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Analysis of Longitudinal Stress Imparted to Fibers in Twisting an Optical Communication Cable Unit

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In the exploratory fiber optic cables used in the Atlanta Fiberguide System Experiment, 12 optical fiber ribbons each containing 12 fibers are stacked one on top of the other to form a rectangular array of 144 optical fibers. Just prior to sheathing, the array is twisted to a given period (lay) in order to improve its bending properties. Moreover, good cable bending properties dictate short lay lengths. However, short lay lengths result in high longitudinal (axial) stresses and strains in the optical fibers. To obtain high fiber yield in cable manufacturing, such strains must be well below 0.3 percent for current 35 ksi fibers. A model which assumes that each fiber follows a helical space curve is used to calculate an upper bound on the axial stress imparted by the twisting operation. The intent was to use the results to choose a lay length short enough to give acceptable bending properties yet long enough to avoid endangering fiber survival in cable manufacture. Model predictions based on a cable design similar to the one in the Atlanta Fiberguide System Experiment lead to the conclusion that the twist period should be not less than 4 inches.

I. INTRODUCTION

In the exploratory Fiber Optic (FO) cables used in the Atlanta Fiberguide System Experiment, 12 optical fiber ribbons each containing 12 fibers are stacked one on top of the other to form a rectangular array of 144 optical fibers.[1,2] Figure 1 shows a representative cross section of a fiber ribbon and of the 144-fiber optical cable core unit. Just prior to sheathing, the unit is twisted to a given period (lay) in order to improve its bending properties. Moreover, good cable bending properties dictate short lay lengths. However, a short lay length results in high longitudinal (axial) stresses and strains on the optical fibers. In order to obtain high

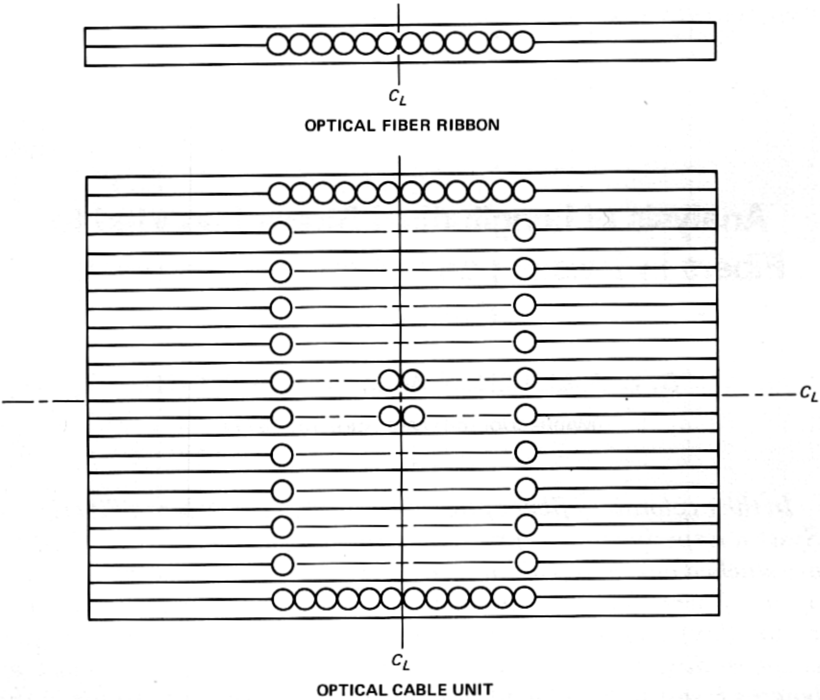

Fig. 1—Representative cross section of fiber ribbon and optical cable core unit.

yield in the cable manufacturing, using current fibers, such strains need to be well below 0.01 (1 percent). A different strain is experienced by each individual fiber when the unit is twisted because the helical paths followed by the fibers differ in length. The type and amount of strain depend on the positions of the fiber within the ribbon and the ribbon within the unit.

Strakhov[3] outlined a model by which the strain introduced in a fiber due to twisting the stacked ribbons can be predicted. In this paper, his model is modified to account for slippage between the ribbons; thus, the net predicted strain on the individual fibers is greatly reduced. The model predicts upper-bound, twisting-induced, tensile and compressive stresses. The intent was to use the numerical results to choose a lay length short enough to give acceptable bending properties, yet long enough to avoid endangering fiber survival during cable manufacture.

II. DESCRIPTION OF THE MODEL

The model is geometric in nature in that the helical space curve length of each fiber is directly related to the strain on that fiber. The underlying assumptions that accompany this model are as follows:

(i) With the cable axis straight, the individual fiber axes coincide with helices of appropriate diameter and pitch.

(ii) All the fibers within a given ribbon are completely coupled to each other.

(iii) Induced stresses are supported entirely by the fibers, i.e., other ribbon materials are ignored.

(iv) The twisted unit maintains a rectangular cross section.

(v) The fibers are treated as filaments which follow the geometric axes of the real fibers.

(vi) The tensile and compressive moduli of the fibers are equal.

The rectangular array assumption ignores both geometric distortion of the cable cross section due to twisting and also dynamic distortion resulting from imparted stresses. We will return to these distortions later.

Polymeric materials of the ribbon can play either an implicit or explicit role in the model. Their role is implied in the assumption of complete fiber coupling within a given ribbon. Explicit participation occurs when the actual stresses-developed in the polymers are considered. In order to calculate these stresses, the basic model assumptions can be extended from the discrete fiber case to the polymer continuum. However, questions about plastic deformation of the polymers and about dynamic distortion of the cross section suggest caution against reading more than upper-bound significance into these results. In any event, the presence of the polymers tends to decrease the maximum tensile stress on the fibers for the ribbons under consideration. That is, ignoring the polymers does not disturb the upper-bound nature of the model's results.

With this in mind, let us refer to Fig. 2 where a ribbon is shown with respect to the center of the unit. We will now develop analytic expressions for the stresses in a twisted array consisting of N ribbons with M fibers per ribbon for a total of $M \cdot N$ fibers. To simplify our expressions M and N are considered to be even numbers. Also, if only one quadrant is considered (here the upper left-hand quadrant), symmetry arguments can be used for the others. From the formula for the distance along a helix, the lengths of the individual fibers can be computed:

$$L_{mn} = L_0[1 + \omega^2 C_{mn}^2]^{1/2} \qquad \begin{array}{l} m = 1, \ldots, M/2 \\ n = 1, \ldots, N/2 \end{array} \tag{1}$$

where L_{mn} is the length of the mth fiber in the nth ribbon, along a given length of cable L_0, C_{mn} is the radius of the helical path of the mth fiber in the nth ribbon, and ω is the twisting rotation rate. That is,

$$\omega = \frac{2\pi}{T} \tag{2}$$

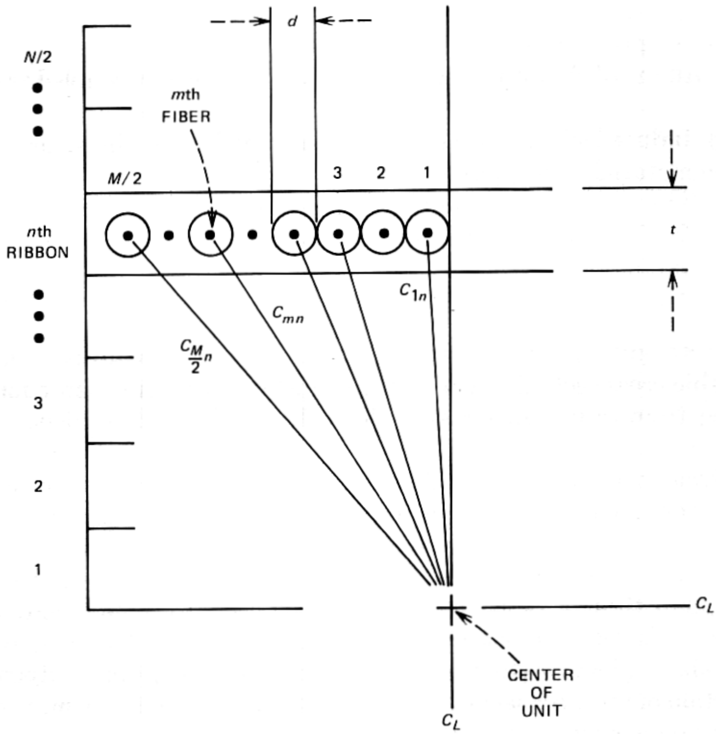

Fig. 2—Position of a ribbon with respect to center of cable.

where T is the twisting lay length. The strain and stress on each of the fibers is given by:

$$\left.\begin{aligned} \epsilon_{mn} &= \frac{L_{mn} - L_{on}}{L_{on}} \\ \sigma_{mn} &= E_f \epsilon_{mn} \end{aligned}\right\} \qquad \begin{aligned} m &= 1, \ldots, M/2 \\ n &= 1, \ldots, N/2 \end{aligned} \tag{3}$$

$$\tag{4}$$

where ϵ_{mn} and σ_{mn} are the strain and stress on the mth fiber of the nth ribbon, respectively, E_f is Young's modulus for the fiber material, and L_{on} is the paid-out ribbon length. L_{on} can also be interpreted as the unstressed ribbon length in a cable length L_o.

The $N/2$ values of L_{on} are obtained by dynamically balancing the forces at each reel at the time of payout. Therefore, the sum of the tensions on all the fibers within one-half the nth ribbon must be equal to one-half the back tension on the ribbon as it is paid out, that is:

$$A_f E_f \sum_{m=1}^{M/2} \epsilon_{nm} \cos\theta_{mn} = T_n/2 \qquad n = 1, \ldots, N/2 \tag{5}$$

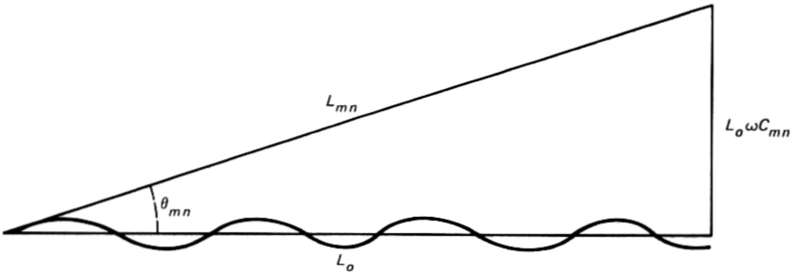

Fig. 3—Geometrical relationships of a cable.

where A_f is the area of each fiber and T_n is the back tension at the payout for the nth ribbon. The cos θ_{mn} factor takes into account the θ_{mn} pitch angle imparted to each fiber by the twisting operation. This factor gives the fiber tension component opposing the back tension. The torque produced by each twisted ribbon is ignored and it is assumed to be balanced by the tape binder and/or sheath. An alternate approach is to balance the torques generated by the individual fibers against the applied torque while assuming that the axial force is balanced through sheath friction. We forego this approach because it is mathematically cumbersome. With reference to Fig. 3,

$$\cos \theta_{mn} = \frac{L_o}{L_{mn}} \qquad \begin{array}{l} m = 1, \ldots, M/2 \\ n = 1, \ldots, M/2 \end{array} \tag{6}$$

We now have all the relations necessary to determine the σ_{mn} as functions of cable geometry. Note that all the explicit dynamical considerations are contained in eqs. (3), (4), and (5). As stated earlier, the fibers in a given ribbon are assumed fully coupled and the ribbons uncoupled. We start our derivation of the twisting stresses σ_{mn} by substituting eqs. (1) and (3) into (5) and solving for L_{on}. This yields:

$$L_{on} = L_o \left[\frac{M/2}{\sum_{m=1}^{M/2} (1 + \omega^2 C_{mn}^2)^{-1/2} + \frac{T_n}{2A_f E_f}} \right] \qquad n = 1, \ldots, N/2 \tag{7}$$

Next we substitute eqs. (7) and (1) into (3) and (4) to obtain

$$\epsilon_{mn} = \left[\frac{(1 + \omega^2 C_{mn}^2)^{1/2}}{M/2} \left(\sum_{m=1}^{M/2} (1 + \omega^2 C_{mn}^2)^{-1/2} + \frac{T_n}{2\, A_f\, E_f} \right) - 1 \right] \tag{8}$$

$$m = 1, \ldots, M/2$$
$$n = 1, \ldots, N/2$$

$$\sigma_{mn} = E_f \left[\frac{(1 + \omega^2 C_{mn}^2)^{1/2}}{M/2} \left(\sum_{m=1}^{M/2} (1 + \omega^2 C_{mn}^2)^{-1/2} + \frac{T_n}{2\, A_f\, E_f} \right) - 1 \right] \tag{9}$$

In practice, T_n is developed by braking at the payout reels, and its magnitude is in the vicinity of 0.11 pounds. To this, a small but unmeasured increment must be added due to friction in the core unit organizer. In practice, the sum of these two components remains negligible, then $T_n = 0$, and eq. (9) reduces to the simpler form:

$$\sigma_{mn} = \frac{E_f}{M/2} (1 + \omega^2 C_{mn}^2)^{1/2} \times \sum_{m=1}^{M/2} (1 + \omega^2 C_{mn}^2)^{-1/2} - E_f \qquad \begin{matrix} m = 1, \ldots, M/2 \\ n = 1, \ldots, N/2 \end{matrix} \qquad (10)$$

In general, $\omega^2 C_{mn}^2 \ll 1$; therefore, the approximation

$$(1 + \omega^2 C_{mn}^2)^{\pm 1/2} \approx 1 \pm {}^1\!/\!_2 \omega^2 C_{mn}^2 \qquad (11)$$

can be legitimately used to gain insight into the fiber stresses within a ribbon. Equation (10) can now be rewritten in the form:

$$\sigma_{mn} \cong {}^1\!/\!_2 \omega^2 C_{mn}^2 S_n + [S_n - E_f] \qquad \begin{matrix} m = 1, \ldots, M/2 \\ n = 1, \ldots, N/2 \end{matrix} \qquad (12)$$

where S_n is the same constant for each fiber in the nth ribbon and is given by:

$$S_n = \frac{E_f}{M/2} \sum_{m=1}^{M/2} (1 + \omega^2 C_{mn}^2)^{-1/2} \qquad n = 1, \ldots, N/2 \qquad (13a)$$

Or, to the degree of approximation of eq. (11),

$$S_n = \frac{E_f}{M/2} \sum_{m=1}^{M/2} (1 - {}^1\!/\!_2 \omega^2 C_{mn}^2) \qquad n = 1, \ldots, N/2 \qquad (13b)$$

From Fig. 2,

$$C_{mn}^2 = (m - {}^1\!/\!_2)^2 d^2 + (n - {}^1\!/\!_2)^2 t^2 \qquad \begin{matrix} m = 1, \ldots, M/2 \\ n = 1, \ldots, N/2 \end{matrix} \qquad (14)$$

where d and t are the fiber diameter and ribbon thickness, respectively.

When eq. (14) is inserted into eq. (12),

$$\sigma_{mn} \approx {}^1\!/\!_2 \omega^2 (m - {}^1\!/\!_2)^2 d^2 S_n + [{}^1\!/\!_2 \omega^2 (n - {}^1\!/\!_2)^2 t^2 S_n + S_n - E_f] \qquad \begin{matrix} m = 1, \ldots, M/2 \\ n = 1, \ldots, N/2 \end{matrix} \qquad (15a)$$

Only the first term varies with m, and it shows a parabolic dependence of σ_{mn} on m. Since $T_n = 0$, the concave upward orientation of σ_{mn} indicates a longitudinal compression of the inner fibers of a ribbon and a tension on the outer fibers. Also, from eq. (13a) we see that S_1 is the largest S_n. We can conclude, then, that the parabolic dependence shown in eq. (15a) is steepest at the center of the core unit; therefore, the stresses will be most severe in the center ribbons.

If we use eq. (13b) to take a closer look at the bracketed term in eq. (12), we find that E_f drops out and the only term remaining is proportional to ω^2. After some algebra, we find

$$\sigma_{mn} \cong \tfrac{1}{2}\omega^2 C_{mn}^2 S_n - \frac{2}{M}\sum_{m=1}^{M/2} \tfrac{1}{2}\omega^2 C_{mn}^2 \tag{15b}$$

Here we see that longitudinal stresses go up as ω^2 to lowest order.

Let us return our attention now to the question of ribbon deformation. Without a detailed structural analysis involving fibers and ribbons, numerical predictions of this effect are impossible. Such an analysis is beyond the scope of this paper. As we shall see, the model still provides an upper bound for stranding stresses because the ribbon deformation tends to relieve stresses. The argument goes as follows. The helical path of each fiber has a curvature, K_{mn}, given by

$$K_{mn} = \frac{\omega^2 C_{mn}}{1 + \omega^2 C_{mn}^2} \qquad \begin{array}{l} m = 1, \ldots, M/2 \\ n = 1, \ldots, N/2 \end{array} \tag{16}$$

This curvature, together with the longitudinal stress, results in a transverse force by the fiber on its local environment. This force is given by

$$f_{mn} = -A_f \sigma_{mn} K_{mn} \qquad \begin{array}{l} m = 1, \ldots, M/2 \\ n = 1, \ldots, N/2 \end{array} \tag{17}$$

where f_{mn} is the transverse force per unit length exerted by the mth fiber of the nth ribbon. The sign convention was chosen such that a negative force means it is directed toward the center of curvature and a positive force is away from the center of curvature. It turns out that for the outside (tensioned) fibers on a ribbon, the force is directed towards the center of the stack while for the inside (compressed) fibers, the force is away from the center of the stack. These are the very directions which lead to a stress-relieving barrel-shaped distortion. An important caveat must be added, however. Sheath forces may play even a larger role in transverse stresses. From eqs. (15b), (16), and (17) it can be seen that, to the lowest order, these transverse forces are proportional to ω^4. The component of these forces in the local plane of the ribbon is cumulative and tends to deform the ribbon from its planar shape. This component can be obtained from eqs. (16) and (17) and Fig. 4.

$$f_{mn_x} = A_f \sigma_{mn} \frac{\omega^2 (m - \tfrac{1}{2}) d}{1 + \omega^2 C_{mn}^2} \qquad \begin{array}{l} m = 1, \ldots, M/2 \\ n = 1, \ldots, N/2 \end{array} \tag{18}$$

where f_{mn_x} is the component of f_{mn} along the plane of the ribbon. The

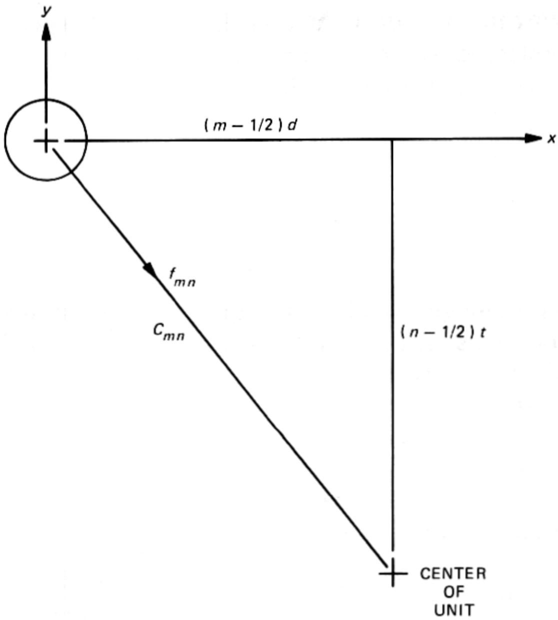

Fig. 4—Forces acting on a fiber in a ribbon.

sign convention has been chosen so that a force toward the right is positive. Because of the cumulative nature of these forces, their sum acting on the individual fiber contact boundaries can be obtained using

$$F_{nk} = \sum_{m=k}^{M/2} f_{mn_x} \qquad \begin{array}{l} n = 1, \ldots, N/2 \\ m = 1, \ldots, M/2 \\ k = 1, \ldots, M/2 \end{array} \tag{19}$$

where F_{nk} is the sum of the x components of the f_{mn} acting on the fiber contact boundaries. A pictorial representation is given in Fig. 5. These forces induce ribbon deformation which only serves to reduce stress levels.

III. APPLICATION TO A SPECIFIC DESIGN: RESULTS AND CONCLUSIONS

Now that a general model has been set forth, it can be used to compute an upper-bound longitudinal stress and strain of any fiber within any optical unit due to its twisting lay, subject to the assumptions and constraints mentioned before. A computer program was written to perform the necessary calculations outlined in Section II. The program employed the exact relations rather than the approximate ones used to gain insight into the model's predictions. The following inputs are needed by the

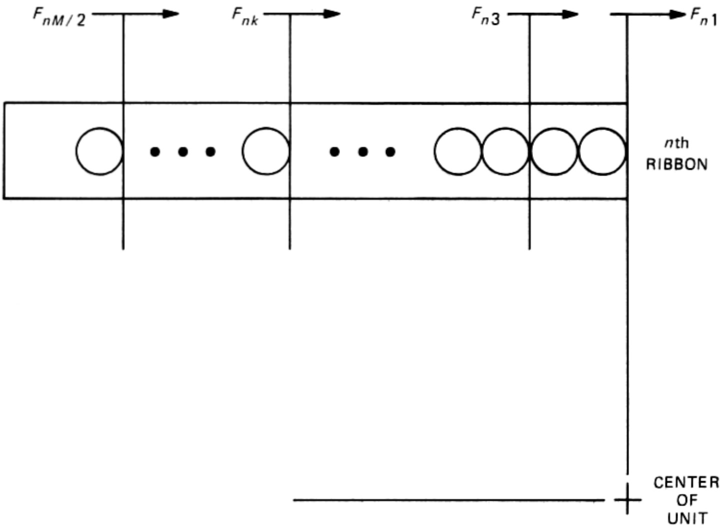

Fig. 5—Cumulative forces acting on a ribbon.

program: the twisting lay, the number of ribbons and the number of fibers in each ribbon, the thickness of the ribbons, the payout back tension, the outside diameter of the fibers, the fiber spacing within each ribbon, and the elastic modulus of the glass.

The original intent in developing this model was to use its numerical results to optimize the twisting lay length with respect to cable bending properties and fiber survival in cable manufacturing. With this in mind, the model was applied to a cable design having a geometry representative of the experimental cables later used in the Atlanta Fiberguide System Experiment.[1] The cross section of the design chosen had 12 optical fiber ribbons, each one containing 12 fibers for a total of 144 fibers in the optical unit. The parameter values were:

$T = 1, 2,$ and 4 inches
$N = M = 12$
$t = 0.007$ inch
$d = 0.004$ inch
$T_n = 0$
$E_f = 10^7$ psi

Because the most severe stresses are experienced by the outside and center fibers of the center ribbons, let us confine our discussion to them. In Table I, a summary of the stresses experienced by these fibers is given. One conclusion that can be drawn from the summary in Table I is that, to avoid significant strain in the fibers, the shortest twisting lay that should be used in this cable design is 4 inches. Although the model pre-

Table I — Stress and strain on the end and center fibers of the center ribbon due to twisting lays of 1, 2, and 4 inches

	T = 1 inch		T = 2 inches		T = 4 inches	
	ϵ	σ, psi	ϵ	σ, psi	ϵ	σ, psi
Outside fiber	.00574	57,400	.00144	14,400	.00036	3600
Center fiber	−.00364	−36,400	−.00092	−9,200	−.00023	−2300

dicts a strain which is above the proof test strain of the current optical fibers when a 1-inch lay is used, it should be pointed out that under such high strains the fibers will start displacing from their original position in the ribbon seeking a stress-relieved barrel-shaped cross section.

In conclusion, from the numerical results obtained in this section, it can be concluded that the twisting lay or period should not be less than 4 inches for the basic cable design used in the Atlanta Fiberguide System Experiment.

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