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Microbending Losses of Single-Mode, Step-Index and Multimode, Parabolic-Index Fibers

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We present formulas for the microbending losses of fibers that are caused by random deflections of the fiber axis. We consider single-mode (or almost single-mode), step-index fibers and multimode, parabolic-index fibers and compare their losses. Loss formulas for the single-mode fiber are derived from coupled-mode theory using radiation modes. Simple empirical approximations of the general formulas are also presented. The losses of the parabolic-index, multimode fiber have been derived earlier. The losses of both fiber types are compared by assuming either that each fiber samples the spatial Fourier spectrum of the distortion function at the same spatial frequency, or by comparing typical fibers of each type with each other regardless of any similarity between them. It is found that the multimode, parabolic-index fiber has lower losses if it supports a sufficient number of guided modes.

I. INTRODUCTION

Two types of optical fibers appear to be promising for wideband communications, the monomode step-index fiber and a multimode fiber with parabolic (or nearly parabolic) refractive index distribution.[1] A third fiber type, the W-fiber,[2] is so similar to the step-index fiber that its properties are easily included in the discussion of single-mode fibers. However, single and multimode fibers suffer radiation losses caused by unintentional random bends of the fiber axis.[3,4] Thus, it is of interest to compare the two types of fibers and investigate which of them is better from the point of view of microbending losses. This loss comparison is the objective of this paper.

Radiation losses of optical fibers are caused by coupling between the guided modes and radiation modes.[5] Two modes with propagation constants β_1 and β_2 are coupled via a spatial frequency θ of the coupling function when[4]

$$|\beta_1 - \beta_2| = \theta. \tag{1}$$

A guided mode with propagation constant β_g couples directly to the radiation modes using the entire spatial frequency spectrum in the range

$$\beta_g - n_2 k < \theta < \beta_g + n_2 k \qquad (2)$$

($n_2 k$ is the plane-wave propagation constant of the cladding). However, most practical, randomly bent fibers have spatial Fourier spectra that drop off very rapidly with increasing spatial frequency so that only a narrow region of the spatial frequency spectrum near

$$\theta = \beta_g - n_2 k = \Omega \qquad (3)$$

is actually responsible for radiation losses in single-mode fibers.

In parabolic-index, multimode fibers, the random curvature of the fiber axis causes coupling among the guided modes. Adjacent guided modes in parabolic-index fibers have almost equal spacings in β space, so that only a very narrow band of spatial frequencies is instrumental in coupling the guided modes among each other.[6] Since most practical Fourier spectra drop off rapidly with increasing spatial frequency, we may assume that only the guided modes immediately adjacent (in β space) to radiation modes couple directly to radiation. Radiation losses in multimode fibers are thus less direct than in monomode fibers. The energy is exchanged among guided modes and is lost only when it reaches the guided modes of highest order. We treat this problem mathematically by assuming that the highest-order modes always carry zero power, because they lose their power directly to radiation.[6] In addition, we assume throughout this discussion that the modes are coupled so tightly that the steady-state power distribution has been established.[5] This assumption allows us to use a definite radiation loss of the multimode fiber. If steady state is not reached, the fiber loss cannot be characterized by a single number independently of the input conditions.

Our discussion has made it clear that essentially one single spatial frequency is responsible for radiation losses in fibers. In multimode, parabolic-index fibers, it is the spatial frequency that couples the guided modes among each other; in single-mode fibers, it is the spatial frequency Ω of (3). However, even though we can specify a definite spatial frequency for the single- as well as the multimode fiber, we still face the possibility that these frequencies may be different in either case. This raises the problem: what criteria are to be used by which a single-mode and a multimode fiber may be compared? In this paper, we shall use two different criteria. First, we asume that the spatial frequencies of interest are identical for both types of fibers. This requires us to assume that the core radius as well as the maximum refractive index difference between core center and cladding is different for each fiber type. When compared on this basis, the parabolic-index, multimode fiber is superior in its loss performance to the single-mode fiber, provided that a sufficiently large number of guided modes can exist. This result is intuitively apparent, since we have seen that the power has to "dribble" down through all the guided modes before it reaches the region of radiation modes and becomes lost.

A second way of comparing single-mode and multimode fibers consists in considering configurations that are typical for each type of fiber regardless of the spatial coupling frequencies and determining quantitatively what their radiation losses are. It is clear that we must assume that the form of the spatial frequency spectrum is identical for each fiber type, otherwise no comparison would be possible. Compared on this basis the typical multimode, parabolic-index fiber has lower loss than the typical single-mode fiber.

A step-index fiber operates in a single guided mode if its characteristic V-number has values $V < 2.405$. However, some fibers are made with a relatively thin inner cladding that is surrounded by an outer cladding whose refractive index is similar to that of the fiber core. An example of such a structure is the W-fiber.[2] Such fibers do not support guided modes in the strict mathematical sense; each mode loses power by leakage through the thin inner cladding and may be regarded as a leaky mode. The loss of the lowest-order leaky mode may be kept so small that it is negligible for practical purposes. The loss of the mode of next higher order may be considerably larger, causing its power to be lost in a relatively short distance. Such a fiber behaves as a single-mode fiber for V values exceeding $V = 2.405$. The radiation losses caused by random bends can be calculated by computing the power-coupling coefficient of the lowest-order mode to the next higher mode and by assuming that the higher-order mode is so lossy that it does not carry any power. We include such quasi-single-mode fibers in our discussion. Since it is impossible to anticipate the actual shape of the power spectra of the coupling function, we use simple power laws as models. Because only limited spatial frequency ranges need to be compared in any case, it should always be possible to approximate an actual power spectrum by a suitable power law.

II. LOSS FORMULA FOR THE SINGLE-MODE FIBER

Random bends of an optical fiber can be described by assuming that the core-cladding boundary is changing as a function of the length coordinate z. Thus, we describe the core boundary of a fiber with random bends by the equation[5]

$$r(\phi, z) = a + f(z) \cos \phi. \tag{4}$$

In this equation, a is the core radius and $f(z)$ describes the shape of the randomly deformed fiber axis; ϕ is the angular coordinate of a cylindrical coordinate system.

Using the standard coupled-mode formalism,[5] we derive the following formula for the radiation loss coefficient of any LP mode (for definition of LP, see Ref. 7) of the step-index fiber with randomly deformed axis,

$$2\alpha_\nu = \frac{(n_1^2 - n_2^2)\gamma^2 J_\nu^2(\kappa a)}{2(n_2\pi a)^2 |J_{\nu-1}(\kappa a) J_{\nu+1}(\kappa a)|} \cdot \int_{-n_2k}^{n_2k} |\beta| \langle F^2(\beta_\nu - \beta)\rangle \left\{ \frac{J_{\nu-1}^2(\sigma a)}{|\sigma J_\nu(\sigma a) H_{\nu-1}^{(1)}(\rho a) - \rho J_{\nu-1}(\sigma a) H_\nu^{(1)}(\rho a)|^2} + \frac{J_{\nu+1}^2(\sigma a)}{|\sigma J_\nu(\sigma a) H_{\nu+1}^{(1)}(\rho a) - \rho J_{\nu+1}(\sigma a) H_\nu^{(1)}(\rho a)|^2} \right\} d\beta. \quad (5)$$

The Fourier spectrum of the distortion function $f(z)$ appearing in (4) is defined as

$$F(\theta) = \lim_{L\to\infty} \frac{1}{\sqrt{L}} \int_0^L f(z) e^{-i\theta z} dz. \quad (6)$$

The ensemble average of the square of $F(\theta)$ can be expressed as the Fourier transform of the correlation function $R(u)$ of $f(z)$,[8]

$$\langle F^2(\theta)\rangle = \int_{-\infty}^{\infty} R(u) e^{-i\theta u} du. \quad (7)$$

The other symbols appearing in (5) are defined as follows:

2α = power attenuation coefficient
a = core radius
n_1 = refractive index of fiber core
n_2 = refractive index of cladding
ν = azimuthal mode number of LP mode ($\nu = 0$ for HE_{11} mode)
β_ν = propagation constant of guided LP mode
$k = \omega(\epsilon_0\mu_0)^{\frac{1}{2}} = 2\pi/\lambda$ free-space propagation constant
$V = (n_1^2 - n_2^2)^{\frac{1}{2}} ka$
$\kappa = (n_1^2 k^2 - \beta_\nu^2)^{\frac{1}{2}}$
$\gamma a = [V^2 - (\kappa a)^2]^{\frac{1}{2}}$
$\sigma = (n_1^2 k^2 - \beta^2)^{\frac{1}{2}}$
$\rho = (n_2^2 k^2 - \beta^2)^{\frac{1}{2}}$
$J_\nu(x)$ = Bessel function
$H_\nu^{(1)}(x)$ = Hankel function of the first kind.

Equation (5) was derived by using the approximate radiation modes of the fiber. This formula is, thus, accurate only if the radiation escapes at small angles relative to the fiber axis.[5] This condition is satisfied for power spectra that drop off rapidly with increasing spatial frequency, an assumption that is made throughout this paper.

It was mentioned in the introduction that certain step-index fibers have claddings that are sufficiently narrow to provide high-leakage losses to all but the lowest-order mode. We obtain the microbending radiation loss for such fibers by assuming that two modes can exist in the fiber, but that only the lowest-order mode carries power. This mode is coupled to the next-higher-order leaky mode as well as to the radiation modes. However, since we assume that the amplitude of the spatial power spectrum of the coupling function decreases rapidly with increasing spatial frequencies, it is sufficient to consider only coupling to the leaky guided mode and ignore the radiation mode spectrum. The power-coupling coefficient between guided modes 0 and 1 is designated as h_{01}. The power loss coefficient for the HE_{11}-mode in the presence of a second but leaky mode may thus be expressed as*

$$2\alpha = h_{01} = \frac{\gamma_0^2\gamma_1^2 J_0^2(\kappa_0 a) J_1^2(\kappa_1 a)[k^3\langle F^2(\bar{\Omega})\rangle]}{2a^2 n_2^2 k^5 J_1^2(\kappa_0 a)\,|J_0(\kappa_1 a) J_2(\kappa_1 a)|}. \tag{8}$$

The subscript 0 refers to the HE_{11} mode while the subscript 1 indicates the next higher LP mode.[7] The spatial frequency $\bar{\Omega}$ is defined as

$$\bar{\Omega} = \beta_0 - \beta_1 \approx \frac{\kappa_1^2 - \kappa_0^2}{2n_2 k}. \tag{9}$$

Equation (8) is applicable in the range

$$2.405 < V < 3.832. \tag{10}$$

A discussion of the validity of using radiation modes to calculate the microbending losses is given in the Appendix.

III. APPROXIMATE LOSS FORMULAS

It is possible to approximate formulas (5) and (8) by expressions that show the relative influence of the fiber parameters and are simple enough to allow evaluation by pocket calculators.

In the remainder of this paper, we restrict ourselves to power spectra of the form

$$\langle F^2(\theta)\rangle = \frac{m\bar{\sigma}^2 \sin\dfrac{\pi}{m}}{\Delta\theta\left[1 + \left(\dfrac{\theta}{\Delta\theta}\right)^m\right]}. \tag{11}$$

* See eqs. (3.6–10) and (5.2–20) in Ref. 5.

This simple power law may serve as a model for every practical power spectrum, because we are usually interested only in a narrow spatial frequency region of the spectrum, which can always be approximated by a function of the form (11). The variance $\bar{\sigma}^2$ is defined by (28); m is the exponent of the power law, and $\Delta\theta$ is the width parameter of the distribution (11).

For large values of the spatial frequency, we can express (11) in the simpler form,

$$\langle F^2(\theta)\rangle = \frac{A_m}{\theta^m}, \tag{12}$$

with

$$A_m = m(\Delta\theta)^{m-1}\bar{\sigma}^2 \sin\frac{\pi}{m}\cdot \tag{13}$$

If we use (12) and the transformation

$$\theta = \Omega x, \tag{14}$$

we can express (5) for $\nu = 0$ (HE_{11}-mode) in the form

$$2\alpha_0 = \frac{(n_1^2 - n_2^2)k\gamma^2\Omega J_0^2(\kappa a)}{\pi^2 n_2 a^2 J_1^2(\kappa a)} \langle F^2(\Omega)\rangle \cdot \int_1^\infty \frac{1}{x^m} \frac{J_1^2(\sigma a)}{|\sigma J_0(\sigma a)H_1^{(1)}(\rho a) - \rho J_1(\sigma a)H_0^{(1)}(\rho a)|^2} dx. \tag{15}$$

This expression is not much simpler than (5), but it shows clearly that the radiation loss depends indeed only on the spatial frequency $\theta = \Omega$ defined by (3). The upper limit ∞ on the integral in (15) is an approximation that is valid for sufficiently large values of m. Equation (15) is useful only in the narrow interval

$$1 < V < 2.405. \tag{16}$$

For smaller V values, the HE_{11} mode is so loosely guided that its field reaches very far into the cladding and suffers excessive bending losses. For $3.832 > V > 2.405$, the step-index fiber is no longer supporting only one mode and (8) must be used. We have plotted (15) for a number of m values in the range (16) and used this plot to obtain the following empirical approximation of (15):

$$2\alpha_0 = \frac{n_2 k^4\langle F^2(\Omega)\rangle}{(m-1)(m-2)}\left(\frac{\Omega}{k}\right)^3\left(V + 7.58 \times 10^{-5}\frac{m^9 V^m}{e^{1.7m}}\right)\cdot \tag{17}$$

This formula holds only for $m \geqq 4$. The accuracy of (17) will be discussed in connection with a discussion of Fig. 3.

A good approximation [less than 0.5 percent error in the range (16)] of the solution of the eigenvalue equation of the HE_{11} mode can be expressed empirically as follows:

$$\kappa_0 a = 2.405 \exp\left(-\frac{0.8985}{V}\right), \tag{18}$$

and (3) can be approximated by

$$\frac{\Omega}{k} = (n_1 - n_2)\left[1 - \left(\frac{\kappa a}{V}\right)^2\right]. \tag{19}$$

Equations (18) and (19) enable us to calculate loss values from (17) without having to solve the eigenvalue equation for the HE_{11} mode.

A similar empirical procedure allows us to approximate (8) as follows:

$$2\alpha_0 = \frac{\langle F^2(\bar{\Omega})\rangle}{2n_2^2 a^6 k^2}\left[9(V - 1.87) - \frac{0.616}{(V - 2.405)^{\frac{1}{4}}}\right]. \tag{20}$$

The spatial frequency $\bar{\Omega}$ is defined by (9) with $\kappa_1 a$ approximated by

$$\kappa_1 a = 3.832 \exp\left(-\frac{0.8492}{V - 0.55}\right). \tag{21}$$

In the range (10), this formula deviates from the exact solution of the eigenvalue equation by less than 0.5 percent (only very close to the endpoint at $V = 2.41$ is the error 0.8 percent).

IV. MULTIMODE LOSSES

The radiation loss formula for multimode parabolic-index fibers, whose mode distribution has reached steady state, has been derived earlier,[6]

$$2\alpha_p = 5.8\,\frac{\langle F^2(\theta_p)\rangle}{a^4}\,\Delta. \tag{22}$$

The refractive-index distribution inside the fiber core is given by the expression

$$n(r) = n_1\left[1 - \left(\frac{r}{a}\right)^2 \Delta\right]. \tag{23}$$

The spatial frequency instrumental for mode coupling is[6]

$$\theta_p = \frac{(2\Delta)^{\frac{1}{2}}}{a}. \tag{24}$$

The total number of guided modes of both polarizations and orientations supported by the fiber is[9]

$$N = (n_1 k a)^2 \frac{\Delta}{2}. \tag{25}$$

Since the core radius a appearing in (22) is different for single-mode, step-index fibers and multimode, parabolic-index fibers, we must express it in terms of some other quantity. We have mentioned in the introduction that the spatial frequency Ω of eq. (3) characterizes coupling of the step-index fiber mode to radiation. One way of comparing single- and multimode fibers is to insist that the critical coupling (spatial) frequency is the same for both fiber types. By eliminating a and Δ, we obtain from (22), (24), and (25),

$$2\alpha_p = 1.45 \frac{n_1}{\sqrt{N}} \left(\frac{\theta_p}{k}\right)^3 k^4 \langle F^2(\theta_p) \rangle. \tag{26}$$

The number of modes carried by the fiber can be varied by varying the core radius a, the relative refractive index Δ, or the free-space propagation constant k. Keeping θ_p and k constant requires us to vary a and Δ as N is varied.

V. COMPARISON OF SINGLE-MODE AND MULTIMODE FIBER LOSSES

We represent the radiation loss coefficients in normalized form by dividing 2α by the product $k^4\langle F^2 \rangle$. According to its definition (6) or (7), the power spectrum has the dimension cm³. Since k has the same dimension as 2α, the ratio is dimensionless. The spatial frequency θ and the shape of $\langle F^2(\theta) \rangle$ are always chosen to be the same for the single-mode or multimode fibers that are to be compared. In Figs. 1 and 2, we have plotted the normalized loss coefficient of the single-mode

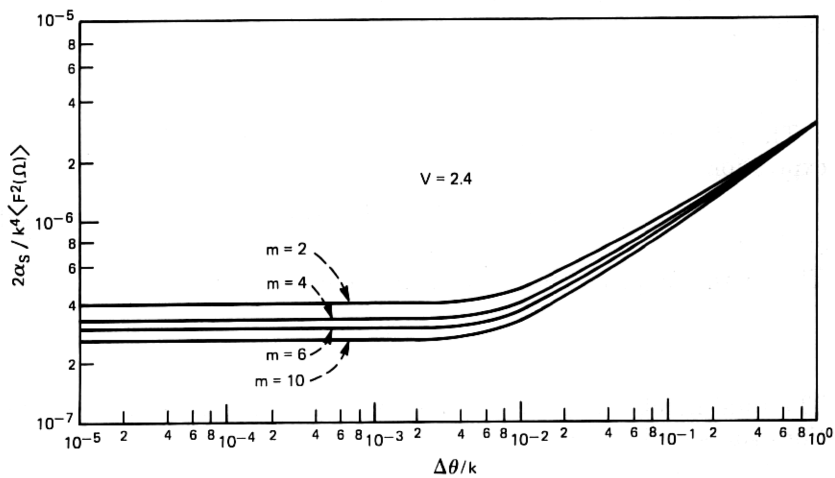

Fig. 1—Normalized loss coefficient of the single-mode, step-index fiber with power spectrum defined by (11) as a function of the width parameter $\Delta\theta/k$ of the power spectrum. The curve parameter m indicates the exponent of the power law. $n_1 = 1.515$, $n_2 = 1.5$, $V = 2.4$.

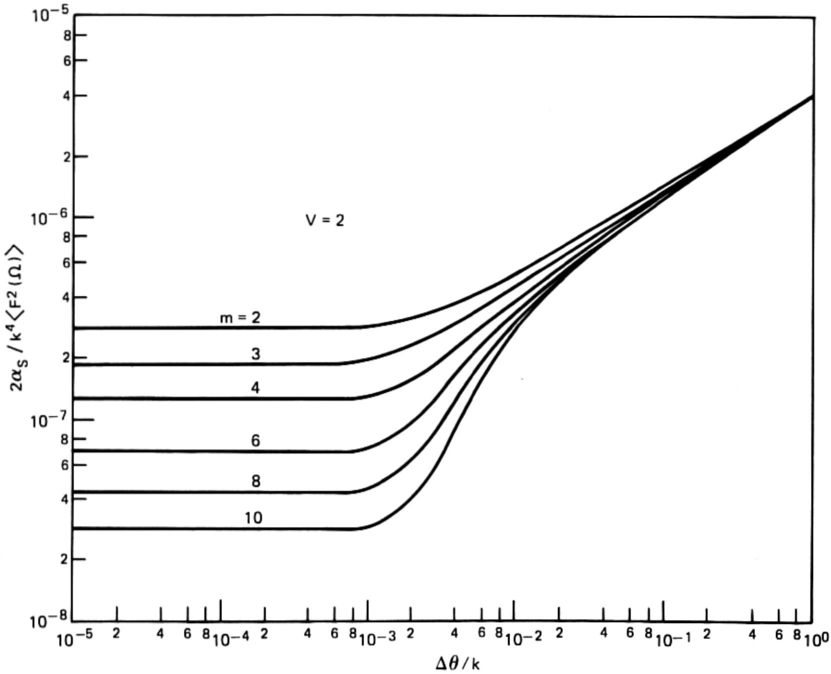

Fig. 2—Similar to Fig. 1 with $V = 2$.

fiber as a function of the normalized width parameter $\Delta\theta/k$ of the power spectrum (11) for $n_2 = 1.5$ and $n_1 = 1.01\ n_2$. The loss curves were computed from (5) and the frequency Ω appearing in the argument of the power spectrum is defined by (3). Figures 1 and 2 show that the loss coefficient is independent of the width parameter for small values of $\Delta\theta/k$. The dependence of the normalized loss on the normalized frequency V is illustrated in Fig. 3 for the case that the power spectrum (11) can be approximated in the form (12). The solid curves are computed from (5) while the dotted curves are calculated from the approximate formulas (17) through (19). It is apparent that the approximation is quite good, particularly for large values of the exponent m. The dash-dotted curves in Fig. 3 illustrate how the approximation deteriorates if the term in (17) containing V^m is omitted. It is apparent that this term provides an important correction for large values of V. The range of V values on the abscissa of Fig. 3 coincides with the single-mode range of (16). Formulas (5) or (17) do not hold for larger V values. For $2.405 < V < 3.83$, eq. (8) or its approximation (20) apply. The agreement of the approximation (20) with (8) is at least as good as that of the solid and dotted curves with $m = 10$ in Fig. 3; therefore, only a

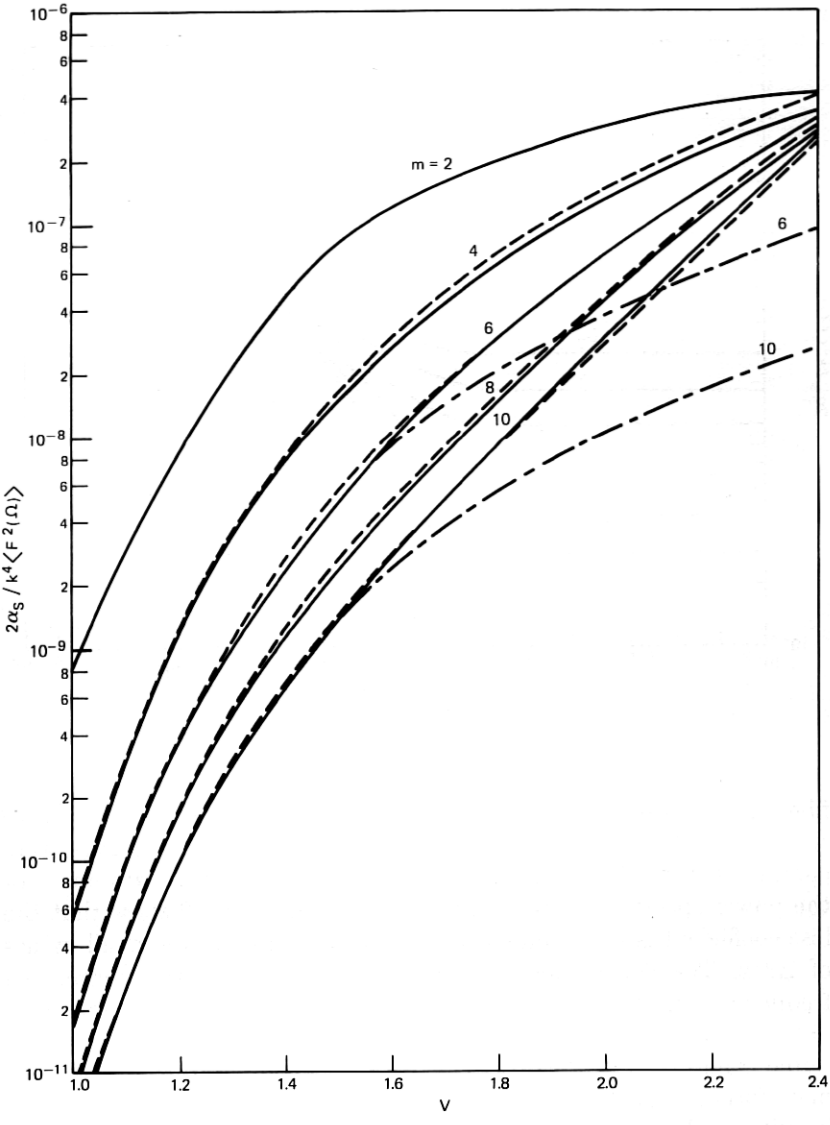

Fig. 3—Normalized loss coefficient of the single-mode, step-index fiber as a function of the normalized frequency parameter V for several values of the power law exponent m. The loss values in this curve correspond to the flat portions on the left-hand side of Figs. 1 and 2. The solid curves correspond to eq. (5) while the dotted curves were computed from the approximate equation (17). The dash-dotted curves represent (17) without the term V^m. $n_1 = 1.515$, $n_2 = 1.5$.

single solid curve was drawn in Fig. 4 to represent these formulas. In the vicinity of $V = 2.4$, the simple equation (8) and consequently its approximation (20) are not exact. As long as the value of the propa-

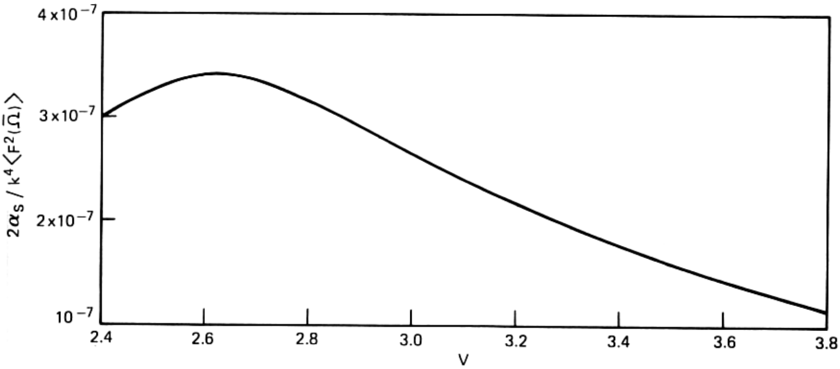

Fig. 4—Plot of eq. (20). This curve represents the normalized radiation loss of a quasi-single, HE_{11} mode in the presence of a leaky (next-higher-order) mode. The curve in this figure continues the curves in Fig. 3 to higher values of V, $n_1 = 1.515$, $n_2 = 1.5$.

gation constant β_1 is very close to n_2k, coupling to the radiation modes is not entirely negligible. It is apparent from Fig. 3 that the loss curves for different values of m have a tendency to converge to one curve. In Fig. 4 this convergence has taken place, but the transition from the many curves to one single curve is lost.

A comparison between the loss coefficients of the single-mode, step-index fiber and the multimode, parabolic-index fiber is possible only if certain assumptions are made. Figure 5 presents plots obtained from the loss formula (26) of the multimode fiber. We have drawn these curves by requiring $\theta_p = \Omega$ with Ω given by (3) and θ_p defined by (24). Multimode and single-mode fibers now sample the power spectrum at the same spatial frequency. The independent variable N in Fig. 5 represents the total number of modes that can propagate in the multimode fiber. As the number of modes changes, the fiber dimensions must change accordingly, keeping the value of θ_p constant.

The curves in Fig. 5 are labeled by the values of the normalized spatial frequency Ω/k; the corresponding values of the V number of the single-mode fiber are written in parenthesis. These V values are needed for a comparison of the loss values in Figs. 3 and 5. The normalized loss values are directly comparable since the same power spectrum at the same spatial frequency has been used in both cases. However, each curve of Fig. 5 must be compared with the curves in Fig. 3 at the proper V value. It is apparent that the loss of the multimode fiber can always be reduced below the loss of the single-mode fiber if the number of modes, N, is made large enough. For example, a single-mode fiber will normally be operated near $V = 2.4$. The corresponding curve (with $V = 2.4$) in Fig. 5 lies below the single-mode loss value

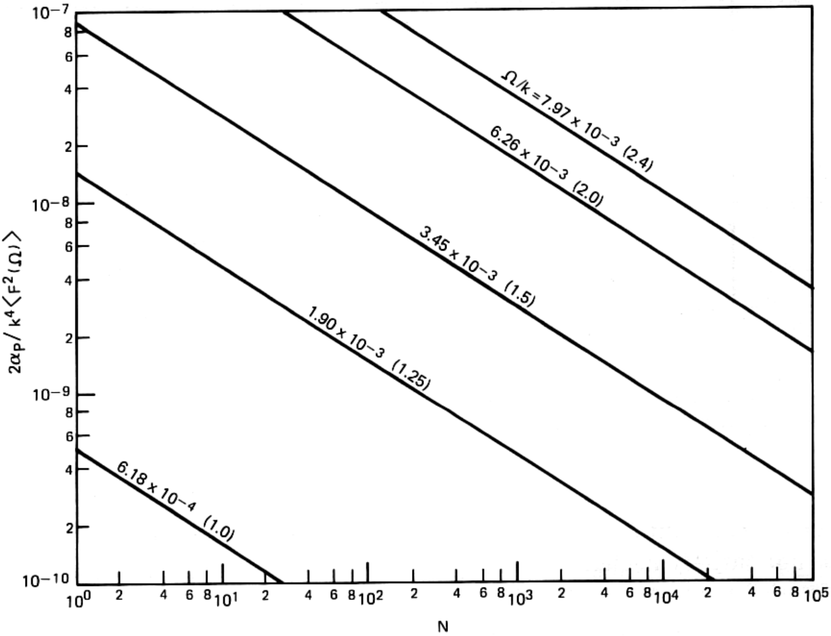

Fig. 5—Normalized loss coefficients of the multimode, parabolic-index fiber which is designed so that its spatial frequency θ_p [eq. (24)] coincides with the spatial frequency Ω [eq. (3)] of the single-mode fiber. The values of Ω/k label the curves; the values shown in parenthesis are the corresponding V values of the single-mode fiber.

for $N > 30$. For a fiber with $n_2 = 1.5$ and $\Delta = 0.01$, $N = 30$ corresponds to $ka = 52$, for $\lambda = 1\ \mu m$ we have a core radius of $a = 8\ \mu m$.

Next we compare the microbending losses of single- and multimode fibers by assuming typical fiber parameters. Single-mode fibers must be designed to have a V value near $V = 2.4$. If the fiber has a narrow inner cladding surrounded by an outer cladding, whose refractive index is similar to that of the core, the leaky mode losses of modes of order higher than HE_{11} may be so large that the fiber behaves effectively as though it supported only one guided mode and its V value may be chosen to be above $V = 2.405$. After the V value has been chosen, it is desirable to make the core radius a as large as possible to ease the splicing problem. We assume that fiber core radius values in the range $a = 5\ \mu m$ to $10\ \mu m$ are of interest. Thus, with $\lambda = 1\ \mu m$, we are interested in ka values in the range $40 < ka < 60$. With $V = 2.4$, the corresponding index difference would be in the range $0.0005 < n_1 - n_2 < 0.002$; for $V = 3.5$, we have instead $0.001 < n_1 - n_2 < 0.0045$.

For a typical multimode, parabolic-index fiber, we use $\Delta = 0.01$ and let the core radius fall in the range $16 < a < 35\ \mu m$ so that with

$\lambda = 1\ \mu m$, we have $100 < ka < 220$. The cladding index is assumed to be that of fused silica, $n_2 = 1.457$, for both types of fibers. Since there is now no fixed spatial frequency that we may use for normalization purposes, we use the power spectrum at the spatial frequency $\theta = n_2 k \Delta$ to normalize the loss coefficient. This normalization uses the Δ value of the multimode fiber ($\Delta = 0.01$) and is certainly quite arbitrary, particularly for the single-mode fiber. However, a common normalization for both fiber types is necessary if we want to compare the loss values.

Figures 6 through 9 present normalized loss values for the single- and multimode fibers for $m = 4$, 6, 8, and 10. The single-mode losses are plotted as solid lines for different values of V; the dotted lines indicate the multimode loss values. The upper scale in the abscissa ranging from $ka = 30$ to $ka = 60$ applies to the single-mode curves, while the lower abscissa from $ka = 100$ to $ka = 220$ applies to the multimode curves. We have included two curves with $V = 3.1$ and

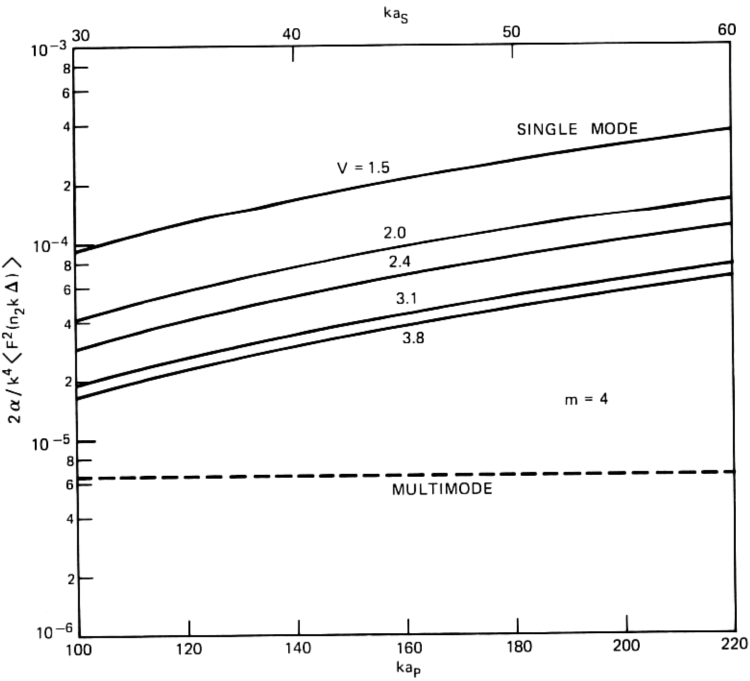

Fig. 6—Normalized radiation losses of single-mode (solid curves) and multimode (dotted curve) fibers. The upper scale on the abscissa applies to the single-mode case while the lower scale belongs to the multimode fiber. The power spectrum used for normalization of the loss factor is taken at the spatial frequency $\theta = n_2 k \Delta$ with $\Delta = 0.01$, $n_2 = 1.457$. This figure uses the exponent $m = 4$ of the power spectrum (12).

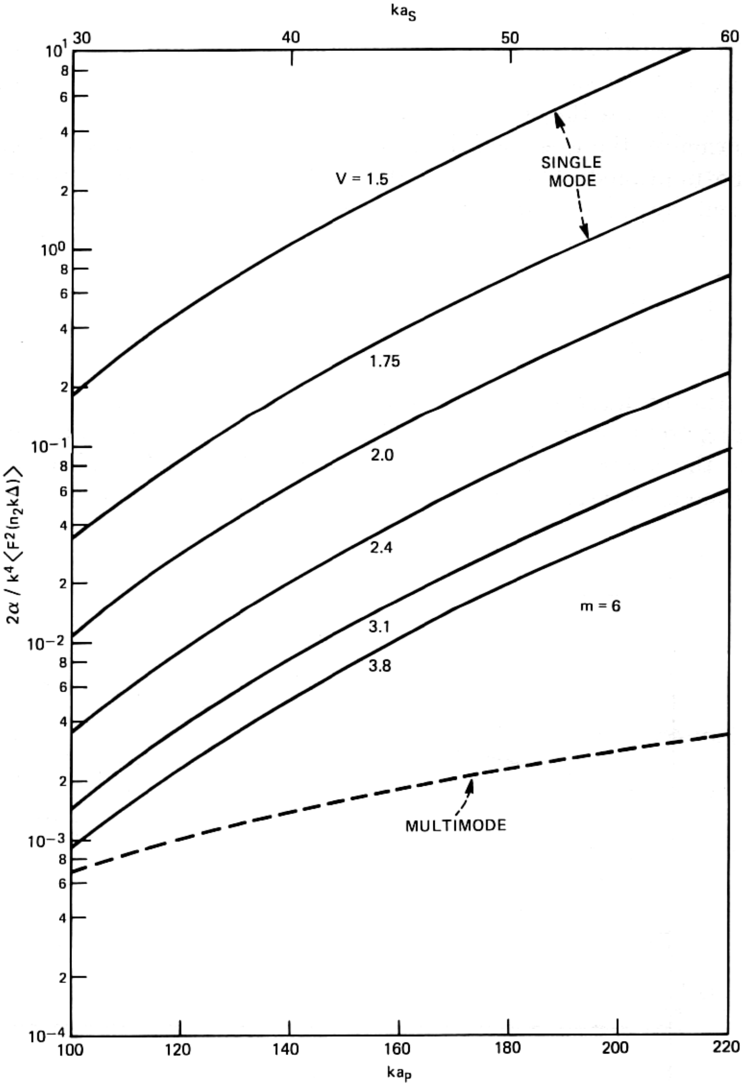

Fig. 7—Same as Fig. 6 with $m = 6$.

$V = 3.8$ in Figs. 6 through 9 to illustrate the case of a fiber that is not strictly speaking "single mode," but supports an additional leaky mode. The curves for V in the range $1.5 < V < 2.4$ were computed from (17) through (19) while the curves with $V = 3.1$ and $V = 3.8$ were computed from (20). The dotted curve for the parabolic-index fiber was obtained from (22).

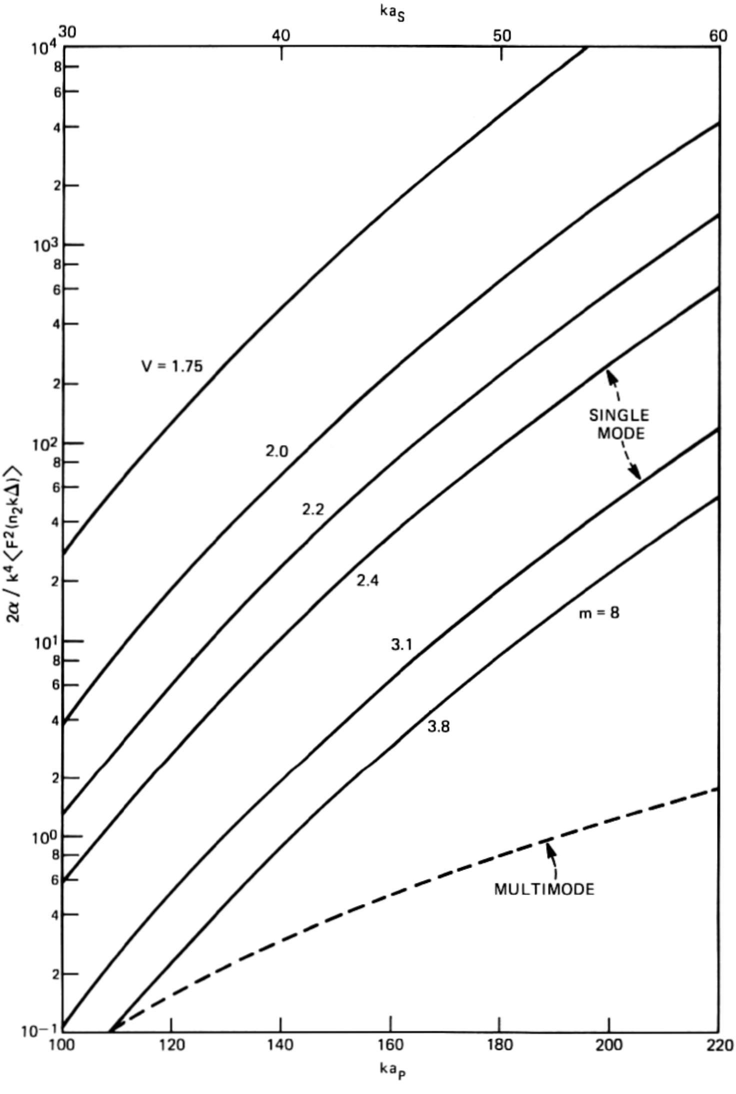

Fig. 8—Same as Fig. 6 with $m = 8$.

It is apparent that typical multimode fibers have lower losses than typical single-mode fibers.

VI. CONCLUSIONS

We have presented loss formulas for the single-mode, step-index fiber and for the multimode, parabolic-index fiber. The step-index fiber

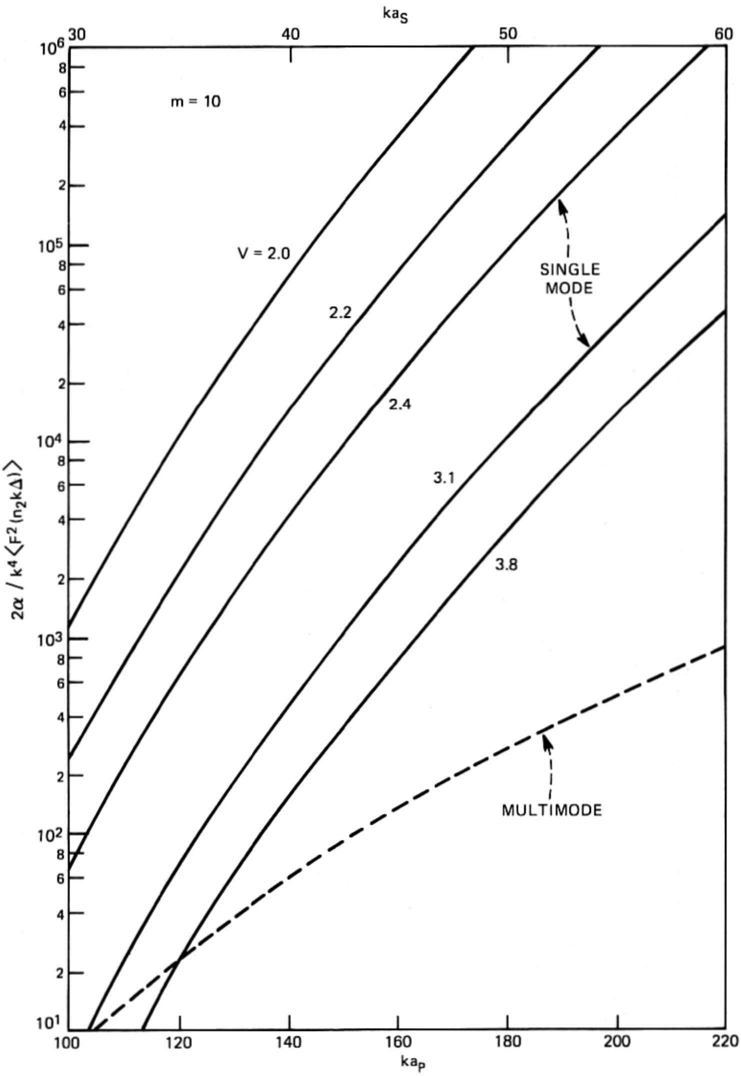

Fig. 9—Same as Fig. 6 with $m = 10$.

formulas were approximated by simple empirical approximations to obtain analytical expressions that can be easily evaluated with the help of a pocket calculator.

We compared the radiation losses of both fiber types that are caused by random deflections of the fiber axis. It was our objective to determine which fiber type is more sensitive to these microbending losses.

Our conclusion is that the single-mode fiber is more susceptible to microbending losses than the parabolic-index, multimode fiber. Since both fiber types are useful for wideband communications purposes using GaAs injection lasers or LEDs (in the case of the multimode fiber only), a knowledge of their respective microbending losses is important for system considerations.

VII. ACKNOWLEDGMENT

I am grateful to D. Gloge who suggested this study and contributed to it through several illuminating discussions. My thanks are also due to E. A. J. Marcatili who made many helpful suggestions.

APPENDIX

Validity of the Loss Formula in Equation (5)

Equation (5) was derived under the assumption that the fiber core is surrounded by an infinite cladding. Actually, cladding modes play the role of our radiation modes and the question arises: how accurate is a description of mode losses in terms of radiation modes? To answer this question, we compare the results of numerical evaluations of (5) with a theory published by Kuhn,[4] who uses cladding modes in a fiber with lossy jacket to calculate mode losses caused by random bends. Kuhn assumes that the curvature of the fiber can be described by either an exponential or gaussian correlation function. Since our formulation uses the function $f(z)$ to describe the fiber axis, we must first transform our equations into a form that allows us to compare it to a description in terms of fiber curvature. If we denote the power spectrum of the curvature function by $\langle C^2(\theta)\rangle$, the following relation holds:

$$\langle F^2(\theta)\rangle = \frac{\langle C^2(\theta)\rangle}{\theta^4}. \tag{27}$$

The variance of $f(z)$ is defined as

$$\bar{\sigma}^2 = \frac{1}{\pi}\int_0^\infty \langle F^2(\theta)\rangle d\theta. \tag{28}$$

The variance of the curvature spectrum is similarly defined as

$$\left\langle\left(\frac{1}{R}\right)^2\right\rangle = \frac{1}{\pi}\int_0^\infty \langle C^2(\theta)\rangle d\theta. \tag{29}$$

If we define the power spectrum $\langle F^2\rangle$ with variance $\bar{\sigma}^2$, we can compute the corresponding variance for the curvature spectrum only if the integral (29) exists. For an exponential correlation function of the curvature, the power spectrum $\langle C^2\rangle$ has a Lorentzian shape[8] that remains finite at $\theta = 0$ so that the integral in (28) does not exist. Correspondingly, (29) does not exist for a Lorentzian shaped $\langle F^2 \rangle$. Thus, a comparison between Kuhn's theory and ours is possible only for the gaussian correlation function.

Kuhn uses the following autocorrelation function for the fiber curvature:[4]

$$R_c(u) = \left\langle \left(\frac{1}{R} \right)^2 \right\rangle \exp\,(-u^2/D^2). \tag{30}$$

D is the correlation length. The power spectrum of the distortion function $f(z)$ is, thus,

$$\langle F^2(\theta) \rangle = \sqrt{\pi} \left\langle \left(\frac{1}{R} \right)^2 \right\rangle \frac{D}{\theta^4} \exp\,[-(\theta D/2)^2]. \tag{31}$$

Numerical integration of (5) yields the solid curve for the normalized radiation power loss shown in Fig. 10. This curve was computed for

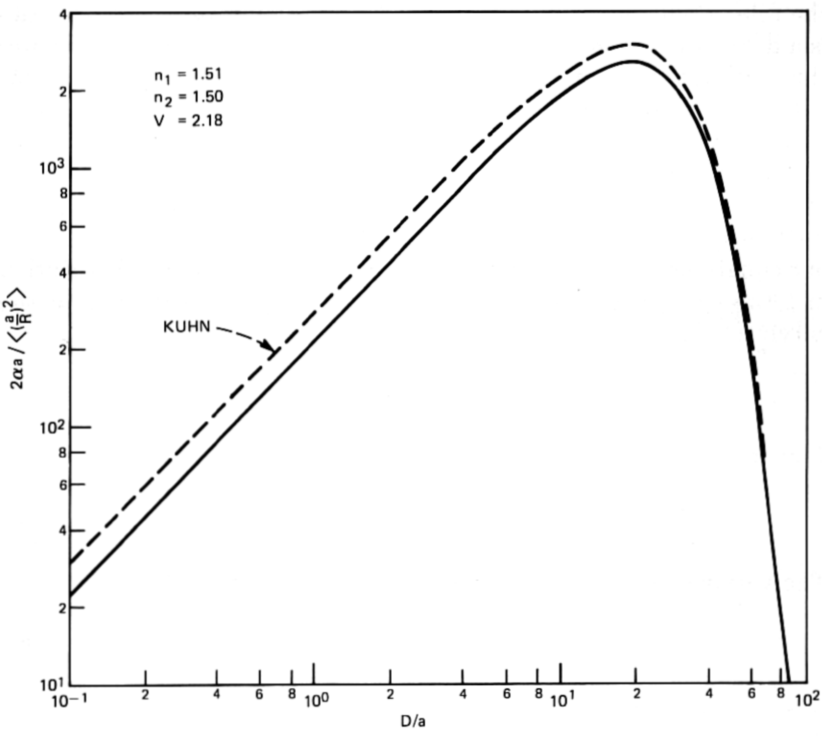

Fig. 10—Comparison between Kuhn's theory and our theory for $n_1 = 1.51$, $n_2 = 1.5$, and $V = 2.18$. The autocorrelation function of the fiber curvature is gaussian.

the following conditions:

$$n_1 = 1.51$$
$$n_2 = 1.5$$
$$ka = 12.56531.$$

These values lead to $V = 2.18$. The eigenvalue equation for LP modes[7] allows us to calculate $ka = 1.5851486$.

The dashed curve in Fig. 10 was obtained from Kuhn's publication.[4] It is apparent that the two theories agree very well for large correlation length corresponding to small spatial frequencies. The agreement is not quite as good for short correlation length or high spatial frequencies. Coupling spectra with large spatial frequencies lead to radiation escaping from the fiber core at large angles. Our eq. (5) is limited to small angle radiation. The solid curve in Fig. 10 was computed by using (5) for large correlation length, but using a corresponding formula derived with the help of free-space radiation modes for small correlation length. However, the longitudinal components of the guided and radiation modes were ignored, which led to an underestimation of the loss for radiation escaping at large angles. In the main part of this paper, we are interested only in coupling functions whose Fourier spectra drop off rapidly with increasing spatial frequency, so that only small spatial frequencies are important; eq. (5) is thus applicable. The comparison of our theory with Kuhn's shows that no large error results from using radiation modes instead of cladding modes to compute microbending losses.

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