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Derivative Measurements of Light-Current-Voltage Characteristics of (Al,Ga)As Double-Heterostructure Lasers

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A modulation and detection scheme for applying derivative techniques to the investigation of stripe geometry (Al, Ga)As double-heterostructure lasers is described. Modulating at constant modulation index allows the quantities $I(dV/dI)$ and $I^2(d^2V/dI^2)$ to be directly obtained in the same apparatus at the first and second harmonic of the modulation frequency, respectively. Particularly strong indications of laser action and other optical interactions in the laser are contained in the second harmonic voltage response. The same apparatus may be used to obtain derivatives of the light-current relation. These are found to sensitively reveal light-current nonlinearities that are believed caused by filaments and other spatial inhomogeneities and instabilities.

I. INTRODUCTION

Derivative techniques are employed in many fields of science and engineering. Reasons for their use range from the enhancement of signal-to-noise ratio, as in nuclear magnetic resonance and optical reflectivity investigations of band structure, to the convenience of direct measurement of dynamic transistor characteristics. Usually the techniques measure system response to a small ac modulation, often using phase detection for signal enhancement.

These techniques have recently been successfully applied to an analysis of the current-voltage characteristics of GaAs injection lasers. The first derivative dV/dI has been shown to be useful, both in homojunction[1] and in double-heterostructure lasers,[2] for measuring the parameters entering the I–V characteristic as well as for extracting lasing threshold and other features intimately related to the lasing process.

This paper describes a modulation and detection technique which has the advantage of allowing the direct measurement of $I(dV/dI)$, which is often the desired quantity (rather than dV/dI). The technique also allows direct measurement of $I^2(d^2V/dI^2)$. Very strong second-derivative signals near and above lasing threshold are observed. Additionally, derivatives of the light-current characteristic are displayed and, as anticipated, show much sharper detail, near lasing threshold, and when filamentary instabilities occur, than does the light-current characteristic itself. All of these properties can be measured in the same apparatus.

II. SIMPLE THEORY

Much of the value of the experimental technique results from the modulation of the laser current at constant modulation index, $m = \Delta I/I =$ constant, rather than alternatives such as constant modulation amplitude, ΔI. To appreciate the reasons for this advantage consider a laser modeled as an ideal *p–n* junction in series with a resistance of value R_s.[1,2] The current-voltage characteristic of this idealized device would be

$$I = I_0[e^{\beta(V-IR_s)} - 1], \tag{1}$$

where conventionally $\beta \equiv q/nkT$, and the other parameters have their usual meanings. Solving eq. (1) for V, neglecting in this context the -1 term, and the current dependences of the parameters of eq. (1), and differentiating yields[1,2]

$$\frac{dV}{dI} = \frac{1}{I\beta} + R_s \tag{2}$$

and

$$\frac{d^2V}{dI^2} = -\frac{1}{\beta I^2}. \tag{3}$$

For many purposes,[2] the preferred forms of eqs. (2) and (3) are

$$I\frac{dV}{dI} = \frac{1}{\beta} + IR_s \tag{2a}$$

and

$$I^2\frac{d^2V}{dI^2} = -\frac{1}{\beta}, \tag{3a}$$

thus allowing the direct extraction of the parameters β and R_s without having to deal with the inverse current dependences. (See Appendix.) Also, the forms (2a) and (3a) lend themselves to the investigation of the small I-V changes which occur because of the optical-electrical interactions in the laser. It is worth emphasizing that direct measurements of the quantities $I(dV/dI)$ and $I^2(d^2V/dI^2)$ are desired as opposed to the synthesis of these quantities from other measured parameters. This improves both the measurement accuracy and the experimental simplicity.

To see how this is possible consider that a current of the form, $I_T = I + \Delta I \cos \Omega t$ is applied to the diode, the response voltage is expanded in a Taylor series, and coefficients in Ω and 2Ω collected. The result is

$$V(\Omega) = mI \frac{dV}{dI} \tag{4}$$

and

$$V(2\Omega) = \left(\frac{m}{2}\right)^2 I^2 \frac{d^2V}{dI^2}, \tag{5}$$

where $m \equiv (\Delta I/I)$ is the modulation index, and only lowest-order terms have been retained.* Thus, if the laser can be modulated keeping the modulation index constant, the desired quantities may be obtained directly at the frequencies Ω and 2Ω.

III. EXPERIMENTAL

3.1 Detection at the modulation frequency (Ω)

The experimental apparatus used is shown in Fig. 1. The key component is the current source. It has, as an offshoot of the feedback circuitry that maintains a constant current for varying load impedance, the property that it can be easily modified so that modulation *at constant m* occurs when an appropriate low-frequency (<1 kHz) ac signal is applied. When the phase detector is tuned to the modulation frequency Ω [mode (a) in Fig. 1], laser response voltages like that labeled $I(dV/dI)$ in Fig. 2 are obtained. This curve has features analogous to those seen previously.[1,2] These include y-axis intercept [equal to $1/\beta$ on the simple theory, eq. (2a)], the slope (proportional to series resistance R_s on the simple theory), and an indication of voltage saturation near lasing threshold. System response was checked both with resistors and with a silicon diode replacing the laser, and the known values of R and β were reproduced very well. The signal $V(\Omega)$ was also confirmed to be linearly proportional to m as expected from eq. (4). Many different lasers were measured and their results seemed generally similar although major deviations from slope linearity below threshold were sometimes observed, as were significant differences among the forms of the $I(dV/dI)$ responses near and above lasing threshold. Voltage saturation was almost never complete, and

* Higher-order corrections depend on higher powers of m and are not significant for m small.

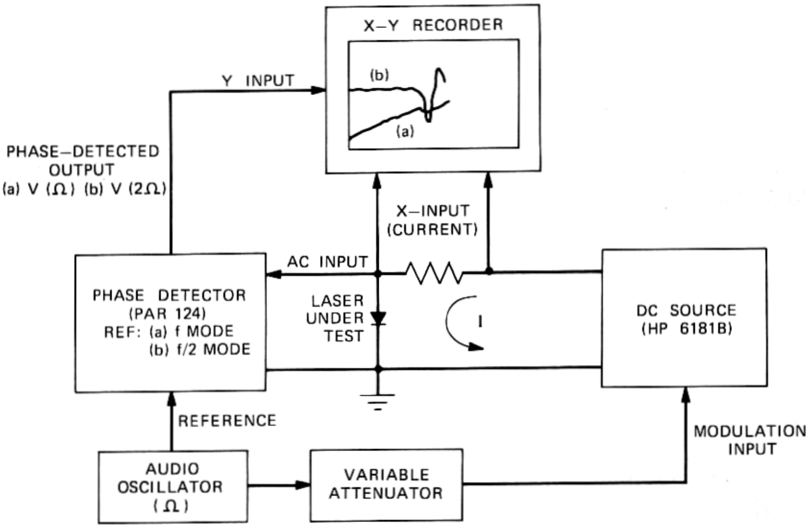

Fig. 1—Schematic diagram of apparatus used to obtain voltage and light derivatives in (Al, Ga)As double-heterostructure lasers. Mode (a) detects $V(\Omega) \propto I(dV/dI)$ and mode (b) detects $V(2\Omega) \propto I^2(d^2V/dI^2)$.

filament changes resulted in complex $I(dV/dI)$ responses. The experimental technique, however, seems accurate, easily calibrated, and simple to use.

3.2 Detection at twice the modulation frequency (2Ω)

An important advantage of this technique is that it can also be used to measure the voltage response at frequency 2Ω and thus to obtain the second derivative $I^2(d^2V/dI^2)$. Detection at frequency 2Ω was accomplished by operating the phase detector in the $f/2$ mode [mode (b) in Fig. 1]. In this mode, the instrument produces an internal reference frequency at exactly twice the frequency of the externally applied reference, thus making the detection system sensitive to $V(2\Omega)$.

Trace (b) of Fig. 2 shows a typical signal detected with the system operated in the 2Ω mode. As anticipated, the signal-to-noise ratio is very large. The initial shape (with current increasing) seems quite reproducible from laser to laser while the structure at higher currents is laser-dependent. Again, the signal appears greatly affected by filamentary laser operation. When the laser was shorted or replaced by a resistance, a horizontal straight line resulted, as expected. As an additional check, the dependence of $V(2\Omega)$ on modulation index was measured below threshold over the range $0 \leqq m \leqq 0.10$ and confirmed to be accurately m^2, consistent with theory [eq. (5)]. Thus, there is

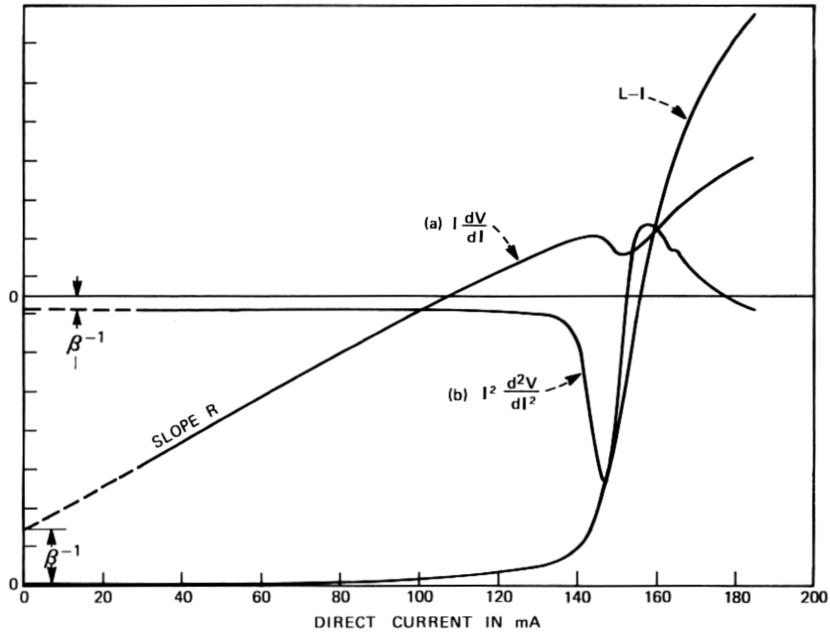

Fig. 2—Typical $V(\Omega)$ and $V(2\Omega)$ responses [curves (a) and (b)] as functions of laser current measured using apparatus of Fig. 1. The zero current intercept of $I(dV/dI)$ and the initial negative offset of $I^2(d^2V/dI^2)$ are proportional to $\beta^{-1} \equiv (nkT/q)$ when the simple model described in the text is valid. Note voltage saturation in $I(dV/dI)$ and the very strong second-derivative signal near threshold.

little indication of feedthrough from the first harmonic or of harmonic distortion of the modulating frequency affecting the detected voltage at the second harmonic. If very accurate measurements were contemplated, this would merit more serious investigation (see, for example, Korb and Holonyak[3*]).

3.3 Light-current derivatives

Using the circuit depicted in Fig. 1, the derivatives $I(dL/dI)$ and $I^2(d^2L/dI^2)$ can also be directly obtained. This was accomplished by placing the laser in a holder in which the optical emission from each mirror face could be separately monitored with silicon photodiodes. The photodiode current then served as the input to the phase detector. This more traditional application of derivative techniques may prove to be a useful adjunct to $L(I)$ measurements, not only to provide

* Also, T. L. Paoli has considered a more complex technique wherein modulation is provided at two frequencies Ω_1 and Ω_2 and the system response is sought at frequency $|\Omega_1 - \Omega_2|$, thus eliminating the harmonic relationship with the interfering signals.

accurate measurement of parameters, such as differential quantum efficiency [$\propto (dL/dI)$], but also to investigate filaments and other spatial inhomogeneities in the laser cavity. Typical derivative signal responses are shown in Fig. 3 along with a light-output-current characteristic measured in the same apparatus. A device with significant curvature in the *L-I* was used as an example to illustrate the enhancement of such slope changes using the derivatives. As expected from earlier results,[4] the emission from the two laser mirrors was often unsymmetric and was usually very unsymmetric when *L-I* kinks[5] were present. These kinks are thought[4] to be associated with spatial inhomogeneities in the laser cavity. This nonsymmetry was also present in the light-current derivatives. Very sharp derivative structure was seen near the light-current kinks. These observations strongly suggest that light-current derivatives, either alone or in combination with voltage derivatives, will prove very useful for investigating light-current nonlinearities, spatial inhomogeneities, and their relationship in injection lasers.

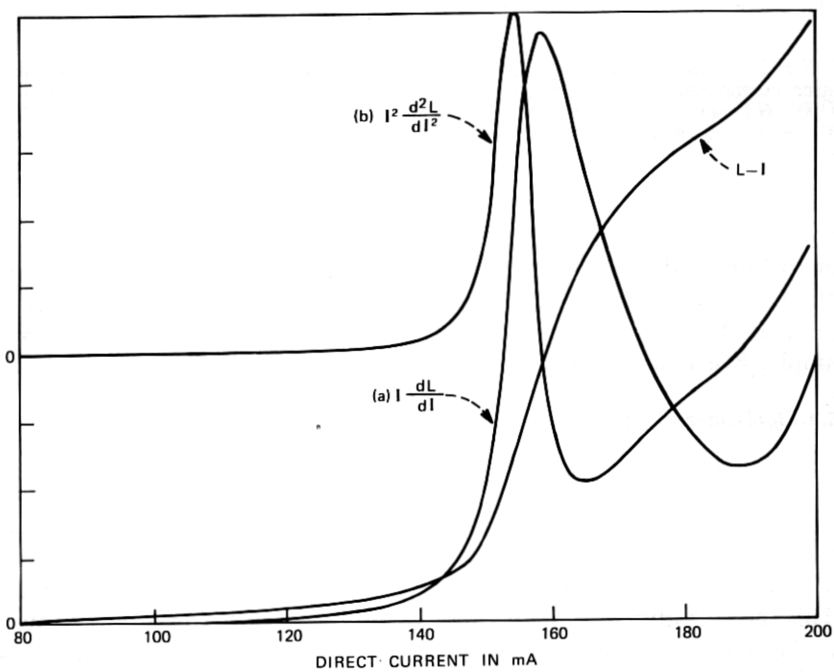

Fig. 3—Typical results for $I(dL/dI)$ and $I^2(d^2L/dI^2)$ vs current in an (Al, Ga)As double-heterostructure laser taken with the apparatus shown in Fig. 1, except that the signal input to the phase detector was obtained from a silicon detector monitoring the light emission from the laser.

IV. SUMMARY AND CONCLUSION

An experimental technique has been described which allows modulation of an (Al, Ga)As double-heterostructure laser at constant modulation index. Derivative techniques were then discussed which illustrate the usefulness of this property in measuring the derivatives $I(dV/dI)$ and $I^2(d^2V/dI^2)$ to extract the parameters describing a laser's I-V relation and to investigate lasing-related voltage changes in this relation. Derivatives of light with respect to current were also obtained in the same apparatus. These results support the conclusion that derivative techniques will be very helpful in studying (Al, Ga)As lasers and similar devices.

V. ACKNOWLEDGMENTS

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APPENDIX

The neglect of the -1 term is not thought to be significant. However, the assumptions of β, R, and i_0 independent of current may sometimes be unjustified. If β depends on current through the parameter n, the first-order correction to the derived value of n is

$$\Delta n(I) = \frac{q}{kT}(V - IR_s)\frac{I}{n}\frac{dn}{dI}.$$

This correction can be large for quite reasonable I-V characteristics. Equivalently we find instead of eq. (2a)

$$I\frac{dV}{dI} = \frac{1}{\beta} + V\frac{I}{n}\frac{dn}{dI} + IR_s\left(1 - \frac{I}{n}\frac{dn}{dI}\right). \tag{2b}$$

Thus, both of the derived parameters β and R_s are sensitive to the assumption of β independent of current.

Analogous comments apply in the second derivative case where (3a) is replaced by

$$I^2\frac{d^2V}{dI^2} = -\frac{1}{\beta}\left(1 - 2\frac{I}{n}\frac{dn}{dI}\right) + (V - IR_s)\frac{I^2}{n}\frac{d^2n}{dI^2}. \tag{3b}$$

Expressions applicable when $R = R(I)$ or $I_0 = I_0(I)$ are easily derived.

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