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Offset and Tilt Loss in Optical Fiber Splices

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Transverse offset and angular misalignment (tilt) are serious causes of loss in multimode fiber splices. Our computation of these losses in multimode graded-index fibers reveals that the loss depends strongly on the power distribution in the fiber. We find that offsets of 0.1 of the core radius (or tilts of 0.1 of the fiber numerical aperture) cause a loss of 0.1 dB in the case of steady-state conditions, and between 0.34 and 0.38 dB if the power distribution is assumed as uniform. We compare these results with measured offset loss values and conclude that the steady-state distribution better reflects transmission line conditions than the uniform distribution.

I. INTRODUCTION

Fiber transmission lines will frequently have more than 10 splices between terminal points; to estimate the overall line loss, we need a reliable figure for the splice loss to be expected under varying line conditions. The loss in multimode fiber splices depends on the power distribution among the modes in the fiber. The difficulty of correctly simulating line conditions in laboratory measurements or computations has so far prevented the establishment of a reliable splice-loss estimate. Calculations have predicted as much as 0.4 dB for a 10-percent relative offset (offset divided by the core radius). These calculations assumed an equal or uniform power distribution in all modes.[1] Techniques used to measure the offset loss have attempted to simulate the equal-power condition by illuminating the full numerical aperture of the input fiber by a coherent source,[1,2] but the resulting loss values were between 0.1 and 0.2 dB for a 10-percent offset. The question has been raised whether some of the discrepancies are attributable to leaky rays.

The following study considers lateral offset and angular misalignment (tilt) of splices made from perfectly similar fibers by joining them without air gap. As an introduction to the subject, we derive simplified offset-loss formulae for power-law profiles, considering both the absence and presence of leaky rays. We find that leaky rays are most likely not the cause of the discrepancies. The main part of this study concerns the misalignment loss in square-law fibers when the mode-power distribution is nonuniform as a result of mode-conversion effects. Special attention is given to the steady-state power distribution.[3] We find that the discrepancies mentioned earlier can be explained in terms of differences in the relevant mode-power distributions.

II. UNIFORM DISTRIBUTION

To compute the offset loss, Miller[1] compares the number of rays that exit and enter at coinciding points of the two fiber end faces of an offset fiber joint (see Fig. 1a). The percentage of rays not accepted on entrance is the offset loss. As a first case, we assume all rays to be uniformly excited. Both fibers have the same dimensions and are of the same kind. In the case of step-index fibers, all rays traversing the overlapping core areas continue to propagate and the only loss occurs outside these areas. Offset graded-index fibers show a loss also in the overlapping areas where the local numerical aperture (LNA) of the outgoing fiber is smaller than the corresponding LNA of the incoming

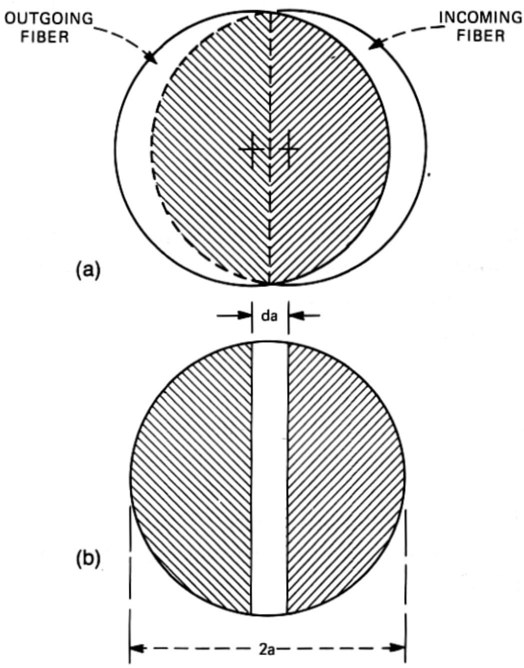

Fig. 1—Areas of transmission in a fiber splice. (a) End-on view of the splice. (b) Area of overlap rearranged for computation of offset loss.

fiber.* This condition exists for example in the right-hand half of the area of overlap in Fig. 1a. In this area, the outgoing fiber accepts only those rays that fall within its LNA; all others are lost. On the other hand—and this is important for our derivation—*all* rays in the outgoing fiber are uniformly excited in this area since we have assumed that all rays of the incoming fiber were excited.

In the left-hand half of the area of overlap, the incoming LNA is smaller than the outgoing LNA. Thus, all incoming rays are accepted there. On the other hand, the outgoing fiber is *not* fully excited. Since we assume that both fibers have the same characteristics, the number of incoming rays in the left half equals that of the accepted rays in the right half. If we arrange the two halves as in Fig. 1b, we find that the number of rays lost equals the number of those rays that traverse the blank area of Fig. 1b in a uniformly excited fiber. The width of this area is equal to the offset.

Assume a fiber core of diameter $2a$ and define a radial coordinate r that is so normalized that $r = 1$ at the core periphery. Let $n(r)$ be the core index and assume a constant index $n(1)$ in the cladding. In the case of full and uniform excitation of all trapped modes, the rays traversing a point in the core are uniformly distributed over all angular directions within a cone that has the LNA as its apex. The number of these rays is proportional to $(\text{LNA})^2 = n^2(r) - n^2(1)$. The integration of this quantity over the blank area of Fig. 1b, properly normalized, yields the offset loss. The integration can be performed analytically for step-index and parabolic profiles and leads to the results of Ref. 1.

If the offset is a small fraction d of the fiber radius, we can approximate the blank area by a rectangle of length $2a$ and width da. In addition, we can ignore the variation of the LNA across the width of that area. These assumptions simplify the integration so that the offset loss becomes simply

$$L_t = \frac{d}{\pi} \frac{\int_0^1 [n^2(r) - n^2(1)]dr}{\int_0^1 [n^2(r) - n^2(1)]rdr}. \tag{1}$$

If the index follows a power law

$$n^2(r) = n_0^2(1 - 2\Delta r^\alpha) \tag{2}$$

in the core, the evaluation yields

$$L_t = \frac{2d}{\pi} \frac{\alpha + 2}{\alpha + 1}. \tag{3}$$

* The LNA is proportional to the square root of the index difference at that point.

A comparison with the exact expressions for $\alpha = 1$, 2, and ∞ shows that (3) represents the first term of an expansion of L_t in powers of d. Remember that the result applies in the case of full and uniform excitation of all trapped rays, that is, those rays that are totally reflected inside the core.

Leaky rays possess an evanescent cladding field in a region extending from the core periphery to a circle of radius A in the cladding.[4] The smaller A, the larger the radiative (leakage) loss $l(A)$. Stewart[4] has found that the solid angle, which includes all rays having a loss smaller than $l(A)$, equals the solid angle comprising the trapped rays divided by $(A^2 - r^2)^{\frac{1}{2}}$. To obtain an upper bound to the influence of the leaky rays on the offset loss, we assume $A = a$ in the following although, by doing so, we include leaky rays that have a significant leakage loss.

Arguments analogous to the previous derivation lead to an offset loss

$$L_f = \frac{d}{\pi} \frac{\displaystyle\int_0^1 \frac{1 - r^\alpha}{(1 - r^2)^{\frac{1}{2}}} dr}{\displaystyle\int_0^1 \frac{1 - r^\alpha}{(1 - r^2)^{\frac{1}{2}}} r dr} \tag{4}$$

for a uniform power distribution in both trapped and leaky modes. We obtain

$$L_f = \frac{d}{2} \frac{1 - 1/K}{1 - K/(\alpha + 1)} \tag{5}$$

with

$$K = 2^{\alpha - 2} \alpha \Gamma^2 \left(\frac{\alpha}{2}\right) \Big/ \Gamma(\alpha). \tag{6}$$

Table I lists loss values for some exponents α. Figure 2 shows loss versus offset for $\alpha = 2$. In that case, an offset of one tenth of the core radius causes a loss of 8.5 percent or 0.38 dB if only the trapped modes are considered and 7.5 percent or 0.34 dB if all leaky modes are taken into account as well. The discrepancy is small.

Equations (1) and (4) are based on the assumption that at least one of the fibers of the joint is fully excited at any given point in the splice cross section. This assumption requires some scrutiny in the

Table I — Offset loss coefficients

α	1	2	4	∞
L_t/d L_l/d	0.95 0.85	0.85 0.75	0.76 0.67	0.64 0.5

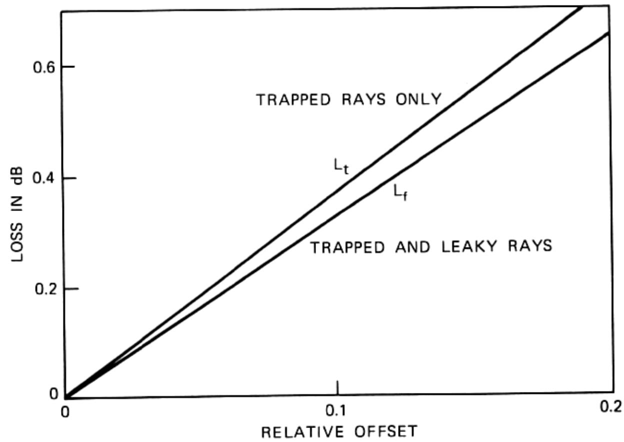

Fig. 2—Offset loss vs relative offset for uniform power distribution according to eqs. (3) and (5).

case of the leaky modes. In this case, the cone of rays traversing a given point in the core becomes elliptical, with the major axes oriented in radial and tangential direction. The offset juxtaposes cones with slightly different orientation. For (4) to hold, the larger of the cones must completely overlap the smaller cone. Without presenting details, we merely mention here that this condition is not satisfied, but that the resulting error is within the linear approximation in d.

III. NONUNIFORM DISTRIBUTIONS

Consider a ray that propagates in a fiber that has an index profile of the form (2). Assume that the ray forms an angle θ with the fiber axis at a distance ra from the axis. The principal ray optics variables r and θ have no physical meaning in the wave optics picture; the combination

$$R = \left(r^\alpha + \frac{\sin^2\theta}{2\Delta}\right)^{\frac{1}{2}}, \tag{7}$$

however, is directly related to the propagation constant of a fiber mode.* Given the power in each fiber mode, we can plot the power distribution $p(R)$. Although the following computations can be performed for general power-law profiles, we restrict the derivation to the square-law profile in the interest of simplicity. The results for near square-law profiles are very similar. We introduce a normalized

* $R^2 = (1 - \beta^2/k^2n_0^2)/2\Delta$, where β is the propagation constant and k the vacuum wave number. R is the magnitude of a vector coordinate in phase space pointing to the locus of the mode. We call R, therefore, the mode coordinate.

angular coordinate

$$\rho = \sin\theta/(2\Delta)^{\frac{1}{2}}, \tag{8}$$

so that, for square-law profiles,

$$R^2 = r^2 + \rho^2. \tag{9}$$

The fact that r and ρ are interchangeable in this relationship implies that offsets and tilts can be treated in the same way.* More specifically, in case of a tilt angle δ and a lateral offset d, we can introduce a single offset variable D so that

$$D^2 = d^2 + \sin^2\delta/2\Delta. \tag{10}$$

Before we calculate the offset loss, let us compute the total transmitted power P for a given power distribution $p(R)$. We note from (9) that $\rho d\rho = RdR$ and that $p(R) = 0$ for $r > R$. Hence,

$$P = \int_0^1 p(R) 2\pi R dR \int_0^R 2\pi\, r dr \tag{11}$$

or

$$P = 2\pi^2 \int_0^1 p(R) R^3 dR. \tag{12}$$

If the power distribution depends on the angle ϕ, as indicated in Fig. 3, we obtain

$$P = 4\pi \int_0^1 R^3 dR \int_0^\pi q(R, \phi)\sin^2\phi d\phi \tag{13}$$

by introducing $rdr = ydx$ with $x = r\cos\phi$ and $y = r\sin\phi$ in (11).

Now consider a lateral offset d. Assume the power distribution in the incoming fiber to be $q(R_i)$ for $R_i < 1$ and zero everywhere else. If we retain only linear terms of d as before, $R_i = R - d\cos\phi$, where R is the mode coordinate of the outgoing fiber. The power distribution in terms of this coordinate becomes

$$p(R) = \frac{2}{\pi}\int_0^{\operatorname{arcos}[(R-1)/d]} q(R - d\cos\phi)\sin^2\phi d\phi. \tag{14}$$

This expression is a modification of the second integral in (13), which takes into account that $q = 0$ for $\phi > \operatorname{arcos}[(R-1)/d]$. A similar derivation holds for a tilt angle δ or a combination of offset and tilt. The result has the form (14) with d replaced by D of (10). It is convenient to normalize $q(R)$ for unit total power P; this is done in the following analyses.

* The modes of square-law fibers occupy a phase-space volume of rotational symmetry.

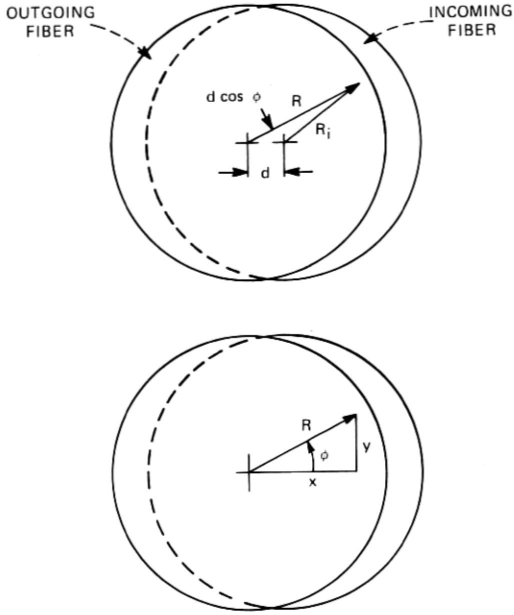

Fig. 3—Coordinate system used to compute offset loss.

As a first example, assume a uniform input distribution

$$q_1(R_i) = \begin{cases} 2/\pi^2 & \text{for} \quad R_i < 1 \\ 0 & \text{for} \quad R_i > 1. \end{cases} \tag{15}$$

Because of (14), the distribution in the splice becomes

$$p_1(R) = \frac{2}{\pi^3} \arccos \frac{R-1}{D} - \frac{2}{\pi^3} \frac{R-1}{D} \left[1 - \frac{(R-1)^2}{D^2} \right]^{\frac{1}{2}}. \tag{16}$$

Figure 4a shows q_1 and p_1 in the vicinity of $R = 1$ for $D = 0.1$. The offset loss is indicated by the cross-hatched area in Fig. 4a. It is

$$L_1 = 2\pi^2 \int_1^{1+D} p_1(R) R^3 dR = \frac{8D}{3\pi} \tag{17}$$

in accordance with (3), which yields $L_t = 8d/3\pi$ for $\alpha = 2$.

Power transfer among the modes as a result of guide imperfections modifies the step distribution (15). If we assume uniform excitation at the input and model the power exchange by a diffusion process, we find that the step function (15) assumes the form

$$1 - \exp[(R_i - 1)/0.17(z/z_c)^{\frac{1}{2}}] \tag{18}$$

a short distance z from the input. In this expression, z_c is the coupling

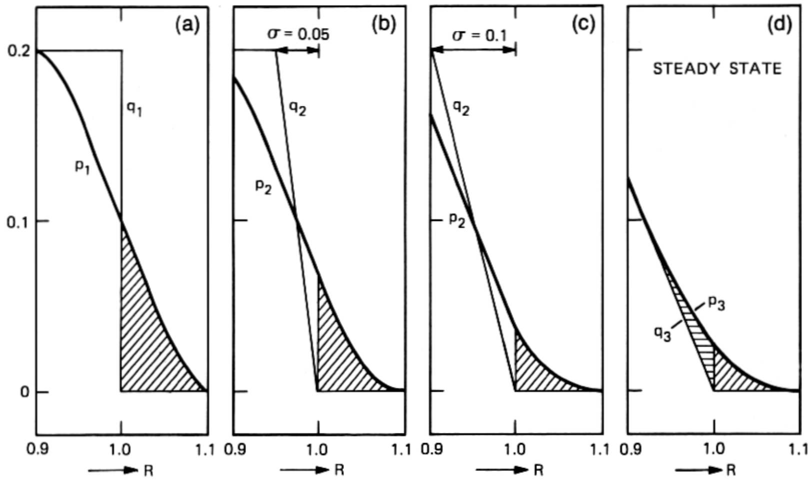

Fig. 4—Power distribution immediately ahead of the splice (q) and immediately behind the splice (p) for various power distributions plotted vs the mode coordinate R. The cross-hatched areas indicate splice loss; $D = 0.1$.

length.[3]* For our purposes, the approximation

$$q_2 = \frac{2(1+2\sigma)}{\pi^2}\begin{cases} 1 & \text{for} \quad R_i < 1-\sigma \\ (1-R_i)/\sigma & \text{for} \quad 1-\sigma < R_i < 1 \\ 0 & \text{for} \quad R_i > 1 \end{cases} \tag{19}$$

is satisfactory and valid as long as

$$\sigma = 0.17(z/z_c)^{\frac{1}{2}} \tag{20}$$

is small compared to unity. Figures 4b and c show plots of q_2 for $\sigma = 0.05$ and $\sigma = 0.1$ as well as the distribution $p_2(R)$ calculated from (14). An integration of the cross-hatched areas similar to (17) yields the loss L_2. Although an analytical solution exists, we list here only the simpler approximate form

$$L_2 = \frac{2D}{\pi}(1+2\sigma)\left(\frac{3}{4-1.3\sigma/D} + \frac{8}{\pi}\frac{\sigma}{D}\right)^{-1}, \tag{21}$$

which is more useful for design consideration. Figure 5 shows L_1 and L_2 for $\sigma = 0.05$ and $\sigma = 0.1$ plotted vs the offset D. If, for example, $z_c = 200$ m, a power distribution resembling that of Fig. 4b ($\sigma = 0.05$)

* The coupling length can be obtained by measuring the impulse response at a fiber length where the steady state has been reached. The coupling length is then equal to that length, multiplied by the square of the rms pulse width measured, and divided by the square of the rms width expected without mode conversion.

would exist 17.3 m from the input; similarly, a distribution resembling that of Fig. 4c would exist 34.6 m from the input (provided the input excitation is uniform). A 10-percent offset (or tilt) at these points would cause a loss of 0.14 dB or 0.09 dB respectively compared to 0.38 dB if the distribution were uniform at the splice. The reason for the drastic decrease in offset loss can be found in the fact that the modes close to cutoff suffer the main part of the offset loss, but cease to carry a significant portion of the total power a short distance away from the (uniformly excited) input.

For $\sigma > D$, the last term in (21) dominates. The loss can then be written in the form

$$L = \frac{\pi^2}{8} q'(1) D^2, \tag{22}$$

where $q'(1)$ is the derivative of q with respect to R at $R = 1$. In this case, the loss depends on the square of the offset and $L(D)$ has zero slope at $D \to 0$.

As z approaches z_c, the power distribution in the fiber resembles more and more the steady-state distribution.[3] Properly normalized, this distribution assumes the form

$$q_3 = \frac{u^3}{2\pi^2(u^2 - 4)} \begin{cases} J_0(uR)/J_1(u) & \text{for} \quad R < 1, \\ 0 & \text{for} \quad R > 1, \end{cases} \tag{23}$$

where J_0 and J_1 are Bessel functions and $u = 2.405$ is the first root of

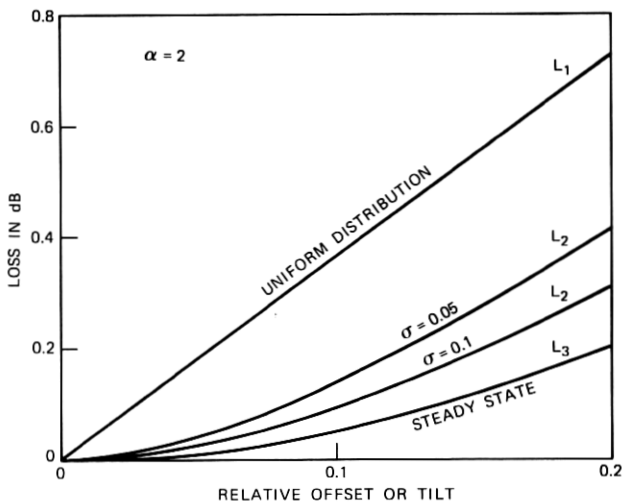

Fig. 5—Offset loss in dB vs relative offset or tilt for the four mode-power distributions sketched in Fig. 4.

J_0. Figure 4d shows q_3 as well as the distribution p_3 in the vicinity of $R = 1$. The fraction of p_3 that extends beyond $R = 1$ indicates the loss in the splice; it can be obtained from (22) by computing $q_3'(1)$ with the help of (23). The resulting loss L_3 is also plotted in Fig. 5. The part of the power distribution p_3 that deviates from the steady-state q_3 in the region $R < 1$ is accepted by the outgoing fiber, but is dissipated later as a result of mode conversion until the steady state is reestablished. An integration of $p_3 - q_3$ analogous to (17) but extending from $R = 1 - D$ to $R = 1$ reveals that this loss contribution is equal to the loss suffered in the splice. Thus, the total loss can be obtained by inserting $q_3'(1)$ into (22), as mentioned earlier, and by doubling the result of this integration; the total loss caused by an offset under steady-state conditions is, therefore,

$$L_s = \frac{u^4 D^2}{8(u^2 - 4)}. \tag{24}$$

This loss is plotted in Fig. 6. A 10-percent offset or a tilt angle equal to one tenth of the fiber NA causes 0.1 dB loss compared to 0.38 dB if the power is uniformly distributed in all trapped modes or 0.34 dB if leaky modes are present as well (Fig. 2).

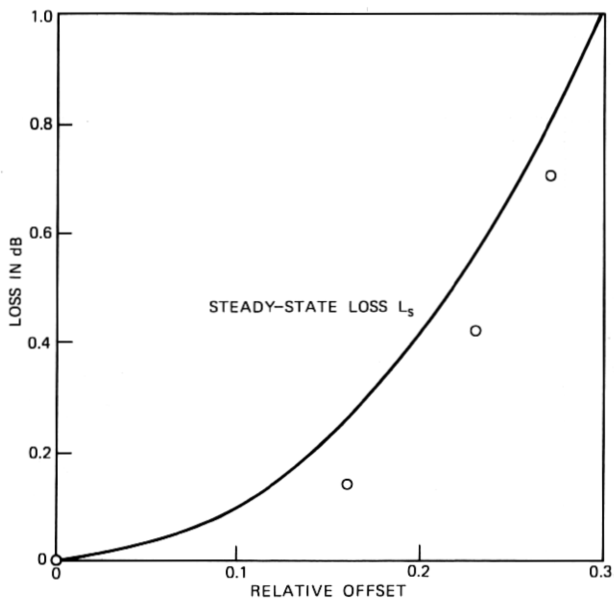

Fig. 6—Steady-state loss vs relative offset. Points show loss measured for offset between two ½-km fibers.

IV. COMPARISON WITH MEASUREMENTS

Figure 6 also shows loss values measured in a splice connecting two $\frac{1}{2}$-km fibers. Details concerning the measurement are presented in Ref. 1. The coupling length characterizing these fibers was not measured, but was probably of the order of the fiber length. The fact that the loss values are consistently lower than the predicted steady-state loss may indicate that the $\frac{1}{2}$-km length was not quite sufficient for the steady-state power distribution to establish itself before and after the splice.

Figure 7 shows loss values measured in a splice connecting two 3-m fiber pieces.[2] The fibers had an outside diameter of 125 μm, a core diameter of 75 μm, and an NA of 0.155. They were part of a cable structure that may have introduced some mode conversion. The fiber input was excited by a HeNe laser. Focused on the core, the gaussian laser beam filled a numerical aperture of 0.25 (at $1/e^2$ intensity). The solid line in Fig. 7 is a repetition of L_2 for $\sigma = 0.05$ plotted in Fig. 5. According to (20), this curve corresponds to the distribution present at a distance of 8.6 percent of the coupling length from the (uniformly excited) input.

It is beyond the purpose of this paper to determine under what circumstances a coherent source produces a uniform power distribution at the fiber input. The results depicted in Fig. 7 suggest that the use of a laser beam for splice loss measurements is acceptable as long as the

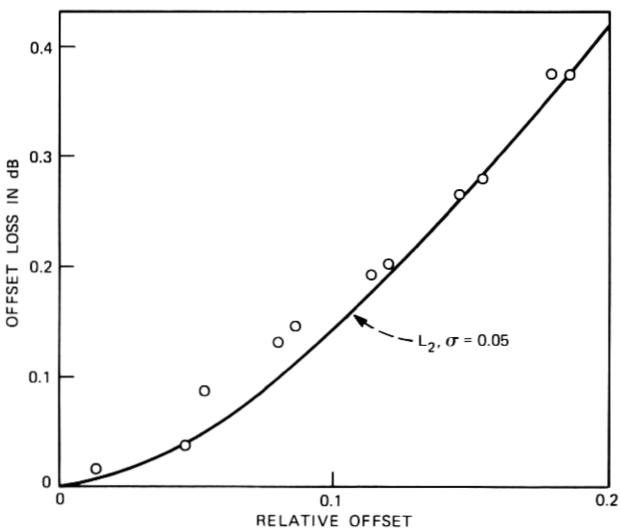

Fig. 7—Loss measured for offset between two 3-m fibers; solid line applies to conditions of Fig. 4b.

fiber NA is adequately filled by the beam and a modest amount of mode conversion is present (or applied) between input and splice. Measurements under the conditions described yield loss values that are about 50 percent too high at 10-percent offset and about equal to the steady state at 20-percent offset (or tilt).

V. CONCLUSIONS

We have attempted to show that the assumption of a uniform power distribution among all modes (or rays) is an idealization rarely obtained in fibers. The power distribution and particularly its behavior at the critical angle strongly affect the offset loss. If the offset loss calculation is based on the steady-state distribution, the loss at 10-percent relative offset (or tilt) is 3.8 times smaller than that predicted for a uniform distribution. Although the steady-state distribution does not necessarily exist at all splice points, we believe that it reflects transmission line conditions much better than a uniform distribution. We have compared these results with measurements by Miller[1] and Chinnock[2] and found that coherent excitation can provide offset loss values representative of line conditions if care is taken that the source fills the NA of the fiber and a modest amount of mode conversion is effective ahead of the splice point.

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