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Loop Plant Modeling:

Cost Models for Loop Plant Work Operations Using Semi-Markov Processes

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An important consideration in making economic evaluations of proposed loop plant relief, rearrangement and rehabilitation projects is the cost of loop plant work operations performed by splicers, installers, repair personnel, and support personnel. Using a semi-Markov process with states corresponding to activities performed during the work operation, probability distributions of the cost of work operations are obtained as a function of various plant conditions such as record error rates and defective pair rates. Transition probabilities and state delays are estimated using various plant reports and field data. A computer program calculates the distribution of the cost. A numerical example illustrates how the model can be used by determining the dependence of cable transfer costs on the number of pairs transferred and the percentage of working circuits transferred.

I. INTRODUCTION

An important consideration in making economic evaluations of proposed loop plant relief, rearrangement, and rehabilitation projects is the cost of loop plant work operations incurred to provide or maintain service. Such operations are performed by installers, cable maintenance and repair personnel, splicers, and support personnel including assignment and test bureau clerks, testers, frame personnel, and engineers.

The traditional method of determining these costs is by direct measurement. Direct measurement is usually limited, however, to a fixed set of conditions whereas a model can show how the costs vary with changing conditions. Therefore, a model can be more useful in estimating the change in costs caused by altering current work procedures, for example, or by introducing some new technology. Also, a model can be tailored to fit local conditions such as a high defective pair rate, for example.

This article presents a method for obtaining cost distributions of loop plant work operations as a function of plant conditions such as record error rates and defective pair rates. The method views a work operation, such as a cable pair transfer or the completion of an inward service order, as a semi-Markov process with constant state delay times and an absorbing state corresponding to the completing step of the operation. The states of the process correspond to activities performed during the work operation. The constant state delay times, i.e., the times required to perform the activities, are defined to be the costs of the activities. The transition probabilities from one activity to the next are the probabilities of various contingencies that arise in the course of performing the activities. The transition probabilities and state delays are estimated using various plant reports such as the assignment pair change summary report and field data. A computer program calculates the distribution of the cost incurred to reach the final (absorbing) state of the process with a probability arbitrarily close to one.

The next section tells in more detail what a cable pair transfer is and why it is needed in the operation of the loop plant. Section 2.1 describes the method of application of the Markov model to the cable pair transfer. Section 2.2 gives a brief description of semi-Markov processes which highlights the properties relevant to the model. Section III contains numerical results illustrating the dependence of cable pair transfer costs on local conditions. In addition, cost estimates obtained using the semi-Markov model are compared to actual cost data. Finally, Section IV concludes that the model gives reasonable cost estimates under a wide variety of conditions.

II. WHAT IS A CABLE PAIR TRANSFER?

Throughout this paper, the particular work operation used to illustrate the method is the cable pair transfer. Cable pair transfers or cable throws are often used in the administration of Multiple Outside Plant (MOP) in conjunction with cable relief or in order to defer relief. Simply stated, a cable transfer involves changing the path by which a cable or portion of cable reaches from the central office to the customer. Two simple illustrations of transfers are given in Fig. 1. Figure 1a illustrates the use of a cable throw in conjunction with relief. Assuming that feeder cables and lateral cables are economically sized,[1] then, referring to Fig. 1a, the number of pairs in the laterals is approximately proportional to $\sqrt{g_1}$ and $\sqrt{g_2}$ where g_1 and g_2 are the lateral growth rates. Since the feeder growth rate is equal to $g_1 + g_2$, its size is proportional to $\sqrt{g_1 + g_2}$. The times to exhaustion are thus proportional to $1/\sqrt{g_1}$, $1/\sqrt{g_2}$, and $1/\sqrt{g_1 + g_2}$,

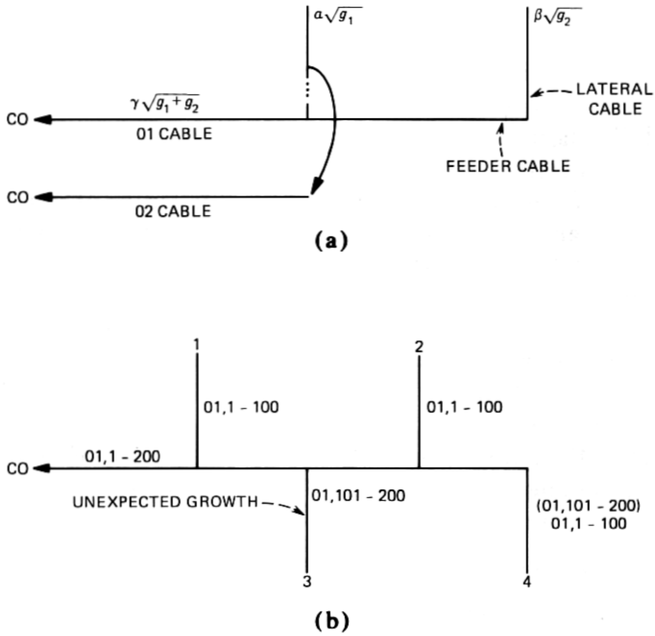

Fig. 1—(a) A cable throw in conjunction with relief. (b) A cable throw to defer relief.

for the two laterals and the feeder, respectively. Therefore, the feeder section (cable 01) exhausts before the lateral resulting in a mismatch. Relief of the feeder cable requires both placing new pairs and transferring some existing customers to the new cable. In Fig. 1a this is accomplished by transferring the pairs from one lateral to the new relief cable (cable 02). Transfers of this type are inherent in the MOP configuration and comprise a significant part of the cost of relief.

Cable throws are also used to defer relief in the event growth patterns are irregular and not as forecasted (see Ref. 2). This situation is illustrated in Fig. 1b. In this case, the feeder cable 01, pairs numbered 1–200, still has spare capacity, but due to an unforeseen growth spurt on lateral 3 the pairs of cable 01, numbered 101–200 have exhausted. In order to defer having to place new cable, lateral 4 is reconnected from cable 01, pairs 101–200 to 01, 1–100 so that pairs previously working in lateral 4 in count 01, 101–200 are now working in count 01, 1–100. This creates spare pairs in 01, 101–200 which can be used in lateral 3.

2.1 Defining the work operation in terms of a semi-Markov process

A semi-Markov process is a stochastic process which may be in any one of a set of states S_i, $i = 1, 2, \ldots$. The process governing the transitions between states is Markov, but the length of stay or delay in any given state is a continuous random variable. In general, the length of stay in any given state may depend on the state entered next in the process. In order to simplify exposition and parameter estimation, however, it will be assumed that the state delays are independent of the next state entered. Furthermore, for our purposes the state delays are assumed to be of fixed duration.

The goal of this work is to estimate the distribution of time required to perform a given work operation. To use the semi-Markov approach, it is necessary to define the steps of the process in such a way that the transition probabilities depend only on the present state of the process (the Markov property) and estimate the transition probabilities and state delays. The final step of the work operation is defined to be an absorbing state in the semi-Markov process, i.e., the probability of a transition from this state to any other state is zero. Therefore, the distribution of time to complete the required operation is the distribution of time to reach the final absorbing state. The required theory of semi-Markov processes[3] is given in the appendix.

2.2 Method of application to a work operation

The approach used in developing cost models of loop plant work operations is first to define the basic tasks required of all departments involved in the operation and to determine the interrelationships between these tasks. The interrelationships between tasks and departments can be illustrated simply by use of a flow diagram of the entire operation.

An example of a flow chart for a portion of the cable pair transfer work operation is given in Fig. 2. Figure 2 shows the beginning of the splicing activity associated with a cable throw. This part of the operation begins with the identification of the "TO" count, i.e., determining (at the transfer location) the central office number of each pair in the new count which the cable pairs are to assume upon completion of the job. The task of identification involves transmitting a tone from the central office to the location of the transfer in the case of pulp-insulated cable and using color-code if PIC (polyethylene insulated conductor) cable. If a TO pair is defective and is to be part of a working circuit, then another pair in the TO count must be found to which the working circuit can be transferred. After identifying the TO pairs, the splicer proceeds to identify the "FROM" count, i.e., the pairs which are to be transferred to the new count (TO count). See Table I for definitions.

The flow-diagram (Fig. 2) consists of rectangular boxes which represent the steps in the work operation and diamond-shaped boxes referred to as decision diamonds that are used to represent the possible decisions that must be made at each step. Note that each step requires a given completion time that corresponds to a delay time.

At each decision diamond there are two possible paths by which the

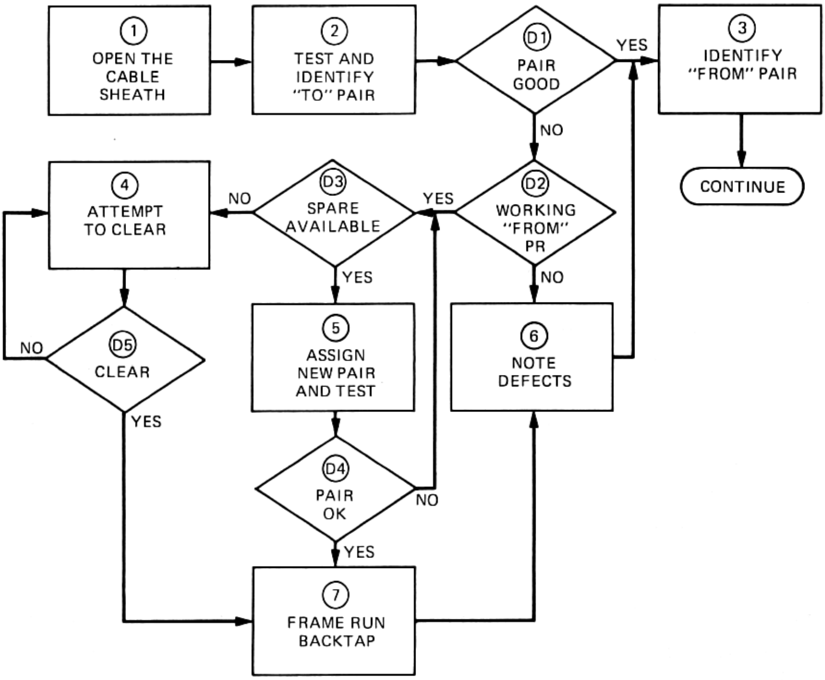

Fig. 2—Flow chart of splicing activity.

process can proceed. The probabilities of moving in the "YES" direction at each decision diamond are given in Table II. The parameters in Table II are defined in Table I. The probability of moving in the "NO" direction is, of course, one minus the probability of moving in the "YES" direction. Except for the steps which are connected directly (such as steps 1 and 2, for example), the paths from one step to another proceed by way of the decision diamonds. The probability of the process moving from one state to another by way of a given path is the probability that at each

Table I — Definitions associated with Fig. 2

$\gamma_t = \begin{cases} 0, & \text{if the sum of the fractions of working and defective pairs in the FROM and TO counts is greater than or equal to one} \\ 1, & \text{otherwise} \end{cases}$

$\gamma_\delta = \begin{cases} 0, & \text{if there are no known defective pairs} \\ 1, & \text{if known defective pair rate is non-zero} \end{cases}$

δ_s = fraction defective among spares in TO count (assumed to be 0.025 in examples)

δ_t = fractional rate of known defective pairs in TO count (assumed to be 0.05 in examples)

W_f = fraction working in FROM pairs being transferred

W_t = fraction working in TO pairs being transferred

S_t = fraction spare in TO count = $1 - W_t - \delta_t$

N = number of pairs being transferred

Table II — Decision probabilities

Decision diamond	Probability of "yes"
D1	$1 - \delta_s S_t$
D2	W_f
D3	γ_t
D4	$1 - \delta_s$
D5	γ_δ

decision diamond the process proceeds in the direction of the given path.

As an example, consider the path from step 2 to step 4 in Fig. 2. This path includes three decision diamonds. Let E_{24} denote the event that given the process is in step 2, then the process will proceed to step 4 next. It is assumed that at each decision diamond the probability of moving next along a particular path is independent of the paths chosen at previous decision boxes. Therefore, letting p_{24} denote the probability of event E_{24}, then, referring to Fig. 2 and the probabilities in Table II,

$$p_{24} = \delta_s S_t W_f(1 - \gamma_t)$$

where the variables are defined in Table I. The probability p_{24} is referred to as the transition probability between steps 2 and 4. In calculating p_{24} it has been implicitly assumed that the probability of proceeding to a given step depends only on the step in which the process resides at present and not on any step in the path which led to the present step, i.e., the process governing the transitions between steps is a Markov process (see Ref. 4). The remaining probabilities are calculated in a similar manner and are summarized in Table III.

In the preceding paragraph, the transition probabilities which govern the process of moving from one step to another were described. The time to move through the entire process, however, depends not only on the transition probabilities but also on the time delay at each step. As stated

Table III — Non-zero transition probabilities

Probability	Expression or value
p_{12}	1
p_{23}	$1 - \delta_s S_t$
p_{24}	$\delta_s S_t W_f(1 - \gamma_t)$
p_{25}	$\delta_s S_t W_f \gamma_t$
p_{26}	$\delta_s S_t(1 - W_f)$
p_{44}	$1 - \gamma_\delta$
p_{47}	γ_δ
p_{54}	$\delta_s(1 - \gamma_t)$
p_{55}	$\delta_s \gamma_t$
p_{57}	$1 - \delta_s$
p_{63}	1
p_{76}	1

previously it is assumed that there is a fixed delay t_i associated with each state. The procedure for estimating these delays is discussed in the next section.

To compute the average time (and subsequently the total cost) of a cable transfer, a set of interstep probabilities p_{ij} and delays t_i are calculated for the entire cable transfer process in the manner discussed in the preceding paragraphs. The probabilities and the delays estimated as described below can then be input to a computer program called MCHART that calculates the cost distribution using the equations given in the appendix.

2.3 Estimation of parameters

Estimates of the frequency and duration of each task have been obtained using several sources including time and motion studies conducted by personnel at Bell Laboratories and operating telephone companies, interviews with craftspeople in N.J. Bell Telephone Company, and from various operating company records. For example, in the cable pair transfer process, the times associated with tasks required by engineering, splicing and the test bureau are based primarily on field estimates. The slicing operations comprise the largest portion of the cost of a cable pair transfer, however, and actual data gathered on times to complete the splicing portion are consistent with model predictions as will be discussed in Section III. Estimates of time to complete the tasks required of assignment bureau personnel, repair clerks, and frame personnel were obtained from time and motion studies. Various plant statistics such as fills (i.e., percent of cable pairs in use) and defective pair rates were obtained from plant assignment sheets prepared at the time of the cable pair transfer.

Table IV gives the estimated delay times for the states in Fig. 2. It should be noted that these values may vary significantly depending on local conditions such as whether or not the plant is aerial or underground.

III. NUMERICAL RESULTS

As discussed in the preceding section, the semi-Markov model has been applied to develop a cost model of a cable pair transfer. A primary goal of this work is to be able to use the model to predict the average cable pair transfer cost as a function of local loop plant parameters. This would permit systems studies of the costs of various strategies for engineering the loop plant.

In the following examples normalized transfer times are used instead of costs since labor rates vary significantly throughout the Bell System. The major point illustrated is not the actual time required to make a cable pair transfer but rather the significant variation in time as a function of the various plant parameters.

Table IV — State delays

State	Task	Delay (hrs/pair) Constr.	PAO	CO	Test
1	Open cable sheath—includes travel and any necessary preparation	$\frac{2.5}{N}$			
2	Identify pair to which transfer is to be made and check it	0.01			
3	Identify pair to be transferred	0.01			
4	Any action necessary to be able to use a pair currently classified as defective	2.0			0.5
5	Assignment office issues a new pair and test bureau tests it		0.2		0.2
6	Discovery of a defective pair requires that it be noted by splicer for entry into PAO records	0.02			
7	Frameman run a new backtap and splicer change drop wire to new pair	0.3		0.2	

Abbreviations
Constr. construction or splicing force
PAO plane assignment office personnel
CO central office force
Test test desk personnel

3.1 Example 1: transfer time as a function of pairs transferred

Figure 3 shows the model calculation of the normalized hours per pair transferred as a function of the number of pairs transferred. The times in Fig. 3 are average or expected times and unless otherwise stated, all estimates shown in this section are average times. The example assumes 25 percent of the pairs transferred are working circuits and the TO count has no working circuits. Note the sharp decrease in both total time and construction time per pair as the number of pairs transferred increases.

It is important to identify the percentage of working circuits involved in the transfer since work time increases as the number of such working circuits increases. For example, transferring a working circuit requires that backtaps be placed at the central office, old jumpers be removed after the throw, line cards updated, and the circuit verified by the splicer at the time of transfer. Furthermore, if the circuit fails when tested, then more time must be expended to fix the cause of the failure. Even when all goes well in the field, it is sometimes necessary to make a rearrangement in the network in order to avoid transferring a working circuit to a pair which is defective or contains another working circuit.

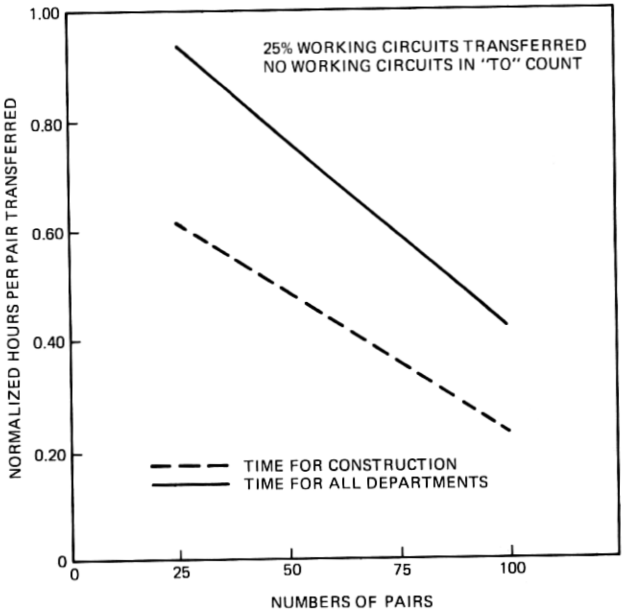

Fig. 3—Variation of cable pair transfer time as a function of the number of pairs transferred.

3.2 Example 2: transfer time as a function of working circuits

As an illustration of the increase in time when working circuits are involved, Fig. 4 shows the normalized construction time and total time for all departments for a 100 pair transfer as a function of the percentage of pairs transferred that contain working circuits. Note first the case in which there are no working pairs in the count to which the transfer is made. In this case it is seen that the construction hours increase as the percentage of working circuits increases but note the even sharper increase in the total hours. In the case where 20 percent of the count to which the pairs are thrown are working pairs, the increases are more rapid.

A major point illustrated by this example is that as the percentage of working circuits increases, the transfer costs increase significantly.

3.3 Prediction of time for the splicing force

Although normalized times have been used in the preceding examples, engineering studies in specific areas require actual times to be calculated. Consequently, it is important to know if the model can predict these times accurately.

The hours attributed to the assignment, repair, and frame forces are

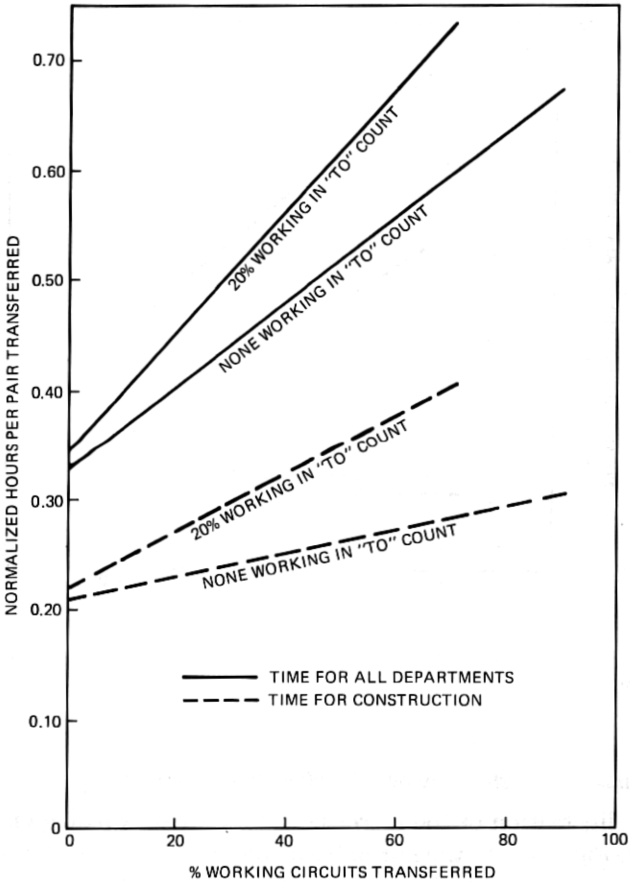

Fig. 4—Transfer time for a 100-pair cable throw as a function of the percentage of working circuits transferred.

based on actual time and motion studies. The times associated with the steps in the splicing operations, however, are based on interviews with splicers and splicing foremen and from unpublished studies and are therefore more subject to potential error. To check the accuracy of these times and the ability of the model to use them to estimate splicing work operation times, which comprise the major portion of the total time, data were gathered from a district in the New Jersey Bell Telephone Company on approximately 100 cable pair transfers involving 20–100 pairs each, the majority being 50 pairs each. The median splicing times are plotted with +'s in Fig. 5 as a function of the number of pairs transferred. The circled points represent the time estimates based on the semi-Markov model. The percentages of working circuits used in the model were taken to be equal to the median percentages in the data itself.

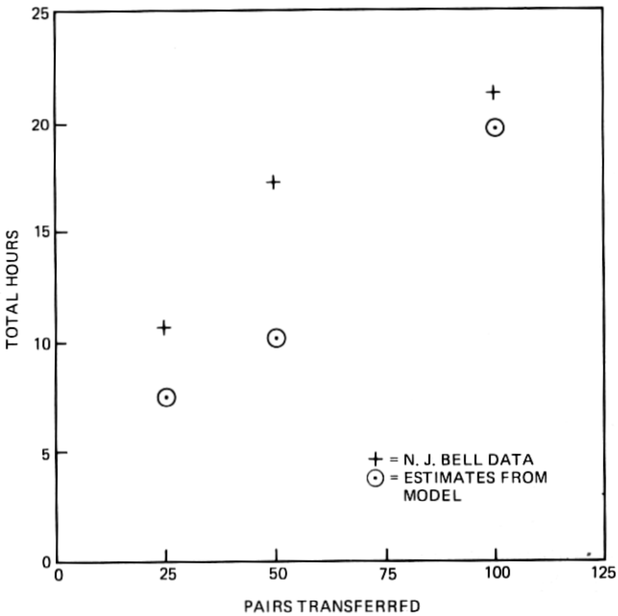

Fig. 5—Comparison of observed total splicing time and model estimates.

From Fig. 5 it appears that the model estimates reasonably well the rate of increases in splicing times with pairs transferred. The model does, however, seem to be biased low. This bias could be explained by underestimation of the time required to perform tasks which are not affected by the number of pairs transferred such as transportation time and opening and closing splice cases.

IV. SUMMARY AND CONCLUSIONS

This paper has presented a technique for applying semi-Markov processes to model the total costs of complex loop plant work operations. The use of such models permits cost studies which can consider the effect of a wide range of local plant conditions and designs. This is a distinct advantage over direct measurement of these work operations which apply only to operations carried out under the same conditions present when the measurements were made. Successful application of the semi-Markov approach to developing the cost of a cable pair transfer was discussed.

It is evident from the results presented that the semi-Markov model can provide reasonably accurate cost estimates of loop plant work operations. The estimates are necessary in order to carry out economic evaluations of different loop plant designs and methods of loop plant administration. In addition, these models have the potential for use in predicting changes in work load requirements and productivity as a function of plant conditions.

ACKNOWLEDGMENT

The particular version of the MCHART program used to construct the numerical examples was written by J. S. Parsons. Mr. Parsons also applied the semi-Markov model to the telephone installation process in unpublished work. The original version of MCHART was designed and written by Mrs. N. L. Basford.

APPENDIX

In the text it was assumed that the states of the semi-Markov process are denoted by S_i, $i = 1, 2, \ldots, n$, and the delays in each state by t_i, $i = 1, 2, \ldots, n$. It is assumed that the final state is an absorbing state corresponding to the final step in the work operation. Therefore, the time to complete the work operation corresponds to the first-passage time to the final (absorbing) state.

To obtain the distribution of first-passage time to the absorbing date, let

$p_j(t)$ = probability of entering state j at time t

Assuming n states, the nth being the absorbing state, then

$$p_j(t) = \sum_{i=1}^{n-1} p_{ij} p_i(t - t_i) \tag{1}$$

where

p_{ij} = probability of a transition from state i to state j

and

t_i = delay in state i

In words, eq. (1) states that the probability of entering state j at time t is equal to the probability of entering some state i at time $t - t_i$, remaining in state i for the constant delay time t_i and then making the transition from state i to state j at time t. Note that

$p_n(t)$ = probability of being absorbed in state n at time t and corresponds to the probability of completing the given work operation at time t

To develop a computational algorithm for $p_j(t)$, $j = 1, 2, \ldots, n$, it is assumed that

$$p_1(0) = 1,$$
$$p_j(0) = 0, \qquad j = 2, 3, \ldots, n, \tag{2}$$

and that time t is counted in integer values. A computer program called MCHART has been developed which computes eq. (1) as a function of time. The maximum value of t is reached when $p_n(t)$ reaches a predetermined value arbitrarily close to one. The distribution of T_n, the time to reach state n, can be computed by noting that

$$P(T_n \leq T) = \sum_{t=0}^{T} p_n(t) \tag{3}$$

The moments of T_n are calculated by the formula

$$E[T_n^m] = \sum_{t=0}^{T} t^m p_n(t), \qquad m = 1, 2, \ldots \tag{4}$$

A typical work operation in the loop plant often involves participation by several different departments. Therefore, the distribution of total time required of each department is also of interest. Letting

$$\begin{aligned} X_j(t) &= 1, \text{ if the process enters state } j \text{ at time } t \\ &= 0, \text{ otherwise} \end{aligned}$$

then

$$v_j(T) = \sum_{t=0}^{T} X_j(t)$$

represents the number of visits to state j during $(0,T)$. Assuming constant delay times, $t_j v_j(T)$ is then equal to the total time spent in state j during $(0,T)$. The average time spent in state j during $(0,T)$ is then $t_j E[v_j(T)]$ where

$$\begin{aligned} E[v_j(T)] &= \sum_{t=0}^{T} E[X_j(t)] \\ &= \sum_{t=0}^{T} p_j(t) \end{aligned} \tag{5}$$

Now let t_{jk} denote the time required by department k while the process is in state j. Then in the same manner, the average time spent by department k in state j can be shown to be equal to

$$t_{jk} \sum_{t=0}^{T} p_j(t)$$

and thus the average time required of department k during $(0,T)$ is obtained by adding the time required in each state, i.e., by the sum

$$\sum_{j=1}^{n-1} t_{jk} \sum_{t=0}^{T} p_j(t)$$

Note that while the foregoing argument was stated in terms of time, the same argument applies to costs or any other quantity that can be expressed as a linear function of time.

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