

Efficient Utilization of Satellite Transponders via Time-Division Multibeam Scanning

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The space segment of a satellite system is proposed wherein a fixed number of identical transponders are shared among a larger number of spot beam regions which completely span a large total service area. Time-division multiple-access techniques are employed, and each transponder is rapidly scanned over appropriately defined group pairs of spot beam regions, thereby establishing full coverage and full interconnectivity. The service is matched to the nonuniform traffic requirements exhibited among the various spot beam regions, reliability can be optimized since all transponders are identical, and each transponder is utilized with an efficiency of 100 percent. A mathematical proof is presented which shows that the traffic can always be assigned on a nonconflicting basis, and an efficient assignment technique is described.

I. INTRODUCTION

The trend toward higher frequency communication satellites employing multiple spot beams affords significant capacity advantages relative to lower frequency, wide-coverage area systems, since the allocated spectral band can be reused in the various spot beams.^{1,2} When used in conjunction with digital modulation techniques and time-division multiple-access, the various coverage regions are readily interconnected via an onboard satellite switch operating in the time-division multiplex mode. In addition to the frequency reuse capability, the down-link transmitter power requirements are generally reduced because the antenna gain is higher than for a wide coverage area system.

Despite these advantages, however, multiple spot beam satellites have several distinct drawbacks. These are generally associated with conflicting requirements concerning reliability,³ coverage and blackout areas,⁴ efficient transponder utilization, and nonuniform traffic density demands.

In the following sections we explore these conflicting requirements and review some partial solutions proposed to date. Then we present a space segment configuration for a satellite communication system which provides high reliability with a minimum of redundancy, access from any location within a wide service area, and up to 100-percent efficiency in transponder capacity utilization matched to an arbitrary nonuniform density of traffic demand over the entire service region. This system employs N identical transponders which are shared on a time-division basis among $M \geq N$ antenna ports spanning the entire service region. Starting with the traffic demand matrix, we give a mathematical proof that the desired arrangement is always possible, and present an assignment algorithm.

Such a system might find applicability to a geosynchronous satellite operating in the 12/14-GHz band. From synchronous orbit, the 3-dB contour of a beam radiated from a $2\frac{1}{4}$ -m antenna would cover about 1 percent of the continental United States. Total United States coverage, then, would require about 100 such beams. Not only is the offered traffic nonuniformly distributed over the subregions, but within most such subregions the traffic is far too small to justify deployment of a dedicated wideband transponder to each. Moreover, from a practical viewpoint, the number of onboard satellite transponders is limited to the range of 10 to 20 by weight, power, and cost constraints. Through proper time-division assignment, these 10 to 20 transponders can efficiently provide service to the entire United States.

II. PROBLEMS OF MULTIPLE SPOT BEAM SATELLITES

A major problem in multibeam satellite design is one of transponder reliability. Unlike area coverage systems wherein the allocated band is divided among several transponders and service is provided via frequency division multiple access, it is desirable to serve each spot beam of a multibeam satellite system with a single transponder. With this approach, the required number of transponders is kept from becoming prohibitive, and the weight of the communications subsystem is minimized. However, sufficient redundancy must be provided to ensure high reliability for each transponder since single failures would preclude continuing service to the area serviced by that transponder. By contrast, for area coverage systems using frequency division multiple access, isolated failures merely cause a slight increase in the demand presented to the surviving transponders.

A second problem in multibeam satellite systems concerns efficient utilization of the satellite transponders. In general, the traffic demands from the various coverage areas (or footprints) are nonuniform. Thus, to utilize each transponder fully, the capacity of each must be tailored to the traffic demand of the area covered by that transponder. A tech-

nique for achieving such a custom fit has been reported,⁵ wherein the bit rate of each beam is selected as a fixed multiple of some basic rate. At the satellite, each uplink beam is demultiplexed into several basic rate bit streams, switched, and then remultiplexed into downlink beams. One disadvantage of this scheme is that onboard demodulation and remodulation is required. However, a more serious disadvantage in such a system is the need for nonidentical transponders, which precludes sharing a common pool of spare transponders among all beams, and the reliability of the system suffers.³

A third problem of multibeam satellites involves means of accessing traffic from areas not within the footprint of some spot beam. Several solutions have been proposed,⁴ involving sharing the spectrum between spot beams and an area coverage beam. These have the disadvantage that the area coverage transponders are different from the spot beam transponders and have higher power requirements to compensate for the loss of antenna gain. Also, the fixed spot beam transponders (assumed identical) are not matched to traffic requirements of the area served.

Another solution to the access problem involves the use of a steerable spot beam which can be rapidly scanned across the entire service region via a phased array antenna, thereby providing universal coverage.⁶ When used in conjunction with a multitude of fixed spot beams, the resulting hybrid system has the advantages of frequency reuse, high antenna gain, and identical transponders, and hence is the most attractive proposal among those reported to date. A similar system which provides for beam scanning by appropriate excitation of feedhorn clusters has also been proposed.⁷ However, such systems do not utilize the transponders efficiently, because of nonuniform traffic demands from the various areas covered.

III. TIME DIVISION MULTIBEAM SCANNING SATELLITE

To enable frequency reuse via a multibeam satellite system employing identical transponders, so that all transponders are used at maximum efficiency and a uniform grade of service is provided over the service area, we propose to generalize upon the scanning-beam approach.

Consider a satellite employing N identical wideband transponders, each with a capacity or throughput of C units. The diameter of the satellite antenna and the resulting beamwidth determine the number M of distinct footprints needed to provide service anywhere throughout the required service area. In general, M may be much greater than N , but in what follows we only require that $M \geq N$.

The system traffic can be represented by an $M \times M$ matrix $[t_{ij}]$ as shown:

$$[t_{ij}] = \begin{bmatrix} t_{11} & t_{12} & \cdots & t_{1M} \\ t_{21} & t_{22} & \cdots & t_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ t_{M1} & t_{M2} & \cdots & t_{MM} \end{bmatrix}. \quad (1)$$

The element t_{ij} represents the traffic originating in beam i and destined for somewhere in beam j . Each footprint might contain several ground stations, so t_{ij} represents the sum of the traffic from all stations within beam i which is directed to stations within beam j .

It is not necessary that the traffic matrix be symmetric, and a loop-back feature is possible, i.e., we do not require $t_{ij} = t_{ji}$, nor do we require $t_{ii} = 0$. Of course, $t_{ij} \geq 0$.

Two requirements must be imposed on the traffic matrix $[t_{ij}]$. First, since the total capacity of the satellite is equal to NC (N transponders each of capacity C), we require that

$$T = \sum_{i=1}^M \sum_{j=1}^M t_{ij} \leq NC. \quad (2)$$

The second requirement is that the traffic originating from or destined for a particular beam should not exceed the capacity of one transponder, i.e.,

$$\text{Row sum } R_i = \sum_{j=1}^M t_{ij} \leq C \quad i = 1, 2, \dots, M \quad (3)$$

$$\text{Column sum } S_j = \sum_{i=1}^M t_{ij} \leq C \quad j = 1, 2, \dots, M. \quad (4)$$

The transponders are utilized with 100-percent efficiency when (2) is satisfied as an equality. This equation may be interpreted as establishing the minimum number N of transponders required. Conditions (3) and (4) are necessary because no two transponders can be connected to a common spot beam (either uplink or downlink) on a noninterfering basis.

If the total offered traffic equals the sum of the transponder capacities, we have the potential for 100-percent utilization. We will show that it is possible to interconnect the various uplink beams, transponders, and downlink beams such that this is achieved. We do this on a time-division basis by enabling each of the N transponders to access any of the M receive (uplink) antenna ports and any of the M transmit (downlink) antenna ports. Figure 1 shows the use of two $M \times N$ crossbar type switches which enable any required interconnection. Alternatively, the appropriate interconnections could be achieved by using N phased array antennas as shown in Fig. 2.

It remains to be shown that all the offered traffic can be allocated among the N transponders on a noninterfering basis, i.e., at any instant of time, the N transponder inputs are each connected to a different receive port, and the N transponder outputs are each connected to a dif-

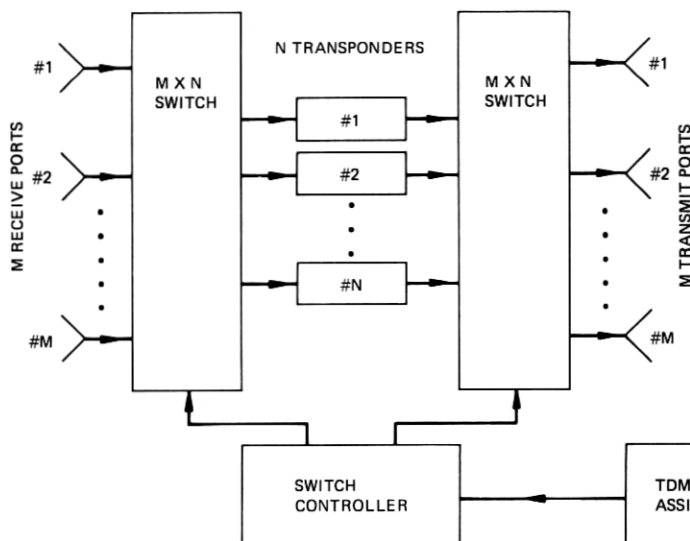


Fig. 1—Satellite communication subsystem for rapid TDMA scanning of multiple transponders using two $M \times N$ crossbar switches.

ferent transmit port. The theorem below guarantees that such an assignment is always possible.

Definition: A diagonal of a matrix $[t_{ij}]$ is a K -tuple $D = \{d_1, d_2, \dots, d_K\}$, where each member is a nonzero element of the matrix and no two elements appear in the same row or same column of the matrix. The length of the diagonal is K (the number of elements) and the diagonal is said to cover the K rows and K columns from which the elements are taken.

Theorem 1: In a traffic matrix $[t_{ij}]$ for which $T = \sum_{i=1}^M \sum_{j=1}^M t_{ij}$ equals NC and for which no row or column sum exceeds C , a diagonal of length N exists which covers all rows and columns which sum to C exactly (if any).

The proof of this theorem is somewhat lengthy and is presented in the appendix.

For convenience, we will assume that the elements t_{ij} of the traffic matrix are integers, representing the traffic as multiples of some basic unit such as, for example, one voice channel.

We shall assign traffic to the various transponders as follows: Let the TDMA frame consist of C time slots, each representing one unit of traffic. There are N such frames, one belonging to each transponder. In the traffic matrix $[t_{ij}]$, find any diagonal of length N which covers all rows and columns summing to C (if any). Theorem 1 guarantees this is always possible. From these N diagonal elements, extract one unit of traffic from each and assign one unit to each of the N transponders. Since the traffic assigned to the transponders (for this time slot) originates from different

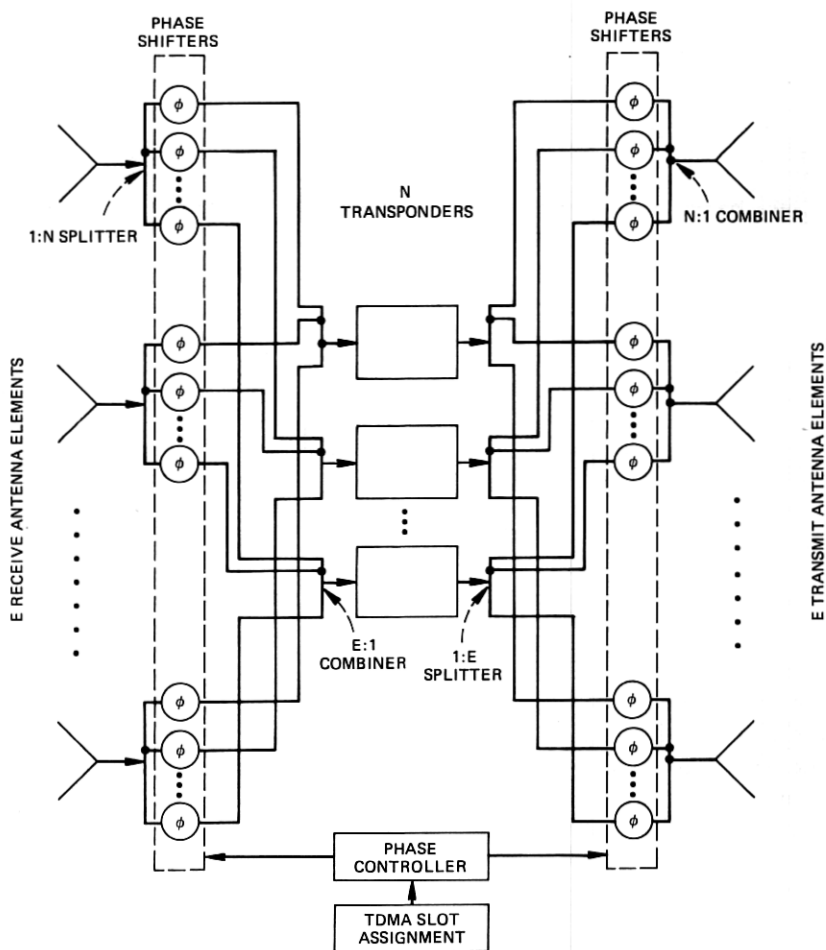


Fig. 2—Satellite communication subsystem for rapid TDMA scanning of multiple transponders using receive and transmit phased-array antennas. Each transponder can be steered independently to M transmit and M receive spot beam regions.

uplink beams and are directed to different downlink beams, then the traffic has been assigned on a noninterfering basis.

Now since N units of traffic have been removed from the matrix, the reduced matrix has a total traffic of $NC - N = N(C - 1)$ units. Furthermore, each transponder has $C - 1$ units of traffic carrying capacity left, and no row or column of the reduced matrix sums to more than $C - 1$. The latter is true because every row and column which summed to C in the original matrix has had one unit of traffic removed (because of the way the diagonal was constructed).

At this stage, we have the same situation as we started with, except $C - 1$ replaces C . By the same technique, we can assign another N units

of traffic to the next time slot in each transponder, and end up with a matrix with remaining traffic $N(C - 2)$ in which no row or column sums to more than $C - 2$. Each transponder has then $C - 2$ time slots unallocated. Hence, we can repeat this procedure until all transponder time slots are used and no traffic remains unallocated.

Thus, the nonuniform demands of a traffic matrix can be met by N identical transponders each operating at 100-percent utilization efficiency. We also note that, although the method described was for a matrix for which eq. (2) was satisfied as an equality (i.e., $T = NC$), it also applies to a matrix for which $T < NC$, because we can always pad such a matrix with dummy traffic⁸ until $T = NC$. The assignments corresponding to the dummy traffic can be ignored, and simply reflect the fact that the available transponder capacity exceeds the demand.

The assignments are not unique, and it may be possible to extract more than one unit of capacity per diagonal element at a time. This is desirable from a practical point of view as it minimizes the number of times the $M \times N$ switches have to be reconfigured during one frame period. To achieve this, it seems desirable to choose the N diagonal elements from large elements in the rows and columns with the largest sums, if possible. The maximum traffic extractable is $t = \min(t_1, t_2)$, where $t_1 =$ smallest element on the diagonal and $C - t_2$ is the largest row or column sum among the rows and columns not covered by the diagonal.

As an example, consider the matrix below with $N = 3$ and $C = 13$.

		Downlink beam j					
		t_{ij}	1	2	3	4	R_i
Uplink beam i	1	3	6	2	1	12	
	2	6	4	0	0	10	
	3	0	1	6	2	9	
	4	2	0	2	4	8	
S_j		11	11	10	7	39 = T	

In Fig. 3 we show the successive reductions of the matrix as the traffic is assigned to transponders. The diagonal elements chosen are circled, and rows and columns which sum to (the reduced value of) C are marked with an asterisk.

The corresponding traffic assignments to the transponders are shown in Fig. 4. The switch must be reconfigured six times per frame for this solution.

IV. CONCLUSIONS

Although the system described has been presented in terms of subdividing the transponder capacity by time division, it is applicable to any other method of subdividing the transponder, e.g., by frequency division or by a combination of time and frequency division. In a frequency-division system, the smallest subdivision unit of capacity would

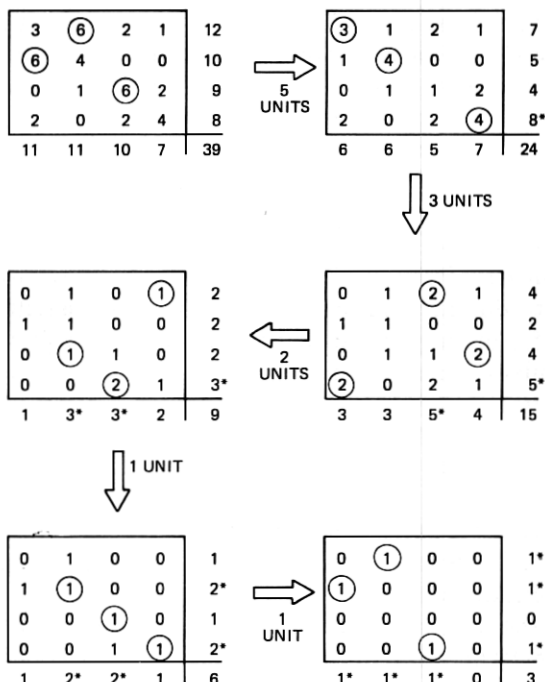


Fig. 3—Illustrative reduction of a 4×4 traffic matrix. The matrix contains 39 units of traffic, and there are three transponders, each of capacity 13.

usually be larger than for a time-division system, and transponder linearity would be an important consideration as far as crosstalk is concerned. In a time-division system, transponder nonlinearities are more tolerable.

For the system proposed, reliability of the transponders could be provided by the usual method of having a standby transponder for every transponder in use or perhaps sharing a standby with two operational transponders. However, an interesting alternative is to provide $N' > N$ transponders and use $M \times N'$ switches at input and output. In this way, failed transponders can be excluded by simply modifying the switching sequence, and we have a pool of spare transponders which can be used to supply replacements for any that fail.

The output switch at the satellite will generally operate at a high power level, and switching time may be a significant factor. In the phased-array realization, the equivalent problem is the time taken to steer the beam from one area to another. In either case, reducing the number of switch reconfigurations per frame will minimize any overhead time due to switching delays. Since the switching sequence, once decided upon, is not changed from frame to frame, a search for an efficient switching sequence is worthwhile.

In practice, other considerations besides switching delays would also

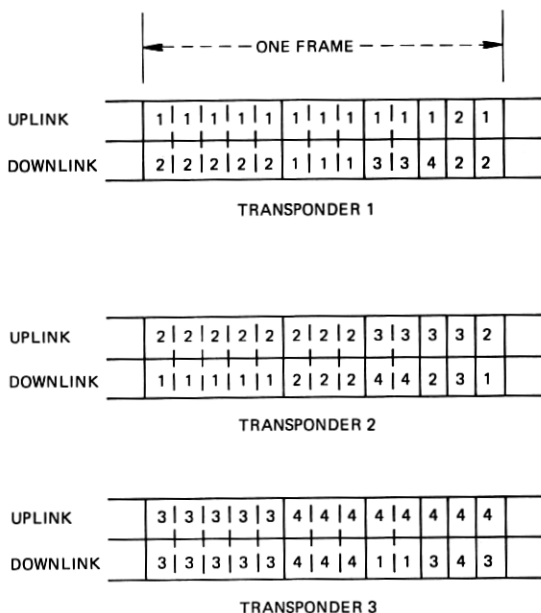


Fig. 4—TDMA frame assignment for the example of Fig. 3. The numbers 1 through 4 appearing within each frame correspond to the spot beam coverage areas 1 through 4 of the 4 × 4 traffic matrix.

be of importance. One additional consideration would be the interference between stations in adjacent beam-coverage areas. This interference would be reduced by ensuring that the stations did not transmit or receive during the same time slot, or by using different polarizations. Alternatively, adaptive interference cancellation can be performed for the phased-array implementation. Constraints imposed by considerations such as these, however, may result in more than the minimum number of transponders being required to transmit the traffic.

We have described a system which enables efficient utilization of transponder capacity, while at the same time providing service over a wide area with a uniform grade of service, identical transponders in the satellite which can be coupled with a very efficient standby method for maintaining transponder reliability, and a system which can be adapted to changing traffic demands by simply altering the switching sequence at the satellite.

APPENDIX

Proof of Theorem 1

The definition of a diagonal and its length are found in the main text.

The proof is approached by first establishing a number of lemmas which can then be used in the proof of the theorem.

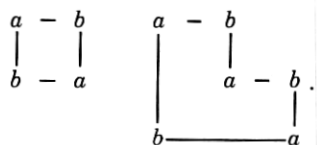
The theorem as proved here is slightly more general than that presented in the main text, since the existence of a diagonal of the required type is demonstrated for $(N - 1)C < T \leq NC$ rather than just for $T = NC$.

Lemma 1: If in a matrix $[t_{ij}]$ there exist diagonals $D_1 = \{a_1, a_2, \dots, a_N\}$ of length N and $D_2 = \{b_1, b_2, \dots, b_L\}$ of length $L \geq N$, then it is possible to construct:

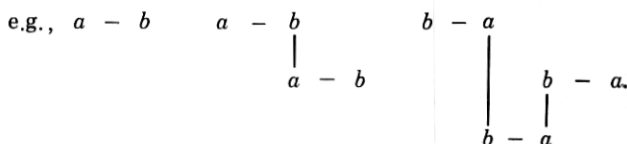
- (i) A diagonal D_3 of length L which covers all the rows and columns covered by D_1 .
- (ii) A diagonal D_4 of length $\geq L$ which covers the rows D_1 and the columns of D_2 .
- (iii) A diagonal D_5 of length $\geq L$ which covers the columns of D_1 and the rows of D_2 .

Proof:

- (i) Form disjoint sets $S_m, m = 1, 2, \dots$ from the elements of D_1 and D_2 as follows:
 - (a) Once an element has been allocated to a set, it is not considered further.
 - (b) To form set S_m , choose an initial element from among those not yet allocated to a set.
 - (c) If b_j is on the same row or column as an $a_i \in S_m$, then $b_j \in S_m$.
 - (d) If a_r is on the same row or column as a $b_t \in S_m$ then $a_r \in S_m$.
 - (e) Continue adding elements to S_m using (c) and (d) until no more can be added. If unallocated elements still remain, form a set S_{m+1} starting at step (b).
- (ii) The sets have the property that an element from one set cannot share a row or column with an element from another set.
- (iii) The sets are of the following varieties:
 - V1: Sets with equal number of elements from D_1 and D_2 which cover the same rows and columns e.g., a, b coincident,



- V2: Sets with equal numbers of elements from D_1 and D_2 which cover the same rows, but not the same columns,



V3: Sets with equal numbers of elements from D_1 and D_2 which cover the same columns, but not the same rows,

e.g., $\begin{array}{c} a \\ | \\ b \end{array} \quad \begin{array}{c} a \\ | \\ b \end{array} - \begin{array}{c} a \\ | \\ b \end{array} \quad \begin{array}{c} b \\ | \\ a-b \\ | \\ b-a \\ | \\ a \end{array} .$

V4: Sets with one more element from D_1 than from D_2 , in which the elements from D_1 cover all the rows and columns of the elements from D_2 ,

e.g., $a \quad a - \begin{array}{c} b \\ | \\ a \end{array} \quad \begin{array}{c} a-b \\ | \\ b-a \\ | \\ a \end{array} \quad \begin{array}{c} a \\ | \\ b \end{array} - \begin{array}{c} a \\ | \\ b-a \end{array} .$

V5: Sets with one more element from D_2 than from D_1 , in which the elements from D_2 cover all the rows and columns of the elements from D_1 ,

e.g., $b \quad b - \begin{array}{c} a \\ | \\ b \end{array} \quad \begin{array}{c} b-a \\ | \\ b \end{array} - \begin{array}{c} b \\ | \\ a \end{array} \quad \begin{array}{c} b \\ | \\ a \end{array} - \begin{array}{c} b \\ | \\ a-b \end{array} .$

Not all varieties need be present, but it is easily seen that there must be at least $L - N$ sets in V5.

$$\text{Let } V5' = \{\text{any } L - N \text{ sets from } V5\}.$$

Then

$$D_3 = \{a_i \notin V_5', b_j \in V_5'\}$$

$$D_4 = \{a_i \in V_1, b_j \in V_2, a_i \in V_3, a_i \in V_4, b_j \in V_5\}$$

$$D_5 = \{a_i \in V_1, a_i \in V_2, b_j \in V_3, a_i \in V_4, b_j \in V_5\}$$

are diagonals of the type required.

Lemma 2: If the maximum length of a diagonal in a matrix $[t_{ij}]$ is N , and the row and column sums do not exceed C , then

$$T_N \triangleq \sum_i \sum_j t_{ij} \leq NC.$$

Proof: If all the nonzero elements are in at most N rows (or columns), then by summing over these rows (or columns) we obtain $T_N \leq NC$.

We therefore need only consider the case where there are more than N rows and more than N columns which contain nonzero elements.

Let $D_1 = \{a_1, a_2, \dots, a_N\}$ be a diagonal of length N covering columns j_1, j_2, \dots, j_N . There must be another column j_{N+1} with a nonzero element x . One of the elements of D_1 , say a_r , must be on the same row as x , otherwise $\{a_1, a_2, \dots, a_N, x\}$ would form a diagonal of length $N + 1$.

Remove the row containing x and a_r from the matrix. We will show that the reduced matrix has a maximum diagonal length of $N - 1$.

In the reduced matrix we have $D'_1 = \{a_1, \dots, a_{r-1}, a_{r+1}, \dots, a_N\}$ of length $N - 1$ and suppose there is also a diagonal $D_2 = \{b_1, b_2, \dots, b_N\}$ of length N (in the reduced matrix). By Lemma 1 we can find from D'_1 and D_2 a diagonal D_3 of length N which covers all the rows and columns covered by D'_1 . Since $N - 1$ of the elements of D_3 are in columns $j_1 \dots j_{r-1}, j_{r+1} \dots j_N$, then both columns j_r, j_{N+1} cannot be covered by the N th element. Hence, D_3 augmented by either a_r or x would form a diagonal of length $N + 1$ in the original matrix. Hence, no such diagonal D_2 exists.

The reduced matrix satisfies the same conditions as the original matrix except $N - 1$ replaces N .

$$T_N = T_{N-1} + R \leq T_{N-1} + C,$$

where

T_N = sum of elements in original matrix

T_{N-1} = sum of elements in reduced matrix

R = sum of elements in row removed $\leq C$.

Hence, $T_N \leq NC$ if $T_{N-1} \leq (N - 1)C$. Since $T_0 = 0$, an inductive argument establishes the result.

Lemma 3: In a matrix $[t_{ij}]$ for which the row and column sums do not exceed C , and for which $T \triangleq \sum_i \sum_j t_{ij}$ satisfies $(N - 1)C < T \leq NC$ for some integer N , there exists a diagonal of length $L \geq N$.

Proof: Let L be the maximum diagonal length.

By Lemma 2, $T \leq LC$

But $T > (N - 1)C$, so $L > N - 1$,

Hence, a diagonal of length $L \geq N$ exists.

Lemma 4: In a matrix $[t_{ij}]$ for which the row and column sums do not exceed C , and for which $T = \sum_i \sum_j t_{ij}$ satisfies $(N - 1)C < T \leq NC$ for some integer N , there exists a diagonal of length $N'' \geq N$ which covers all rows and columns which sum to C exactly.

Proof: The submatrix consisting only of the P rows which sum to C has, by Lemma 3, a diagonal $D_1 = \{a_1, a_2, \dots, a_P\}$ of length P , because its el-

elements sum to PC . This diagonal covers all the P rows summing to C . Note that $P \leq N$.

By Lemma 3, the original matrix has a diagonal $D_2 = \{b_1, b_2, \dots, b_N\}$ of length N . By Lemma 1, we can construct from D_1 and D_2 a diagonal D'_2 of length $N' \geq N$ which covers all the columns of D_2 and all the rows of D_1 .

Let $D'_1 = \{a'_1, \dots, a'_Q\}$ be a diagonal of length Q of the submatrix consisting only of the Q columns which sum to C exactly. Note that $Q \leq N \leq N'$.

Then by Lemma 1 we can construct from D'_1 and D'_2 a diagonal D''_2 of length $N'' \geq N'$ which covers all the columns of D'_1 and all the rows of D'_2 (and hence all the rows of D_1).

Hence, D''_2 covers all the rows and columns which sum to C exactly. *Theorem: In a matrix $[t_{ij}]$ for which the row and column sums do not exceed C , and for which $T \triangleq \sum_i \sum_j t_{ij}$ satisfies $(N-1)C < T \leq NC$ for some integer N , there exists a diagonal of length N which covers all rows and columns which sum to C exactly.*

Proof: By Lemma 4, a diagonal $D_1 = \{a_1, a_2, \dots, a_L\}$ of length $L \geq N$ exists which covers the P rows and Q columns which sum to C exactly.

Divide D_1 into disjoint subdiagonals S_1, S_2 , and S_3 with L_1, L_2 , and L_3 elements, respectively, with $L_1 + L_2 + L_3 = L$.

$S_1 = \{\text{elements of } D_1 \text{ in both a row and a column summing to } C\}$

$S_2 = \{\text{elements of } D_1 \text{ in either a row or a column summing to } C, \text{ but not both}\}$

$S_3 = \{\text{elements of } D_1 \text{ in neither a row nor a column summing to } C\}$

If $L_1 + L_2 \leq N$, then a diagonal consisting of the $L_1 + L_2$ elements from S_1 and S_2 plus any $N - L_1 - L_2$ elements of S_3 is a diagonal of length N covering all rows and columns summing to C .

Hence, we need only consider the case $L_1 + L_2 > N$. Note that $P + Q = 2L_1 + L_2 > N$ also.

Consider the submatrix consisting of P rows and Q columns containing only those elements (including zero elements) which lie in both a row and a column summing to C . We know S_1 is a diagonal of length L_1 of this submatrix. The sum of the elements in the submatrix $= T' = \{\text{sum of elements of original matrix in those } P \text{ rows}\} - \{\text{sum of elements of original matrix in the same } P \text{ rows, but which do not lie in the columns summing to } C\}$. Hence, $T' \geq (P + Q - N)C$, since the first sum is PC and the second cannot exceed $(N - Q)C$.

By Lemma 3, the submatrix has a diagonal S_4 of length $P + Q - N = 2L_1 + L_2 - N > L_1$ (since $L_1 + L_2 > N$). By Lemma 1 we can construct from S_1 and S_4 a diagonal S'_1 of length $2L_1 + L_2 - N$, which covers all the rows and columns covered by S_1 .

Now S'_1 covers $L_1 + L_2 - N$ rows summing to C and $L_1 + L_2 - N$ columns summing to C not covered by S_1 and which must have been cov-

ered by S_2 . Hence, form a new subdiagonal S'_2 by deleting from S_2 the elements in these rows and columns. Then S'_1 and S'_2 cover different rows and columns.

Now S'_1 is a diagonal of length $L'_1 = 2L_1 + L_2 - N$ and S'_2 is a diagonal of length $L'_2 = L_2 - 2(L_1 + L_2 - N)$. Thus, the elements of S'_1 and S'_2 form a diagonal which covers all the rows and columns which sum to C and its length is $L'_1 + L'_2 = N$.

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