THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 55, No. 8, October 1976
Printed in U.S.A.

Digital Coding of Speech in Sub-bands

By R. E. CROCHIERE, S. A. WEBBER, and J. L. FLANAGAN

(Manuscript received March 26, 1976)

A rationale is advanced for digitally coding speech signals in terms of sub-bands of the total spectrum. The approach provides a means for controlling and reducing quantizing noise in the coding. Each sub-band is quantized with an accuracy (bit allocation) based upon perceptual criteria. As a result, the quality of the coded signal is improved over that obtained from a single full-band coding of the total spectrum. In one implementation, the individual sub-bands are low-pass translated before coding. In another, "integer-band" sampling is employed to alias the signal in an advantageous way before coding. Other possibilities extend to complex demodulation of the sub-bands, and to representing the sub-band signals in terms of envelopes and phase-derivatives. In all techniques, adaptive quantization is used for the coding, and a parsimonious allocation of bits is made across the bands. Computer simulations are made to demonstrate the signal qualities obtained for codings at 16 and 9.6 kb/s.

I. DIVISION OF SPEECH SPECTRUM INTO SUB-BANDS

For digital transmission a signal must be sampled and quantized. Quantization is a nonlinear operation and produces distortion products that are typically broad in spectrum. Because of the characteristics of the speech spectrum, quantizing distortion is not equally detectable at all frequencies. Coding the signal in narrower sub-bands offers one possibility for controlling the distribution of quantizing noise across the signal spectrum and, hence, for realizing an improvement in signal quality. In earlier work, splitting of the spectrum by high-pass and low-pass filtering has been used advantageously for video and speech transmission.[1,2]

A question, then, is what design of sub-bands makes sense for speech coding? A choice based upon perceptual criteria is suggested, namely, band-partitioning such that each sub-band contributes equally to the so-called articulation index (AI).[3] The AI concept is based upon a nonuniform division of the frequency scale for the speech spectrum. Twenty nonuniform contiguous bands are derived in which each elemental band contributes 5 percent to the total AI.

Appealing to this notion, one partitioning of the frequency range 200 to 3200 Hz into four "equal-contribution" bands is given below and shown in Fig. 1.

Sub-band Number	Frequency Range (Hz)
1	200–700
2	700–1310
3	1310–2020
4	2020–3200

Each sub-band in its original analog form contributes 20 percent to AI. The total AI, therefore, is 80 percent, which corresponds to a word intelligibility of approximately 93 percent.[4]

II. LOW-PASS TRANSLATION OF SUB-BANDS

A straightforward approach to processing the sub-bands is to make a low-pass translation before coding. This facilitates sampling-rate reduction and realizes any benefits which might accrue from coding the low-pass signal.

The low-pass translation can be accomplished in a variety of ways. One method is shown in Fig. 2. The input speech signal is filtered with a bandpass filter of width W_n for the nth band. W_{1n} is the lower edge of the band and W_{2n} is the upper edge of the band. The resulting signal $s_n(t)$ is modulated by a cosine wave, cos $(W_{1n}t)$, and filtered

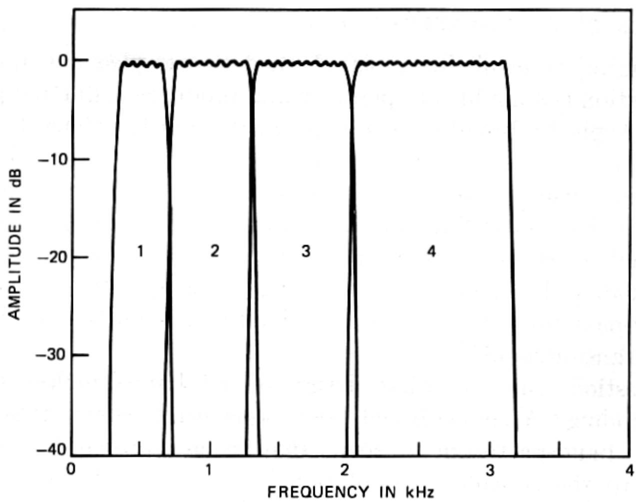

Fig. 1—Partitioning of the speech spectrum into four contiguous bands that contribute equally to articulation index. The frequency range is 200 to 3200 Hz.

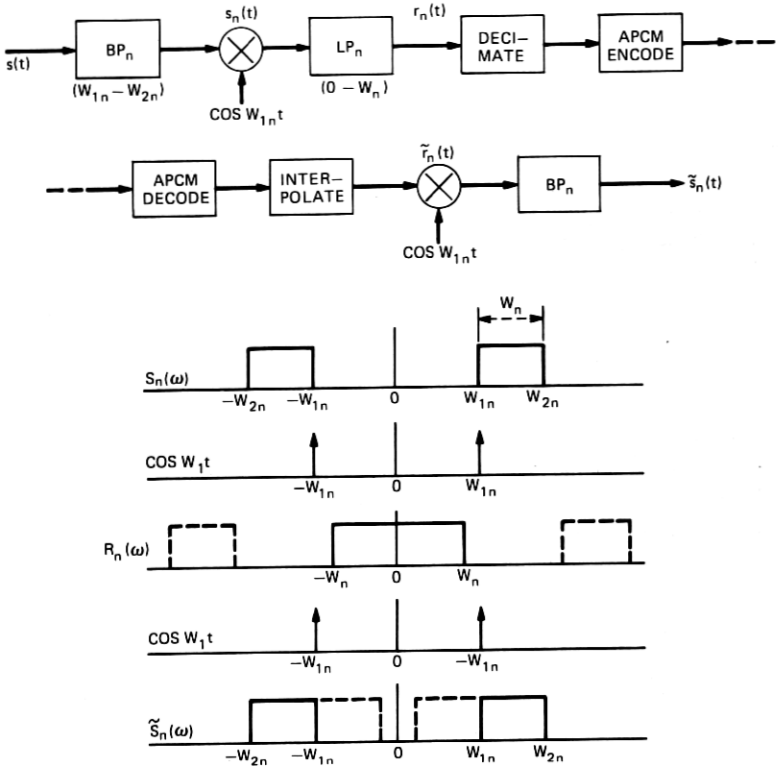

Fig. 2—Sequence of operations for low-pass translation of speech sub-bands, adaptive PCM encoding, transmission, decoding, and band restoration.

by a low-pass filter $h_n(t)$ with bandwidth $(0 - W_n)$. This filter is necessary to remove the unwanted signal images above $2W_{1n}$, as shown in Fig. 2. The resulting signal $r_n(t)$ corresponds to the low-pass translated version of $s_n(t)$ and can be expressed in the form:

$$r_n(t) = [s_n(t) \cos (W_{1n}t)]^* h_n(t). \tag{1}$$

Notice, in this instance, that a constraint is implied by the convolution, namely, that the passband width $W_n \leqq 2W_{1n}$, or that $W_{2n} \leqq 3W_{1n}$. Practically this poses no problem.*

The signal $r_n(t)$ is sampled at rate $2W_n$. If it is already in digital form, the sampling rate is decimated (reduced) to the rate $2W_n$. This signal is digitally encoded and multiplexed with encoded signals from other channels as shown in Fig. 3. At the receiver the data is demulti-

* For example, this constraint requires that W be increased slightly, from 200 to 233 Hz, for $n = 1$ in Fig. 1.

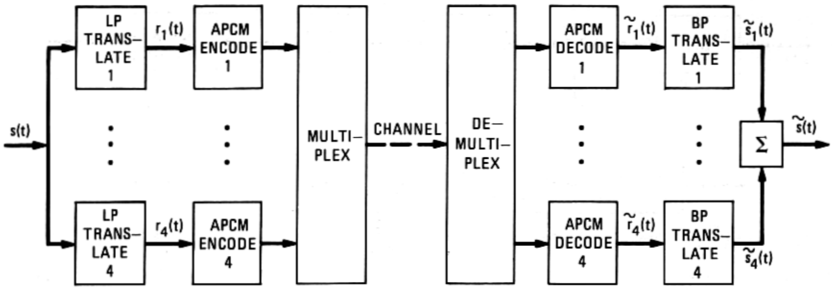

Fig. 3—Four-band encoder using low-pass translation and APCM encoding in each band.

plexed into separate channels, decoded, and interpolated to give the estimate $\tilde{r}_n(t)$ for the nth channel. Reconstruction of the detected signal is simply done by the reverse band translation. That is, it is modulated by $\cos(W_{1n}t)$ and bandpass filtered to the original passband, as shown in Fig. 2. The sub-band signal $\tilde{s}_n(t)$ is then summed with the other bands to give the full-band signal $\tilde{s}(t)$.

An alternate implementation of the low-pass translation method, which avoids the above-mentioned restriction on W_n, follows from a modification of the complex demodulation process. In this approach, $s(t)$ is complex modulated by $e^{j\omega_n t}[\omega_n = (W_{1n} + W_{2n})/2 =$ center frequency of band $n]$ and filtered by a low-pass filter $h_n'(t)$ with bandwidth $(0 - W_n/2)$. The resulting complex signal $a_n(t) + jb_n(t)$,

$$a_n(t) = [s(t) \cos \omega_n t]^* h_n'(t) \tag{2a}$$

$$b_n(t) = [s(t) \sin \omega_n t]^* h_n'(t) \tag{2b}$$

corresponds exactly to the output of the phase vocoder.[5] The conjugate of this signal $a_n(t) - jb_n(t)$ corresponds to a modulation of $s(t)$ by $e^{-j\omega_n t}$. If the complex signal $a_n(t) + jb_n(t)$ is complex modulated by $e^{-j(W_n/2)t}$ and its conjugate complex modulated by $e^{j(W_n/2)t}$, the two resulting complex signals correspond to the negative and positive frequency components of the low-pass translated signal $r_n(t)$, as shown in Fig. 4. The sum of these two signals gives a real signal corresponding to the desired low-pass translated signal $r_n(t)$; i.e.,

$$r_n(t) = [a_n(t) + jb_n(t)]e^{-j(W_n/2)t} + [a_n(t) - jb_n(t)]e^{+j(W_n/2)t}, \tag{3}$$

or

$$r_n(t) = 2\left[a_n(t) \cos\left(\frac{W_n}{2}t\right) + b_n(t) \sin\left(\frac{W_n}{2}t\right)\right]. \tag{4}$$

For reconstruction, it can be shown that $a_n(t)$ and $b_n(t)$ can be recovered from the low-pass translated signal $r_n(t)$ by the following relations

$$a_n(t) = [r_n(t)\cos(W_nt/2)]^*h_n'(t) \tag{5a}$$

$$b_n(t) = [r_n(t)\sin(W_nt/2)]^*h_n'(t). \tag{5b}$$

Equations (4) and (5) suggest a method of implementation of the low-pass translation and reconstruction with a phase vocoder. For a digital implementation of the low-pass translation, this approach is particularly appealing. For example, at the sampling rate $f_s = 2W_n/2\pi$, the sequences corresponding to $\cos(W_nt/2)$ and $\sin(W_nt/2)$ are 1, 0, -1, 0, 1, $\cdots$, and 0, 1, 0, -1, 0, $\cdots$, respectively. Therefore, an efficient way to generate $r_n(t)$ is to sample a_n and b_n (or decimate if they are in digital form) to one half of this sampling rate (i.e., $W_n/2\pi$) and form $r_n(t)$ by interleaving samples of a_n and b_n (with appropriate sign changes). A similar approach can be used in the reconstruction process by recognizing that alternate samples of $r_n(t)\cos(W_nt/2)$ and

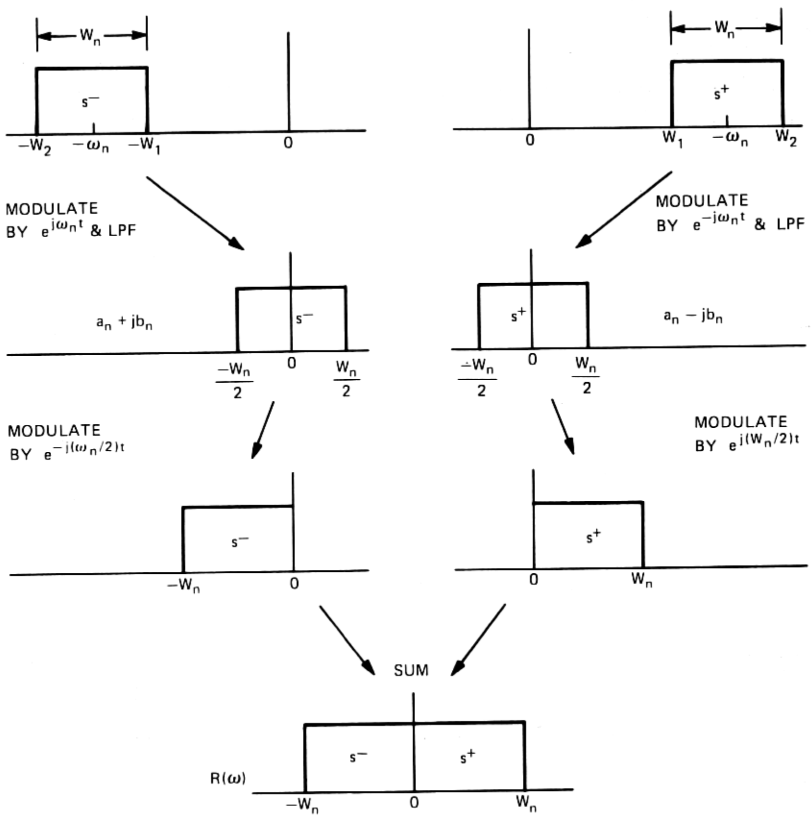

Fig. 4—Frequency-domain interpretation of complex demodulation method for low-pass translation.

$r_n(t) \sin (W_n t/2)$ (at sampling rate $2W_n/2\pi$) are zero valued. Thus, the two input sequences to the interpolators (which can be sampled at half of this rate or $W_n/2\pi$) can be generated by selecting alternate samples of $r_n(t)$ (with appropriate sign changes).

A further modification on this approach can be made by noting that, since adaptive coding is used to encode $r_n(t)$, the sign changes in the construction and separation of $r_n(t)$ are not necessary. That is, an alternate sequence $r_n'(t)$ can be generated by interleaving samples of a_n and b_n without sign changes. This sequence can be encoded and decoded and inputs to the interpolators can be formed from alternate samples of $\tilde{r}_n'(t)$ (without sign changes). Figure 5 shows an implementation of this method. The signal $s(t)$ is modulated by $\cos \omega_n t$ and $\sin \omega_n t$, where ω_n is the center frequency of band n. These signals are filtered with low-pass filters $h_n'(t)$ with bandwidth $(0 - W_n/2)$. The outputs are decimated (if they are in digital form) or sampled (if analog) at a sampling rate W_n. The low-pass translated signal $r_n'(t)$ is obtained (at sampling rate $2W_n$) by interleaving samples of a_n and b_n. $r_n'(t)$ is encoded, transmitted, and decoded as in Fig. 3. On reconstruction a_n and b_n are recovered by selecting alternate samples of $\tilde{r}_n'(t)$. These signals are then interpolated, filtered, modulated, and

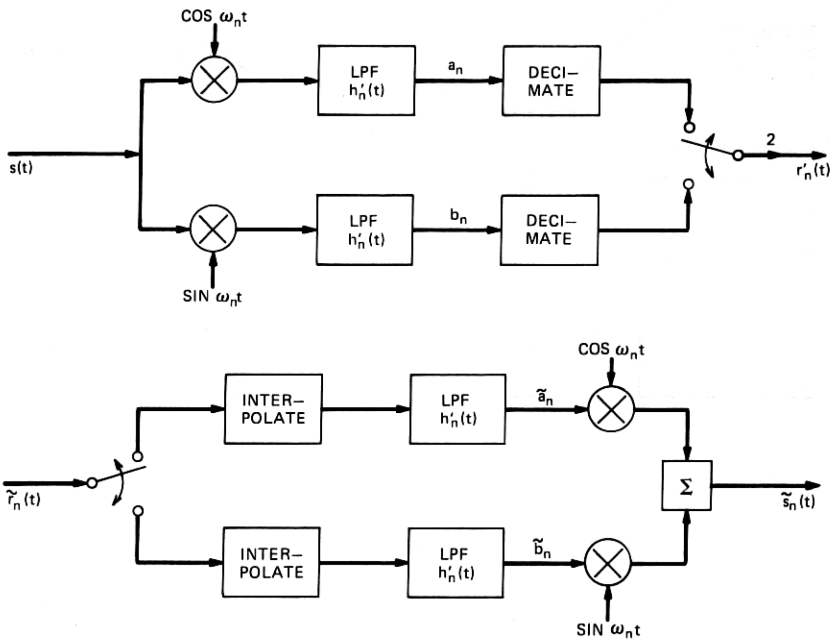

Fig. 5—Implementation of complex demodulation for low-pass translation with interleaving of samples of a_n and b_n.

summed as shown in Fig. 5 to give the reconstructed sub-band signal $\tilde{s}_n(t)$.

For digital implementation h_n' can be realized with a digital filter. Decimation, or sampling-rate reduction by an integer factor M, can be achieved by retaining only one out of every M samples of the output of the filter. The filter is necessary to avoid aliasing. Interpolation by an integer factor M is accomplished by increasing the sampling rate by filling in $M - 1$ zero-valued samples between each pair of input samples. The filter h_n' then removes the unwanted harmonic images of the base-band signal and smooths (i.e., interpolates) these samples to appropriate values of the base-band waveform. Efficient methods for implementing digital decimators and interpolators are discussed in Ref. (6).

III. ENCODING OF THE SUB-BAND SIGNALS

Digital encoding of the low-pass translated signal $r_n(t)$ is best accomplished using adaptive-PCM (APCM).[7,8] APCM encoding is preferred over adaptive-differential PCM (ADPCM) methods in this case due to the low sample-to-sample correlation of the low-pass-translated, Nyquist-rated, sampled signals.

For computer simulations, APCM coders based on a one-word step-size memory were used according to methods proposed by Jayant, Flanagan, and Cummiskey.[7–9] Step-size adaption is achieved according to the relation

$$\Delta_r = \Delta_{r-1} \times M, \tag{6}$$

where Δ_r is the quantizer step-size used for the rth sample and Δ_{r-1} is the step-size of the $(r - 1)$th sample. M is a multiplication factor whose value depends on the quantizer level at the $(r - 1)$th sample. For example, in a two-bit quantizer, two magnitude levels and the sign can be represented. If the smaller magnitude level is used at time $r - 1$, M is chosen to have a value $M = M_1 < 1$, and if the larger magnitude level is chosen, $M = M_2 > 1$ is used. For a three-bit quantizer, four magnitude levels and the sign can be represented. In this case, there are four choices for M. Through simulations, appropriate values of M for a two-bit quantizer were found to be $M_1 = 0.845$ and $M_2 = 1.96$. For a three-bit quantizer, they are $M_1 = 0.845$, $M_2 = 1.0$, $M_3 = 1.0$, and $M_4 = 1.4$. Note that the three-bit quantizer does not change its step-size at time r unless the largest or smallest quantizer level is encountered at time $r - 1$. The above values of M are in approximate agreement with values proposed by Jayant[7] for full-band APCM encoding.

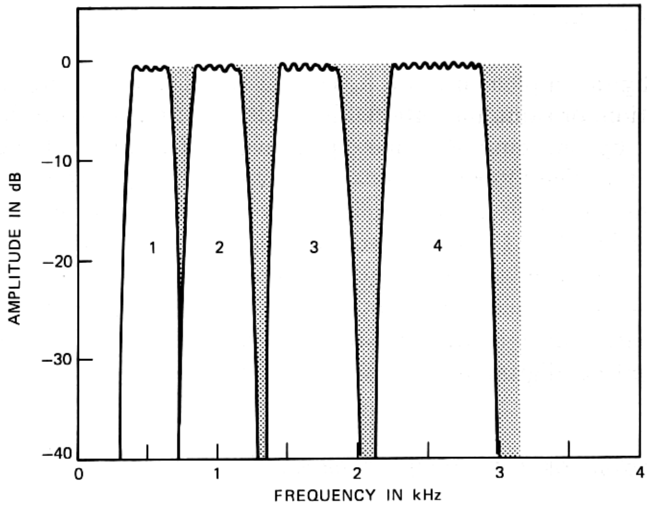

Fig. 6—Partitioning of the speech spectrum into four noncontiguous bands to achieve reduced bit-rate coding.

IV. SUB-BAND CODING FOR TRANSMISSION AT DATA RATES

The transmission bit rate of the sub-band coder can be reduced into the range of conventional data speeds by further limiting the sub-bands in width and tolerating some spectral gaps as shown in Fig. 6. Carried to excess, the noncontiguous bands produce a reverberant quality in the signal, such as one gets from comb filtering. In moderation, however, some highly useful compromises can be achieved between transmission bit rate and quality. The coded bands still cover a respectable range of the speech spectrum, and provide a quality considerably better than coding a single full-band signal.

V. INTEGER-BAND SAMPLING AND HARDWARE CONSIDERATIONS

Another attractive alternate implementation of these ideas is to use "integer-band" sampling to code a signal that is aliased in an advantageous way. The technique is illustrated in Fig. 7.

The signal sub-bands $s_n(t)$ are chosen to have a lower cutoff frequency of mf_n and an upper cutoff frequency of $(m + 1)f_n$, where m is an integer and f_n is the bandwidth of the nth band. This bandpassed signal is sampled at $2f_n$ to produce the sampled spectrum shown in Fig. 7 (for $m = 2$). The received signal is recovered by decoding and bandpassing to the original signal band. Typically, values of m from 1 to 3 are most useful for coder applications with lower bands using values of $m = 1$ and upper bands using $m = 2$ or $m = 3$. This integer-band sampling technique achieves the theoretical maximum efficiency in sampling.[10]

A very attractive advantage of the integer-band sampling approach is that it does not require the use of modulators. A slight disadvantage is that the above restrictions prevent the choice of bands strictly on the basis of equal contribution to AI. However, little loss in performance is observed if this equal contribution to AI condition is only approximate (within a factor of 2). This implementation was used for perceptual comparisons, which will be discussed later.

This approach is especially attractive for implementing the bandpass filters as charge-coupled-device (CCD) transversal filters. The analog to discrete-time conversion is inherently accomplished by the CCD filter with little or no analog prefiltering or post filtering required for the prevention of aliasing. The initial signal sampling can be conveniently high, say 15 kHz, to realize the CCD filter, and the filter output can be decimated to the $2f_n$ rate for coding. After transmission and decoding, the $2f_n$ rate can be interpolated to the 15-kHz rate for the final bandpass filtering, again by the analog CCD filter.

Another advantage of CCD filters (and also digital filters) is that the filter cutoff frequencies are inherently normalized to the initial sampling frequency. Therefore, the sampling frequency and, consequently, the bit rate of the coder, can be varied over a limited range by

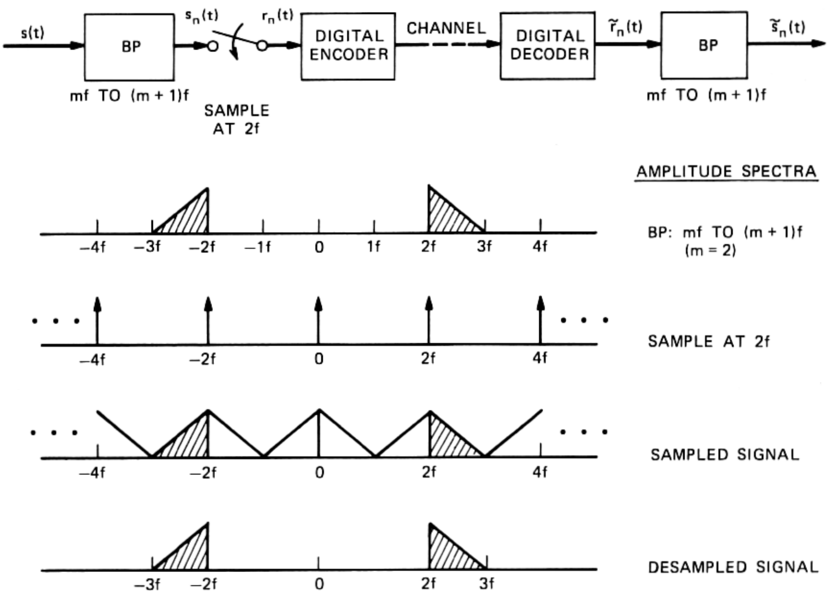

Fig. 7—Integer-band sampling technique for digital encoding of speech sub-bands.

Table I — Frequencies and sampling rates for the 16-kb/s coder

Sub-band No.	ω_n Center Freq (Hz)	$r_n'(t)$ Sampling Rate (s^{-1})	Decimation (From 10 kHz)	Quantization (Bits)
1	448	1250	16	3
2	967	1429	14	3
3	1591	1667	12	2
4	2482	2500	8	2

varying the master clock frequency. This cannot be achieved with analog filters.

Present technology is able to provide four 100-tap CCD transversal filters on a single integrated-circuit chip or one 200-tap filter on a chip with all necessary drivers and control logic.

VI. COMPUTER SIMULATIONS OF SUB-BAND ENCODERS

The sub-band coder has been implemented by computer simulation for transmission bit rates of approximately 16 kb/s and 9.6 kb/s. The complex demodulation approach in Fig. 5 was used for low-pass translation of the bands. An initial sampling rate of 10 kHz was employed in both cases.

The 16-kb/s coder was implemented with the band center frequencies and sub-band sampling rates shown in Table I. Bandwidths are equal to one half of the sampling rates and correspond to those shown in Fig. 1. Three-bit coders were used in the two lower bands, and two-bit coders were used for the upper bands. The filters were 125-tap FIR filters. As can be observed in Fig. 1, the filters overlap in their transition bands and give an overall flat frequency response from 200 Hz to 3100 Hz.

The 9.6-kb/s coder was implemented with the bands given in Table II and illustrated in Fig. 6. In this case gaps were allowed between bands. Larger filter orders, 175-tap (FIR), were used to reduce transition bands and conserve bandwidth. Only the lower band used a three-bit coder. Upper bands used 2-bit coders.

Table II — Frequencies and sampling rates for the 9.6-kb/s coder

Sub-band No.	ω_n Center Freq (Hz)	$r_n'(t)$ Sampling Rate (s^{-1})	Decimation (From 10 kHz)	Quantization (Bits)
1	448	800	25	3
2	967	952	21	2
3	1591	1111	18	2
4	2482	1538	13	2

Illustrations of the signal coded for 16 kb/s and 9.6 kb/s by the above-band-translation technique are given by the spectrograms of Figs. 8 and 9, respectively. In each figure, the upper spectrogram corresponds to the original sentence. The middle spectrogram corresponds to the signal played through the filters, decimators, and interpolators—but without coders. The bottom spectrogram illustrates the sub-band encoded speech at the designated bit rate.

Other simulations have also been made for encoding the signals $a_n(t)$ and $b_n(t)$ directly and also for encoding the magnitude and phase derivative (as in the phase vocoder). Similar quality results were found in these simulations.

VII. SUBJECTIVE COMPARISONS WITH OTHER ENCODING METHODS

Informal listening tests were made to compare the quality of the sub-band coder simulations with that of full-band encoding. For the 16-kb/s coder, comparisons were made with 2- and 3-bit ADPCM. For the 9.6-kb/s coder, comparisons were made with adaptive delta modulation (ADM) (i.e., 1-bit ADPCM). Results for the 16-kb/s coder comparisons are given in Table III.

Twelve listeners were asked to compare pairs of sentences for signal quality and indicate which was better. Two speakers were used in the experiments and sentence pairs were played in a randomly selected order. Each listener made a total of 16 comparisons in each of the experiments.

In comparing 16-kb/s sub-band encoding to 16-kb/s (2 bits/sample) ADPCM, listeners rated the sub-band encoded sentence as having higher quality in 94 percent of the sentence pairs. When the bit rate of the ADPCM coder was increased to 24 kb/s (3 bits/sample), they rated the sub-band encoded sentence as having higher quality in 34 percent of the sentence pairs. Experiment I demonstrates that the quality of the 16-kb/s sub-band coder is clearly preferred over that of ADPCM at the same bit rate. In Experiment II listeners exhibited much greater indecision, indicating that the quality of the 16-kb/s sub-band coder is close to that of 24-kb/s ADPCM, but that preference leans slightly in favor of the ADPCM.

Also included in Table III are signal-to-quantizing-noise ratios (s/n) measured on the speech signals, averaged for the two speakers for each of the coding methods. s/n data is not found to be a reliable indicator of listener preference. This observation is not surprising and has been previously recognized in the speech coding literature.[7,8]

A second series of listening experiments compared 9.6-kb/s sub-band coding with ADM. The sub-band encoder in this case is implemented with the integer-band method described earlier. The ADM coder is a

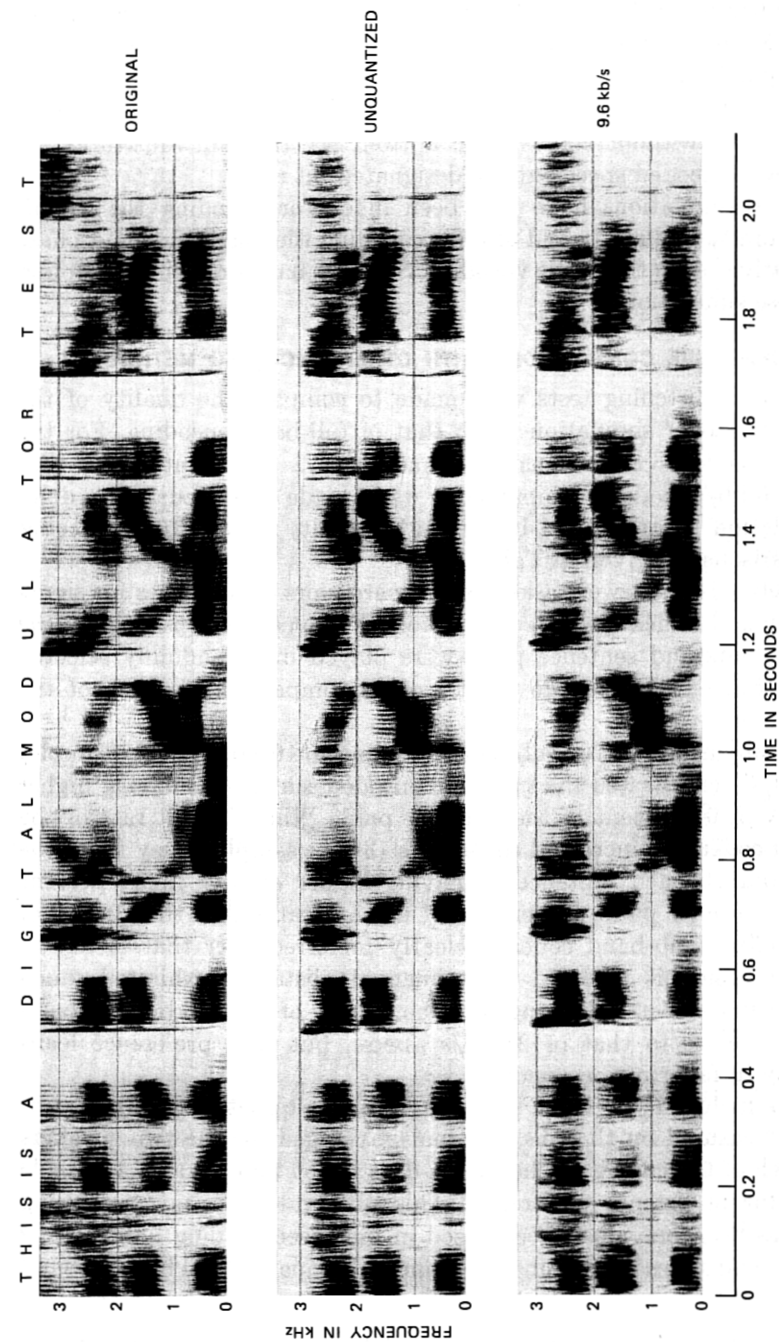

Fig. 8—Sound spectrograms comparing original, unquantized speech output and the 16-kb/s output from the sub-band coder.

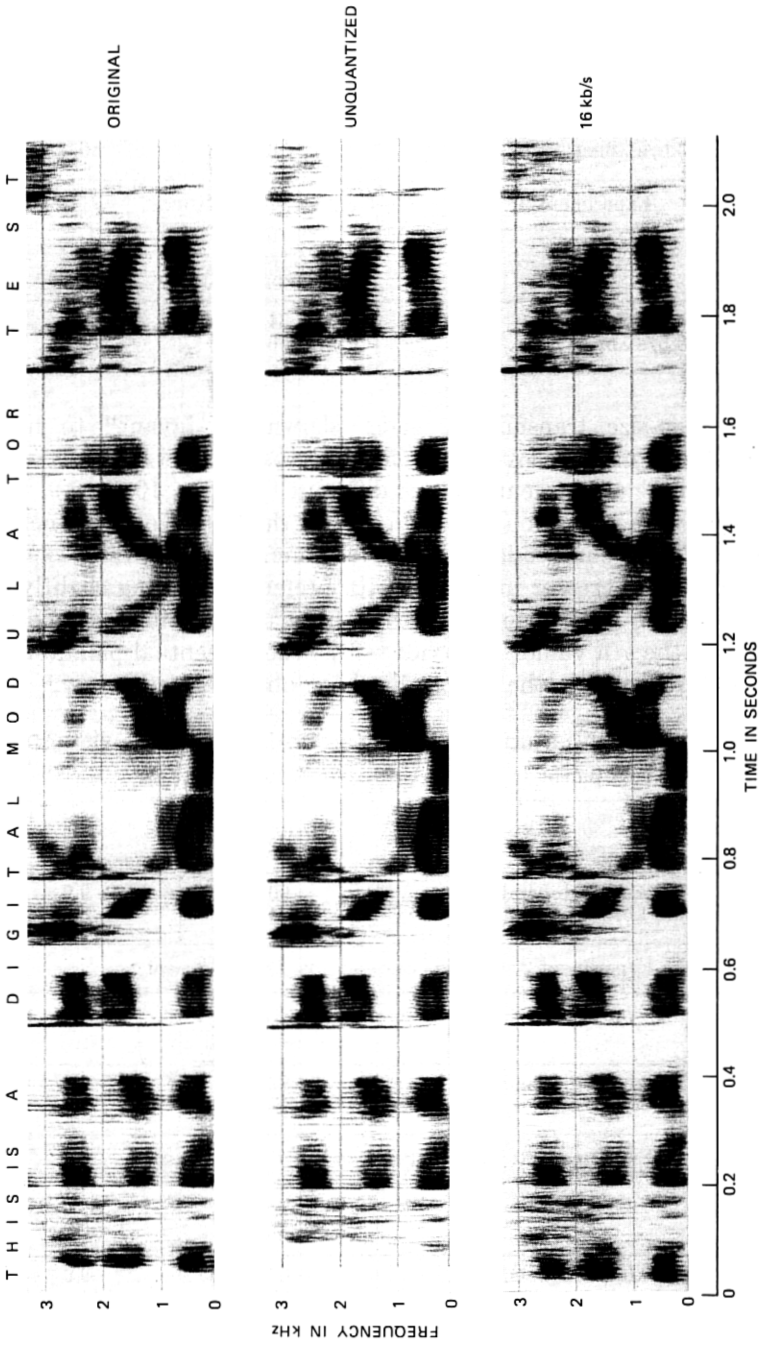

Fig. 9—Sound spectrograms comparing original, unquantized speech output and the 9.6-kb/s output from the sub-band coder.

Table III — Comparison of 16-kb/s sub-band coder with ADPCM

Experiment 1: 16-kb/s Sub-band vs 16-kb/s ADPCM

	Listener Preference (%)	S/N (dB)
16-kb/s Sub-band	94	11.1
16-kb/s ADPCM (2 Bits)	6	10.9

Experiment 2: 16-kb/s Sub-band vs 24-kb/s ADPCM

	Listener Preference (%)	S/N (dB)
16-kb/s Sub-band	34	11.1
24-kb/s ADPCM	66	14.5

forward step-size transmitting coder shown by Jayant[11] to have improved performance over conventional ADM. Table IV shows the results of these experiments. Three different bit rates, 10.3, 12.9, and 17.2 kb/s, were used for the ADM coder. In the first two experiments, the 9.6-kb/s sub-band coder was clearly preferred. In the third experiment, there was greater indecision with preference leaning slightly in favor of the sub-band coder. Note that this is true despite the opposite ordering of the s/n values! In other words, the perceptual palatability is not well reflected in the s/ns as has been observed previously.[8]

Table IV — Comparison of 9.6-kb/s sub-band coder with ADM

Experiment 1: 9.6-kb/s Sub-band vs 10.2-kb/s ADM

	Listener Preference (%)	S/N (dB)
9.6-kb/s Sub-band	96	9.9
10.3-kb/s ADM	4	8.2

Experiment 2: 9.6-kb/s Sub-band vs 12.9-kb/s ADM

	Listener Preference (%)	S/N (dB)
9.6-kb/s Sub-band	82	9.9
12.9-kb/s ADM	18	9.7

Experiment 3: 9.6-kb/s Sub-band vs 17.2-kb/s ADM

	Listener Preference (%)	S/N (dB)
9.6-kb/s Sub-band	61	9.9
17.2-kb/s ADM	39	11

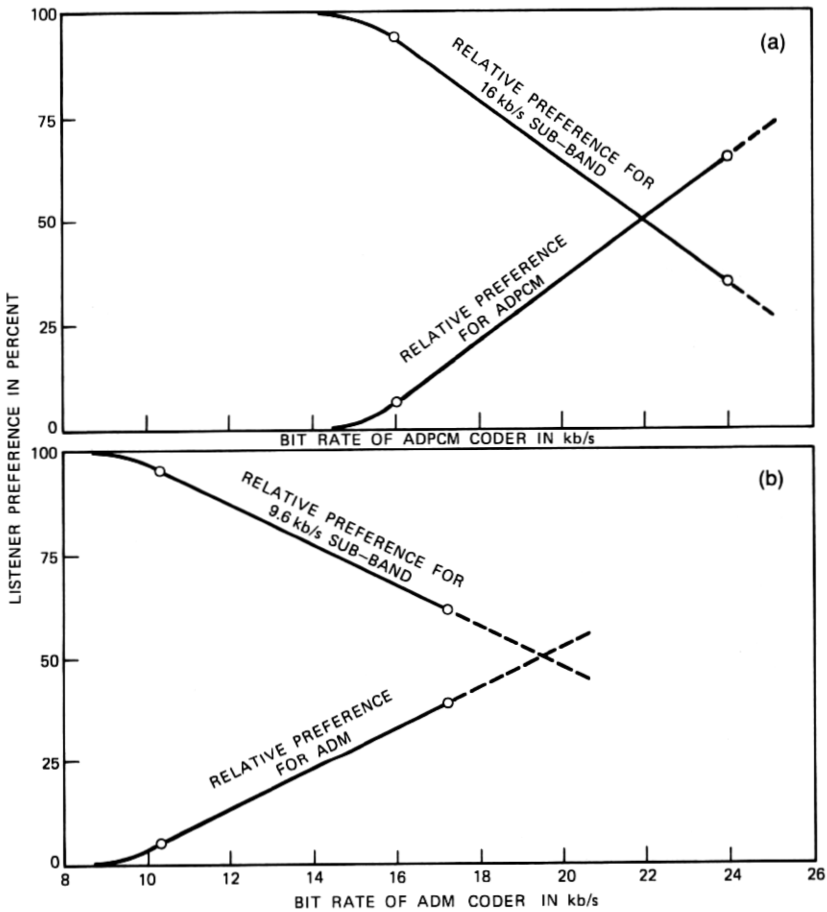

Fig. 10—(a) Relative comparison of quality of 16-kb/s sub-band coding against ADPCM coding (based on listener preference) for different ADPCM coder bit rates. (b) Relative comparison of quality of 9.6-kb/s sub-band coding against ADM coding for different ADM coder bit rates.

Figure 10 summarizes the results of the listener preference tests in Tables III and IV. Listener preference is plotted against the ADPCM and ADM coder bit rates. The crossover points of the curves in the two comparisons determine the point at which the two types of coders have approximately equal subjective quality. In the first comparison, the quality of the 16-kb/s sub-band coder is seen to be comparable to that of 22-kb/s ADPCM; i.e., it has a 6-kb/s advantage over the ADPCM coder. In the second comparison, the 9.6-kb/s coder has a subjective quality that is comparable to the 19-kb/s ADM and, therefore, has a 9.4-kb/s advantage over ADM.

It is clear from the listener preference tests that the sub-band coding technique is considerably better in quality than full-band ADPCM or ADM coding methods. We have carried this coding down to 7.2 kb/s and find that the quality is only slightly poorer than that at 9.6 kb/s. We have also pressed the coding rate down to 4.8 kb/s and find that the quality becomes considerably poorer owing to the increased band limiting and gaps between bands.

VIII. CONCLUSION

We have described a method for digitally coding speech in sub-bands of the total signal spectrum. Partitioning into sub-bands has several distinct advantages. Bit allocations for quantization of each band can be made on a perceptually palatable basis. Quantization products in a given band are confined to that band and do not "spill over" into adjacent frequency ranges. Selection of sub-band widths can also be made according to perceptual criteria, namely, for equal contributions to AI (and hence to signal intelligibility). As a result, the sub-band coding produces a quality signal that is better than a single full-band coding at the same total bit rate. The price paid is the band-filtering and the individual coding.

"Integer-band" sampling is demonstrated to be an economical and effective method for implementing the sub-band coder. Emerging technologies in device fabrication (such as CCDs) suggest economical implementations of the band filtering in terms of analog transversal filters.

The sub-band coder, implemented by integer band sampling, is demonstrated for speech transmission at rates of 16, 9.6, and 7.2 kb/s. The latter two transmission rates push down into the data range and are attractive for "voice-coordination" over data channels.

Informal perceptual experiments demonstrate that the signal quality of speech coded at 9.6 kb/s by the sub-band method is approximately equivalent to a 19-kb/s coding of the full-band signal. For a given transmission bit rate, therefore, the sub-band technique provides a significant improvement in signal quality. Or alternatively, for a given signal quality, the sub-band system can provide the transmission at a significantly reduced bit rate.

REFERENCES

1. E. R. Kretzmer, "Reduced-Alphabet Representation of Television Signals," IRE Convention Record, *4* (1956), pp. 140–147.
2. E. E. David and H. S. McDonald, "A Bit-Squeezing Technique Applied to Speech Signals," IRE Convention Record, *4* (1956), pp. 148–153.
3. L. L. Beranek, "The Design of Communications Systems," Proc. IRE, *35* (September 1947), pp. 880–890.

4. K. D. Kryter, "Methods for the Calculation and Use of the Articulation Index," J. Acoust. Soc. Amer., *34* (1962), pp. 1689–1697.
5. J. L. Flanagan and R. M. Golden, "Phase Vocoder," B.S.T.J., *45* (November 1966), pp. 1493–1509.
6. R. E. Crochiere and L. R. Rabiner, "Optimum FIR Digital Filter Implementations for Decimation, Interpolation, and Narrowband Filtering," IEEE Trans. Acoust., Speech, and Signal Proc., *ASSP-23*, No. 5, October 1975, pp. 444–456.
7. N. S. Jayant, "Digital Coding of Speech Waveforms: PCM, DPCM, and DM Quantizers," Proc. IEEE, *62* (May 1974), pp. 611–632.
8. P. Cummiskey, N. S. Jayant, and J. L. Flanagan, "Adaptive Quantization in Differential PCM Coding of Speech," B.S.T.J., *52* (September 1973), pp. 1105–1118.
9. N. S. Jayant, "Adaptive Quantization With a One-Word Memory," B.S.T.J., *52* (September 1973), pp. 1119–1144.
10. C. B. Feldman and W. R. Bennett, "Band Width and Transmission Performance," B.S.T.J., *28* (July 1949), pp. 490–595.
11. N. S. Jayant, "Step-Size Transmitting Differential Coders for Mobile Telephony," B.S.T.J., *54* (November 1975), pp. 1557–1581.