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More on Rain Rate Distributions and Extreme Value Statistics

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A new methodology is described for estimating the 5-minute rain rate distribution from yearly 5-minute maximum rain rate data and yearly accumulated rainfall data published by the National Climatic Center for U.S. locations. The method previously described gives the high rain rate portion of the distribution, whereas the extended methodology yields the complete distribution, which is assumed to be approximately lognormal. The three parameters characterizing the lognormal distribution can be calculated by application of the theory of extreme value statistics. The calculated results agree well with the 20-year data. The accuracy of the calculated results is limited by the instability of extreme rain rate data with a finite time base. Two-year rain rate data measured by a tipping bucket rain gauge at Palmetto, Georgia, are used to demonstrate that the time variation of rainfall process obeys a proportionate relationship, supporting the lognormal hypothesis.

I. INTRODUCTION

Reference 1 has described a methodology for calculating long-term distributions of high rain rates by applying the theory of extreme value statistics to the yearly maximum 5-minute rain rate data published by the National Climatic Center.[2,3] The obtained high rain rate distributions cover the range of interest to the engineering of terrestrial microwave radio links. However, for other applications, such as earth-satellite radio engineering, the rain rate distributions in the moderate and low rain rate ranges are also needed. This paper describes a methodology to obtain rain rate distributions covering the entire range (i.e., from below 5 mm/hr to greater than 200 mm/hr). The rain rate distributions are assumed to be approximately lognormal. The three parameters characterizing the lognormal distribution can be calculated from the yearly maximum 5-minute rain rate data and the yearly total accumulated

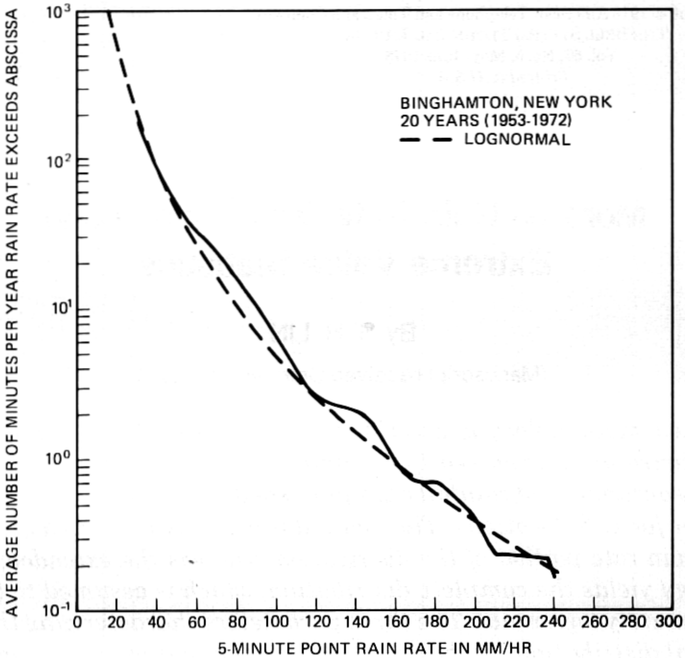

Fig. 1—Binghamton, New York: Comparison of 20-year distribution of 5-minute rain rate calculated by extreme value theory and lognormal hypothesis (dashed line) with 20-year data (solid line).

rainfall data by applying the theory of extreme value statistics. The calculated results agree well with 20-year data as shown in Figs. 1 to 13 and Figs. 17 to 20.

Sections II and III describe the method and discuss the results. Section IV discusses characteristics of measured time variations of rain rates in support of the proportionate effect described by Aitchison and Brown.[10] The proportionate variation of rain rates is simply another manifestation of the lognormality of rain rate statistics.

In this paper, a "5-minute rain rate" corresponds to the average value of the randomly varying rain rate in a 5-minute interval and is calculated as $\Delta H/\tau$ where ΔH is the 5-minute accumulated depth of rainfall and $\tau = 5$ minutes or $1/12$ hour is the rain gauge integration time. The methodology is also applicable to integration times other than 5 minutes.

II. EXTREME VALUE STATISTICS AND LOGNORMAL RAIN RATE DISTRIBUTION

Many sets of rain rate data indicate that rain rate distributions can be closely approximated by the lognormal distribution (see Refs. 4 to

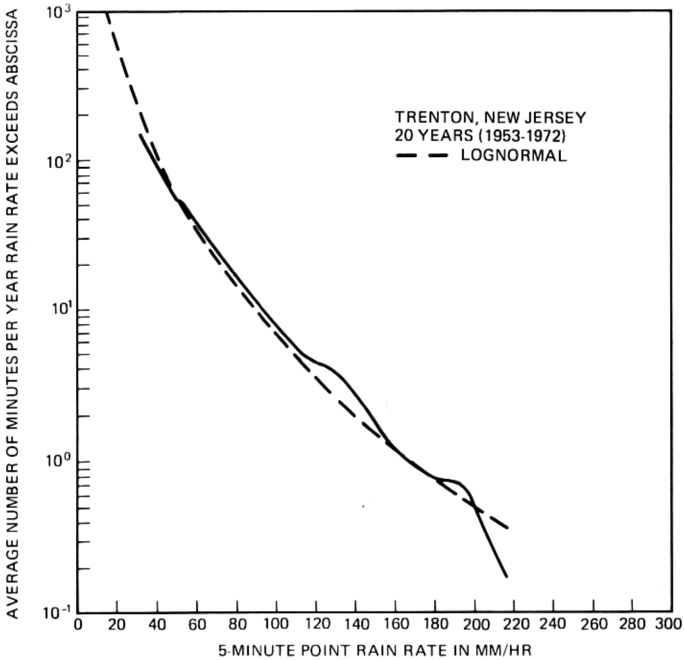

Fig. 2—Comparison for Trenton, New Jersey.

9, 23, 24 and Figs. 1 to 13 and 17 to 20 in this paper)*:

$$P(R \geq r) \approx P_0 \cdot \frac{1}{2} \operatorname{erfc}\left[\frac{\ln r - \ln R_m}{\sqrt{2}\, S_R}\right] \tag{1}$$

where R is the randomly varying 5-minute point rain rate, erfc($\sim$) denotes the complementary error function, ln($\sim$) denotes natural logarithm, S_R is the standard deviation of ln R during the raining time,[4] R_m in mm/hr is the median value of R during the raining time and P_0 is the probability that rain will fall at the point where the rain rate R is measured. Rain rate data usually emphasize high rain rate statistics with the result that the value of P_0, and hence the total raining time per year, are not directly available. In the following, it is demonstrated that the values of P_0, R_m and S_R, and hence the entire distribution $P(R \geq r)$, can be determined from the yearly maximum 5-minute rain rate data and the yearly total accumulated rainfall data.

Let W denote the long-term average value of the yearly accumulated depth of rainfall.† The relationship between W and the parameters in

* Figures 14, 15, 16, 21, and 22 are discussed later in Sections II and III.
† Excluding snowfall.

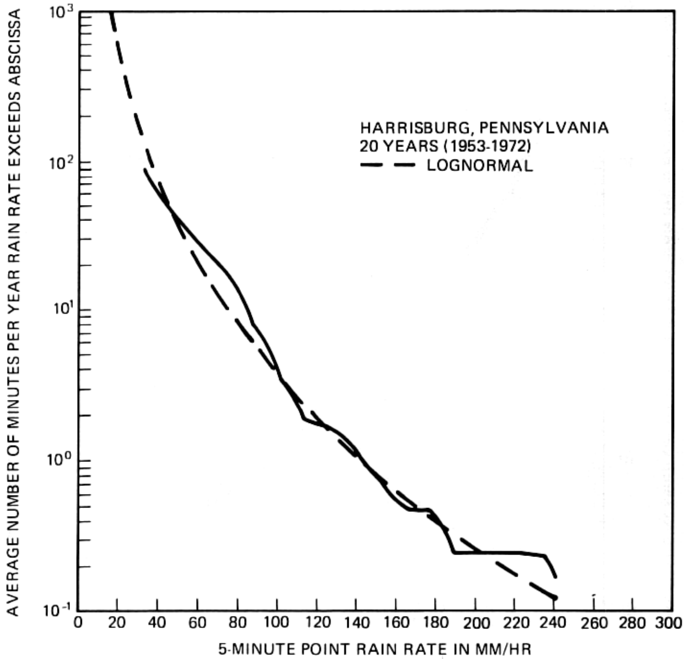

Fig. 3—Comparison for Harrisburg, Pennsylvania.

eq. (1) is

$$
\begin{aligned}
W &= \langle R \rangle \times \text{total raining time/year} \\
&= \langle R \rangle \times P_0 \times (8760 \text{ hours/year}) \\
&= R_m \times e^{S_R{}^2/2} \times P_0 \times (8760 \text{ hours/year})
\end{aligned}
\tag{2}
$$

where

$$\langle R \rangle = R_m \times e^{S_R{}^2/2} \tag{3*}$$

is the mean value of R during the raining time.[4] Long-term ($\geq$30 years) data on W for U.S. locations can be found in Refs. 2 and 11.

Let R_1 denote the yearly maximum 5-minute rain rate which varies from year to year. The distribution of R_1 is[1]

$$P(R_1 \geq r) = 1 - e^{-(e^{-y})} \tag{4}$$

where

$$y = \alpha(\ln r - U) \tag{5}$$

is called the reduced variate, α and U are scale and location parameters

* This relationship among $\langle R \rangle$, R_m and S_R holds if R is lognormal.[4,10]

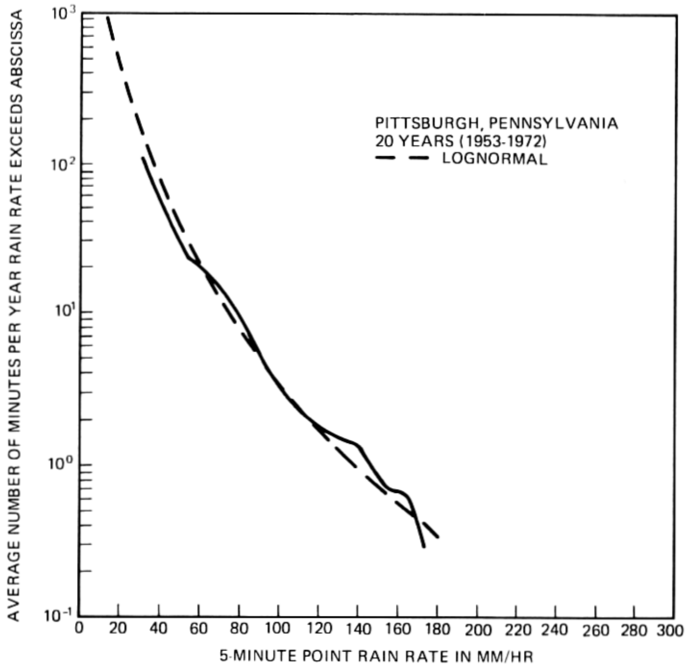

Fig. 4—Comparison for Pittsburgh, Pennsylvania.

respectively. Notice that the lognormal rain rate distribution (1) is uniquely determined by the three parameters P_0, R_m and S_R; whereas the distribution (4) of the yearly maximum 5-minute rain rate R_1 is uniquely determined by the two parameters α and U. Gumbel[12,13,22] has given the following approximate relationships among α, U and the parent distribution (1):

$$\Phi\left(\frac{U - \ln R_m}{S_R}\right) \simeq 1 - \frac{1}{P_0 \cdot N} \tag{6}$$

$$\alpha = \frac{P_0 \cdot N}{S_R}\,\phi\left(\frac{U - \ln R_m}{S_R}\right) \tag{7}$$

where

$$\Phi\left(\frac{U - \ln R_m}{S_R}\right) = 1 - \frac{1}{2}\,\mathrm{erfc}\left[\frac{U - \ln R_m}{\sqrt{2}\,S_R}\right] \tag{8}$$

is the standard unit normal distribution function,

$$\phi(z) = \frac{d}{dz}\,\Phi(z) \tag{9}$$

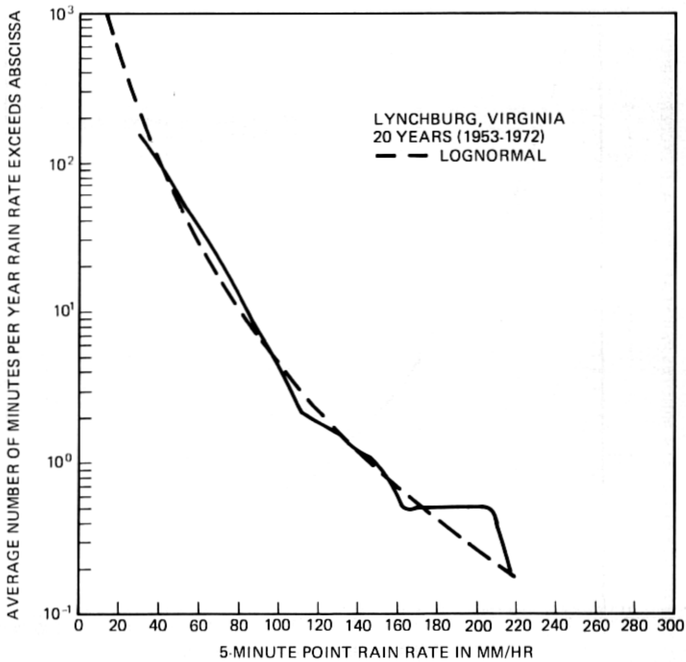

Fig. 5—Comparison for Lynchburg, Virginia.

is the normal probability density function, and

$$
\begin{aligned}
N &= \text{total number of 5-minute intervals per year} \\
&= (525600 \text{ minutes/year})/5 \text{ minutes} \\
&= 105120. \qquad (10)
\end{aligned}
$$

From eqs. (4) and (5) it is easily shown[12,13] that U is the most probable value (i.e., the mode) of $\ln R_1$ where R_1 is the randomly varying yearly maximum 5-minute rain rate. Let us define

$$R_u = e^U. \qquad (11)$$

Equation (6) states that, on long-term average, the randomly varying rain rate R will exceed R_u by approximately 5 minutes per year.* Equation (7) further specifies the slope (i.e., the derivative or probability density) of the rain rate distribution at $R = R_u$. Solving eqs. (6) and (7)

* From eqs. (1) and (6), it is easily shown that

$$P(R \geq R_u) = P_0 \cdot \left\{1 - \Phi\left(\frac{U - \ln R_m}{S_R}\right)\right\} = \frac{1}{N}.$$

Multiplying this probability by the total time per year yields 5 minutes per year.

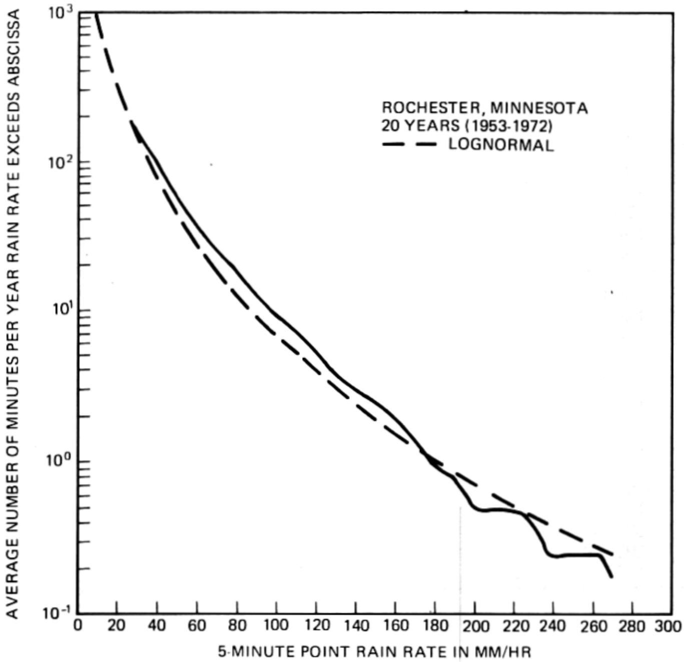

Fig. 6—Comparison for Rochester, Minnesota.

yields

$$S_R = \frac{P_0 \cdot N}{\alpha} \cdot \phi \left[\Phi^{-1} \left(1 - \frac{1}{P_0 \cdot N} \right) \right] \tag{12}$$

and

$$R_m = \exp \left[U - S_R \cdot \Phi^{-1} \left(1 - \frac{1}{P_0 \cdot N} \right) \right] \tag{13}$$

where $\Phi^{-1}(\sim)$ denotes the inverse normal probability function.

Reference 1 has given a set of formulas for calculating the parameters α and U from the yearly maximum 5-minute rain rate data. For completeness, this set of formulas is included in Appendix A. Knowing the values of W, α and U allows us to solve* the three equations (2), (12), and (13) for the three unknowns P_0, R_m and S_R. Substituting these three parameters into eq. (1) then yields the entire rain rate distribution.

For example, Table I lists the yearly maximum 5-minute rain rate R_1 measured at Binghamton, New York, for the 20-year period from 1953 to 1972.[2] Applying the formulas in Appendix A to the data in Table I

* These transcendental equations are solved numerically by a computer iteration process.

Table I — Yearly maximum 5-minute rain rates at Binghamton, New York

Year	Yearly maximum 5-minute rain rate, mm/hr
1953	103.63
1954	100.58
1955	161.54
1956	152.40
1957	91.44
1958	103.63
1959	201.17
1960	243.84
1961	112.78
1962	67.06
1963	115.82
1964	188.98
1965	91.44
1966	134.11
1967	85.34
1968	91.44
1969	106.68
1970	91.44
1971	97.54
1972	76.20

yields

$$\alpha = 3.224$$

$$U = 4.5736.$$

The 30-year (1941–1970) average value of W at Binghamton[2] is

$$W = 762 \text{ mm/year.}$$

Substituting this set of W, α and U into eqs. (2), (12), and (13) yields

$$P_0 = 0.018 \text{ (i.e., 1.8 percent),}$$

$$R_m = 2.631 \text{ mm/hr,}$$

$$S_R = 1.1015 \text{ nepers.}$$

The lognormal distribution (1) of the 5-minute rain rates calculated from this set of P_0, R_m and S_R agrees closely with the 20-year data[14] as displayed in Fig. 1. Similarly, Figs. 2 to 13 show the close agreement between the calculated result and the 20-year data at 12 other locations.

However, high rain rate statistics require a very long time base to yield stable results. The sensitivity of the high rain rate distribution with respect to time base measured at Newark, New Jersey, is shown in Fig. 14. It is seen that increasing the time base from 19 years to 21 years significantly alters the distribution for rain rate beyond 150 mm/hr. J. W. King[15] and J. Xanthakis[16] have presented approximately 100 years of

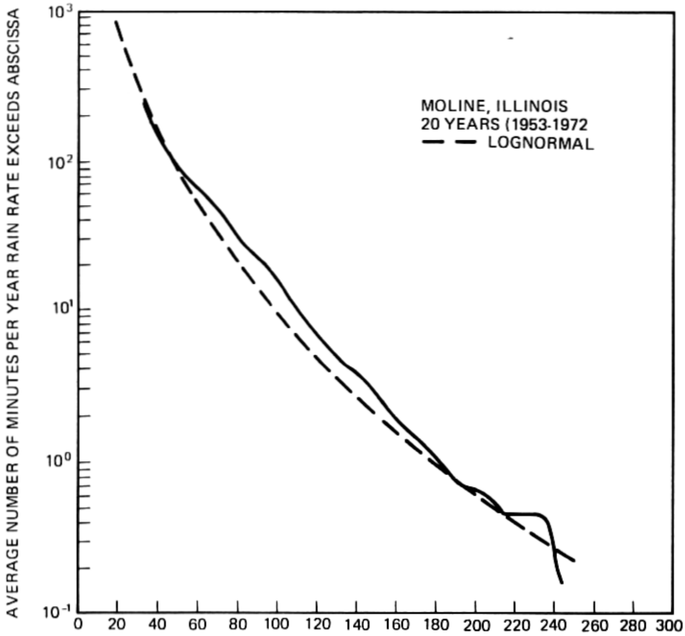

Fig. 7—Comparison for Moline, Illinois.

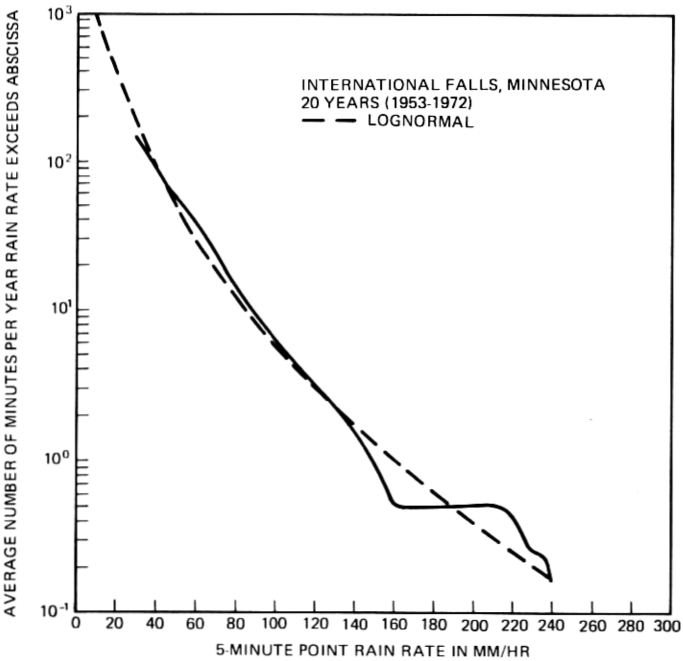

Fig. 8—Comparison for International Falls, Minnesota.

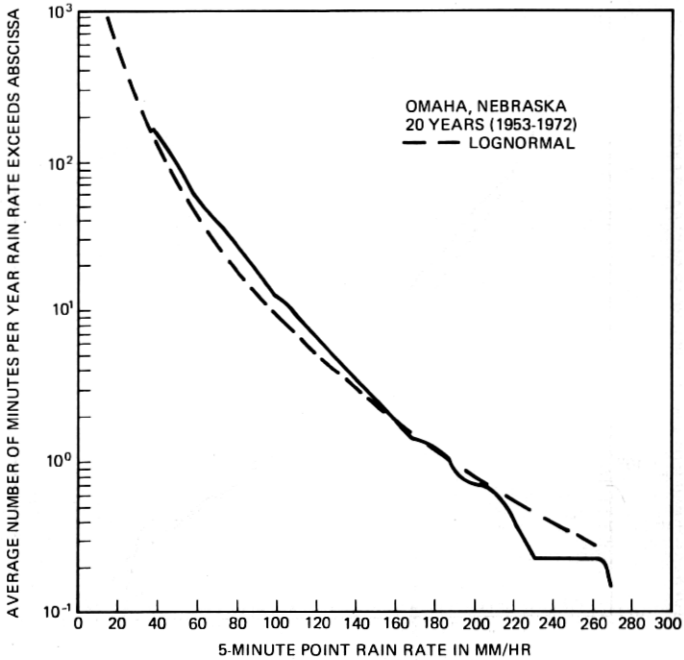

Fig. 9—Comparison for Omaha, Nebraska.

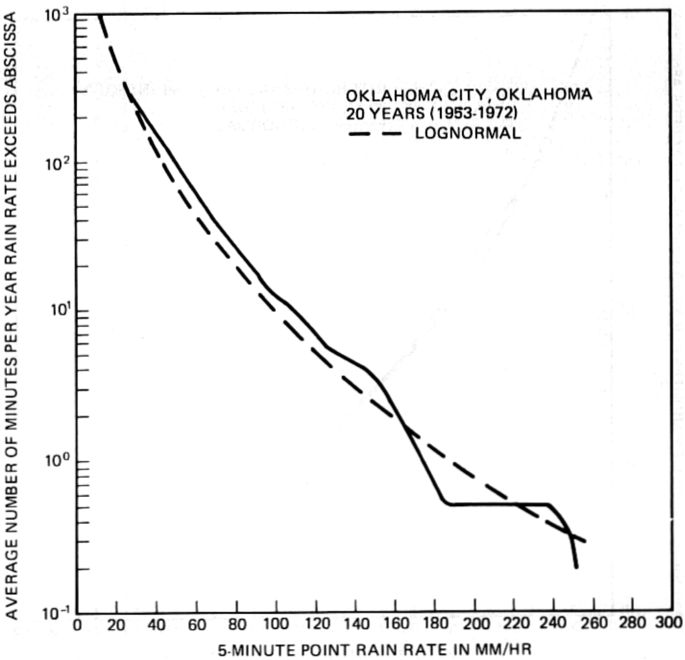

Fig. 10—Comparison for Oklahoma City, Oklahoma.

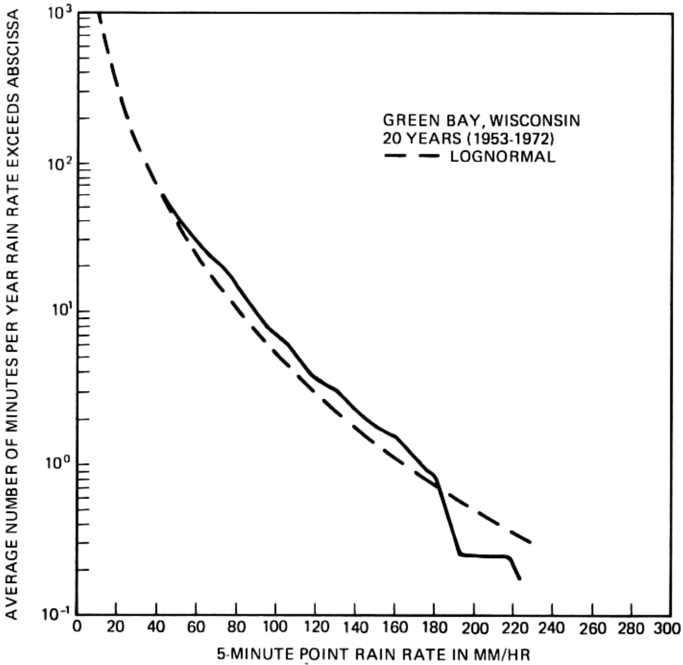

Fig. 11—Comparison for Green Bay, Wisconsin.

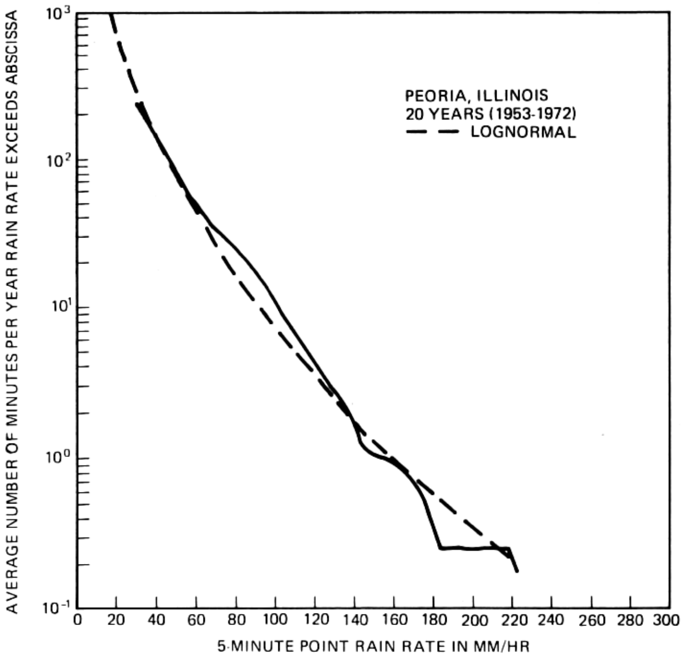

Fig. 12—Comparison for Peoria, Illinois.

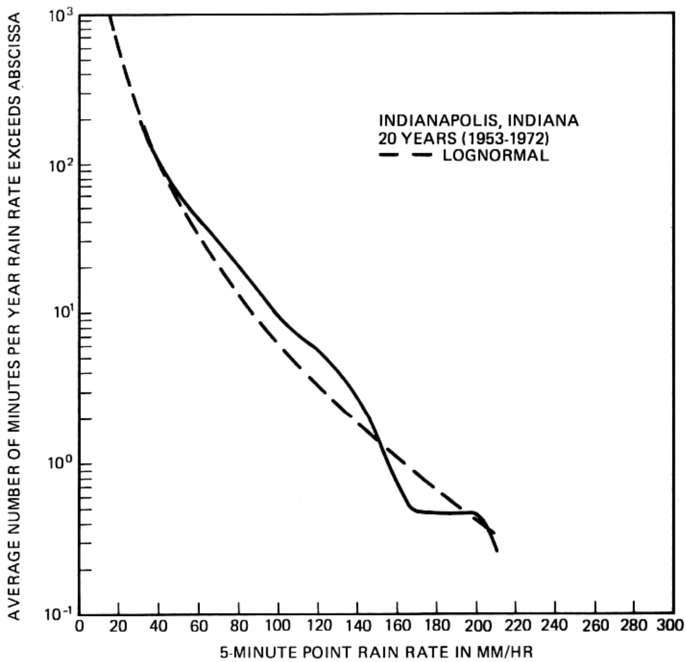

Fig. 13—Comparison for Indianapolis, Indiana.

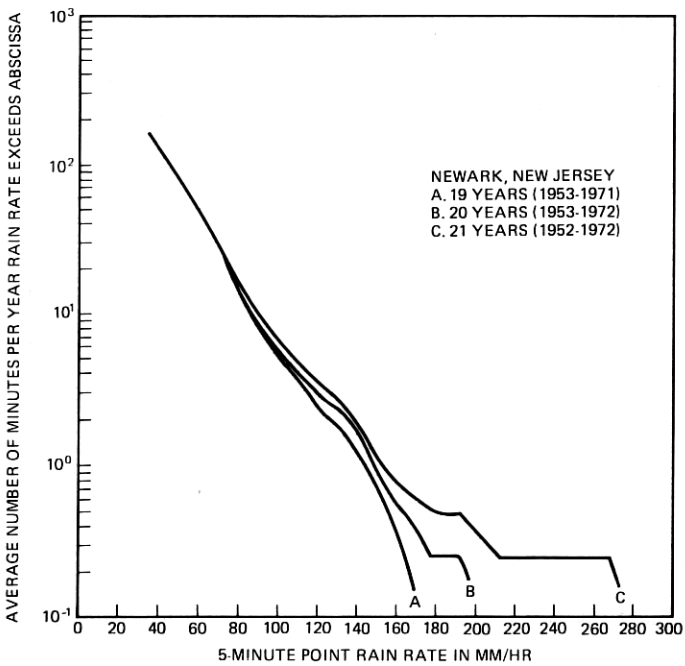

Fig. 14—The sensitivity of rain rate distribution with respect to time base.

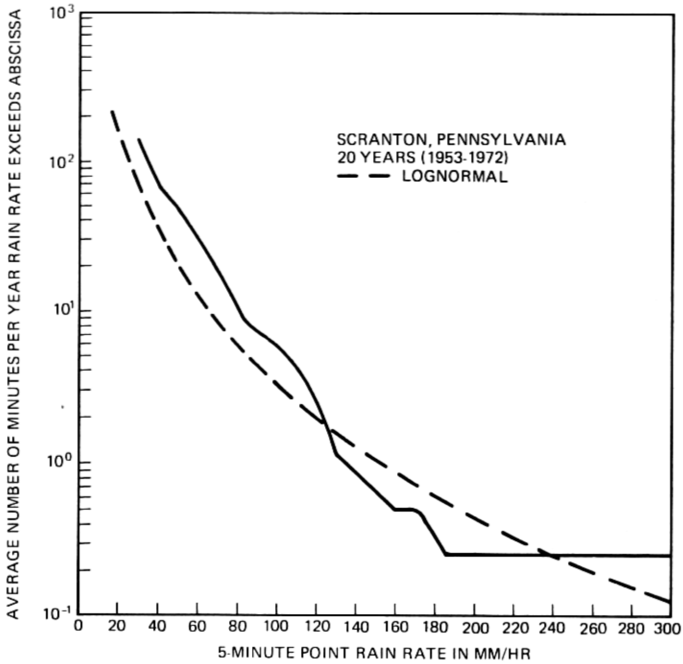

Fig. 15—Comparison for Scranton, Pennsylvania.

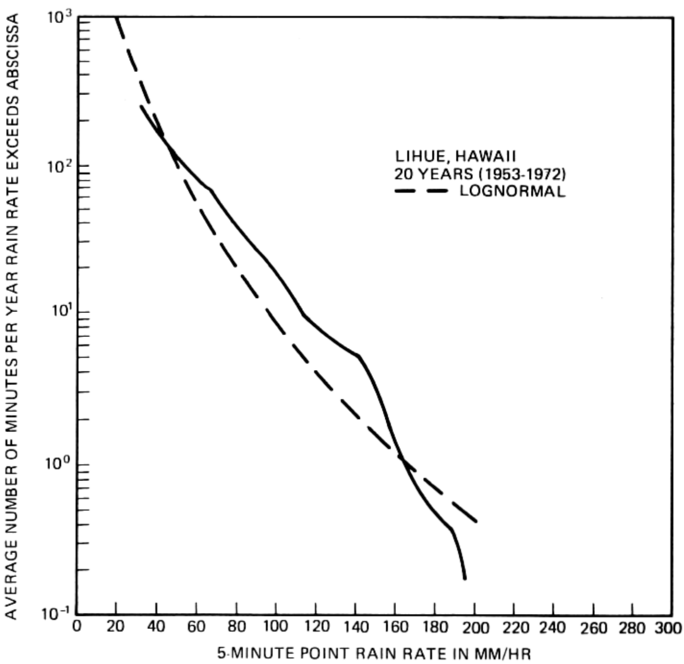

Fig. 16—Comparison for Lihue, Hawaii.

rainfall data to show that the variations of annual accumulated rainfall from year to year is correlated with the 11-year cyclic variations of sunspot numbers. R. J. Talbot and D. M. Butler[17] have discussed the climatic effects during passage of the solar system through nonuniform interstellar dust clouds. Therefore, rainfall activity is influenced not only by many terrestrial environmental factors but also possibly by extra-terrestrial sources. The required time base for stable rainfall statistics may be longer than 20 years. Since the parameters of the lognormal distribution are estimated from the yearly maximum rain rate data, the instability noted in Fig. 14 limits the accuracy of the calculated results.* Figures 15 and 16 give two examples of the effects of unstable high rain rate data on the estimated rain rate distributions.

III. FIFTY-YEAR DISTRIBUTIONS

Section IV of Ref. 1 describes a set of formulas for calculating the parameters α and U from rainfall intensity-duration-frequency curves for U.S. locations published by the Weather Bureau.[3] These curves are derived by the Gumbel method[12,13] using the theory of extreme value statistics and are based on approximately 50 years (1900–1950) of rainfall data. From this data source, we need only the following three numbers for a given location to calculate α and U:

M = the number of years of rainfall data from which rainfall-intensity-duration frequency curves are derived,

r_a = the extreme rain rate with 2-year return period, i.e., the rain rate which is exceeded once in 2 years, on the average, by the yearly maximum 5-minute rain rates,

r_b = the extreme rain rate with 10-year return period, i.e., the rain rate which is exceeded once in 10 years, on the average, by the yearly maximum 5-minute rain rates.

Therefore, in principle, long term distribution (1) of 5-minute rain rates for U.S. locations can easily be obtained by this method. The only input required are the four parameters W, M, r_a and r_b for each location read from Refs. 2 and 3.

For example, for San Francisco, California, the four numbers are

$$W = 115 \text{ mm/year}$$

$$M = 48 \text{ years (1903–1950)}$$

$$r_a = 1.9 \text{ inches/hr} = 48.3 \text{ mm/hr}$$

$$r_b = 3.05 \text{ inches/hr} = 77.5 \text{ mm/hr.}$$

* The accuracy of the calculated results for the very low rain rate region (i.e., ≤ 10 mm/hr) may be also limited because the parameters P_0, R_m and S_R are estimated from the extreme, high rain rate data.

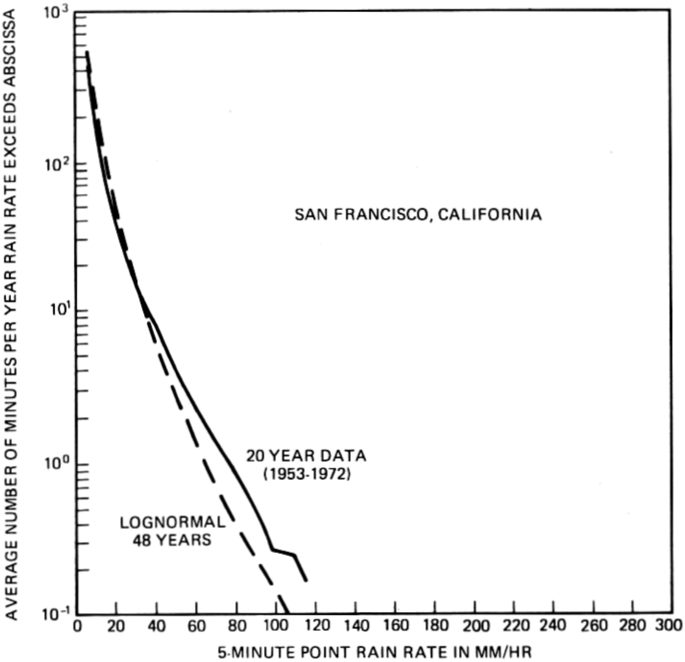

Fig. 17—Comparison of 48-year (1903–1950) distribution of 5-minute rain rate calculated by extreme value theory and lognormal hypothesis (dashed line) with 20-year (1953–1972) data (solid line) at San Francisco, California.

The formulas for calculating α and U are given in Appendix B for completeness. By substituting these values of M, r_a and r_b into eqs. (26) to (31) we obtain

$$\alpha = 3.6297$$

$$U = 3.7786.$$

Substituting this set of W, α and U into eqs. (2), (12) and (13) yields

$$P_0 = 0.0016 \qquad \text{(i.e., 0.16 percent)}$$

$$R_m = 6.23 \text{ mm/hr}$$

$$S_R = 0.7771 \text{ neper.}$$

Figure 17 shows that the calculated lognormal distribution of 5-minute rain rates for the 48-year period (1903–1950) is reasonably close to the 20-year data (1953–1972). Similarly, Figs. 18, 19, and 20 show the agreement between calculated results (≥43 years) and the 20-year data. On the other hand, Figs. 21 and 22 give two examples of appreciable differences due to the instability of high rain rate data.

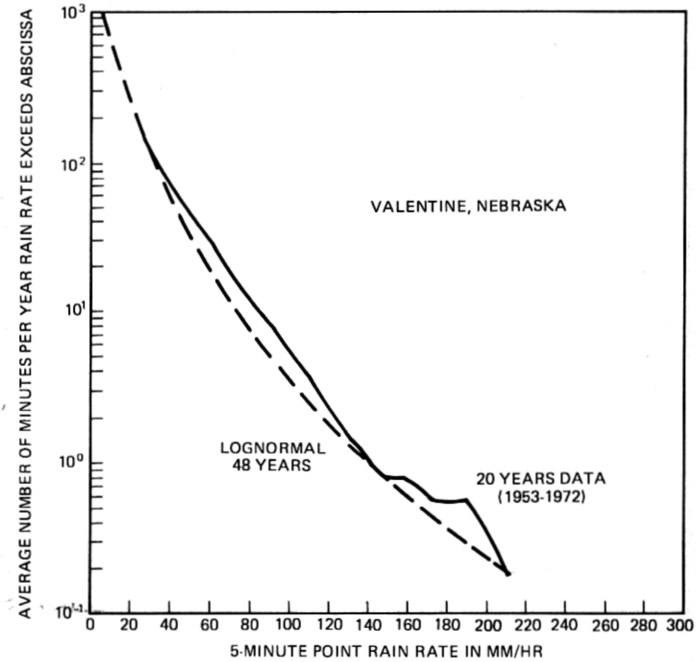

Fig. 18—Comparison of 48-year (1903–1906, 1908–1951) distribution of 5-minute rain rate calculated by extreme value theory and lognormal hypothesis (dashed line) with 20-year (1953–1972) data (solid line) at Valentine, Nebraska.

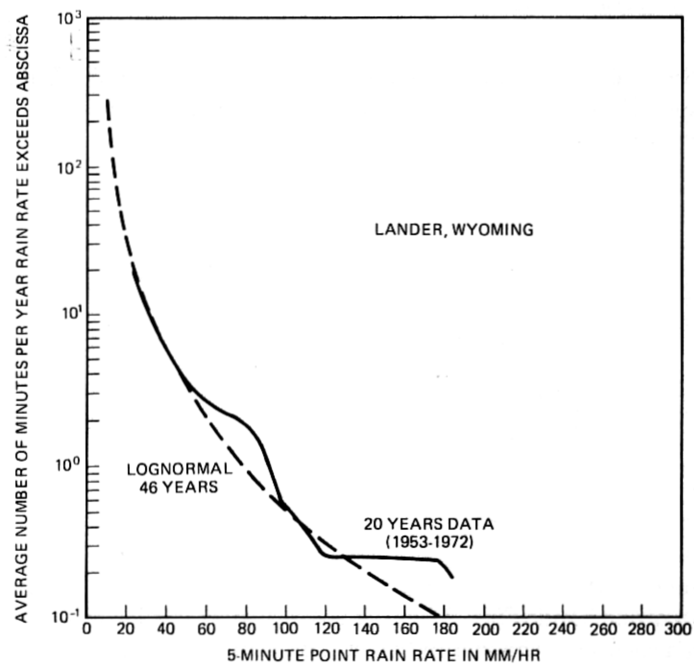

Fig. 19—Comparison of 46-year (1905–1950) distribution of 5-minute rain rate calculated by extreme value theory and lognormal hypothesis (dashed line) with 20-year (1953–1972) data (solid line) at Lander, Wyoming.

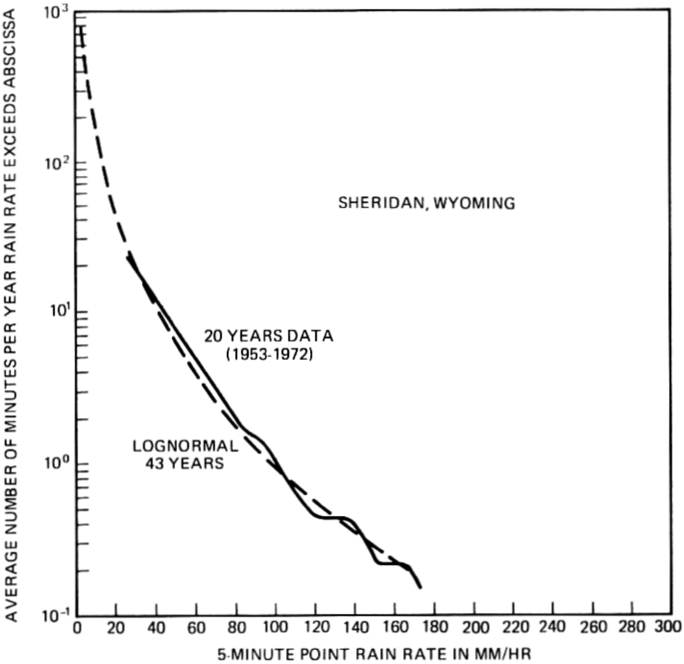

Fig. 20—Comparison of 43-year (1908–1950) distribution of 5-minute rain rate calculated by extreme value theory and lognormal hypothesis (dashed line) with 20-year (1953–1972) data (solid line) at Sheridan, Wyoming.

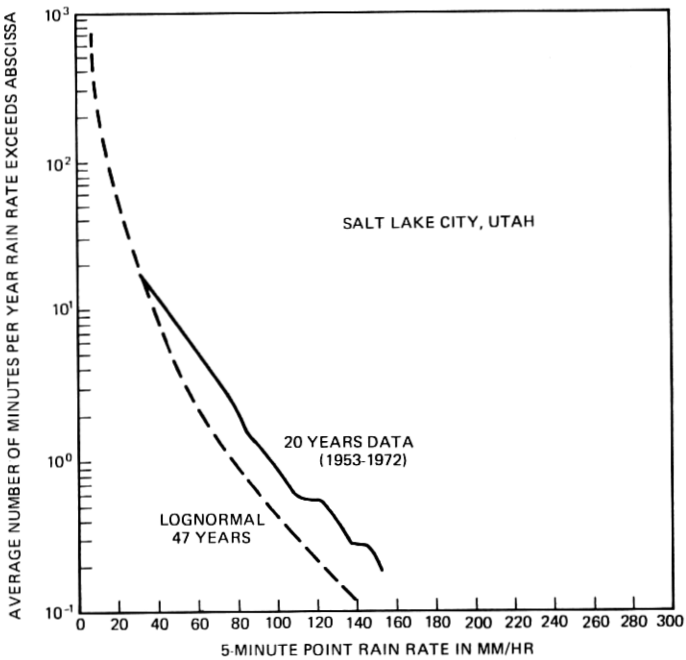

Fig. 21—Comparison of 47-year (1903–1907, 1909–1920, 1922–1951) distribution of 5-minute rain rate calculated by extreme value theory and lognormal hypothesis (dashed line) with 20-year (1953–1972) data (solid line) at Salt Lake City, Utah.

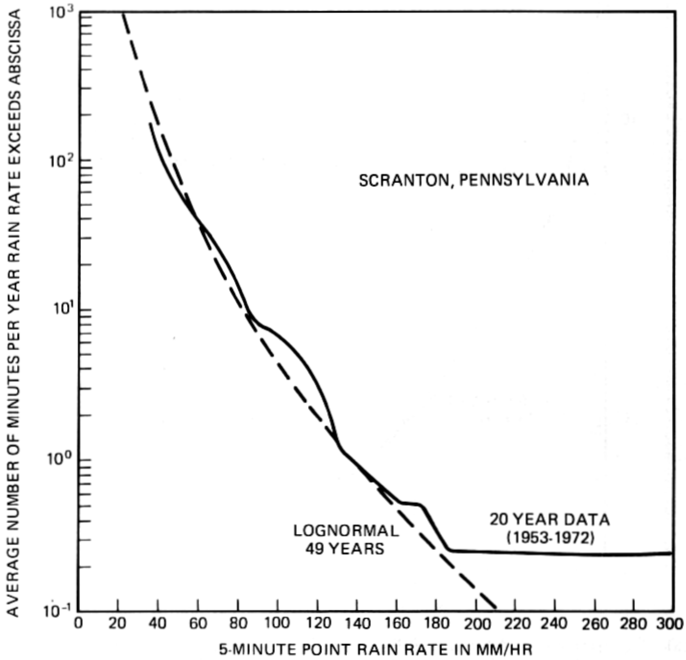

Fig. 22—Comparison of 49-year (1903–1951) distribution of 5-minute rain rate calculated by extreme value theory and lognormal hypothesis (dashed line) with 20-year (1953–1972) data (solid line) at Scranton, Pennsylvania.

IV. PROPORTIONATE EFFECT AND LOGNORMAL RAIN RATE DISTRIBUTION

The rainfall process is influenced by many environmental parameters. An important question is whether the environmental parameters affect the rain rate in a proportional fashion or in an additive fashion. It is well known[5,10,21] that a proportional fashion leads to a lognormal distribution whereas an additive fashion leads to a normal distribution. The following rain rate data will shed some light on this question.

Rain rate data measured in Illinois,[18] New Jersey,[19] and Canada[20] indicate that the short term mean rain rate $\langle R \rangle_s$ and the deviations, ΔR, from the short term mean $\langle R \rangle_s$ appear to be correlated. The subscript s in this section denotes "short term" mean value. These data indicate that the magnitude of the deviations, ΔR, tends to increase with the short term mean $\langle R \rangle_s$. In the following, we present 2 years of rain rate data measured by a tipping bucket rain gauge at Palmetto, Georgia, to confirm this correlation between ΔR and $\langle R \rangle_s$.

Figure 23 displays the time-varying rain rate, $R(t)$, in two 1-hour periods measured by the tipping bucket rain gauge at Palmetto, Georgia. In Fig. 23a, the hourly mean rain rate $\langle R \rangle_s$ is 12.6 mm/hr and the hourly

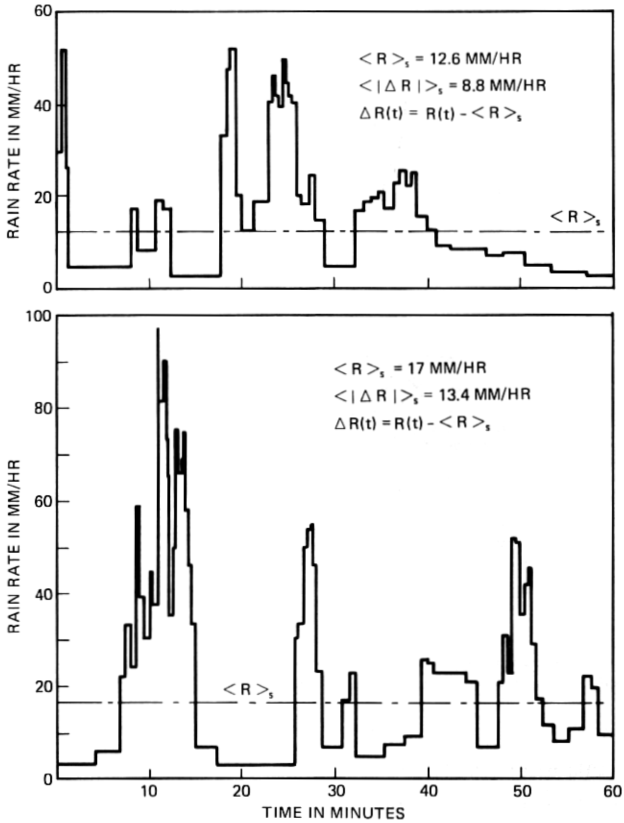

Fig. 23—Time-varying rain rate measured by a tipping bucket rain gauge at Palmetto, Georgia.

mean deviation $\langle |\Delta R| \rangle_s$ is 8.8 mm/hr where

$$|\Delta R(t)| = |R(t) - \langle R \rangle_s| \tag{14}$$

In Fig. 23b, the values of $\langle R \rangle_s$ and $\langle |\Delta R| \rangle_s$ are 17 and 13.4 mm/hr, respectively. Two years of rain rate data at Palmetto have been processed in this fashion and all the hourly $\langle R \rangle_s$ and $\langle |\Delta R| \rangle_s$ pairs are plotted in Fig. 24. It is seen that $\langle R \rangle_s$ and $\langle |\Delta R| \rangle_s$ are indeed correlated and the average relationship is approximately a straight line with a 45 degree slope on the log × log graph paper. To examine this proportional relationship more closely, let

$$X(t) = \ln R(t), \tag{15}$$

$$|\Delta X(t)| = |X(t) - \langle X \rangle_s| \tag{16}$$

The relationship between the hourly mean value $\langle X \rangle_s$ and the hourly

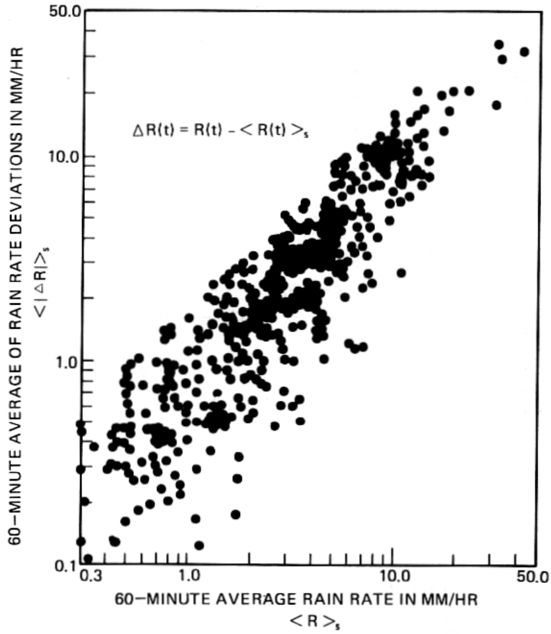

Fig. 24—Two-year data on correlation between hourly mean rain rate, $\langle R \rangle_s$, and hourly mean deviation, $\langle |\Delta R| \rangle_s$, measured by the tipping bucket rain gauge at Palmetto, Georgia.

mean deviation $\langle |\Delta X| \rangle_s$ processed from the same 2-year rain rate data are plotted in Fig. 25. It is seen that $\langle |\Delta X| \rangle_s$ is practically independent of $\langle X \rangle_s$. In Figs. 23 to 25, we use 1-hour period for short-term mean only as an example. The 2-year data were processed by several different "short-term periods" ranging from 5 minutes to 1 hour and showed essentially the same correlation between ΔR and R. Figures 24 and 25 indicate that ΔR is approximately linearly proportional to R:

$$\Delta R = h \cdot R \tag{17}^*$$

where h is a proportional parameter. The scattering of the data in Fig. 24 and the random variations of ΔR in Fig. 23 indicate that the proportional parameter, h, is not a constant, but is a time-varying random variable. Equation (17) can be interpreted in that the change, ΔR, in the rain rate is proportional to the product of the rain rate R and the intensity of the cause, h. In other words, the environmental parameters affect the rain rate in a proportional fashion. Therefore, the data of Figs. 24 and 25 are another manifestation of the lognormal rainfall process and support the lognormal hypothesis (1). Readers interested in the

* Equations (15) and (17) imply that ΔX is independent of X and is consistent with the data in Fig. 25.

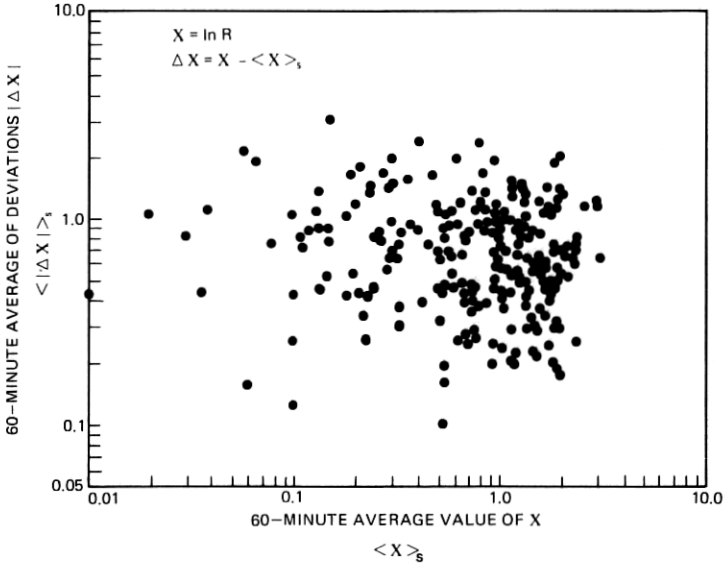

Fig. 25—Two-year data measured by the tipping bucket rain gauge at Palmetto, Georgia, demonstrating that the hourly mean deviation, $\langle|\Delta X|\rangle_s$, and the hourly mean value, $\langle X\rangle_s$ are uncorrelated, where $X = \ln R$.

derivation of the lognormal distribution from the proportionate relationship (17) are referred to Refs. 10 and 21.

V. CONCLUSION

A new method has been described for calculation of 5-minute rain rate distributions from yearly maximum 5-minute rain rate data and yearly total accumulated rainfall data which are available from National Climatic Center[2,3] for U.S. locations. By applying the theory of extreme value statistics and the lognormal hypothesis, the obtained rain rate distribution covers the entire range of rain rates (i.e., from below 5 mm/hr to greater than 200 mm/hr) for wide application. The calculated results agree well with long term (20 to 50 years) data as shown in Figs. 1 to 13 and 17 to 20. The accuracy of the calculated results is limited by the instability of the high rain rate distribution with finite time base.

VI. ACKNOWLEDGMENTS

W. C. Y. Lee,[25] R. A. Semplak,[26] P. L. Rice, and N. R. Holmberg[27] have separately described three different approximate methods for obtaining rain rate distribution from rainfall data published by the National Climatic Center. In an unpublished work, W. Y. S. Chen and R. L. Lahlum applied the theoretical distribution of yearly maximum 5-minute rain rates and an empirical extrapolation to obtain the distribution of high

rain rates. The methods described in Ref. 1 and this paper are inspired from the pioneer work of Lee, Semplak, Rice, Holmberg, Chen, and Lahlum. The work of W. C. Y. Lee provided an impetus for more sophisticated approaches taken by Semplak, Chen, Lahlum, and the author.

APPENDIX A

Formulas for Calculating Extreme Value Parameters α and U from Yearly Maximum Five-Minute Rain Rates

Let $R_1(j)$, $j = 1,2,3, \ldots, M$ be the measured yearly maximum 5-minute rain rate in M years of measurements, and let

$$x_1(j) = \ln [R_1(j)] \tag{18}$$

The formulas for calculating α and U are:

$$\alpha = \frac{\sigma_z}{\sigma_x}, \tag{19}$$

and

$$U = \overline{x}_1 - \frac{\overline{z}}{\alpha} \tag{20}$$

where

$$\overline{x}_1 = \frac{1}{M} \sum_{j=1}^{M} x_1(j) \tag{21}$$

is the sample mean of x_1,

$$\sigma_x = \left\{ \frac{1}{M-1} \sum_{j=1}^{M} [x_1(j) - \overline{x}_1]^2 \right\}^{1/2} \tag{22}$$

is the sample standard deviation of x_1,

$$z(j) = -\ln \left(-\ln \frac{j}{M+1} \right), \tag{23}$$

$$\overline{z} = \frac{1}{M} \sum_{j=1}^{M} z(j), \tag{24}$$

and

$$\sigma_z = \left\{ \frac{1}{M-1} \sum_{j=1}^{M} [z(j) - \overline{z}]^2 \right\}^{1/2}. \tag{25}$$

APPENDIX B

Formulas for Calculating α and U from Rainfall Intensity-Duration-Frequency Curves

$$\alpha = \alpha_\infty \cdot \sigma_z \cdot \frac{\sqrt{6}}{\pi}, \tag{26}$$

$$U = U_\infty + \frac{1}{\alpha_\infty}\left[\gamma - \frac{\bar{z}}{\sigma_z}\cdot\frac{\pi}{\sqrt{6}}\right], \tag{27}$$

$$\alpha_\infty = \frac{A_a - A_b}{\ln r_a - \ln r_b}, \tag{28}$$

and

$$U_\infty = \frac{A_a \ln r_b - A_b \ln r_a}{A_a - A_b} \tag{29}$$

where

$$A_a = -\ln\left[\ln\frac{Q_a}{Q_a - 1}\right], \tag{30}$$

$$A_b = -\ln\left[\ln\frac{Q_b}{Q_b - 1}\right], \tag{31}$$

$$Q_a = 2 \text{ (years)}, \tag{32}$$

$$Q_b = 10 \text{ (years)}, \tag{33}$$

$$\gamma = \text{Euler's constant} \approx 0.5772, \tag{34}$$

$\bar{z}$ and σ_z are defined by eqs. (24) and (25).

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