to analyze the basic channel, to which amplitude companding can be added to improve the apparent performance by reducing the impairment level at the receiver output when the desired speech is absent.

2.1 Correction distortion

Residual correction distortion in a flat-fading environment is caused by the system's inability to maintain constant channel gain during the deepest fades caused by the imposed correction-gain limit. The correction distortion limits the best-case performance of the mobile communication link in a noiseless, interference-free, flat-fading environment. This impairment is termed distortion because the average impairment power is proportional to the average desired speech power. However, it is not harmonically related to the speech. Also note that after correction, a small amount of distortion energy falls outside the original voiceband and is removed by the final audio filter. This is neglected in this analysis.

The original voice signal can be conveniently represented as

$$s(t) = a(t)e^{i\theta(t)}, \tag{1}$$

where $a(t)$ and $\theta(t)$ are audio envelope and phase functions, respectively. After SSB modulation and transmission, this signal and its pilot are received as

$$\begin{aligned} s_r(t) &= a(t)r(t)\exp[i(\omega_c t + \theta(t) + \phi(t))] + r(t)\exp[i(\omega_p t + \phi(t))] \\ &= z(t)a(t)\exp[i(\omega_c t + \theta(t))] + z(t)\exp(i\omega_p t), \end{aligned} \tag{2}$$

where $z(t) = r(t)e^{i\phi(t)}$ is the complex fading envelope caused by vehicle motion. Reference 7 describes the model of propagation in the mobile environment under consideration. In this standard model, $z(t)$ is a complex random process, the real and imaginary parts of which can be taken as independent real Gaussian processes with identical symmetric power spectra. For the E field vertical-whip antenna, these power spectra, and hence the power spectrum of $z(t)$, are of the form

$$Z(f) = \left[1 - \left(\frac{f}{f_d}\right)^2\right]^{-1/2}, \tag{3}$$

where f_d is the maximum Doppler frequency based on the vehicle speed. Figure 2 shows this spectrum. The above model possesses the widely used Rayleigh statistics for $r(t)$, because of the independent Gaussian real and complex parts of $z(t)$. In the flat-fading or negligible delay-spread environment, $z(t)$ multiplies the original signal compo-

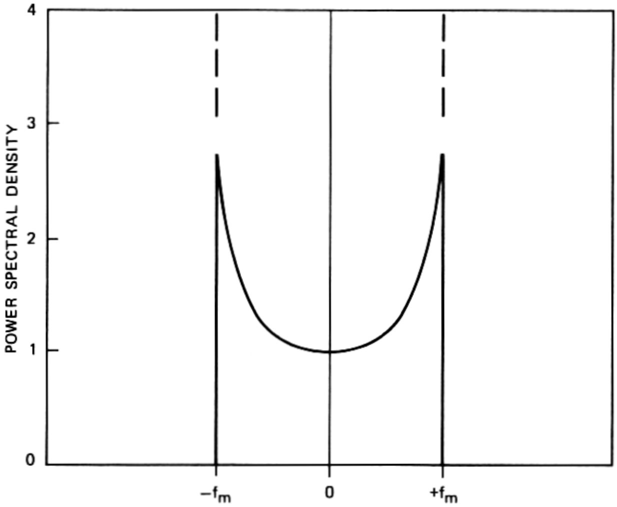

Fig. 2—Pilot spectrum for an E field antenna.

nents independent of their frequency, as in (2). This is assumed for this section. Including significant delay spread yields frequency selective fading and results in the complex fading envelope at one frequency having only partial correlation with the fading envelope at a different frequency. Finally, the model is ergodic; consequently, throughout this paper, averages over time, which are denoted with a bar, are used interchangeably with ensemble averages.

To develop a basis for voice correction, the received pilot, represented by the second term in (2), is separated from the voice by filtering. The ideal filter bandwidth required is twice the Doppler shift associated with the highest expected vehicle speed, for example, 160 Hz at 60 mph for 900-MHz center frequency. The absolute delay in both paths is assumed to be equalized, to allow timely amplitude and phase correction. If the delay is equalized, the magnitude of the delay does not affect the following calculations and hence will not be carried along. Note also, that the pilot signal is only delayed in passing through the pilot filter, because of the assumed existence of delay equalization across the filter passband and the assumption that the filter bandwidth is chosen wide enough to encompass the pilot spectrum. After multiplying by the hard limited pilot and subsequent lowpass filtering to remove the sum component, the phase corrected output is written as

$$s_p(t) = a(t)r(t)e^{i\theta(t)}. \tag{4}$$

The amplitude correction is gain limited to follow fades only as deep as ϵr_0, where r_0 is the RMS average of $r(t)$, and ϵ is the correction-gain limit parameter. The maximum available correction gain is thus defined as $1/\epsilon$. The pilot envelope $r(t)$ is detected in the receiver and used by the gain correction block to develop the final corrected audio output given by

$$s_b(t) = r(t)\min\left(\frac{1}{r(t)}, \frac{1}{\epsilon r_0}\right)a(t)e^{i\theta(t)}, \tag{5}$$

where $\min(\cdot, \cdot)$ is defined as the smaller of the two arguments on an instantaneous basis. The signal-to-correction distortion (S/D) ratio is then defined as

$$\text{SDR} = \overline{s^2}/\overline{(s_b - s)^2}. \tag{6}$$

Substituting (1) and (5) into (6) and simplifying, based on the independence of voice and fading envelope signals, gives the correction distortion in the flat Rayleigh-fading environment as

$$\text{SDR} = \left[\overline{\left(r(t)\min\left(\frac{1}{r(t)}, \frac{1}{\epsilon r_0}\right) - 1\right)^2}\right]^{-1}. \tag{7}$$

The fading envelope is assumed to be characterized accurately by the Rayleigh density function:

$$p(r) = \frac{2r}{r_0^2}\exp(-r^2/r_0^2)u(r). \tag{8}$$

The S/D ratio may now be computed as

$$\text{SDR} = \left[\int_0^{\epsilon r_0}\left(\frac{r}{\epsilon r_0} - 1\right)^2 \frac{2r}{r_0^2}\exp(-r^2/r_0^2)\,dr\right]^{-1}. \tag{9}$$

This reduces to

$$\text{SDR} = \left(\frac{1}{\epsilon^2}(1 - e^{-\epsilon^2}) + 1 - 2\sum_{n=0}^{\infty}\frac{(-1)^n\epsilon^{2n}}{(2n+1)n!}\right)^{-1}, \tag{10}$$

an expression for the best-case distortion for the moving pilot-based receiver with a fixed available gain limit. Figure 3 gives a plot of this expression as a function of $1/\epsilon$, the available correction gain.

The correction process has been simulated by a digital computer program to supplement and check the analytical results. The flat-fading, no-interference version of the simulation is based on (3), (5), and (8). The corrected output and the time average of the power spectrum of the corrected output were obtained from the simulation and are given in Figs. 4 and 5. This trial corresponds to a frequency of 900 MHz in a flat Rayleigh-fading environment, and a vehicle speed of 70 mph. The shape is characteristic of distortion spectra produced at a wide range of correction gain limits and with or without simulated

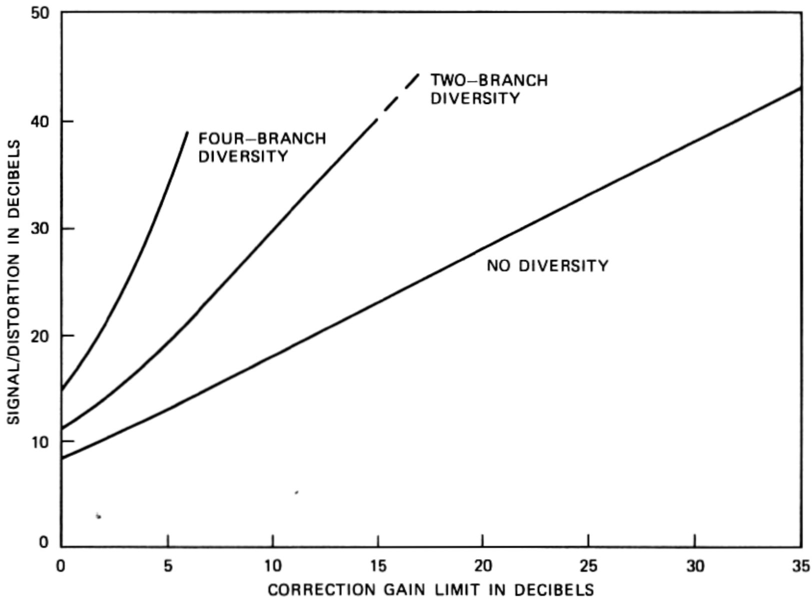

Fig. 3—Gain-limit distortion (flat fading).

urban delay spread (discussed later). Of course, these factors do directly affect the *amount* of correction distortion.

2.2 Effects of correction on interference

The baseband S/I ratio (SIR_b) will be expressed as a function of the cochannel voice signal-to-interference ratio at RF(SIR_r) and the correction gain limit. The pilot signal is assumed to be free of interference, including cochannel pilot interference (which will be discussed in Section 2.4). The receiver input in the case of a desired signal plus a single interferer is given by

$$s_r(t) = a_d r_d \exp[i(\omega_c t + \theta_d + \phi_d)] + r_d \exp[i(\omega_{pd} t + \phi_d)] \\ + \alpha[a_i r_i \exp[i(\omega_c t + \theta_i + \phi_i)] + r_i \exp[i(\omega_{pi} t + \phi_i)]], \quad (11)$$

where

a_d, a_i are the desired and interfering voice envelopes,
θ_d, θ_i are the desired and interfering voice phase functions,
ω_c is the voice virtual carrier frequency,
ω_{pd} is the desired pilot frequency,
ω_{pi} is the interfering pilot frequency—assumed for this analysis to be offset from the desired voice and pilot,
r_d, r_i are the Rayleigh fading envelopes,

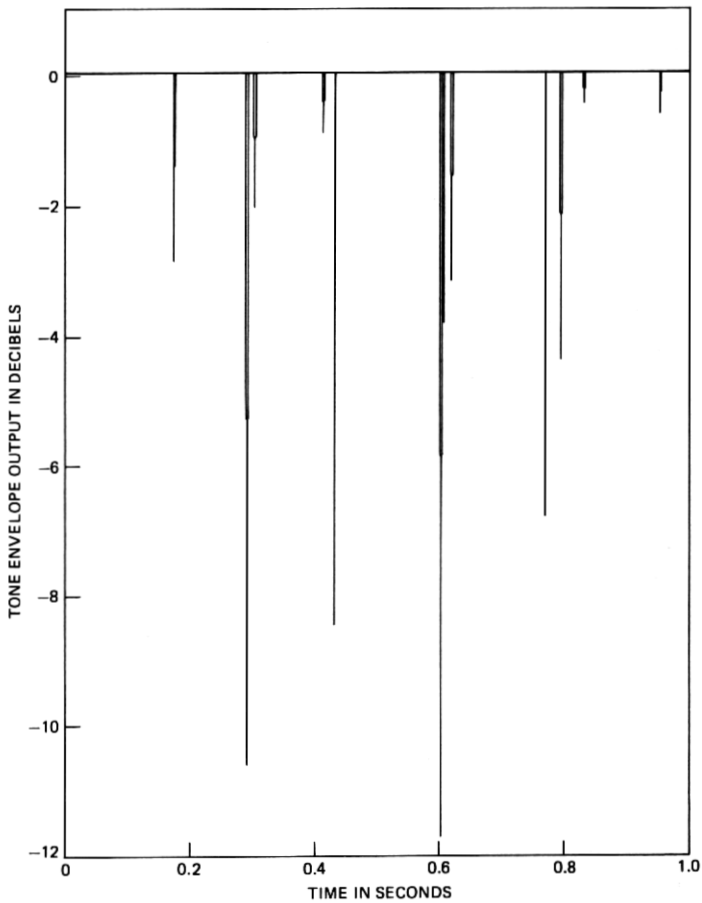

Fig. 4—Tone envelope at the corrected output, 20-dB correction.

ϕ_d, ϕ_i are the rapid phase-variation functions due to motion,

α is a relative propagation constant that determines the S/I at RF (i.e., $\text{SIR}_r = 1/\alpha^2$).

The corrected receiver output can then be written as

$$s_b(t) = s(t) + d(t) + i(t), \tag{12}$$

where

$$s(t) = a_d e^{i\theta_d}, \tag{13}$$

$$d(t) = \left[r_d \min\left(\frac{1}{r_d}, \frac{1}{\epsilon r_{do}}\right) - 1 \right] a_d e^{i\theta_d}, \tag{14}$$

$$i(t) = \alpha r_i \min\left(\frac{1}{r_d}, \frac{1}{\epsilon r_{do}}\right) a_i \exp[i(\theta_i + \phi_i - \phi_d)]. \tag{15}$$

The signal-to-correction distortion ratio remains defined as before. The signal-to-interference ratio at the output is defined conventionally as

$$\mathbf{SIR}_b = \frac{\overline{s(t)^2}}{\overline{i(t)^2}}. \tag{16}$$

For assumed equal power averages for $a_d(t)$ and $a_i(t)$, and independence of the voice and fading envelopes, (13) and (15) can be substituted into (16) and the result reduced to

$$\mathbf{SIR}_b = \left[\overline{r_i^2}\ \overline{\min\left(\frac{1}{r_d}, \frac{1}{\epsilon r_{do}}\right)^2}\right]^{-1} \mathbf{SIR}_r \tag{17}$$

or

$$\mathbf{SIR}_b = \gamma^{-1}\mathbf{SIR}_r, \tag{18}$$

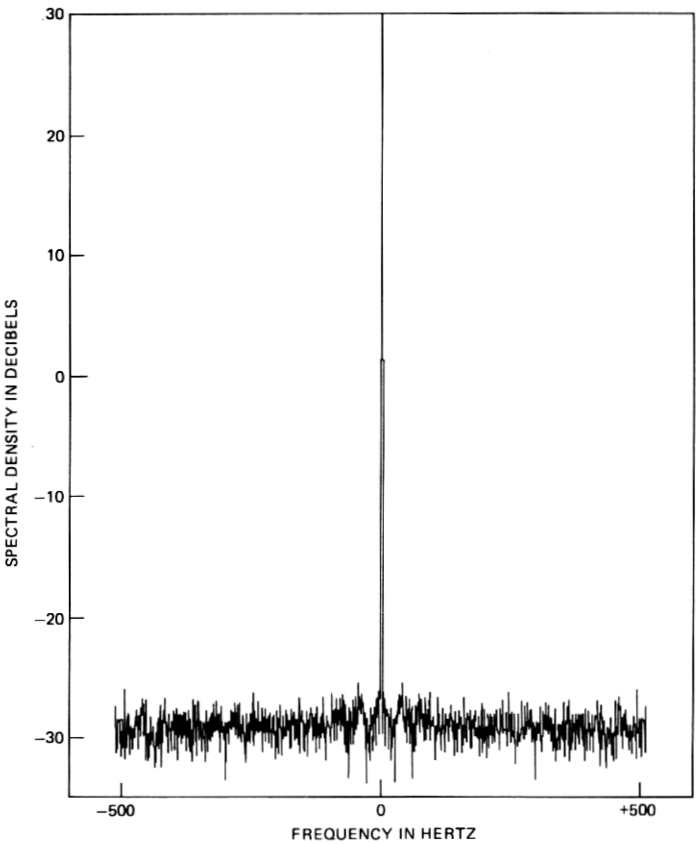

Fig. 5—Tone-power spectrum at corrected output, 20-dB correction flat fading.

where γ is the signal-to-interference disadvantage ratio (SIR$_r$ to SIR$_b$). The expression for γ can be found by performing the indicated averages. This gives

$$\gamma = \int_0^{\infty} r_i(t)^2 p(r_i)\, dr_i \left(\frac{1}{(\epsilon r_{do})^2} \int_0^{\epsilon r_{do}} p(r_d)\, dr_d + \int_{\epsilon r_{do}}^{\infty} \frac{p(r_d)}{r_d(t)^2}\, dr_d \right), \tag{19}$$

where

$$p(r_i) = p(r_d) = 2 \frac{r}{r_0^2} \exp(-r^2/r_0^2) u(r). \tag{20}$$

Using the known series expansion for the last integral above,[8] the expression can be rewritten as

$$\gamma = -2 \ln(\epsilon) - \psi + \frac{1}{\epsilon^2} (1 - e^{-\epsilon^2}) - \sum_{n=1}^{\infty} \frac{(-1)^n \epsilon^{2n}}{n n!}, \tag{21}$$

where ψ is the Euler constant $(0.5772\cdots)$.

The above series converges rapidly for $\epsilon \leq 1$, the region of practical interest. Figure 6 gives the S/I disadvantage, γ, for the pilot-based receiver under consideration, as a function of available correction gain. The disadvantage varies from 6 to 8 dB for the range of gain limit from 15 dB to 25 dB. An analysis of the S/I performance of FM in the

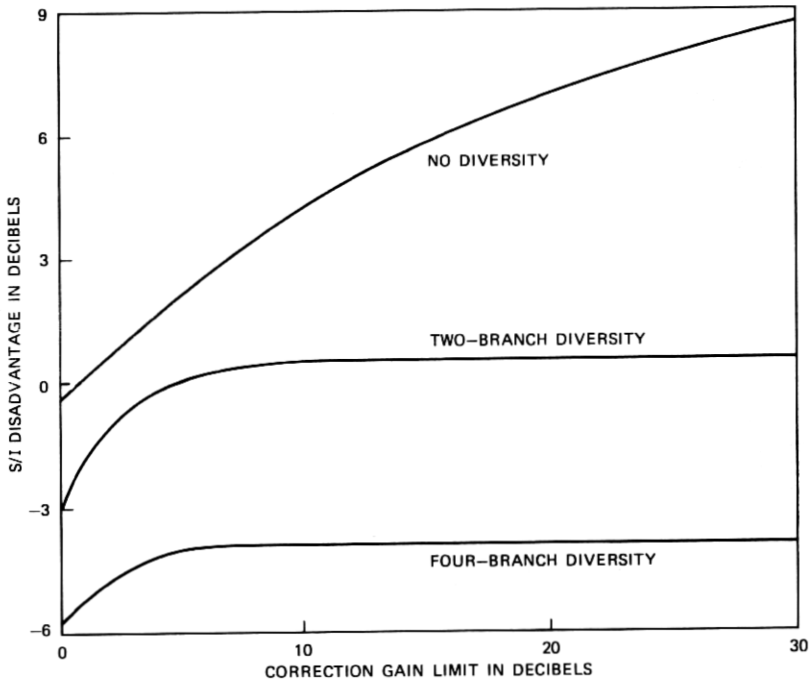

Fig. 6—Signal/interference disadvantage.

flat-fading environment can be taken from Jakes.[2] However, final conclusions from modulation comparisons must also await the experimental comparison of the subjective nature of the two different types of impairments. In addition, other factors, such as talker activity, are expected to have an unequal impact on the performance of different systems and other important impairments may be present due to particular hardware implementations.

The correction effect on the signal-to-noise ratio is identical to the effect on interference, described above, as long as the noise and fading processes are assumed independent.

2.3 Delay spread

The existence of a spread in arrival time of the various multipath components leads to the frequency selective fading phenomenon, for vehicles moving in the multipath field. In this case, the correlation in phase or amplitude between two pilots separated in frequency and transmitted from the same antenna is high for small frequency separation and falls to essentially zero as the separation substantially exceeds the correlation bandwidth. The correlation bandwidth is the frequency separation for a given environment, at which the complex correlation coefficient for the two pilot phasors has dropped to a given value (0.5 is common and is used here). The correlation bandwidth encountered in the mobile environment is quite large compared to 3 kHz, i.e., 90 kHz or more, over 99 percent of the urban area.† However, as was pointed out by Gans[2] for the AM case, the gain-correction process requires a very high degree of correlation between the rapid phase and gain variation imposed on the pilot to that of the phase and gain variation imposed on the voice band, if voice distortion is kept small. The relationship between the RF angular frequency separation (s), the delay spread (σ), and the complex envelope correlation (λ), is given exactly[2] for exponential delay distributions and approximately for typical distributions, as

$$\lambda^2 = (1 + s^2\sigma^2)^{-1}. \tag{22}$$

From this we can see that for a pilot to voice-tone separation of 2 kHz and 3 μs urban delay spread, the pilot to tone correlation is 0.9993. This is thought to be a near worst case condition for the urban environment (1 percentile probability), based on the limited delay-spread measurements reported in Ref. 9. The audio-frequency dependent distortion resulting from the effect of pilot-voice band decorrelation can be evaluated from the signal equations for the receiver:

Audio: $$s(t) = a(t)e^{i\theta(t)}, \tag{23}$$

† Based on a one percentile 3 μs RMS delay spread from Ref. 9 and eq. (22).

Received Input:

$$\begin{aligned} s_r(t) &= ar_v\exp[i(\omega_c t + \theta + \phi_v)] + r_p\exp[i(\omega_p t + \phi_p)] \\ &= z_v a \exp[i(\omega_c t + \theta)] + z_p\exp(i\omega_p t), \end{aligned} \tag{24}$$

Received Output:

$$s_b(t) = ar_v\min\left(\frac{1}{r_p}, \frac{1}{\epsilon r_{po}}\right)\exp[i(\theta + \phi_v - \phi_p)], \tag{25}$$

where

$$z_v = r_v(t)e^{i\phi_v(t)} \qquad \text{and} \qquad z_p = r_p(t)e^{i\phi_p(t)}$$

are correlated complex Gaussian random processes, band limited to twice the vehicle speed Doppler frequency. The spectrum assumed for these processes is appropriate for a vertically-polarized E field antenna as described in Section 2.1. The signal-to-correction distortion ratio is written as before as

$$\text{SDR} = \frac{\overline{(s(t))^2}}{\overline{(s_b(t) - s(t))^2}}. \tag{26}$$

After substituting (23) and (25) into the above and performing the indicated averages, the SDR can be written in terms of the envelope and phase properties as

$$\text{SDR} = \left[\overline{\left| r_v\min\left(\frac{1}{r_p}, \frac{1}{\epsilon r_{po}}\right)e^{i(\phi_v - \phi_p)} - 1\right|^2}\right]^{-1}, \tag{27}$$

where the subscripts p and v denote pilot and voice, and the subscript o indicates the RMS time average. The above process, in conjunction with (22), has been simulated on the digital computer using Monte Carlo techniques. The results, which again depend on the gain available for correction, as well as the audio-pilot frequency separation, are shown in Fig. 7 for the 1-μs urban delay-spread environment and in Fig. 8 for the 3-μs environment. Frequency separations of 2 and 3 kHz were used. The 2-kHz separation case was chosen to give distortion results representative of what might be expected for a 300- to 3000-Hz voice channel employing a pilot at or near the virtual voice-carrier position. The 3-kHz separation gives an upper bound to the distortion in the above case. Figure 9 shows the dependence of distortion on delay spread at a fixed 20-dB correction gain and a 2-kHz frequency separation.

The results indicate that the distortion introduced by delay spread will be relatively modest (typically SDR > 20 dB), and quite limited in occurrence for a particular coverage area, although an experimental

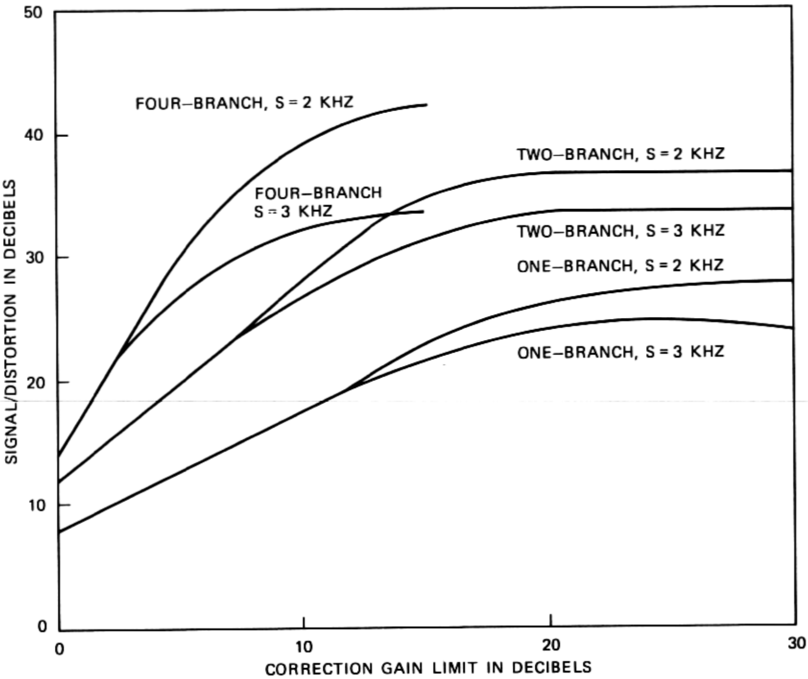

Fig. 7—Delay-spread distortion. N-branch diversity-simulation results. Delay = 1 μs, S = frequency separation.

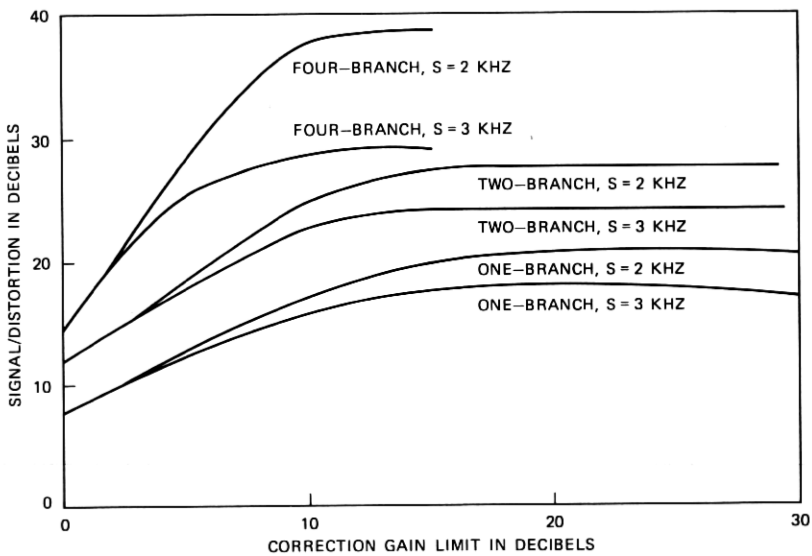

Fig. 8—Delay-spread distortion. N-branch diversity-simulation results. Delay = 3 μs, S = frequency separation.

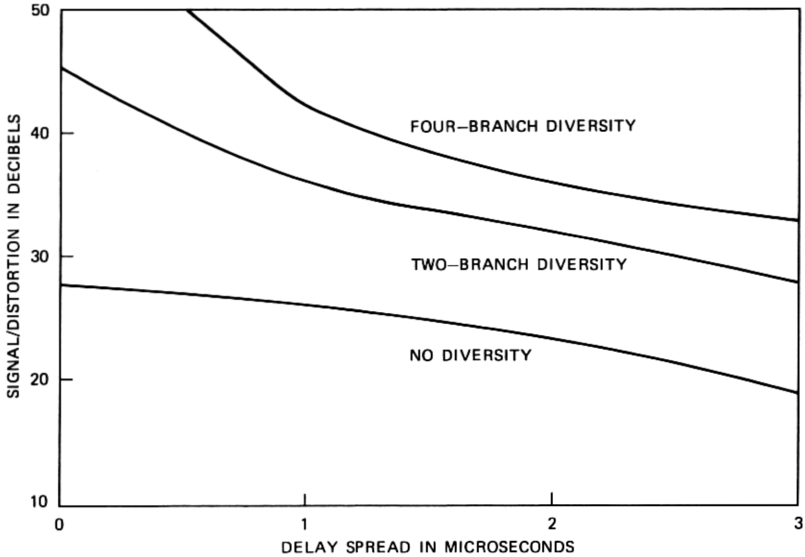

Fig. 9—Delay-spread distortion. Correction limit = 20-dB, 2-kHz frequency separation.

evaluation must be made to determine the subjective nature of this impairment. The audio tone output power spectrum for the 3-μs delay spread, 2-kH separation, case is given in Fig. 10 and is similar in shape to the output spectrum for gain-limit distortion alone (Fig. 5).

In reviewing this paper, M. J. Gans gave a closed-form expression for (27) as

$$\text{SDR} = \left(1 + \frac{1 - e^{-\epsilon^2}}{\epsilon^2} + (1 - \lambda^2)E_1(\epsilon^2) - \frac{\lambda\sqrt{\pi}}{\epsilon}\operatorname{erf}(\epsilon)\right)^{-1}. \qquad (27')$$

The simulation results check well with this exact expression.

2.4 Cochannel pilot interference

The interference effect from a cochannel pilot on the desired channel pilot decorrelates the received pilot and voice-complex envelopes. Thus cochannel pilot interference induces correction distortion in a manner very similar to that induced by operation in a high delay spread environment. Pilot-interference distortion results are obtained by simulating the pilot envelope $z_p(t)$ as the sum of an independent interfering pilot and the desired pilot. Frequency flat fading is assumed, so consequently the desired pilot envelope is taken as identical to the voice envelope $z_v(t)$. The processes $z_p(t)$ and $z_v(t)$ are then used in conjunction with (25) to numerically estimate the distortion due to cochannel pilot interference. Figure 11 presents the results. The distortion induced by cochannel pilot interference is seen to be approximately the same level as the baseband voice interference over the most likely operating conditions, but is present only during the desired speech.

III. IMPAIRMENTS OF A SPACE DIVERSITY RECEIVER

The three impairments discussed above for the nondiversity receiver are heavily dependent on the fading statistics of the complex fading envelope. Consequently, it is reasonable to expect that diversity significantly improves the situation. Indeed, this has been found to be the case. Figure 12 gives the equal-gain N-branch diversity receiver assumed. The individual branch signals are first phase corrected and then summed. The envelope correction signal is formed from the sum of the individual branch-pilot envelopes. Amplitude correction is then applied exactly as in the nondiversity case. Diversity combination in

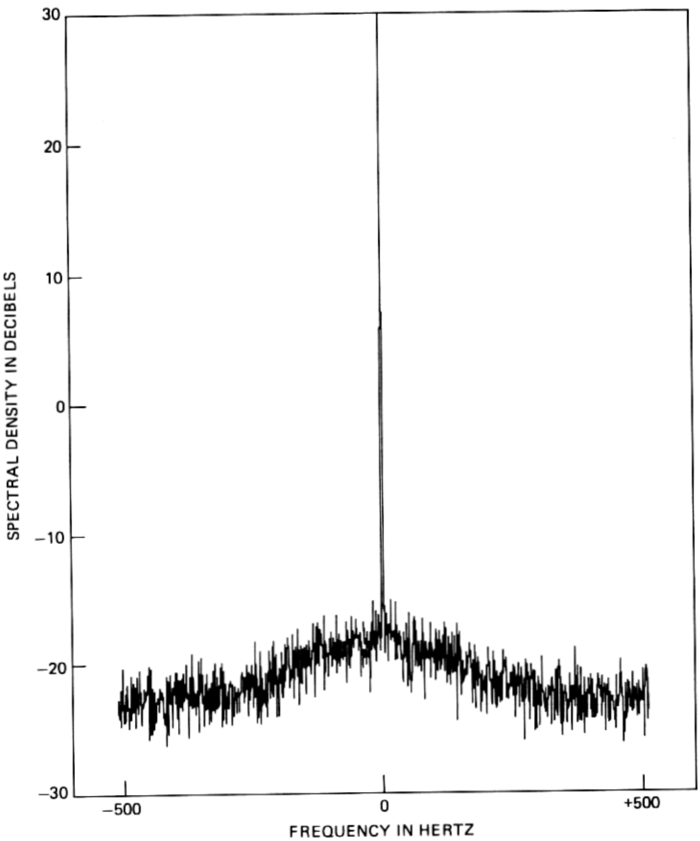

Fig. 10—Tone-power spectrum at corrected output. 20-dB correction, 3-μs delay spread, 2-kHz frequency separation.

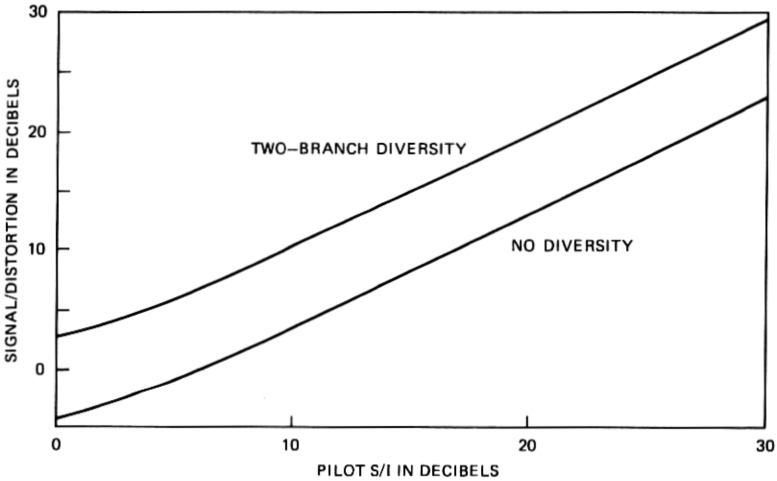

Fig. 11—Pilot-interference distortion. Maximum correction = 20 dB.

this way maintains the necessary voice correction and simultaneously achieves the benefits of having improved signal-envelope statistics prior to the gain correction step.

The simulation program has been extended to include N-branch diversity based on the development in Sections 3.1 to 3.3. Results for equal gain combining will be reported here. Analytical results have

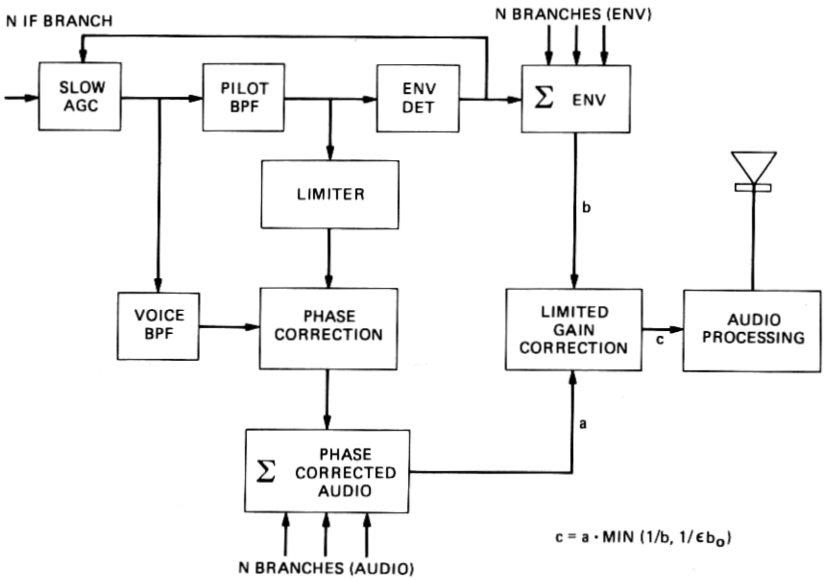

Fig. 12—N-branch diversity-pilot-based receiver.

been obtained for the signal-to-interference disadvantage for the two-branch diversity case. The fading statistics on each diversity branch were taken to be independent, and as described in (20).

3.1 Gain limit distortion—diversity

The signal equations for the diversity receiver in the frequency flat-fading environment are given by

Received Input(nth branch):

$$s_{rn}(t) = ar_n\exp[i(\omega_c t + \theta + \phi_n)] + r_n\exp[i(\omega_p t + \phi_n)], \quad (28)$$

Corrected Output:

$$s_b(t) = r_s\min\left(\frac{1}{r_s}, \frac{1}{\epsilon r_{so}}\right)a_v e^{i\theta}, \quad (29)$$

where the subscript s denotes a summation over N-diversity branches, and the subscript o denotes the RMS time average. The distortion as defined in (6) can be evaluated from (1) and (29) for the N-branch diversity case as

$$\text{SDR} = \left[\overline{\left(r_s\min\left(\frac{1}{r_s}, \frac{1}{\epsilon r_{so}}\right) - 1\right)^2}\right]^{-1}. \quad (30)$$

The fading envelopes were assumed to be independent and Rayleigh as given in (8). Results of the simulation based on (8) and (29) are given in Fig. 3 for two- and four-branch diversity receivers.

3.2 Signal to interference disadvantage—diversity

The signal-to-interference disadvantage as defined in (18) can be evaluated for the N-branch, equal gain, diversity receiver following the previous development of (17) and including diversity as in Section 3.1. This leads to the following expression for S/I disadvantage:

$$\gamma = \overline{Nr_{io}^2\min\left(\frac{1}{r_{ds}}, \frac{1}{\epsilon r_{dso}}\right)^2}, \quad (31)$$

where r_o is the RMS average common to each branch envelope. The subscripts d and i denote the desired and interferer envelopes and the other subscripts are defined in Section 3.1. Signal-to-interference disadvantage results based on the numerical evaluation of (31) are given in Fig. 6. Rayleigh envelope statistics, as given in (20), were again assumed.

For the two-branch case, the probability density function of the sum of two Rayleigh envelopes can be obtained by a series evaluation of the convolution integral involving the individual density functions. This can then be applied to (31) and the indicated averages can be performed to give a series expression for the S/I disadvantage for two-branch equal-gain diversity as

$$\gamma = 2 \sum_{n=0}^{\infty} \frac{(-1)^n(\delta\epsilon)^{2n}}{n!2^n} \left(\frac{1}{2n+1} \frac{1}{2n+3}\right)\left[\sum_{m=0}^{n} \frac{1}{(\delta\epsilon)^{2m}} e^{-(\delta\epsilon)^2/2} G(n, m)\right.$$

$$\left. - \sum_{m=0}^{n+1} \frac{1}{(\delta\epsilon)^{2m}} e^{-(\delta\epsilon)^2/2} G(n+1, m) + \frac{1}{(\delta\epsilon)^{2n+2}} G(n+1, n+1)\right], \tag{32}$$

where $\delta = r_{so}/r_o = \sqrt{2 + \pi/2}$ and

$$G(n, m) = \frac{n!2^m}{(n-m)!}. \tag{33}$$

In the limit of infinite available correction gain, the expression for two-branch diversity S/I disadvantage reduces to a constant given by

$$\gamma = 2 \sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{2n+1} \frac{1}{2n+3}\right)$$
$$= 1.142 \quad \text{or} \quad 0.58 \text{ dB}. \tag{34}$$

Here again, a closed-form expression has been supplied, in review, by M. J. Gans for (32), which gives the S/I disadvantage as

$$\gamma = \frac{1 - e^{-2x^2}}{x^2} - \frac{2\sqrt{\pi}}{x} e^{-x^2} \mathrm{erf}(x) + \pi[1 - \mathrm{erf}^2(x)], \tag{32'}$$

where $x = \epsilon\sqrt{1 + \pi/4}$.

3.3 Delay spread—diversity

Following the analysis of Section 2.3, the diversity signal-to-distortion ratio at the receiver output is found to be

$$\text{SDR} = \left[\overline{\left|\min\left(\frac{1}{r_{ps}}, \frac{1}{\epsilon r_{pso}}\right)\left(\sum_{n=1}^{N} r_{vn} e^{i(\phi_{vn} - \phi_{pn})}\right) - 1\right|^2}\right]^{-1}, \tag{35}$$

where the subscripts p, v, s, and o are as previously defined, and the subscript n denotes the diversity branch number. This expression can be shown to be finite (nonzero) in the case of unlimited available correction gain, completely uncorrelated pilot and voice envelopes, and two-branch diversity, by applying the series expansion for the probability density of r_s in the indicated average and replacing the complex and signed quantities with their respective magnitudes. The complex envelopes of one branch were taken as independent of the complex envelopes of other branches. As in Section 2.3, the complex pilot envelope is assumed to be correlated with the complex fading voice envelope, corresponding to a given delay spread. For the assumed exponential delay spread profile, the relationship is given in (22). Based on these assumptions, the process has been simulated for the same conditions used to produce the nondiversity delay spread results. The diversity delay spread results are given in Figs. 7 and 8.

IV. CONCLUSIONS

This paper has addressed the pilot-based correction of SSB voice signals for the effects of vehicle motion in a Rayleigh-distributed multipath field. Results were presented for the specific, idealized receiver in Fig. 1 which employs feed-forward gain and phase correction. The amount of gain available for correction in the receiver is an important parameter that affects the audio quality of a pilot-based SSB receiver in the fading environment. This gain limit is bounded on the low end by the distortion that results from imperfectly correcting the voice signal during fades. The gain limit is bounded on the high end by the increasingly detrimental effects of correction on cochannel interference and by urban delay spread. Approximately 20 dB of correction relative to the RMS envelope appears reasonable, for a nondiversity receiver. For this degree of correction, the residual distortion is 27 dB below the speech, in the absence of significant delay spread.

The correction process also deteriorates the S/I (S/N) ratio at baseband by 5 to 9 dB compared to the SIR (SNR) at RF for the range of useful correction gain limits of 10 to 30 dB. This S/I disadvantage is about 7 dB for the 20-dB correction limit mentioned above. Cochannel pilot interference would additionally contribute an approximately equal impairment level while the desired speech is present (pilot interference distortion).

The effects of urban delay spread do not appear to represent a major obstacle. The distortion resulting from a relatively rare 3-μs delay-spread environment is still more then 18 dB below the speech.

Both equal gain and maximal ratio space diversity can be applied to the previously described SSB receiver. The latter would be expected to have a slight edge in performance improvement. Equal gain diversity was selected for this analysis for convenience. Two-branch, equal gain diversity was seen to significantly mitigate the previously described impairments. The amount of distortion, and the S/I disadvantage for a given gain limit, is considerably reduced. In addition, the use of unlimited correction gain is feasible without adverse effects on interference and delay spread performance. In fact, correction gain in a diversity receiver would probably be determined by the diminished returns in audio quality and by hardware considerations. This point is likely to be found somewhere between 10 and 20 dB of correction. A two-branch diversity receiver with 20 dB of correction would have distortion that falls 45 dB below the voice in a nondelay spread environment, and more than 24 dB below the voice when the delay spread is as high as 3 μs. The S/I disadvantage for this case would be less than 1 dB.

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