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A Phased, Optical, Coupler-Pair Switch

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We analyze an optical switch consisting of two directional couplers with an intermediate phase-compensating network. The conditional requirements on coupler length, phase mismatch between guides in each coupler, and phase compensation between couplers are established such that two switching states exist. It is shown that, to switch between states,

(*i*) *The couplers must be 3 dB or stronger.*
(*ii*) *Each coupler must be "adjusted" to become a hybrid by controlling the phase mismatch between guides.*
(*iii*) *The differential phase ϕ must be initially set to turn on one state.*
(*iv*) *An additional increment of π in the phase shift causes switching to the second state.*

It is estimated that the controlling voltage required to activate the bipolar switch is of the order of 1 to 2 V for a 3-μm guide with 3-mm-long electrodes in Ti-diffused $LiNbO_3$. We are currently in the process of fabricating such devices.

I. INTRODUCTION

A number of methods[1–4] have been suggested for realizing optical switches in integrated optical format; in many cases, the schemes are extensions of the microwave art.[5,6] Recently, Papuchon et al.[7] and Campbell et al.[8] obtained switching by perturbing the phase synchronism in indiffused strip-guide directional couplers of about one coupling period in length. Kogelnik et al.[9] have proposed a switch consisting of tandem couplers with adjustable phase mismatch between guides in each coupler. This approach results in relaxed dimensional tolerances over previously described coupler switches. However, the adjustment of phase mismatch in both couplers is required to switch between states.

In this paper, we propose and analyze an optical switch consisting of directional coupler pairs with an intermediate phase-compensating network. We show that the switching between states can be achieved by a π change in the intermediate phase shift providing that each coupler has been initially adjusted for hybrid performance, i.e., for 3-dB coupling. Prior to adjustment, the coupling strengths are required to be 3 dB or stronger, implying coupler lengths of $\frac{1}{2}$ to $1\frac{1}{2}$ basic coupling periods.

II. PRELIMINARY ANALYSIS

We assume directional couplers with uniform coupling having transfer matrices of the form[10]

$$\begin{bmatrix} E_3 \\ E_4 \end{bmatrix} = \begin{bmatrix} A & B \\ -B^* & A^* \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix}, \tag{1}$$

where $|A|^2 + |B|^2 = 1$ (see Fig. 1). For coupler 1, the matrix coefficients relating the field amplitudes E_k are given by[11]

$$A_1 = \cos \alpha_1 - j \frac{\gamma_1}{\sqrt{\gamma_1^2 + 1}} \sin \alpha_1, \tag{2}$$

$$B_1 = j \frac{\sin \alpha_1}{\sqrt{\gamma_1^2 + 1}}, \tag{3}$$

and, for coupler 2,

$$A_2 = \cos \alpha_2 - j \frac{\gamma_2}{\sqrt{\gamma_2^2 + 1}} \sin \alpha_2, \tag{4}$$

$$B_2 = j \frac{\sin \alpha_2}{\sqrt{\gamma_2^2 + 1}}, \tag{5}$$

where

$$\alpha_{1,2} = \sqrt{\gamma_{1,2}^2 + 1}\, \frac{\pi l_{1,2}}{2L_0}, \tag{6}$$

$$\gamma_{1,2} = \frac{\Delta\beta_{1,2} L_0}{\pi}, \tag{7}$$

$\Delta\beta_{1,2}$ = propagation constant difference between the guides in couplers 1 and 2, respectively,

$l_{1,2}$ = the interaction length,

L_0 = transfer length for total power transfer for $\gamma = 0$.

If we connect the two couplers in series via a differential phase-shifting network (Fig. 1), then, for unit excitation of port 1 of the first coupler, we obtain for the output signals

$$E_5 = A_1 A_2 - B_1^* B_2 e^{j\phi}, \tag{8}$$

$$E_6 = -A_1 B_2^* - A_2^* B_1^* e^{j\phi}, \tag{9}$$

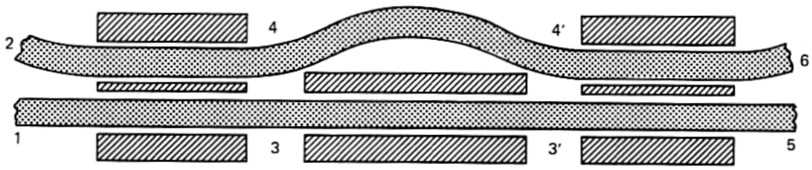

Fig. 1—Two couplers with adjustable $\Delta\beta$, connected in series with an intermediate phase shifter.

where ϕ is the differential phase. The two states of the switch are

$$\begin{aligned} &\text{State I:} & E_5 &= 0, & |E_6| &= 1 \\ &\text{State II:} & |E_5| &= 1, & E_6 &= 0. \end{aligned}$$

Thus, for State I, the condition is

$$e^{j\phi_1} = \frac{A_1 A_2}{B_1^* B_2}, \tag{10}$$

i.e.,

$$\left|\frac{A_1 A_2}{B_1^* B_2}\right| = 1. \tag{11}$$

This simply implies that, if the couplers have coupling coefficients that are the inverse of each other in magnitude, then a unique $\phi = \phi_1$ exists that permits the realization of State I.

For State II, the condition is

$$e^{j\phi_2} = -\frac{A_1 B_2^*}{A_2^* B_1^*}, \tag{12}$$

i.e.,

$$\left|\frac{A_1 B_2^*}{A_2^* B_1^*}\right| = 1. \tag{13}$$

This implies that, if the magnitudes of the coupling coefficients are identical, then there exists a $\phi = \phi_2$ that permits the realization of State II.

It follows, then, that with the parameters of the one coupler specified, there exist values of ϕ that permit the realization of States I or II for each allowed value of γ_2 and α_2. To realize both states by adjustment of ϕ alone, hybrid couplers are necessary, i.e., $|A_1| = |A_2| = |B_1| = |B_2| = 1/\sqrt{2}$.

Low-loss strip waveguides made by diffusion of metals in electro-optic materials[12–14] are available to facilitate the fabrication of these switches. However, the difficulty in forming coupler pairs is the maintenance of uniform guide width, separation, and refractive index differential.* Thus, realization of hybrid pairs is rather difficult. The above analysis suggests that, given couplers with a power-coupling coefficient greater than 3 dB, States I or II can be achieved by appropriately altering the phase mismatch in the couplers (electro-optically or otherwise) and introducing a specific differential phase shift ϕ between couplers.

Thus, our requirements are

(*i*) Each coupler must initially have coupling coefficient $|B_1|$, $|B_2| \geqq 1/\sqrt{2}$.

(*ii*) Direct-current biasing can then be provided in each coupler to alter the phase mismatch in order to achieve $|B_1| = |B_2| = 1/\sqrt{2}$.

(*iii*) A differential phase shift ϕ_1 is then introduced between couplers for State I to exist, and then additional phase shift of π, viz., $\phi_2 = \phi_1 + (2m + 1)\pi$, causes the switch to go to State II. Thus, activation of either state can be achieved by the addition or removal of the control voltage that causes a phase shift of π between couplers.

Section III provides a detailed analysis and establishes the initial conditions to realize the switch.

III. DETAILED ANALYSIS

In this section, we derive the equations governing the states of the switch in terms of the coupler parameters and the differential phase shift.

Substituting eqs. (2) through (5) in (10), we obtain

$$e^{j\phi_1} = \left[\frac{\gamma_1^2 + \cos^2\alpha_1}{\sin^2\alpha_1} \cdot \frac{\gamma_2^2 + \cos^2\alpha_2}{\sin^2\alpha_2} \right] e^j \left\{ \tan^{-1}\left[\frac{-\gamma_1 \tan\alpha_1}{(1+\gamma_1^2)^{\frac{1}{2}}} \right] + \tan^{-1}\left[\frac{-\gamma_2 \tan\alpha_2}{(1+\gamma_1^2)^{\frac{1}{2}}} \right] \right\}. \tag{14}$$

The phase shift ϕ_1 required to null port 5 is

$$\phi_1 = \tan^{-1}\left[\frac{-\gamma_1 \tan\alpha_1}{(1+\gamma_1^2)^{\frac{1}{2}}} \right] + \tan^{-1}\left[\frac{-\gamma_2 \tan\alpha_2}{(1+\gamma_2^2)^{\frac{1}{2}}} \right], \tag{15}$$

and the additional condition established in eq. (14) is

$$\frac{\gamma_1^2 + \cos^2\alpha_1}{\sin^2\alpha_1} = \frac{\sin^2\alpha_1}{\gamma_2^2 + \cos^2\alpha_2}. \tag{16}$$

Equation (16) relates the coupler parameters such that we have

* The effect of dimensional tolerances on individual coupler performance has been treated by several authors (Refs. 15–18). The effect of such tolerances on interference-switch performance has been considered by Shelton (Ref. 19).

couplers whose ratio of the power-coupling coefficients are the inverse of each other in *magnitude*, i.e., $|A_1|/|B_1| = |B_2|/|A_2|$. Inspection of (15) and (16) reveals that, for a given value of γ_1, α_1, there are many combinations of γ_2, α_2, and ϕ_1 that will meet the inverse condition requirement and therefore realize State I.

Equations (2) through (5) can be substituted in eq. (12) to yield the following conditions for the phase shift ϕ_2 required to null port 6:

$$\phi_2 = \pi + \tan^{-1}\left\{\frac{-\gamma_1 \tan \alpha_1}{(1+\gamma_1^2)^{\frac{1}{2}}}\right\} + \tan^{-1}\left\{\frac{-\gamma_2 \tan \alpha_2}{(1+\gamma_2^2)^{\frac{1}{2}}}\right\}. \tag{17}$$

Using eq. (15), (17) can be written as

$$\phi_2 = \phi_1 + \pi, \tag{18}$$

and the additional condition is

$$\frac{\gamma_1^2 + \cos^2 \alpha_1}{\sin^2 \alpha_1} = \frac{\gamma_2^2 + \cos^2 \alpha_2}{\sin^2 \alpha_2}. \tag{19}$$

Equation (19) illustrates the need for the ratio of the coupling coefficients to be identical, i.e., $|A_1|/|B_1| = |A_2|/|B_2|$. Again, we have many combinations γ_2, α_2, and ϕ_2 that will meet this requirement to realize State II for a given value of γ_1, α_1. Although (18) is written in terms of ϕ_1, note that the values of γ_1, α_1 and the associated values of γ_2, α_2, and ϕ_2 are distinctly different from those needed to realize State I.

Obviously, if we set $|B_1| = |B_2| = 1/\sqrt{2}$, i.e., if the couplers become hybrids, then eqs. (16) and (19) are simultaneously satisfied and both states can be simultaneously realized by simply changing ϕ by adding π. In other words, the dc biasing in each coupler is so chosen that (γ_1, α_1) and (γ_2, α_2) satisfy

$$\frac{|\sin \alpha_{1,2}|}{(\gamma_{1,2}^2 + 1)^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}. \tag{20}$$

For illustrative purposes, we consider only the fundamental solution of eq. (20) for positive γ, which is plotted in Fig. 2. The plot shows the normalized length of the coupler as a function of the normalized phase mismatch γ to achieve hybrid performance. In practice, the couplers as fabricated are not usually exact hybrids. For any phase mismatch between guides, the coupler lengths have to be within $\frac{1}{2}$ to $1\frac{1}{2}$ transfer lengths (L_0). In practice, this can be easily achieved.

To illustrate the adjustment for hybrid performance, let us consider two couplers with the following initial specifications:

Coupler x: $\gamma_x = 0.24$, $l/L_0 = 1.2$,
Coupler y: $\gamma_y = 1.1$, $l/L_0 = 1.1$.

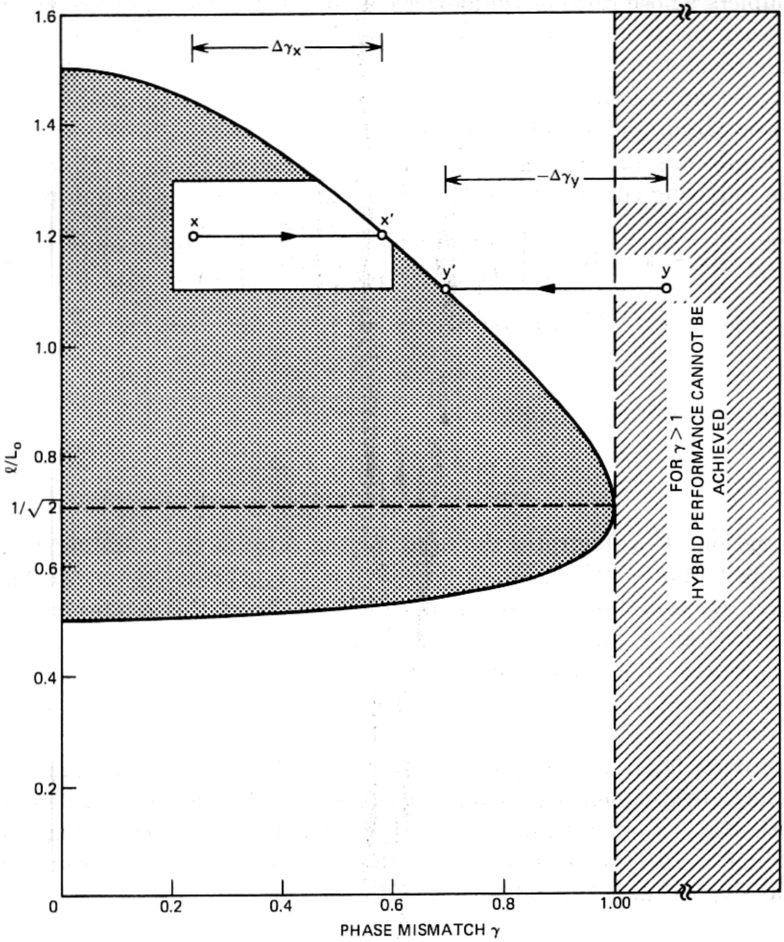

Fig. 2—Required coupler length (l/L_0) to obtain hybrid operation as a function of the phase mismatch parameter γ of the coupler.

As seen from Fig. 2, biasing the two couplers to change γ's by $\Delta\gamma_x$ and $-\Delta\gamma_y$, respectively, results in hybrid performance, with $\gamma_{x'} = 0.58$, $\gamma_{y'} = 0.7$. Then, initializing the phase shifter to a value ϕ_1 given by eq. (15), the switch can then be operated by changing the value of ϕ_1 by adding π.

To obtain the phase change ϕ_1, let us further assume the two couplers are identical in performance, i.e., $\gamma_1 = \gamma_2$, $\alpha_1 = \alpha_2$. (This assumption is easily satisfied, especially with fabrication processes using electron-beam lithography with analog scan or step-and-repeat techniques.)

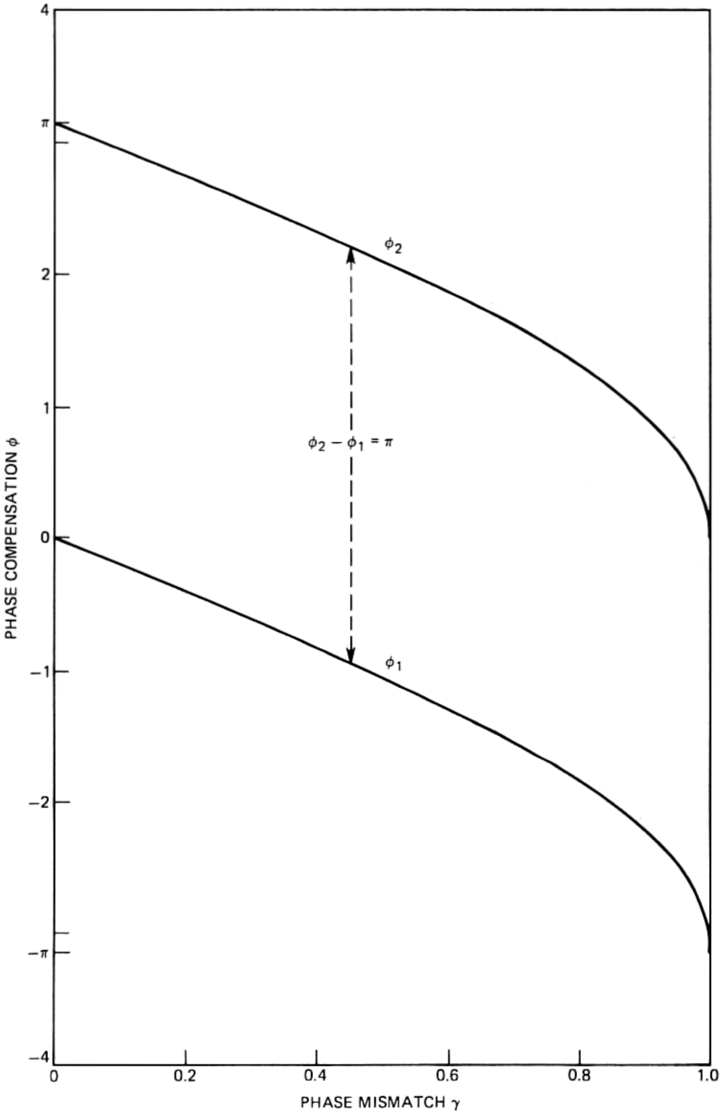

Fig. 3—ϕ_1 and ϕ_2 required to switch on State I and II, respectively, assuming identical coupler pairs and both being adjusted to provide hybrid operation.

For this case, then,

$$\phi_1 = 2 \tan^{-1}\left[\frac{-\gamma_1}{(1-\gamma_1^2)^{\frac{1}{2}}}\right] \tag{21}$$

and

$$\phi_2 - \phi_1 = \pi. \tag{22}$$

Equations (21) and (22) are plotted in Fig. 3. Assuming Ti-diffused $LiNbO_3$ strip waveguides,[20] the control voltage required to produce a phase shift of π is estimated to be 1 to 2 V for a 3-μm guide width and $L = 3$ mm. The special case with $\phi = 0$ is treated in the appendix.

IV. CONCLUSIONS

Equations have been established governing the states of a switching network consisting of two dephased, directional couplers interconnected via a differential phase network. The results imply that, in practice, one would strive to fabricate coupler pairs where power-coupling coefficients are greater in magnitude than 3 dB and, by appropriate bias, adjust each for 3-dB coupling. Two unique values of differential phase will then exist such that both switching states can be realized. The voltage required to switch between states is estimated to be of the order of 1 or 2 V.

APPENDIX

If we set $\phi_1 = 0$, then we have the equations governing the switch described in detail by Kogelnik et al.[21] It can be shown that, in addition to requirements of eq. (16) (State I), we must have

$$-\frac{\gamma_2}{\gamma_1} = \frac{\sin^2 \alpha_1}{\gamma_1^2 + \cos^2 \alpha_1} \tag{23}$$

and

$$\tan \alpha_2 = 2\,\frac{\sqrt{\gamma_1^2 + \cos^4 \alpha_1}}{\sin^2 \alpha_1}. \tag{24}$$

These simply imply that, given γ_1 and α_1, γ_2, and α_2 are uniquely fixed. This is because $\phi = 0$ is a unique state and the second coupler must satisfy both *magnitude* and *phase* conditions.

The graphical data presented by Kogelnik are quite useful in that the magnitude requirement of eqs. (16) and (19) is implied in those results.[21]

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