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Loop Plant Modeling:

An Approach to Modeling Operating Costs in the Loop Network

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A large share of loop network modeling effort is aimed toward characterizing operating costs. These costs arise because of the day-to-day activities associated with providing loop facilities. This paper considers those activities which occur as a result of inward service orders, i.e., requests for a cable pair. The models are designed primarily to reflect the impact of changes in the feeder portion of the network on operating cost. These models provide a basis for systems for administering the loop feeder network. Applications of the models are illustrated by examples.

I. INTRODUCTION

This paper, along with several other papers in this issue (Refs. 2–6), is concerned with the mathematical modeling of operating costs in the loop network. Specifically, models which predict the occurrence of loop network "activities" will be developed. These activities, together with their associated costs, constitute a cash flow which is a major component of the cost of the loop network. The goal of this modeling effort is to determine the effect of network design parameters on the operating cost so that the design of the network can be optimized on a total cost basis.

As is evident from the number of related papers in this issue, the concept of loop network operating cost is quite important. Moreover, there are several approaches to modeling operating cost as well as several areas of application of the models. In a very general sense, a loop network operating cost model predicts the sequence of activities (and the resulting cash flow) in the loop network. Many of these activities occur in direct response to inward service orders (see Ref. 1 for definition of terms).

Other activities, such as those relating to network troubles, may either be spontaneous or related to service order activity. Particular models may differ with regard to the types of activities modeled and the parameters of the loop network included in the model. This paper focuses on modeling activities directly related to inward service orders and the impact of parameters of the feeder network on these activities. These models can be applied in the feeder network design process to answer questions such as how to allocate feeder facilities and when to provide feeder relief.

Refs. 2 and 3 consider a broader class of loop network activities and place more emphasis on the impact of changes in the distribution network, notably conversion to interface design. Ref. 4 is also concerned with distribution design, but the emphasis there is on optimizing the parameters of a particular design: the Serving Area Concept. In Ref. 5, the approach used is similar to the one used here, but the object is to determine the optimum strategy for assigning facilities to inward service orders. Finally, Ref. 6 deals with modeling a particular activity in terms of its fundamental components.

Section II of this paper is an overview of the service order process which illustrates the kinds of loop network activities which may result from an inward service order. Section III presents the basic model for multiple outside plant (MOP). In Section IV, this model is extended to include use of the Connect-Through (CT) plan, which is discussed in Section II. Applications of the models are illustrated by means of examples.

II. INWARD SERVICE ORDERS AND LOOP OPERATING COST

Whenever a request for service, i.e., an inward service order, is received, a cable pair must be provided to connect the customer's premises to the local central office. The provision of this pair may involve one or more "activities" involving Operating Company personnel and equipment. These activities are the basic source of loop network operating cost. In this section, the process of providing a pair will be discussed in some detail in order to show how these activities arise.

2.1 Reassignable plant

Reassignable plant will be considered first. In reassignable plant, any pair which is not actually serving a customer (i.e., "working") is considered available for assignment (i.e., "spare").

Consider an inward order for residential service* at a given address. A particular *serving terminal,* in which several pairs (usually 10 to 50) are terminated, is associated with this address. If one or more pairs in

* Assume POTS unless otherwise indicated.

this serving terminal is spare, one will simply be chosen for assignment to the new customer. The connection is completed by having a *service wire* or "drop" connected from the customer's premises to the spare pair at the serving terminal. These operations (i.e., assign pair and connect drop) are the minimum effort required to provide service in reassignable plant.

If there is no spare pair in the designated serving terminal, the inward order is said to be "blocked." In this case, additional operations will be required. There are several alternatives. Figure 1 illustrates one possibility known as a line and station transfer (LST). Customer B, whose designated serving terminal is T_2, needs service, but T_2 contains no spare pairs. There is a spare pair (P_2) in T_1, however, and pair P_1, which currently serves customer A out of T_1, also appears in T_2. Therefore, A can be transfered to P_2, freeing P_1 to serve B. But what does this involve? Connecting the drop from P_1 to B is unavoidable. However, moving the drop at T_1 (a move which must be carefully coordinated with changes in the central office) is extra work which would not be required if a spare was present.

Another alternative is to connect a drop from B to P_2 at terminal T_1. This is known as wiring out of limits (WOL) and involves the extra effort to secure the drop at the poles adjacent to T_1 and T_2 and any intermediate poles. WOLs are also trouble prone and unsightly. Other alternatives include multiple LSTs, clearing defective pairs, and application of single channel carrier (Ref. 7). All involve extra cost.

From this discussion, it is apparent that *avoidable* operating expenses in reassignable plant are triggered by blocked inward orders or *blockages*. Thus in Section III the emphasis will be placed on modeling blockages and the costs of LSTs, WOLs, etc. necessary to "clear the blockage."

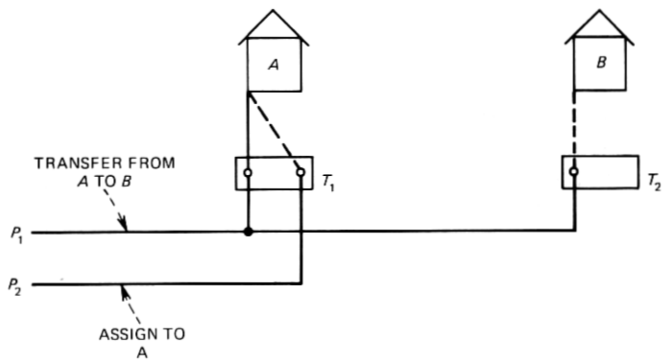

Fig. 1—Line and station transfer.

2.2 Connect-through administration

Connect-through or CT administration is the policy of leaving assigned pairs connected after service is discontinued. This policy is based on the assumption that a vacated premises will be reoccupied in a short time and that the new occupant will request telephone service. If a vacated premises is reoccupied and a pair is connected through to that premises, the installation activity is limited entirely to station work (installing stations, inside wiring, etc.). This is known as reusing a CT or, simply, a reuse. Under CT administration, a reuse is the simplest loop operation which may result from an inward order.

Clearly, it is simpler, and therefore less costly, to reuse a CT rather than assign a spare. Savings due to reuses make the CT plan economically attractive. Moreover, the advent of PhoneCenters increases the potential savings due to reuses. Under the PhoneCenter concept, the customer may elect to obtain station equipment at a local PhoneCenter and install it using previously installed jacks. Thus, in many cases, PhoneCenters will eliminate the need for station work at the customer's premises. If, in addition, it is not necessary for the installer to connect a pair for service, the installer visit is eliminated. Therefore, in a PhoneCenter environment, the savings due to a reuse, relative to the cost of assigning a spare, are greater by approximately the cost of the installer visit (i.e., the travel time).

Under CT administration, a pair may be in any of three states: working, spare, or CT. A CT pair is connected to a premises but not working. Both working and CT pairs are said to be assigned and CT pairs are sometimes called idle assigned pairs. Although CT pairs are available for assignment to the premises to which they are connected, they may or may not be considered available for assignment elsewhere.

Breaking a CT pair, or a BCT, is the process of assigning a CT pair to a new premises. A BCT involves both disconnecting and connecting a drop, either in the same terminal or in different terminals. A BCT is generally more complex than assigning a spare but less complex than an LST or a WOL (note that an LST or a WOL may involve a BCT). The rate of occurrence of BCTs depends not only upon customer movement and the configuration of the network, but upon the specific CT policy which is applied.

Variations on the basic CT plan are defined in terms of the treatment of CT pairs. Generally, CT pairs are divided into two categories: expendable CT pairs (i.e., those which can be reassigned to a new premises) and reserved CT pairs. These categories are recognized in the assignment preference list which reflects the policy for assigning pairs to inward service orders. An assignment preference list might look like the following:

1. Reuse CT pair.
2. Assign a spare pair.

3. Break an expendable CT pair.
4. Perform an LST, WOL, etc.
5. Break a reserved CT pair.

The operating cost under the CT plan depends on how the expendable and reserved CT categories are defined. Thus, the CT model must reflect this categorization.

One way to categorize CT pairs is to establish a *reserve time* such that only pairs which have been in the CT state for a period of time longer than the reserve time are expendable. This convention will be adopted in the derivation of the CT model in Section IV. If the reserve time is zero, then all CT pairs are expendable and a BCT will always be done in preference to an LST, WOL, etc. On the other hand, if the reserve time is infinite, then no CT pairs are expendable and a BCT will occur only as a last resort. The CT model can evaluate the effect of varying the reserve time between these two extremes.

2.3 The Serving Area Concept

The Serving Area Concept (SAC, Ref. 10) is a relatively new way to structure the loop network. Under SAC, a minimum of two distribution cable pairs are provided between each living unit and a serving area interface (SAI), which serves from 200 to 600 living units. Feeder cable pairs are also terminated at the SAI and a facility is provided for service by connecting the appropriate distribution pair to a feeder pair.

SAC operation is quite different than conventional design. For example, nearly all activity occurs at the SAI rather than at individual serving terminals. Although SAC is mentioned here for completeness, it will not be dealt with in detail in this paper. For a detailed discussion of operating costs under SAC, the reader is referred to Ref. 5.

2.4 Operating cost convention

This section will be concluded with a discussion regarding the way operating costs will be expressed in this paper.

There is a certain minimum cost required to provide a pair for service. In reassignable plant, the minimum cost is the cost of connecting to a spare pair in the designated serving terminal. Under the CT plan, the minimum cost activity is a reuse, provided the inward service order results from reoccupancy of a vacated premises. Even in CT plant, first occupancy of a new premises will necessitate at least connection to a spare pair.

It will be the convention of this paper to state the cost of an activity relative to the cost of the simplest (minimum cost) activity required to serve the inward order. This convention will be explained further as it is applied in Sections III and IV.

III. BASIC MODEL FOR REASSIGNABLE PLANT

Although most of the loop network today is operated under some kind of CT policy, reassignable plant is more straightforward and is a better starting point for the development of operating cost models. Moreover, many of the elements of the reassignable plant model carry over to more complex models.

3.1 Allocation areas and pair groups

The models derived here and in Section IV all assume the same geographic organization of the loop network. The geographic area served by the central office is divided into elemental units called *allocation areas*.[1,8] An allocation area generally contains 500–2000 customers. Each allocation area is associated with a *pair group* consisting of those pairs which are either available for assignment to customers in the allocation area or can be made available through simple work operations (e.g., splicing). Ideally, no pair should be available for assignment in more than one allocation area. In practice, a pair which appears in more than one allocation area is associated with one of the allocation areas according to a "tie breaking rule" which will not be discussed here. The relationship between allocation areas and pair groups is illustrated in Fig. 2.

The operating costs in an allocation area are assumed to depend only on parameters of the allocation area and its pair group. Thus, the allocation area is the largest unit which has to be modeled. The operating costs for a larger area are determined by summation.

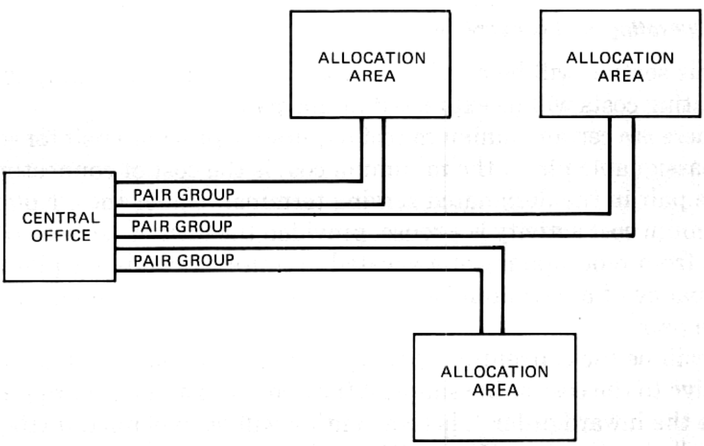

Fig. 2—Allocation areas and pair groups.

3.2 Allocation area model for reassignable plant

In reassignable plant, each inward service order results in either assignment of a spare pair in the designated serving terminal or a more complex operation (LST, etc.), which will be called "clearing a blockage" or, simply, a "blockage." In accordance with the cost convention stated in Section II, the cost of connecting to a spare in the designated serving terminal is assumed to be zero. For further simplification, it is assumed that the cost of clearing a blockage is the same for all blockages and is equal to the average cost of clearing a blockage. Thus, the operating cost incurred as the result of an inward service order is either zero or C'_{BLK}, the cost of clearing a blockage *relative to the cost of connecting to a spare in the designated serving terminal* (the prime is used to emphasize the relative nature of the cost factor).

A blockage is modeled as a probabilistic event and the probability that an inward service order is blocked is denoted $P(\text{BLK})$. It is further assumed that inward service orders occur at a given constant rate, λ. Therefore, the expected operating cost per unit time, b, is given by

$$b = \lambda C'_{\text{BLK}} P(\text{BLK}) \tag{1}$$

The inward service order rate is a forecast quantity which usually must be derived from forecasts at the central office level. The cost of clearing a blockage may be estimated using techniques discussed in Refs. 2 and 6. Both of these quantities are assumed to be given here, leaving the probability of blockage as the key quantity to be derived.

3.2.1 Basic hypergeometric blocking probability model

Figure 3 illustrates a simple allocation area configuration. The allocation area is served by a pair group containing n feeder pairs. It is assumed that all n pairs are available for assignment within the allocation area. If the pair group contains defective pairs or pairs which have not been distributed to serving terminals, these pairs are not included in n.

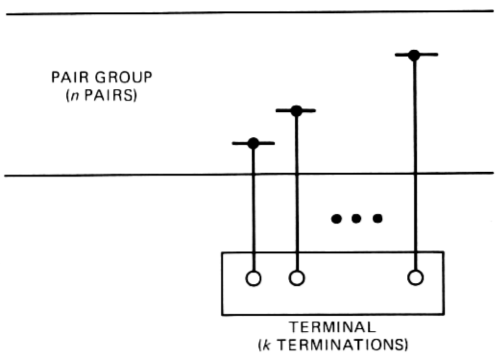

Fig. 3—Structure of hypergeometric blocking probability model.

The n pairs are distributed randomly among an unspecified number of k-pair serving terminals. It is assumed that w of the n pairs are working.

An inward move will be blocked if all k pairs in the designated serving terminal are working. Thus, the probability of blockage is the probability that k pairs, selected at random from n pairs, are all working or

$$P(\text{BLK}) = \frac{\binom{n-k}{w-k}}{\binom{n}{w}} \triangleq H(n,w,k) \tag{2}$$

Figure 4 is a plot of $P(\text{BLK})$ given by eq. (2) as a function of working pair fill f ($f = w/n$) for various values of k with $n = 1000$. Note that the probability of blockage increases sharply with fill in the high fill region and is quite sensitive to terminal size. In fact, if k is much less than n and w (as it usually is), eq. (2) can be approximated by

$$P(\text{BLK}) \approx (w/n)^{k} \tag{3}$$

The basic hypergeometric model [eq. (2)] can be extended to more complex network configurations. At this point, however, a simple example illustrating an important application of the operating cost model will be presented.

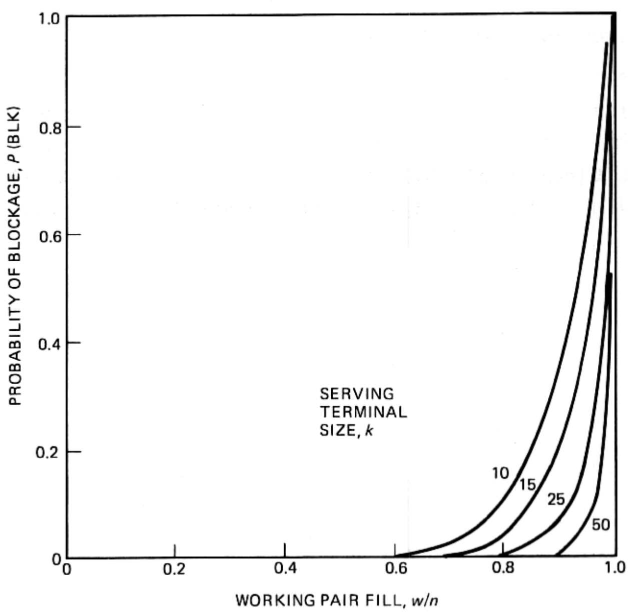

Fig. 4—Probability of blocking in reassignable plant.

Example: economic fill at relief

Suppose that the working pair fill of a given allocation area is increasing monotonically with time so that, at some point, additional pairs must be provided to the allocation area. The optimum fill at which new pairs are added, or the *economic fill at relief* (EFAR), is chosen to minimize the total cost of providing service. It is assumed that the cost of relief can be expressed as a levelized equivalent annual charge (LEAC, see Appendix to Ref. 9) which begins when the new pairs are added and that enough new pairs are added to reduce the operating cost to a negligible level for all future time. Under these assumptions, the total present worth cost of providing facilities is given by

$$\mathrm{PW} = \int_0^T \lambda C'_{\mathrm{BLK}} P(\mathrm{BLK}) e^{-rt} dt + \frac{\mathrm{LEAC}}{r} e^{-rT} \tag{4}$$

where T is the time at which relief is placed and e^{-rt} is the present worth factor. Note that, since fill is increasing with time, $P(\mathrm{BLK})$ is a function of t. A necessary condition for economic fill at relief is the following:

$$\lambda C'_{\mathrm{BLK}} P(\mathrm{BLK}) = \mathrm{LEAC} \tag{5}$$

[Equation (5) is derived from the condition $d\mathrm{PW}/dT = 0$.] Thus, relief should be placed at that point where the operating cost reaches the annual charge for the relief pairs.

For $\lambda = 100$ orders per year, $C'_{\mathrm{BLK}} = \$100$ and LEAC = \$5000 per year, relief should occur when $P(\mathrm{BLK}) = 0.5$. If the pair group size n is 1000 pairs and the terminal size k is 10 terminations, the economic fill at relief is approximately 0.90 as shown in Fig. 4.

This example is quite artificial because of the numerous simplifying assumptions made. In particular, a relief project almost always affects more than one allocation area and the sum of the allocation area operating costs must be compared with the relief cost. However, the example does illustrate the basic idea of economic fill at relief, one of the primary applications of the operating cost model.

3.2.2 Extension to multiple terminal sizes

Up to now, it has been assumed that only one size of serving terminal appears in the allocation area. Suppose, instead, that there are N_i terminals of size k_i for $i = 1,2,\ldots,m$. The probability of blockage given that the inward order occurs at a terminal of size k_i is $H(n,w,k_i)$ (eq. 2) so that the overall probability of blockage is given by

$$P(\mathrm{BLK}) = \sum_{i=1}^{m} H(n,w,k_i)P(k_i) \tag{6}$$

where $P(k_i)$ is the probability that the inward order occurs at a terminal of size k_i.

If it is assumed that an inward order is equally likely to occur at any terminal, then

$$P(k_i) = N_i \Big/ \sum_{j=1}^{m} N_j \tag{7}$$

On the other hand, it may be more reasonable to assume that serving terminals are sized according to demand so that an inward order is more likely to occur at a larger terminal. Thus, eq. (7) may be replaced by

$$P(k_i) = k_i N_i \Big/ \sum_{j=1}^{m} k_j N_j \tag{8}$$

Another approach to modeling an allocation area containing a mix of terminals is to define an *equivalent terminal size* k_{eq} such that

$$P(\text{BLK}) = H(n,w,k_{\text{eq}}) \tag{9}$$

Although no analytic relation between k_{eq} and the k_i and N_i has been derived, k_{eq} can be chosen to fit $P(\text{BLK})$ to observed values or values obtained by computer simulation.

In summary, for reassignable plant designed under the multiple outside plant doctrine, operating costs are the result of blockages. The probability of blockage is the critical factor for determining operating cost [eq. (1)]. The probability of blockage, which has been derived from a simple model of the loop network in an allocation area, depends primarily on working pair fill and terminal size.

IV. EXTENDED MODEL FOR CONNECT-THROUGH PLANT

In this section, the basic model for reassignable plant will be extended to include areas operating under the CT plan. The extension is necessary mainly to include the impact of reusing and breaking CT pairs as discussed in Section II. It is also necessary to distinguish between working and assigned pairs and model their trajectories over time.

4.1 Allocation area model for CT plant

Figure 5 illustrates the "flow" of inward service orders under the CT plan. The inward service orders are sorted into two categories: those which correspond to reoccupancy of a vacated premises* and those which correspond to first occupancy of a new premises. If a vacated premises is reoccupied and a CT pair is assigned to that premises, then the CT pair is reused and no cost is incurred. If no CT pair is assigned to the premises, a new pair must be assigned. This operation is called a reterminate connection (RTC) and incurs a cost, C'_{RTC}. Even though an RTC may be accomplished by connecting to a spare pair in the designated serving

* In this discussion, the term premises is used in a general sense to denote a potential point of demand for service.

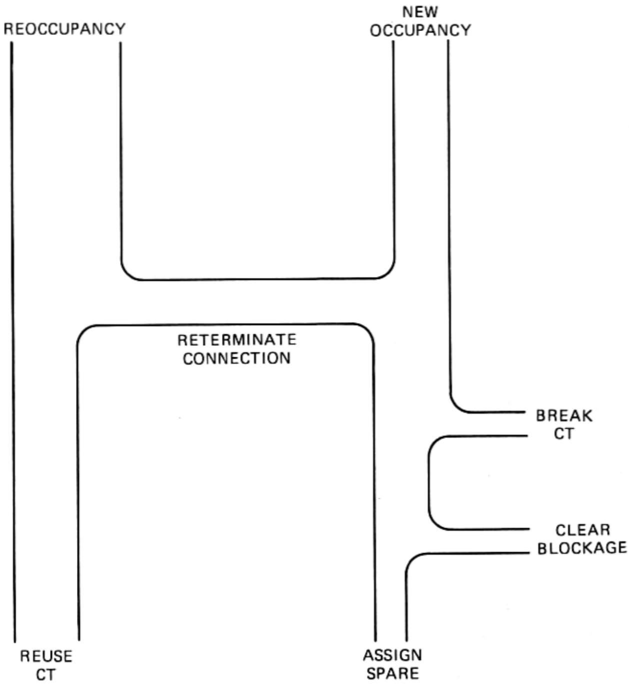

Fig. 5—Inward service order flow in CT plant.

terminal, the cost of this operation *relative to the cost of a reuse* must be counted, since a reuse is the simplest operation required to provide a pair for a reoccupancy.

As shown in Fig. 5, inward orders which result in RTC are lumped with inward orders corresponding to new occupancies. These orders are served by either assigning a spare pair in the designated serving terminal or breaking a CT pair (BCT), or clearing a blockage. No cost is associated with assigning a spare since this is the simplest operation required to provide a pair for a new occupancy and since the cost of assigning a spare to a reoccupancy has been accounted for by C'_{RTC}. The cost of breaking a CT pair, C'_{BCT}, and the cost of clearing a blockage, C'_{BLK}, are given relative to the cost of connecting to a spare in the designated serving terminal. Note that C'_{BLK} is defined the same way for the reassignable plant model.

Let RTC, BCT, and BLK be the rate of occurrence of reterminate connection, break CT, and clear blockage in a given allocation area. Then the operating cost per unit time, b, in this allocation area is given by

$$b = \mathrm{RTC} \cdot C'_{\mathrm{RTC}} + \mathrm{BCT} \cdot C'_{\mathrm{BCT}} + \mathrm{BLK} \cdot C'_{\mathrm{BLK}} \tag{10}$$

Note that eq. (10) is consistent with eq. (1) since, in reassignable plant, $\mathrm{RTC} = \mathrm{BCT} = 0$ and $\mathrm{BLK} = \lambda \cdot P(\mathrm{BLK})$.

4.2 Probability of blockage and BCT

Let λ_{IN} be the rate of new occupancies and let $\lambda_{\text{NET}} = \lambda_{\text{IN}} + \text{RTC}$. BCT and BLK in eq. (10) can be expressed as

$$\text{BCT} = \lambda_{\text{NET}} P(\text{BCT}) \tag{11}$$

and

$$\text{BLK} = \lambda_{\text{NET}} P(\text{BLK}) \tag{12}$$

where $P(\text{BCT})$ and $P(\text{BLK})$ are the probabilities of the BCT and blockage *events*, respectively. These probabilities can be derived by extending the results of Section III.

First consider the probability that there is no spare pair in the designated serving terminal. Since only those pairs which are not *assigned* are considered spare, this probability is given by $H(n,a,k)$, where a is the number of assigned pairs [see eq. (2)].*

If the designated serving terminal contains no spare pair, but contains at least one expendable CT pair (see Section II), a BCT will occur. It is assumed that a CT is expendable if it has been idle for a designated reserve time, τ_R, or longer. It is further assumed that the ages of the CT pairs (i.e., the time they have been idle) are independent, exponentially distributed random variables with parameter τ_V. The parameter τ_V may be interpreted as the mean vacancy time of a premises in the allocation area. The probability, $P(\text{EXP})$, that a CT pair is expendable is given, therefore, by

$$P(\text{EXP}) = e^{-\tau_R/\tau_V} \tag{11}$$

If there is no expendable CT or spare in the designated serving terminal, then it is necessary to either clear the blockage (LST, etc.) or break a reserved CT. In order to simplify the model it is assumed that breaking a reserved CT is equivalent to clearing a blockage. Since reserved CT pairs are broken only as a last resort, the error due to this assumption is only significant at high working pair fill (i.e., $w/n \approx 1$).

Now consider the conditional probability, $P(\text{BLK}/\overline{\text{SPR}})$, of a blockage given that there is no spare pair in the designated serving terminal. This is taken to be the probability that k pairs, selected at random from a population of a pairs, w of which are working and $a - w$ of which are CT with expendability probabilities given by eq. (11) are all either working or nonexpendable. This probability is given by

* It is assumed that a single terminal size, k, is in use. The results can be extended to multiple terminal sizes as in Section III.

$$P(\text{BLK}/\overline{\text{SPR}}) = \prod_{i=1}^{k} \left[\underbrace{\frac{w-i+1}{a-i+1}}_{\substack{\text{probability}\\ \text{that } i\text{th}\\ \text{pair is}\\ \text{working}}} + \underbrace{\left(1 - \frac{w-i+1}{a-i+1}\right)\left(1 - e^{-\tau R/\tau V}\right)}_{\substack{\text{probability that } i\text{th pair}\\ \text{is a reserved CT}}} \right]$$

$$= H(a,w',k) \tag{12}$$

where

$$w' = w + (a - w)(1 - e^{-\tau R/\tau V}) \tag{13}$$

Note that w' is the sum of the working pairs and the expected number of reserved CT pairs. The probability of blockage can now be computed as

$$\begin{aligned} P(\text{BLK}) &= P(\text{BLK}/\overline{\text{SPR}})P(\overline{\text{SPR}}) \\ &= H(a,w',k)H(n,a,k) \\ &= H(n,w',k) \end{aligned} \tag{14}$$

Equation (14) differs from eq. (2) only in the replacement of working pairs, w, with "equivalent working pairs," w'.

The probability that the designated serving terminal contains at least one expendable CT pair, given that it contains no spare pair, is $1 - H(a,w',k)$, so that

$$P(\text{BCT}) = (1 - H(a,w',k))H(n,a,k) \tag{15}$$

Figure 6 is a plot of P(blk) and P(BCT) as a function of the ratio τ_R/τ_V with the other parameters fixed at the values stated in the figure. This figure is a rough illustration of how the reserve time, which is a control variable, can influence the operating cost.

Actually, eq. (15) is the probability that an expendable CT pair in the designated serving terminal is broken. As discussed earlier, however, some blockages may include breaking expendable CT pairs in conjunction with clearing a blockage or breaking reserved CT pairs. The total BCT probability, denoted P_{TOT}(BCT), is taken to be

$$P_{\text{TOT}}(\text{BCT}) = P(\text{BCT}) + P(\text{BLK})(a - w)/(n - w) \tag{16}$$

In deriving eq. (16), it is assumed that, when a blockage occurs, the pair ultimately assigned is selected at random from the $n - w$ nonworking pairs, $a - w$ of which are CT pairs. These additional BCT are treated as blockages for the purpose of computing operating cost. However, they are included with the other BCT in the RTC model discussed in Section 4.3 and in modeling the trajectory of assigned pairs over time in Section 4.4.

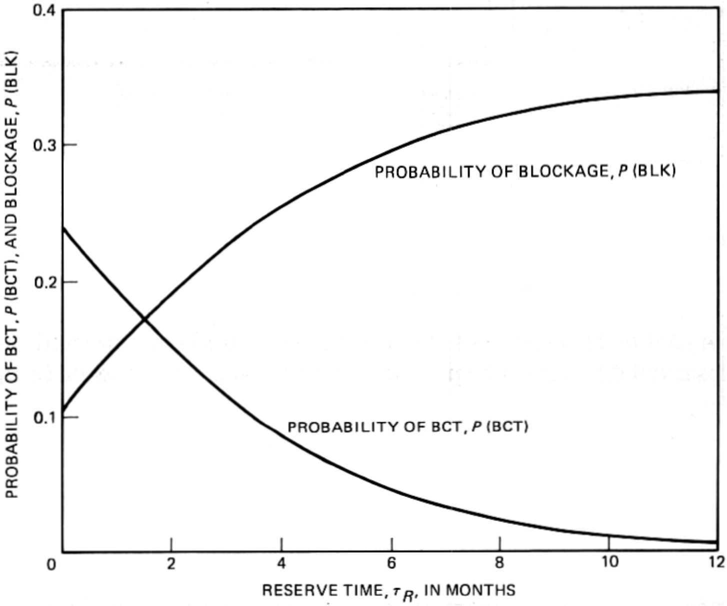

Fig. 6—Probability of BCT and blockage in CT plant.

4.3 Reterminate connection model

As discussed in Section 4.1, a reterminate connection occurs when a vacated premises is reoccupied and no pair is assigned to the premises. Thus, an RTC is the ultimate consequence of a BCT. This observation is the basis of the RTC model developed in this section.

Whenever a BCT occurs, an entity is created which corresponds to a vacated premises which has no pair assigned to it. An RTC occurs when one of these entities becomes reoccupied. An RTC does *not* occur when a vacated premises which has an assigned pair is reoccupied (this is a reuse) or when a new premises is occupied for the first time (see Fig. 5). Let $\mathrm{ENT}(t)$ be the number of entities defined above which exist at time t. The RTC rate is taken to be

$$\mathrm{RTC}(t) = \mathrm{ENT}(t)/\tau_V \tag{17}$$

where τ_V is again interpreted as the mean vacancy time for an unoccupied premises.

Whenever an RTC occurs, an entity is destroyed, i.e., there is one less vacated premises with no pair assigned. Thus, the number of entities at time t satisfies

$$\frac{d}{dt}[\mathrm{ENT}(t)] = \mathrm{BCT_{TOT}}(t) - \mathrm{RTC}(t) \tag{18}$$

where the time variation of the BCT and RTC rate has been explicitly indicated. Note that the total BCT rate, which follows from eq. (16), is used. The RTC model is obtained by combining eqs. (17) and (18)

$$\tau_V \frac{d}{dt}[\mathrm{RTC}(t)] + \mathrm{RTC}(t) = \mathrm{BCT_{TOT}}(t) \tag{19}$$

Figure 7 is a block diagram of the CT model as it stands at this point. The parameter λ_{IN} is the rate of new occupancy (see Fig. 5). New occupancies combine with RTC to form the net inward order rate, λ_{NET} (see Section 4.2). Net inward orders result in either assignment to a spare pair in the designated serving terminal (not shown in Fig. 7), breaking an expendable CT in the designated serving terminal [eq. (15)], or clearing a blockage [eq. (14)]. A fraction of the blockages results in additional BCT [eq. (16)]. The relationship between the RTC rate and the total BCT rate is illustrated in the frequency domain, for convenience.

4.4 Assigned and working pair trajectories

In the reassignable plant model, it is sufficient to model $w(t)$ as a specified function of time. The CT model is more complex, however, since $w(t)$ and $a(t)$ cannot be modeled independently.

Both $w(t)$ and $a(t)$ are modeled as responses to a given driving function, $p(t)$, which can be thought of as the number of premises in the allocation area. It is assumed that a vacant premises becomes occupied at rate $1/\tau_V$ and an occupied premises becomes vacant at rate $1/\tau_O$.* The following state equations are taken to characterize $w(t)$ and $a(t)$:

$$\frac{d}{dt} w(t) = [p(t) - w(t)]/\tau_V - w(t)/\tau_O \tag{20}$$

$$\frac{d}{dt} a(t) = [1 - P_{TOT}(\mathrm{BCT})][p(t) - a(t)]/\tau_V \tag{21}$$

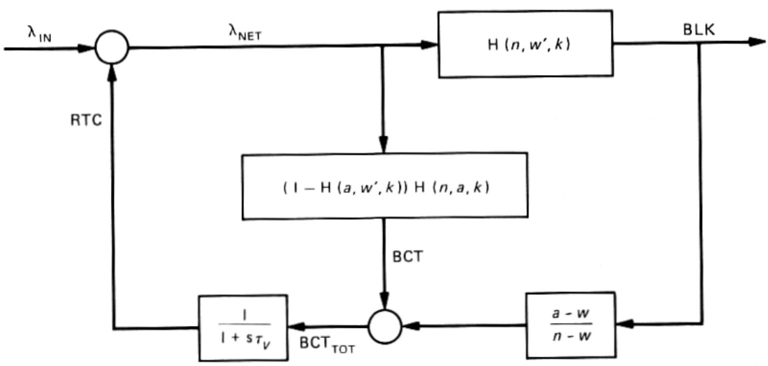

Fig. 7—Block diagram of CT model.

* The same symbol, τ_V, is used elsewhere in the CT model, although it has not been shown that all of these "vacancy times" are identical.

Whereas eq. (20) is a straightforward dynamic model, eq. (21) deserves further explanation. First of all, the quantity $[p(t) - a(t)]/\tau_V$ is the rate at which unoccupied premises which have no pair assigned become occupied. This rate includes new occupancies and RTC, i.e.,

$$[p(t) - a(t)]/\tau_V = \lambda_{\text{IN}}(t) + \text{RTC}(t) \tag{22}$$

or

$$\lambda_{\text{NET}}(t) = [p(t) - a(t)]/\tau_V \tag{23}$$

The rate of increase of $a(t)$ is $\lambda_{\text{NET}}(t)$ less the *total* BCT rate, hence eq. (21).

$P_{\text{TOT}}(\text{BCT})$ is a rather complex, nonlinear function and eq. (21) must be solved numerically. This is done in the following example.

Example: economic fill at relief in CT plant

In this example, the optimal time to relieve an allocation area operating under the CT plan is computed. It is assumed that when relief occurs, the number of available pairs increase such that for time $t > T$, the time of relief, $P(\text{BLK}) = P(\text{BCT}) = 0$. Thus, there are no blockages or BCT for $t > T$. However, there *will* be RTC. For $t > T$, eq. (19) becomes

$$\tau_V \frac{d}{dt}[\text{RTC}(t)] + \text{RTC}(t) = 0$$

so that

$$\text{RTC}(t) = \text{RTC}(T)e^{-(t-T)/\tau_V} \tag{24}$$

for $t > T$. The present worth of the cost of all RTC which occur for $t > T$ is given by

$$\begin{aligned} \text{PWRTC}(T) &= \int_T^\infty \text{RTC}(t)e^{-rt}dt \\ &= \frac{\tau_V C'_{\text{RTC}}\text{RTC}(T)e^{-rT}}{1 + r\tau_V} \end{aligned} \tag{25}$$

The total present worth cost for the allocation area is given by an extended version of eq. (4).

$$\begin{aligned} \text{PW} = &\int_0^T (\text{RTC} \cdot C'_{\text{RTC}} + \text{BCT} \cdot C'_{\text{BCT}} + \text{BLK} \cdot C'_{\text{BLK}})e^{-rt}dt \\ &+ \frac{\tau_V C'_{\text{RTC}}\text{RTC}(T)e^{-rT}}{1 + r\tau_V} + \frac{\text{LEAC}}{r}e^{-rT} \end{aligned} \tag{26}$$

The optimal time to place relief follows from the condition $d\text{PW}/dT = 0$, as in section 3.2.1. Now

$$\frac{d\text{PW}}{dT} = [\text{RTC}(T) \cdot C'_{\text{RTC}} + \text{BCT}(T) \cdot C'_{\text{BCT}} + \text{BLK}(T)C'_{\text{BLK}}]e^{-rT}$$

$$+\frac{\tau_v C'_{\mathrm{RTC}}}{1+r\tau_V}\left(\frac{d}{dt}\,[\mathrm{RTC}(t)]|_{t=T}e^{-rT} - r\mathrm{RTC}(T)e^{-rT}\right)$$

$$- \mathrm{LEAC}\, e^{-rT} \quad (27)$$

so that the optimal T must satisfy

$$\left(\tau_V \frac{d}{dt}\,[\mathrm{RTC}(t)]|_{t=T} + \mathrm{RTC}(T)\right)\frac{C'_{\mathrm{RTC}}}{1+r\tau_V}$$

$$+ \mathrm{BCT}(T)\cdot C'_{\mathrm{BCT}} + \mathrm{BLK}(T)\cdot C'_{\mathrm{BLK}} = \mathrm{LEAC} \quad (28)$$

or, using eq. (19),

$$\mathrm{BCT_{TOT}}(T)\frac{C'_{\mathrm{RTC}}}{1+r\tau_V} + \mathrm{BCT}(T)\cdot C'_{\mathrm{BCT}} + \mathrm{BLK}(T)\cdot C'_{\mathrm{BLK}} = \mathrm{LEAC} \quad (29)$$

Equation (29) is the analog of eq. (5) for the extended model. Note that, through the first term of eq. (29), BCTs are given an additional cost penalty to account for future RTCs.

The optimum relief time, T, is determined by numerically minimizing PW given by eq. (26). Some sample results are listed in Table I. Sample plots of the trajectories of RTC, BCT, $\mathrm{BCT_{TOT}}$, and BLK (Fig. 8) and $w(t)$ and $a(t)$ (Fig. 9) are also shown.

Table I — Example of optimal relief time for CT plant

Allocation area parameters		
Available pairs	n	1200 pairs
Premises (initial)	$p(0)$	1030 prem.
Assigned pairs (initial)	$a(0)$	1000 pairs
Working pairs (initial)	$w(0)$	920 pairs
Premises growth	g	10 prem./mo.
Mean vacancy time	τ_V	3 mo.
Mean occupancy time	τ_O	24 mo.
Serving terminal size	k	10 term.
Cost factors		
Reterminate connection	C'_{RTC}	\$ 25
Break CT	C'_{BCT}	\$ 10
Clear blockage	C_{BLK}	\$100
Convenience rate	r	0.01/mo.

Optimal time of relief

LEAC* ($ per month)	Reserve time (months)	Relief time (months)	Present worth cost ($)
2500	0	28	206433
2500	2	26	209808
2500	12	24	214102
2500	120	24	214271
1250	0	23	108529
1250	2	21	110411
1250	12	19	112740
1250	120	19	112833

* Levelized equivalent annual charge.

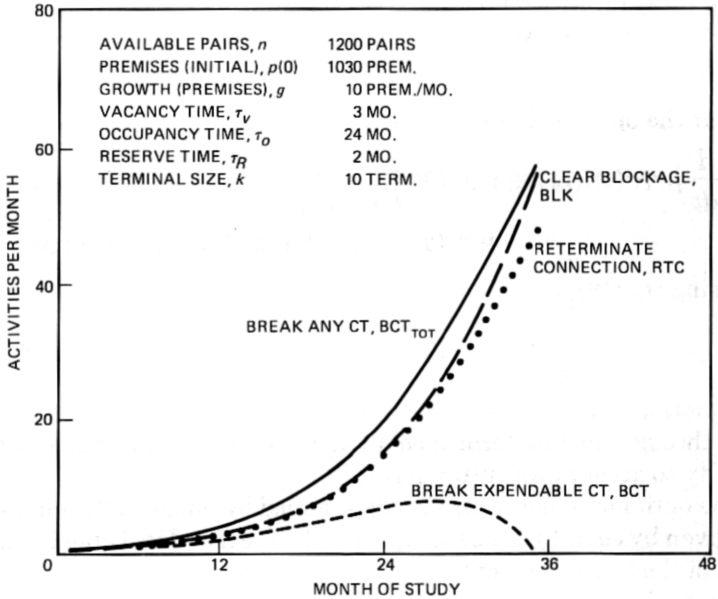

Fig. 8—Record of activity rates for CT model.

V. SUMMARY AND APPLICATION

This paper has presented a basic approach to modeling inward service order related operating costs in the loop network. These models provide

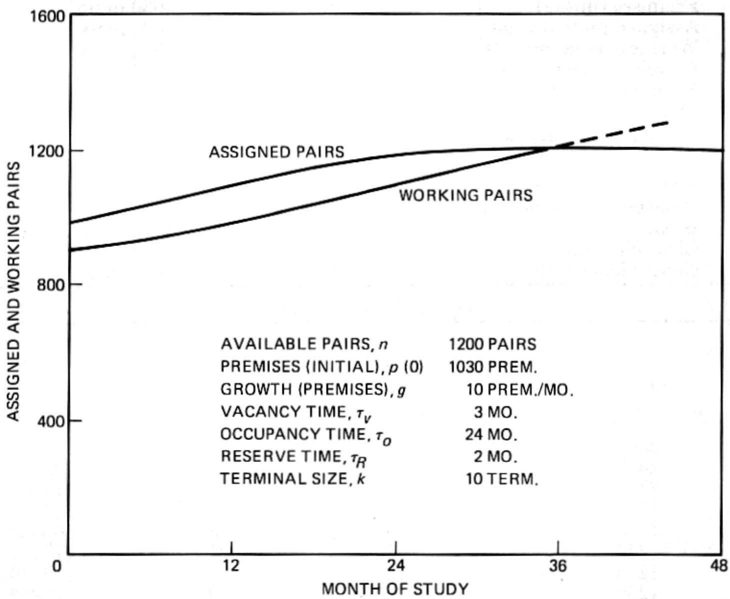

Fig. 9—Assigned and working pair trajectories.

both a theoretical basis and a practical method for the development of systems for administering the loop network. One such system is the Economic Feeder Administration and Relief (EFAR) computer program. EFAR computes the optimal time to place relief feeder cable. EFAR also evaluates the economic impact of transferring available pairs among pair groups. The EFAR algorithm is based on the reassignable plant model developed in Section III. Some variation of the more general CT model of Section IV will be incorporated into future releases of EFAR.

During the initial EFAR field trial, the reassignable plant model was tested by comparing its predictions to observed blockage rates. As a result of this test, heuristic modifications were added to the model. Further tests are proposed for the CT model. Both data collected from actual loop network operation and data derived from computer simulation will be used.

Compared with, say, the cost of placing new cable, loop network operating costs are very difficult to model. This is simply because the models must reflect a large number of small events rather than one large event. Thus, it is unreasonable to expect the kind of accuracy one could achieve in estimating the cost of a major construction project. Nevertheless, it is even more unreasonable to ignore operating costs—they are a significant part of the total cost of providing loop facilities.

VI. ACKNOWLEDGMENT

The original work at Bell Laboratories on loop network operating cost models was done by E. P. Klein, now of AT&T. In addition to the references cited, others who have contributed to the general knowledge in this area include J. O. Bergholm, M. J. Krone, and D. B. Luber.

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