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Some Properties of the Variance of the Switch-Count Load

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Under equilibrium conditions, the load carried by a service system is defined as the average amount of traffic handled per unit of time. An unbiased estimate of this parameter is provided by the switch-count load, which is obtained by recording the number of busy servers at regular time intervals and then taking the arithmetic mean of these observations.

Formulas for the variance of this measurement (which are applicable to delay-and-loss systems with either finite- or infinite-source inputs and arbitrary defection rates) were derived in a previous paper[1]; a program for their computation is now available and has been used to explore effects of parameter changes on the switch-count load variance. The purpose of the present paper is to describe results of this investigation and, in particular, to draw attention to two properties which may be unexpected: (i) the variance of the switch-count load does not always decrease when waiting positions are added, and (ii) the variance of the carried-load estimate obtained from continuous observation over a given time interval is not a lower bound for the variance of load estimates calculated from a finite sequence of recording of the number of busy servers.

I. INTRODUCTION

When statistical equilibrium prevails, the load carried by a group of servers is defined as the average amount of traffic handled per unit of time. In telephony, this parameter is often evaluated by "switch-counting."[2,3] According to this method, the number of servers in use is recorded at regular time intervals; these numbers are then added together and their sum, divided by the number of observations (scans), is an unbiased estimate of the carried load. This measurement is called hereafter the switch-count load to distinguish it from estimates based on continuous observation. The latter are obtained by dividing the aggregated usage of the servers by the length of the measurement interval and can be viewed as limits of switch-count load measurements in which the number of scans tends to infinity while the length of the observation period is kept unchanged.

The problem of finding the variance of the switch-count load in loss systems with exponential service times has attracted a good deal of attention. Description of some earlier contributions to this subject can be found in Ref. 1.

In a recent paper,[1] formulas for the variance of the switch-count load were derived for delay-and-loss systems with state-dependent input rates and exponential service times. (As is customary, the state of the system at some instant, t, is defined as the number of customers who are either being served or are waiting at that time.) More precisely, the assumptions made here are as follows:

(*i*) Calls originate at rate λ_n (>0) whenever the system is in state n. We consider here only the two cases where the λ_n's are either independent of n (Poisson input) or are proportional to the number of idle sources (finite-source input).

(*ii*) Requests which originate when no free server is available are either delayed or lost; they are delayed if a waiting position is available and lost otherwise. While waiting, the requests are allowed to defect from the system at the same constant individual rate, j.

(*iii*) The service times are exponentially distributed; they are also independent of each other and of the state of the system. The average service time is taken throughout as the unit of time.

As in Ref. 1, we make the assumption that the number of servers and the number of waiting positions are finite. In the computations, however, we have to be more restrictive since numerical as well as storage problems may limit their ranges. But this, as it turns out, does not preclude investigations of rather large systems, and even of queues with Poisson input and infinite waiting room, so long as they can be approximated by systems that are computationally manageable. From a practical point of view, this treatment of the delay systems with unboundable queues has been found to be satisfactory even when they are nearly saturated.

A program for the computation of the formulas derived in Ref. 1 has been written and has been used to investigate the effects of parameter changes on the variance of the switch-count load—special attention being paid to the influence of delays on the variability of this measurement. The purpose of this expository paper, which serves as a complement to Ref. 1, is to describe the numerical results obtained thus far; these, in turn, suggest some general qualitative characteristics of the switch-count load variance that may be unexpected at times. These properties are "read off" the graphs and tables; they are stated and explained informally in the following discussions. Their proofs will be presented in a subsequent paper.

The original goal of the computations was to determine the scope of the approach developed in Ref. 1 and, principally, to determine its limits of accuracy as the number of servers and/or waiting positions become large. Some conclusions in this regard appear in Ref. 1. We mention here only that a thorough examination of the numerical stability of the computations was carried out for systems with as many as 400 devices and that, from a practical point of view, no significant loss in accuracy could be detected over this range. (By device, we mean here either a server or a waiting position.)

II. NOTATION AND DEFINITIONS

Following is a list of symbols and definitions used throughout:

c = number of servers,

d = number of devices (=number of servers + number of waiting positions),

s = number of sources (s is considered to be infinite whenever the input is Poissonian),

a = offered load in erlangs (this symbol is used only when the input is Poissonian),

λ = demand rate of an idle source—i.e., of a source that is neither being served nor waiting for service,

$\Lambda = s \cdot \lambda$ (this symbol as well as λ are used only in the case of finite-source inputs),

$h.t.$ = average service-time (used throughout as the unit of time),

j = individual defection rate of the waiting requests (j is Palm's j-factor; it is equal to zero whenever waiting requests do not defect and to infinity when waiting is not allowed),

T = length of the observation period (in multiples of $h.t.$),

n = number of recordings (scans) made during the observation period,

τ = interval between consecutive recordings of the number of busy servers in multiples of $h.t.$,

$N_c(t)$ = number of busy servers at time t ($0 \leqq N_c(t) \leqq c$),

$L_n(T)$ = switch-count load based on n scans spaced τ ($= T/n$) apart over an interval of length T.

Clearly, the n observations which enter in the computation of $L_n(T)$ can be made in many ways. If we take the beginning of the observation period as the time origin, then for any θ such that $0 \leqq \theta \leqq \tau$, the instants θ, $\theta + \tau$, $\theta + 2\tau$, $\cdots$, $\theta + (n-1)\tau$, constitute a possible scanning sequence. Under equilibrium conditions, the statistical properties of $L_n(T)$ are independent of θ and, for the sake of definiteness, we shall set it equal to τ so that

$$L_n(T) = \frac{1}{n}\left[N_c(\tau) + N_c(2\tau) + \cdots + N_c(n\tau)\right], \qquad n\tau = T.$$

When the switch-count load is measured as described above, we shall say, whenever emphasis is needed (and only then), that the measurement is of type I. Sections III through VI as well as VIII pertain only to these measurements. Type II measurements are introduced and dealt with in Section VII, while pertinent numerical examples are presented in Tables I through IV.

III. QUEUING EFFECT

Superficially, it would seem that, as time elapses, the number of busy servers is less volatile when waiting is allowed than when it is not. The reason sometimes advanced to support this view is that a comparison of a loss system with a delay system having the same number of servers would reveal that, for a given offered load, the mean number of busy servers tends to be smaller for the loss system than for the delay system, and that the "holes" in the carried-load process (see Fig. 1) of a loss system would be shortened and partially filled if the blocked calls were allowed to wait. If this were the case, the traffic fluctuations would be dampened and the conclusion could then be drawn that the variance of the switch-count load must decrease as the number of waiting positions increases. (Throughout this and the next three sections, the scanning rate is assumed to be fixed.) Ac-

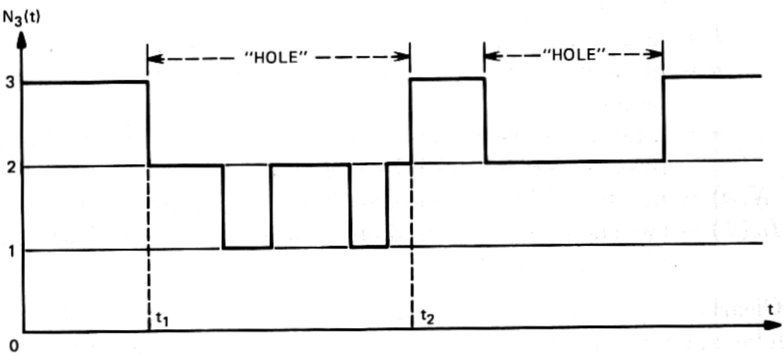

Fig. 1—Carried-load process in a loss system with three servers.

cordingly, for a given offered load, the variance of the switch-count load would be largest for loss systems and could therefore serve as an upper bound for the switch-count load variance in delay-and-loss systems.

Under the present conditions, the preceding argument is readily seen to be fallacious. Indeed, let t_1 be an instant at which an interval of full server occupancy terminates and let t_2 be the instant when, for the first time after t_1, all the servers are again occupied (see Fig. 1). Because of the assumptions made here regarding the input and the disposition of the requests, the behavior of the carried-load process over (t_1, t_2) is unaffected by what took place prior to t_1; hence, there cannot be any tendency for the "holes" to be filled since the distribution of their lengths remains the same and the evolution of the process over such "holes" is unchanged by the occurrence of delays. The only thing that happens is that the intervals of full uninterrupted occupancy tend to be of longer duration when queuing is permitted; the "holes" themselves are merely shifted.

The question of whether or not the variance of the switch-count load always decreases as the number of waiting positions increases has a clear-cut answer: it is "no," since the behavior of the switch-count load regarded as a function of the number of devices depends essentially on the offered load. We state next a few general properties.

So long as the offered load is light, the variance of the switch-count load increases monotonically as the number of waiting positions increases (see Fig. 2a). At higher loads, which, however, still fall below c, the variance first increases and then decreases monotonically toward a positive value as the number of waiting positions increases (see Fig. 2b). A similar behavior can be observed when $a = c$ and at "moderate" loads in excess of c, but with one difference, namely that the variance now tends to zero as the number of waiting positions tends to infinity (see Fig. 2c). Finally, when $a > c$ and is sufficiently high, the variance decreases monotonically and tends to 0 as the number of waiting positions increases (see Fig. 2d). It can be shown that the variance of the switch-count load always behaves in this manner and that, regardless of the number of servers, the lengths of the observation period, and the number of scans, each of the four patterns sketched in Fig. 2 does occur for some values of the offered load. Figure 3 depicts this rather intricate behavior of the switch-count load variance in the case of a four-server system with Poisson input and no defection from the queue.

For all values of T, the variance of $L_1(T)$ is equal to the variance, $\sigma^2_{c,d}$, of the equilibrium distribution of the number of busy servers.

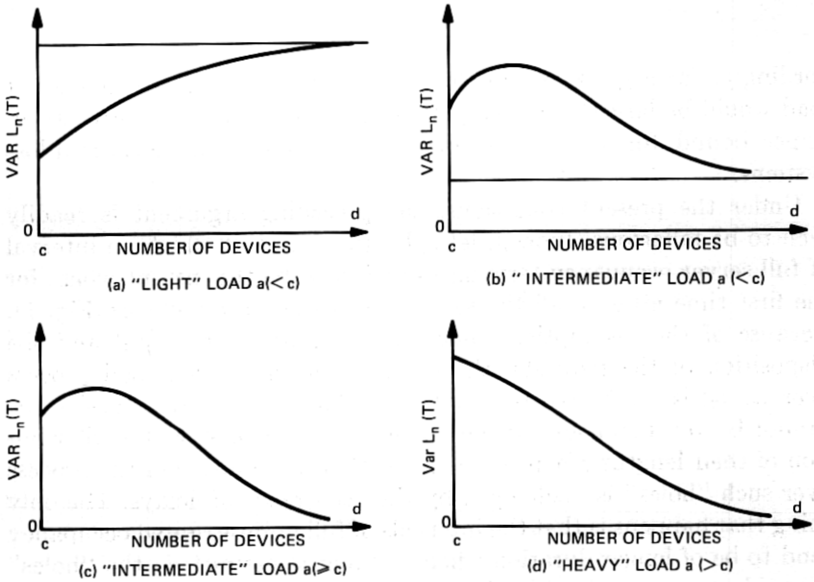

Fig. 2—Variance of switch-count load vs number of devices.

The latter, therefore, also display the characteristic behavior of the switch-count load variance. We can of course proceed in the opposite direction: we can first determine the behavior of $\sigma^2_{c,d}$ and, hence, of Var $L_1(T)$, and then anticipate some of the properties of the variance of $L_n(T)$. This is done next.

Let $P_{c,d}(n)$ be the equilibrium probability of n servers busy in a system with c servers and d devices. Let $\mathcal{P}_{c,d} \equiv \{P_{c,d}(0), P_{c,d}(1), \cdots, P_{c,d}(c)\}$, $\mathcal{P}_c \equiv \mathcal{P}_{c,c}$ and $\mathcal{U}_c$ be the distribution whose total probability mass is concentrated at c. Under the present conditions, the state probabilities are governed by the familiar birth-and-death equations and these imply that the ratios between the probabilities $P_{c,d}(n)$, $n = 0, 1, \cdots, c-1$, are independent of d. Hence,

$$\mathcal{P}_{c,d} = q_d\mathcal{P}_c + (1 - q_d)\mathcal{U}_c, \qquad 0 \leqq q_d \leqq 1,$$

where $(1 - q_d)$ is the probability that at least one waiting position is occupied ($q_c = 1$).

Let m_c be the mean of $\mathcal{P}_c$ and $\sigma^2_c \equiv \sigma^2_{c,c}$ its variance. Simple calculations show that

$$\sigma^2_{c,d} = q_d\sigma^2_c + q_d(1 - q_d)(c - m_c)^2.$$

Now, regarded as a function of q only,

$$V_c(q) \equiv q\sigma^2_c + q(1 - q)(c - m_c)^2$$

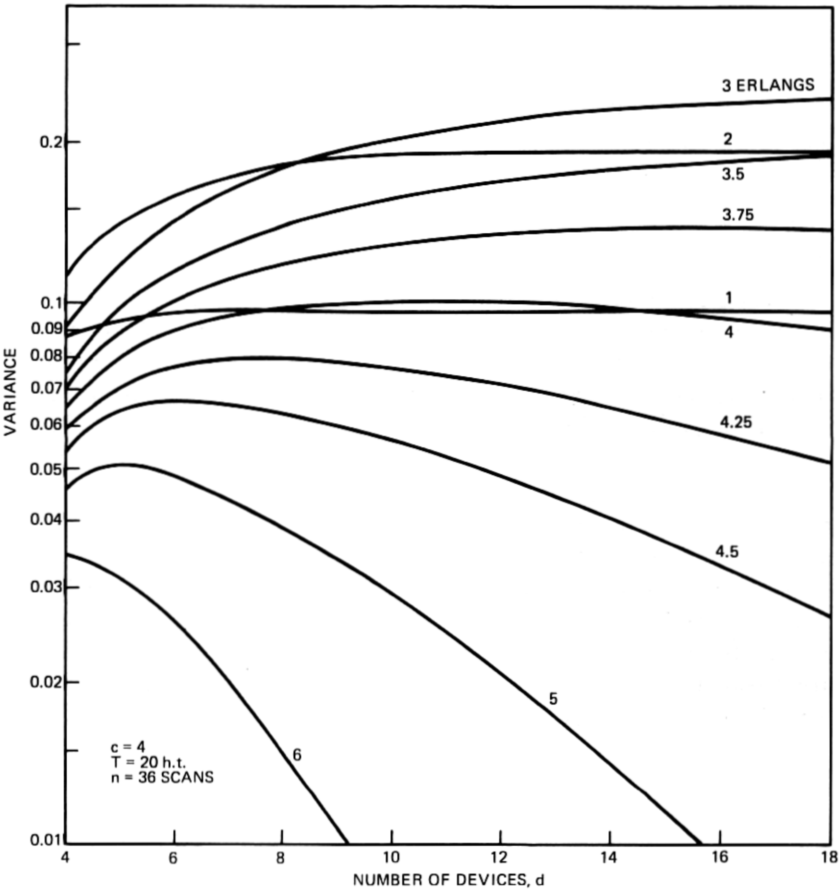

Fig. 3—Variance of the switch-count load (Poisson input).

is increasing whenever

$$q^* \equiv \frac{\sigma_c^2 + (c - m_c)^2}{2(c - m_c)^2} > q \qquad (\geqq 0),$$

and decreasing when $q^* < q$.

As the load tends to ∞ (c being kept fixed), q^* remains bounded away from 0 while q_d and q_{d+1} tend to 0. Hence, for sufficiently large values of a the probabilities q_d and q_{d+1} are both smaller than q^* and, since $q_{d+1} < q_d$ for all d's, we have

$$\sigma_{c,d+k+1}^2 - \sigma_{c,d+k}^2 = V_c(q_{d+k+1}) - V_c(q_{d+k}) < 0, \qquad k = 0, 1, 2, \cdots.$$

Conversely, as the load tends to 0, q^* tends to $\frac{1}{2}$ while q_d and q_{d+1} tend to 1. Hence, for sufficiently small loads, $q^* < q_{d+1} < q_d$ and the preceding inequality is reversed.

These considerations explain how the behavior of $\sigma_{c,d}^2 = \text{Var } L_1(T)$, as d varies, is governed by two simple facts, namely that (i) transfers of "sufficiently small" probability masses to the sample value that is farthest from the mean lead to distributions with greater variances, and that (ii) an increased concentration of the probability mass in the vicinity of the mean is accompanied by a decrease of the variance. As we have just seen, such changes of the probability distribution of the number of busy servers can be induced by changing the number of waiting positions. Thus, so long as the offered load, a, does not exceed a certain bound (a_1 in the example of Fig. 4), Var $L_1(T)$ increases monotonically as d increases. Conversely, whenever a is larger than a specific value (a_2 in Fig. 4), Var $L_1(T)$ decreases monotonically as d increases. However, because of the discreteness of d, this monotonic decrease may occur over a wider range (for $a \geqq a_3$ in Fig. 4; a_3, in this instance, falls just short of 4). Finally, there is an intermediate range ($a_1 \leqq a < a_3$ in Fig. 4) where Var $L_1(T)$ first increases and then decreases monotonically as d increases from c to ∞.

We now turn our attention to Var $L_n(T)$, $n > 1$. The behavior of this variance is more difficult to elucidate because $L_n(T)$ is now the arithmetic mean of n correlated random variables. The informal argument used in the preceding paragraphs may nevertheless be modified so as to cover this new situation. For given a, c, and d ($>c$),

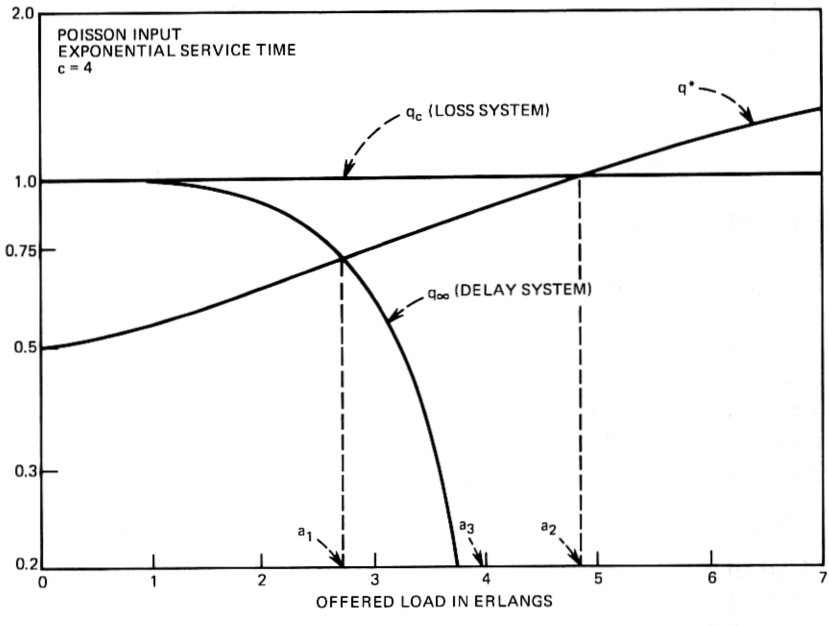

Fig. 4—Parameters q_c, q_∞, and q^* as functions of the offered load.

consider the aggregate (ensemble) of all possible switch-count load measurements over a given time interval of length T. This finite aggregate may be split into two disjoint classes: Class 1 includes all the measurements for which the number of busy devices does not exceed c at any of the scanning instants $\tau, 2\tau, 3\tau, \cdots, n\tau$; Class 2 comprises all the other measurements. [The possible values of Class 1 measurements are $0, 1/n, 2/n, \cdots, c(=cn/n)$, and those of Class 2 are $c/n, (c+1)/n, \cdots, c$.] Given that a measurement is of Class 1, let $\mathcal{P}_c^{(1)}$ be its conditional distribution. Similarly, let $\mathcal{P}_{c,d}^{(2)}$ be the conditional distribution of the Class 2 measurements.

Under the present assumptions, $\mathcal{P}_c^{(1)}$ is identical to the distribution of $L_n(T)$ for $d = c$ and is thus independent of d. The roles played by $\mathcal{P}_c$ and $\mathcal{U}_c$ above are now taken over by $\mathcal{P}_c^{(1)}$ and $\mathcal{P}_{c,d}^{(2)}$, respectively, and the distribution, $\mathcal{Q}_{c,d}$, of $L_n(T)$ is therefore given by

$$\mathcal{Q}_{c,d} = \mathbf{q}_d \mathcal{P}_c^{(1)} + (1 - \mathbf{q}_d)\mathcal{P}_{c,d}^{(2)},$$

where $\mathbf{q}_d$ is the probability that a measurement is of Class 1.

By assumption, the system is in equilibrium at time 0 and the probability that at least one waiting position is occupied at any given scanning instant increases with d. Thus, as d becomes larger, the proportion of Class 2 measurements increases. It also stands to reason, however, that the values of these measurements tend to be larger than those of Class 1 and that their magnitudes also increase with d. Thus, we may expect a greater proportion of relatively high load measurements as d increases. As before, and so long as the offered load is sufficiently small, the appearance of these "more extreme" values produce a scattering of the probability masses and this will tend to magnify the variance of $L_n(T)$. But when the offered load exceeds a certain level, the Class 1 measurements are, on the average, just about as large as those of Class 2 and increases of d make "relatively small" carried-load measurements less likely. This leads to a concentration of the probability masses about the mean of $L_n(T)$ and brings down its variance. These effects are clearly visible in Figs. 3, 5, and 6.

IV. EFFECT OF DEFECTIONS

In the preceding section we have attempted to explain how changes in the number of waiting positions affect the variance of the switch-count load. The arguments advanced depend on the fact that "more queuing" tends to increase the probability of full server-occupancy (the all-server-busy state) at the expense of the probabilities of the states $0, 1, 2, \cdots, c - 1$, while leaving the ratios between the latter unchanged. For fixed values of c and d, similar transfers of probability masses—with analogous consequences for Var $L_n(T)$—can be induced

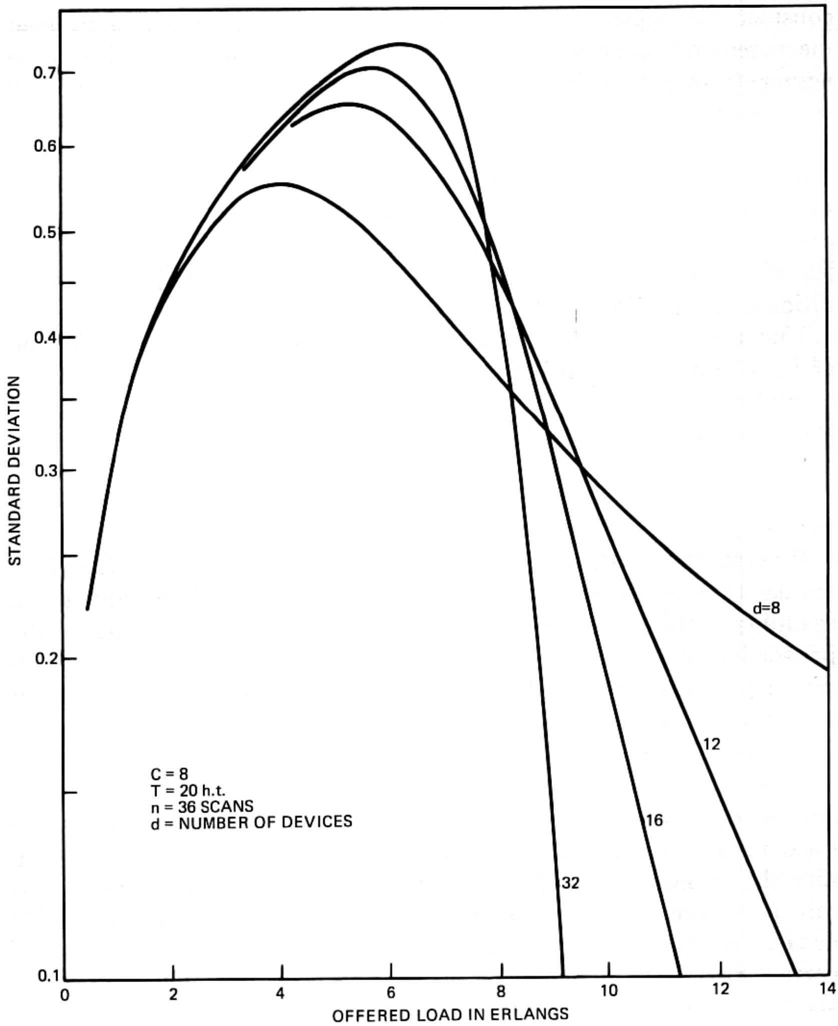

Fig. 5—Standard deviation of the switch-count load (Poisson input).

by varying the rates at which requests may defect from the waiting line.

If we assume, for example, that the requests defect at a constant individual probability rate, j, then the probability of full occupancy increases monotonically as j decreases. (Note that when $j = 0$, all delayed calls wait until served, and that whenever $j = \infty$, no waiting ever occurs and all the blocked calls are "lost." The familiar "blocked-calls-held" assumption corresponds to a j value of 1.) We may therefore expect that, for a given offered load, the variance of $L_n(T)$ will,

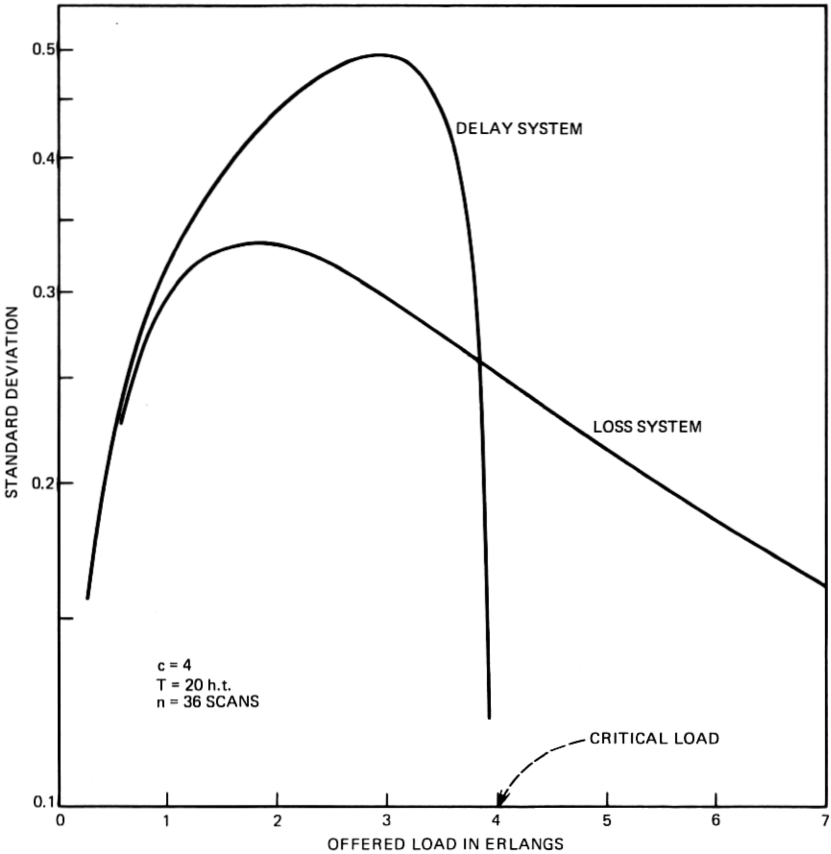

Fig. 6—Standard deviation of the switch-count load, delay vs loss system (Poisson input).

as j varies, display a similar behavior to that observed when the number of waiting positions changes. That this is indeed the case can be seen in Fig. 7. Hence, we may state the following: For given a, c, and d, and decreasing values of j, the variance of the switch-count load increases for sufficiently small values of the offered load and decreases whenever a is large enough. Also, there are values of the offered load for which intermediate defection rates do not imply intermediate values of Var $L_n(T)$.

V. FINITE-SOURCE EFFECT

A similar situation obtains—and admits of a similar explanation—when the input is generated by a finite number of sources (see Fig. 8). In this case, as the number of sources increases, so does the probability

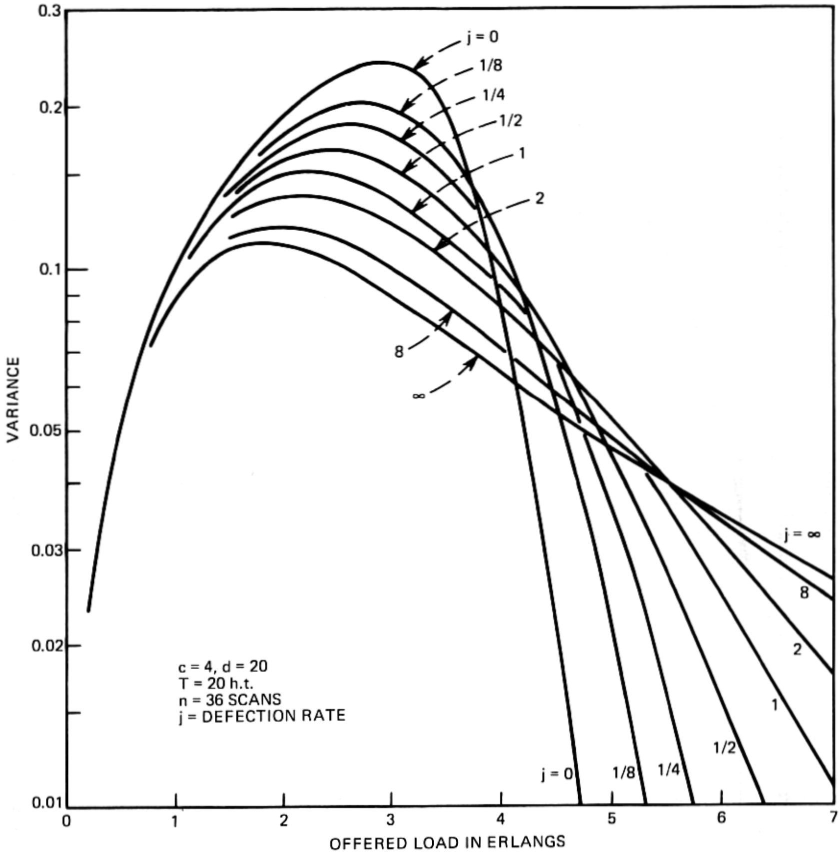

Fig. 7—Effect of defection rate on variance of the switch-count load, delay-and-loss system (Poisson input).

of waiting and Var $L_n(T)$, as a function of a, behaves as above. However, in these systems, the (overall) offered load is somewhat elusive as it depends not only on λ and the number of sources, s, but also on the structural parameters c and d and on the defection rate. Thus, the dependence of Var $L_n(T)$ on the number of sources is shown in Fig. 8 for prescribed values of $\Lambda = s \cdot \lambda$.

Since

$$\lambda_n = (s - n)\lambda = \Lambda \left(1 - \frac{n}{s}\right), \qquad n = 0, 1, \cdots, d - 1,$$

the rate λ_n, for any given n, therefore increases with s. This in turn implies that, for fixed Λ, the offered load increases with s. Hence, if the abscissa in Fig. 8 had been the offered load (instead of Λ), the

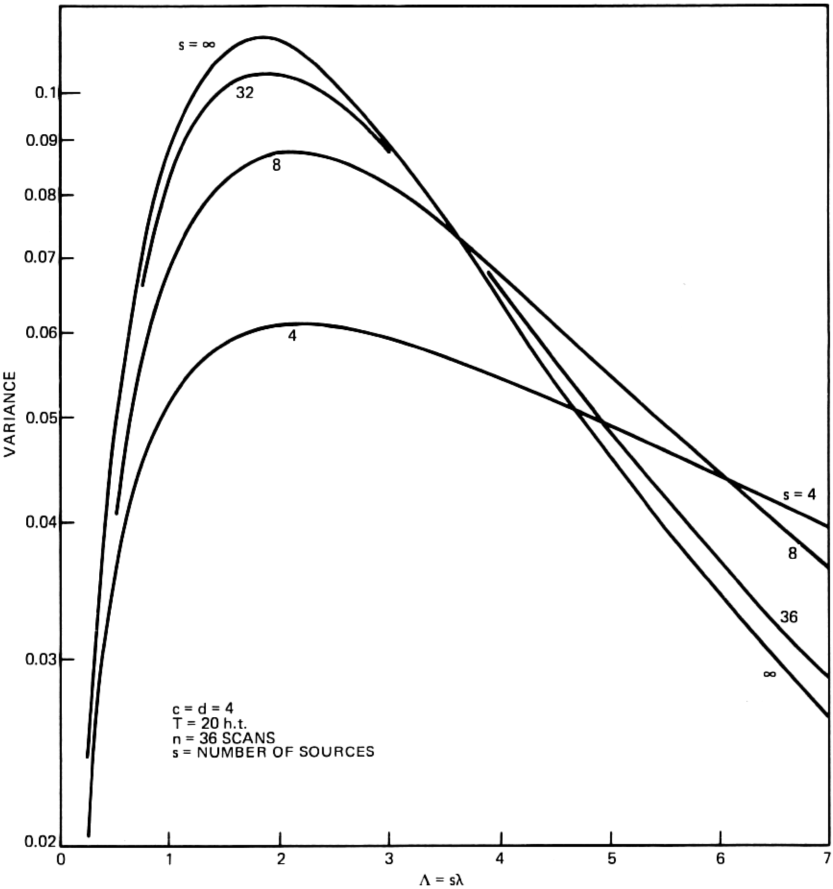

Fig. 8—Variance of the switch-count load, loss system (Poisson and finite-source inputs).

plotted points, with the exception of the origin and of those on the $s = \infty$ curve, would have been moved to the right. It is apparent, however, that such a change of abscissa would not have destroyed the overall incidence pattern depicted in Fig. 8.

VI. ANOTHER VIEW

We have studied thus far the effects of various parameter changes on Var $L_n(T)$ for known values of the (individual or overall) demand rates. This approach is particularly convenient since it gives us a direct handle on the state probabilities and, as we have seen, the behavior of Var $L_n(T)$ was then readily predictable in these terms. However, no general monotonicity property emerged within this framework. But if, instead of the offered load, we use the carried load as primary variable, we obtain a very different and far less intricate picture which, in turn, leads to a simple general rule.

From Figs. 9 to 12, we may infer that:

(*i*) For a given number of servers and a given value of the carried load, Var $L_n(t)$ decreases monotonically as the number of waiting positions decreases (see Fig. 9).

(*ii*) For given c and d, and a prescribed value of the carried load,

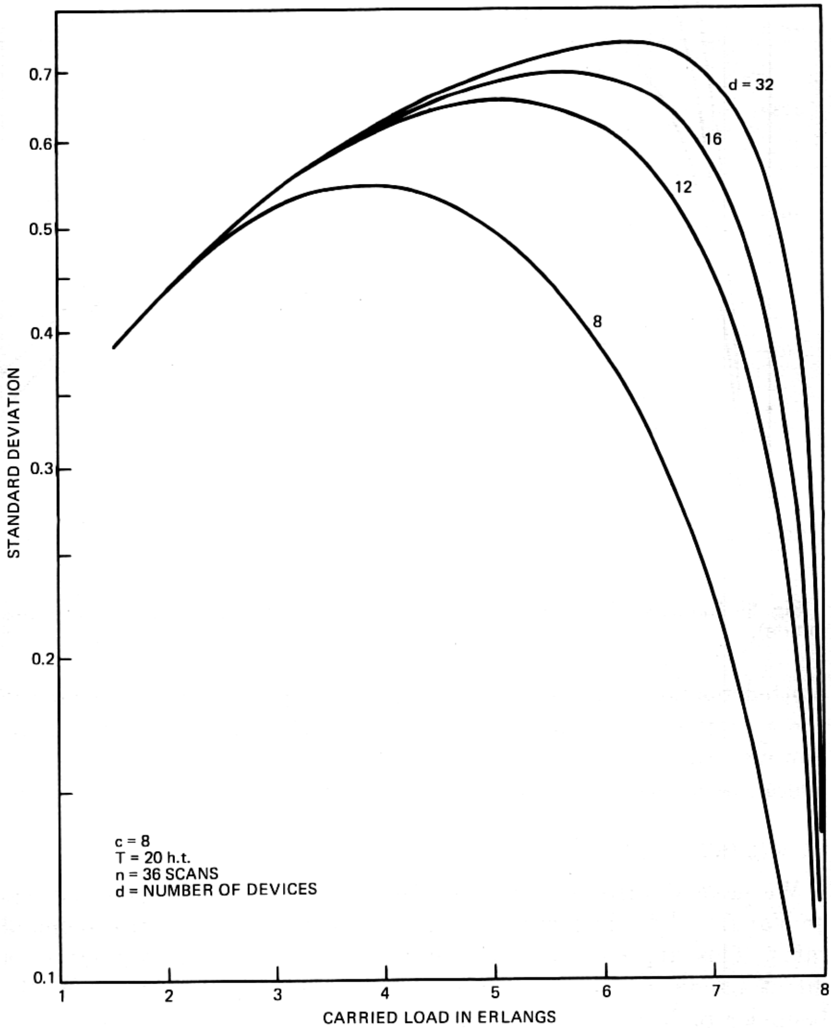

Fig. 9—Standard deviation of the switch-count load (Poisson input).

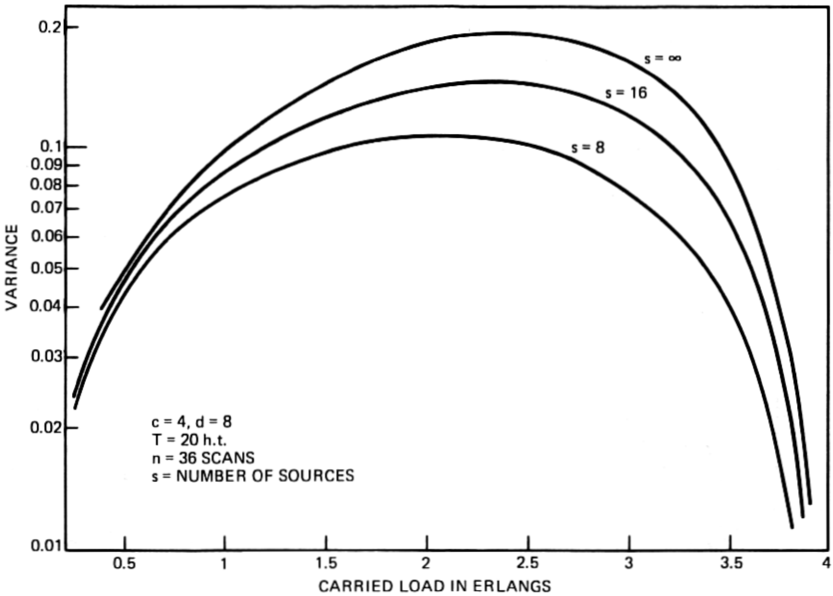

Fig. 10—Variance of the switch-count load, delay-and-loss system (Poisson and finite-source inputs).

Var $L_n(T)$ decreases monotonically as the number of traffic sources decreases (see Figs. 10 and 11).

(*iii*) For given c and d, and a fixed value of the carried load, Var $L_n(T)$ decreases monotonically as the defection rate, j, increases (see Fig. 12).

We note that these decreases of Var $L_n(T)$ are accompanied, in all cases, by increases in the number of calls that must be offered to maintain the carried load at a prescribed level. This brings us back to a "hole-filling" argument (cf. Section III) with a new twist, the prescription of the carried load that makes it operative. And now we see that, at a given occupancy, the "holes" are filled by allowing less rather than more queuing! Indeed, with less queuing, the average length of the busy periods is shortened and it is, therefore, necessary to fill the "holes" partially so as to maintain the carried load at its designated level. And with such compensatory fillings, the variance of the switch-count load can be expected to—and actually does—go down. These arguments provide us with an intuitive justification of inferences (*i*) through (*iii*) since, for a constant carried load, either decreasing the number of waiting positions or the number of sources or, alternatively, increasing the defection rate tends to reduce queuing.

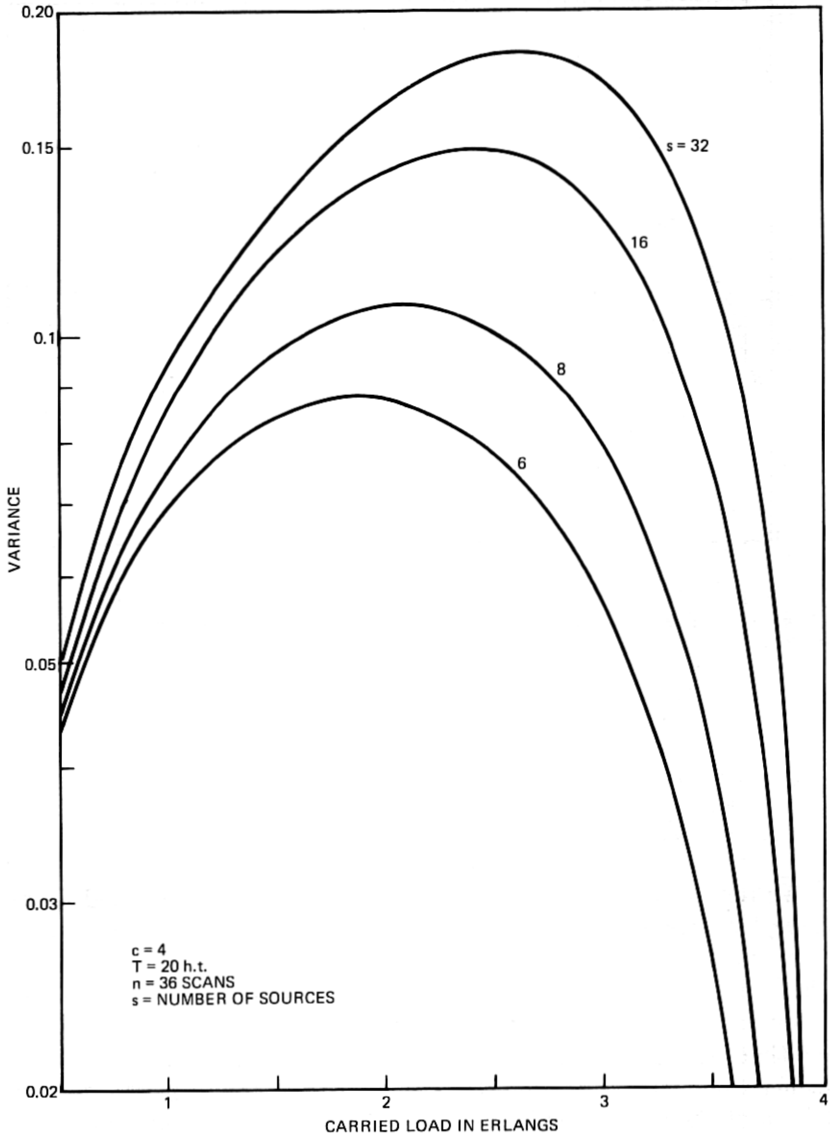

Fig. 11—Variance of the switch-count load, delay system (finite-source input).

The preceding considerations remain valid if one prescribes the blocking probability instead of the carried load. This is borne out by the data presented in Fig. 13.

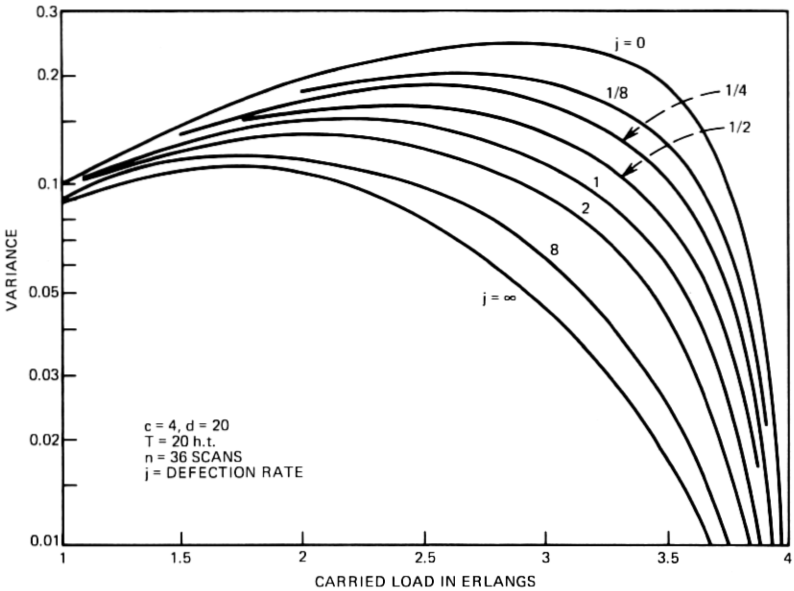

Fig. 12—Effect of the defection rate on the variance of the switch-count load, delay-and-loss system (Poisson input).

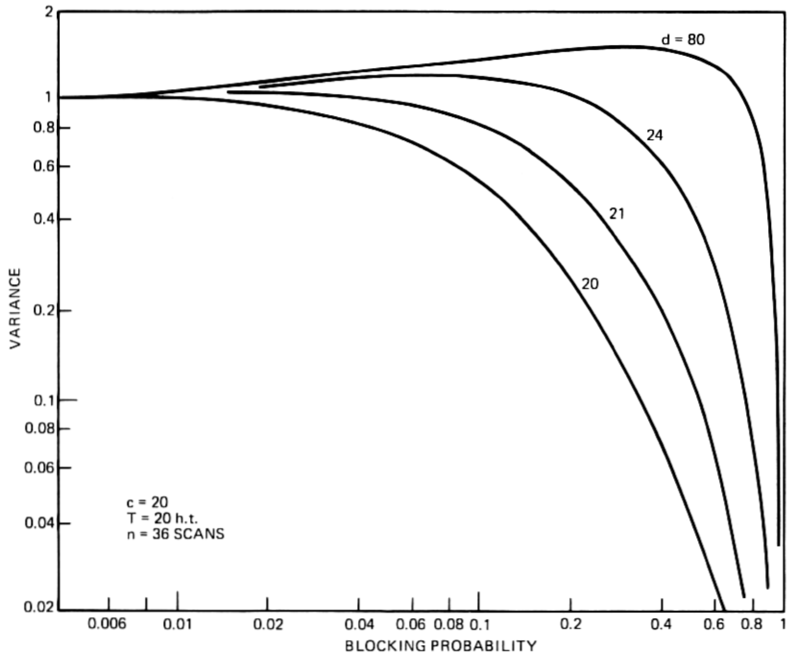

Fig. 13—Variance of the switch-count load vs blocking probability (Poisson input).

VII. EFFECT OF THE SCANNING RATE

We start with the statements of two properties. (In the sequel, c, d, s, and j are assumed fixed.)

(*i*) For a given value of the offered load and a given length of the observation period, the variance of the switch-count load decreases monotonically as the number of scans increases.

(*ii*) For a given offered load and a given number of scans, the variance of the switch-count load decreases monotonically as the length of the observation period increases.

All that is needed to prove the last assertion is a straightforward application of the familiar formula for the variance of a sum of correlated random variables (Ref. 4, pp. 229 ff) and use of the fact that the covariance, $R(t)$, between two observations of the number of busy servers made t apart decreases as t increases.

For given T and n, we have always assumed thus far that the n scans were made $\tau = T/n$ apart. Under this circumstance, it can be proved that (*i*) above is satisfied. But, as we shall see, (*i*) may fail to hold if the n scanning instants are chosen in a different way. We shall make use of this unexpected fact to show that the carried-load measurement obtained by continuous observation is never a minimum-variance estimate of the carried load.

For given T and n, the switch-count load was defined by the relation

$$L_n(T) \equiv \frac{1}{n}[N_c(\tau) + N_c(2\tau) + \cdots + N_c(n\tau)], \qquad n\tau = T.$$

In the following discussion, this measurement is said to be of type I and, accordingly, we shall designate it by $L_n^{\mathrm{I}}(T)$ instead of $L_n(T)$. The switch-count load measurements of type II are defined as follows:

$$L_n^{\mathrm{II}}(T) \equiv \frac{1}{n}\{N_c(0) + N_c(\tau') + \cdots + N_c[(n-1)\tau']\},$$

where $\tau' = T/(n-1)$. As defined here, measurements of type II differ from those of type I in that a recording is made at each end of the observation period (see Fig. 14). To avoid minor qualifications, we also define $L_1^{\mathrm{II}}(T)$ by setting it equal to $L_1^{\mathrm{I}}(T)$.

Figure 14 shows that each measurement of type I may also be regarded as a measurement of type II, and conversely. Thus, for $\tau = T/n$, the two random variables $L_n^{\mathrm{I}}(T)$ and $L_n^{\mathrm{II}}(T-\tau)$ are equidistributed for all n. In the present context, however, it is useful to make a distinction between type I and type II measurements because, as shown next, their respective variances do not behave in the same manner as the number of scans increases.

The load measurements $L_n^{\rm I}(T)$ and $L_n^{\rm II}(T)$ are sums of n correlated, but identically distributed, random variables, and their respective variances are, therefore, given by (see Ref. 4, p. 229):

$$\text{Var } L_n^{\rm I}(T) = \frac{1}{n^2}\{nR(0) + 2(n-1)R(\tau) + 2(n-2)R(2\tau) + \cdots + 2R[(n-1)\tau]\}, \qquad \tau = T/n,$$

and

$$\text{Var } L_n^{\rm II}(T) = \frac{1}{n^2}\{nR(0) + 2(n-1)R(\tau') + 2(n-2)R(2\tau') + \cdots + 2R[(n-1)\tau']\}, \qquad \tau' = T/(n-1),$$

where $R(0) = \sigma_{c,d}^2$.

For given T and n, the spacing between successive scans is greater for type II than for type I measurements ($\tau' > \tau$). Hence, since R decreases monotonically as its argument increases, we have $R(k\tau') < R(k\tau)$ for all k. It is then readily seen from the two preceding variances formulas that

$$\text{Var } L_n^{\rm I}(T) > \text{Var } L_n^{\rm II}(T), \qquad n = 2, 3, \cdots.$$

Furthermore, it is easy to prove that

$$\text{Var } L_\infty(T) = \lim_{n\to\infty} \text{Var } L_n^{\rm I}(T) = \lim_{n\to\infty} \text{Var } L_n^{\rm II}(T),$$

where $L_\infty(T)$ is the observed carried load obtained by continuous measurement over $(0, T)$. But whereas, according to statement (i), above, Var $L_n^{\rm I}(T)$ decreases monotonically towards Var $L_\infty(T)$ as n tends to infinity, Var $L_n^{\rm II}(T)$ first decreases to a value that lies below Var $L_\infty(T)$ and then increases monotonically towards Var $L_\infty(T)$ (see Fig. 15 and Tables I through IV, where all entries for which Var $L_n^{\rm II}(T)$ < Var $L_\infty(T)$ are italicized).

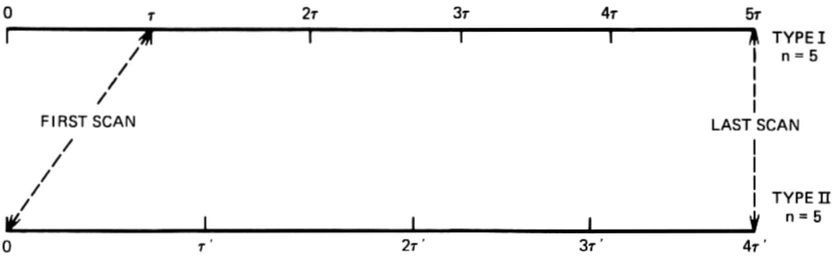

Fig. 14—Type I vs type II measurements.

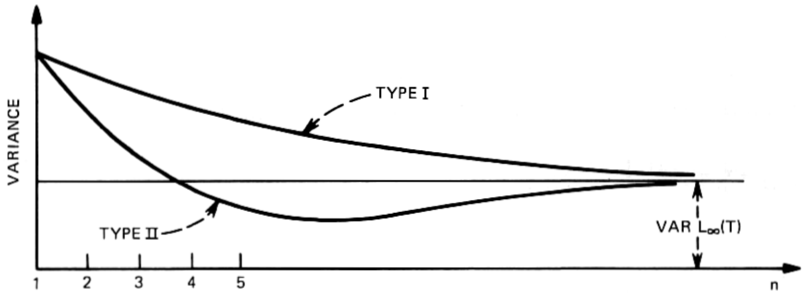

Fig. 15—Variance of type I and type II measurements.

Table I — Variance of the switch-count load for various offered loads
Poisson input; loss system, $c = d = 4$; $T = 15.36$ *h.t.*

No. of Scans	a = 0.25 erlang		a = 0.5 erlang		a = 1 erlang	
	Type I	Type II	Type I	Type II	Type I	Type II
13	0.034332	0.032435	0.067979	0.064228	0.124918	0.118142
17	0.032724	0.031185	0.064770	0.061727	0.118634	0.113110
21	0.031931	0.030650	0.063187	0.060653	0.115531	0.110921
25	0.031484	*0.030391**	0.062294	*0.060132*	0.113778	0.109840
33	0.031025	*0.030184*	0.061377	*0.059713*	0.111976	*0.108942*
49	0.030685	*0.030112*	0.060698	*0.059565*	0.110642	*0.108574*
65	0.030562	*0.030129*	0.060454	*0.059597*	0.110162	*0.108597*
∞	0.030401	0.030401	0.060133	0.060133	0.109529	0.109529
No. of Scans	**a = 1.5 erlangs**		**a = 2 erlangs**		**a = 2.5 erlangs**	
	Type I	Type II	Type I	Type II	Type I	Type II
13	0.156473	0.148318	0.163109	0.155147	0.154579	0.147698
17	0.147517	0.140800	0.152063	0.145398	0.142050	0.136159
21	0.143076	0.137442	0.146553	0.140916	0.135747	0.130709
25	0.140563	0.135736	0.143421	0.138570	0.132147	0.127783
33	0.137974	*0.134244*	0.140186	0.136419	0.128411	0.125000
49	0.136053	*0.133505*	0.137778	*0.135195*	0.125619	*0.123268*
65	0.135362	*0.133431*	0.136909	*0.134950*	0.124609	*0.122822*
∞	0.134449	0.134449	0.135761	0.135761	0.123270	0.123270
No. of Scans	**a = 3 erlangs**		**a = 3.5 erlangs**		**a = 4 erlangs**	
	Type I	Type II	Type I	Type II	Type I	Type II
21	0.119526	0.115301	0.102950	0.099532	0.088157	0.085446
25	0.115590	0.111901	0.098788	0.095772	0.083855	0.081434
33	0.111485	0.108576	0.094420	0.092017	0.079310	0.077355
49	0.108399	0.106382	0.091116	0.089435	0.075847	0.074466
65	0.107279	*0.105741*	0.089910	0.088626	0.074577	0.073517
129	0.106169	*0.105386*	0.088713	*0.088057*	0.073311	*0.072768*
257	0.105885	*0.105491*	0.088406	*0.088076*	0.072986	*0.072712*
∞	0.105790	0.105790	0.088303	0.088303	0.072876	0.072876

*All entries for which Var $L_n^{\text{II}}(T) <$ Var$_\infty$ (T) appear in italics.

Table II — Variance of the switch-count load for various offered loads

Poisson input; delay-and-loss system, $c = 4$, $d = 80$; $T = 15.36$ *h.t.*

No. of Scans	$a = 0.25$ erlang		$a = 0.5$ erlang		$a = 1$ erlang	
	Type I	Type II	Type I	Type II	Type I	Type II
13	0.034364	0.032464	0.068701	0.064901	0.136799	0.129203
17	0.032756	0.031215	0.065492	0.062411	0.130537	0.124388
21	0.031963	0.030680	0.063910	0.061345	0.127445	0.122331
25	0.031515	*0.030421*	0.063017	*0.060829*	0.125698	*0.121338*
33	0.031056	*0.030214*	0.062100	*0.060417*	0.123902	*0.120549*
49	0.030716	*0.030143*	0.061421	*0.060275*	0.122572	*0.120290*
65	0.030594	*0.030160*	0.061177	*0.060310*	0.122093	*0.120367*
∞	0.030432	0.030432	0.060855	0.060855	0.121463	0.121463

No. of Scans	$a = 1.5$ erlangs		$a = 2$ erlangs		$a = 2.5$ erlangs	
	Type I	Type II	Type I	Type II	Type I	Type II
13	0.202358	0.191037	0.261618	0.246937	0.306471	0.289678
17	0.193599	0.184477	0.251394	0.239647	0.296204	0.282872
21	0.189262	0.181690	0.246311	0.236590	0.291074	*0.280080*
25	0.186807	*0.180358*	0.243426	*0.235159*	0.288151	*0.278822*
33	0.184278	*0.179324*	0.240445	*0.234105*	0.285122	*0.277981*
49	0.182401	*0.179031*	0.238225	*0.233917*	0.282856	*0.278012*
65	0.181725	*0.179175*	0.237423	*0.234165*	0.282035	*0.278374*
∞	0.180831	0.180831	0.236361	0.236361	0.280944	0.280944

No. of Scans	$a = 3$ erlangs		$a = 3.5$ erlangs		$a = 3.75$ erlangs	
	Type I	Type II	Type I	Type II	Type I	Type II
13	0.316095	0.300266	0.242082	0.232099	0.149011	0.143657
17	0.307459	0.294979	0.236873	0.229033	0.146173	0.141970
21	0.303119	*0.292863*	0.234234	*0.227809*	0.144733	*0.141286*
25	0.300638	*0.291949*	0.232727	*0.227285*	0.143905	*0.140988*
33	0.298056	*0.291418*	0.231147	*0.226994*	0.143037	*0.140812*
49	0.296116	*0.291620*	0.229954	*0.227144*	0.142380	*0.140874*
65	0.295410	*0.292014*	0.229518	*0.227396*	0.142140	*0.141003*
∞	0.294469	0.294469	0.228934	0.228934	0.141817	0.141817

An immediate consequence of this phenomenon is that $L_\infty(T)$ is *not* a minimum variance estimate of the load carried over intervals of length T.

To shed some light on how this behavior of Var $L_n^{\text{II}}(T)$ comes about, we consider the following simple examples.

Let X_1, X_2, and X_3 be identically distributed random variables with variances σ^2, and assume that Cov (X_1, X_2) = Cov $(X_2, X_3) = \sigma^2\theta$ and that Cov $(X_1, X_3) = \sigma^2\theta^2$, where $0 \leqq \theta \leqq 1$. (It is readily shown that this particular choice of the covariances is legitimate.)

Table III — Variance of the switch-count load for various lengths of observation period

Poisson input, $a = 2$ erlangs; loss system, $c = d = 4$

No. of Scans	$T = 0.48$		$T = 0.96$		$T = 1.92$	
	Type I	Type II	Type I	Type II	Type I	Type II
1	1.392290	1.392290	1.392290	1.392290	1.392290	1.392290
2	1.207470	*1.073203*	1.073203	*0.902186*	0.902186	0.758072
3	1.172790	*1.086189*	1.011540	*0.890833*	0.799280	*0.674767*
5	1.154976	*1.106250*	0.979610	*0.908806*	0.744683	*0.664710*
9	1.148037	*1.122470*	0.967124	*0.929260*	0.723056	*0.678623*
17	1.145806	*1.132783*	0.963103	*0.943647*	0.716056	*0.692850*
33	1.145168	*1.138609*	0.961952	*0.952112*	0.714048	*0.702227*
49	1.145042	*1.140660*	0.961725	*0.955143*	0.713652	*0.705726*
65	1.144997	*1.141707*	0.961644	*0.956700*	0.713510	*0.707550*
∞	1.144938	1.144938	0.961537	0.961537	0.713323	0.713323

No. of Scans	$T = 3.84$		$T = 7.68$		$T = 15.36$	
	Type I	Type II	Type I	Type II	Type I	Type II
1	1.392290	1.392290	1.392290	1.392290	1.392290	1.392290
2	0.758072	0.701766	0.701766	0.696191	0.696191	0.696145
3	0.599079	0.521641	0.489334	0.469113	0.465107	0.464138
5	0.508964	*0.446917*	0.390314	0.320960	0.288003	0.282078
9	0.471908	*0.434391*	0.287193	0.264847	0.190330	0.181270
17	0.459714	*0.439503*	0.265409	*0.252440*	0.152063	0.145398
33	0.456197	*0.445779*	0.259008	*0.252168*	0.140186	0.136419
49	0.455501	*0.448493*	0.257734	*0.253109*	0.137778	*0.135195*
65	0.455252	*0.449974*	0.257277	*0.253786*	0.136909	*0.134950*
∞	0.454923	0.454923	0.256674	0.256674	0.135761	0.135761

Under these conditions, we have

$$\text{Var}\,\frac{X_1 + X_3}{2} = \frac{\sigma^2}{4}\,(2 + 2\theta^2)$$

and

$$\text{Var}\,\frac{X_1 + X_2 + X_3}{3} = \frac{\sigma^2}{9}\,(3 + 4\theta + 2\theta^2).$$

Hence,

$$\text{Var}\,\frac{X_1 + X_2 + X_3}{3} > \text{Var}\,\frac{X_1 + X_3}{2},$$

whenever $3 - 8\theta + 5\theta^2 < 0$ or, equivalently, when $3/5 < \theta < 1$. Thus, the preceding inequality holds so long as the correlation, θ, between X_2 and either X_1 or X_3 is sufficiently large to wipe out the accuracy gains that usually accrue by increasing the sample sizes.

The preceding model applies without alteration to single-server loss systems with Poisson input. If, in this instance, we take $X_1 = N_1(0)$,

Table IV — Variance of the switch-count load for various lengths of observation period

Poisson input, $a = 2$ erlangs; delay-and-loss system, $c = 4$, $d = 80$

No. of Scans	$T = 0.48$		$T = 0.96$		$T = 1.92$	
	Type I	Type II	Type I	Type II	Type I	Type II
1	1.652174	1.652174	1.652174	1.652174	1.652174	1.652174
2	1.481714	*1.352457*	1.352457	*1.173156*	1.173156	*0.987875*
3	1.449704	*1.367446*	1.294822	*1.172862*	1.075209	*0.931136*
5	1.433250	*1.387236*	1.264970	*1.194538*	1.023479	*0.934834*
9	1.426839	*1.402759*	1.253287	*1.215878*	1.002992	*0.954575*
17	1.424778	*1.412527*	1.249524	*1.230361*	0.996354	*0.971246*
33	1.424188	*1.418022*	1.248446	*1.238769*	0.994449	*0.981700*
49	1.424071	*1.419953*	1.248234	*1.241764*	0.994073	*0.985534*
65	1.424030	*1.420938*	1.248158	*1.243298*	0.993938	*0.987519*
∞	1.423975	1.423975	1.248057	1.248057	0.993760	0.993760

No. of Scans	$T = 3.84$		$T = 7.68$		$T = 15.36$	
	Type I	Type II	Type I	Type II	Type I	Type II
1	1.652174	1.652174	1.652174	1.652174	1.652174	1.652174
2	0.987875	0.869191	0.869191	0.831006	0.831006	0.826225
3	0.832771	0.713694	0.649697	0.591226	0.568974	0.555158
5	0.746706	*0.663085*	0.514116	0.459861	0.382168	0.360649
9	0.711598	*0.663341*	0.454556	*0.419120*	0.287082	0.268209
17	0.700048	*0.674474*	0.434212	*0.414579*	0.251394	0.239647
33	0.696712	*0.683611*	0.428232	*0.418021*	0.240445	*0.234105*
49	0.696051	*0.687254*	0.427040	*0.420157*	0.238225	*0.233917*
65	0.695814	*0.689194*	0.426612	*0.424232*	0.237423	*0.234165*
∞	0.695502	0.695502	0.426047	0.426047	0.236361	0.236361

$X_2 = N_1(T/2)$, $X_3 = N_1(T)$, then $\theta = \exp[-(1+a)(T/2)]$ and $\text{Var } L_3^{\text{II}}(T) > \text{Var } L_2^{\text{II}}(T)$, provided T is small enough. Hence, $\text{Var } L_n^{\text{II}}(T)$ does not necessarily decrease as n increases. This simple system is used next to construct examples in which $\text{Var } L_2^{\text{II}}(T)$ is smaller than $\text{Var } L_\infty(T)$.

For $c = d = 1$ and Poisson input, it can be shown that

$$R(t) = \sigma^2 e^{-(1+a)t}.$$

Since, however,

$$\begin{aligned}\text{Var } L_\infty(T) &= \lim_{n\to\infty} \text{Var } L_n^{\text{I}}(T) = \lim_{n\to\infty} \text{Var } L_n^{\text{II}}(T) \\ &= \lim_{n\to\infty} \frac{1}{n^2}\{nR(0) + 2(n-1)R(\tau) + 2(n-2)R(2\tau) \\ &\qquad + \cdots + 2R[(n-1)\tau]\} \\ &= \frac{\sigma^2}{T^2}\int_0^T (T-t)\cdot R(t)dt,\end{aligned}$$

a simple calculation shows that

$$\text{Var } L_\infty(T) = \frac{2\sigma^2}{(1+a)T^2}\left[T - \frac{1}{1+a} + \frac{e^{-(1+a)T}}{1+a}\right].$$

Consequently,

$$\text{Var } L_2^{\text{II}}(T) < \text{Var } L_\infty(T)$$

whenever

$$4(\zeta - 1 + e^{-\zeta}) > \zeta^2(1 + e^{-\zeta}), \qquad \zeta \equiv (1+a)T.$$

The preceding inequality is satisfied so long as

$$0 < \zeta \equiv (1+a)T < 2,$$

and, for any given a (>0), Var $L_\infty(T)$ therefore exceeds Var $L_2^{\text{II}}(T)$ provided $0 < T < 2/(1+a)$. (It is easy to find examples of multi-server systems for which Var $L_2^{\text{II}}(T) <$ Var $L_\infty(T)$. Illustrations of this type can be found in Tables I to IV).

The preceding results admit of the following generalization: For any given n ($\geqq 2$) and a, the inequality Var $L_n^{\text{II}}(T) <$ Var $L_\infty(T)$ holds provided T is small enough (see Tables III and IV).

We note next that the behavior of Var $L_n^{\text{II}}(T)$ as a function of n is an immediate consequence of the following three properties: For any given a and T,

(*i*) There is an n such that Var $L_n^{\text{II}}(T) <$ Var $L_{n+1}^{\text{II}}(T)$.
(*ii*) If Var $L_n^{\text{II}}(T) <$ Var $L_{n+1}^{\text{II}}(T)$ for some n, then Var $L_{n+k}^{\text{II}}(T) <$ Var $L_{n+k+1}^{\text{II}}(T)$ for all k's.
(*iii*) $\lim_{n\to\infty}$ Var $L_n^{\text{II}}(T) =$ Var $L_\infty(T)$.

Only the last of these three properties, which, of course, also holds for type I measurements, can be regarded as evident. The other two do not admit of a simple explanation since they reflect numerical attributes of the covariance function whose impact is hard to anticipate. Hence, the fact that (*i*) is valid for type II but not for type I measurements appears to be essentially fortuitous. That Var $L_n^{\text{I}}(T)$ displays, as n varies, a simpler and probably more frequently observed behavior than Var $L_n^{\text{II}}(T)$ does not invalidate the preceding remarks since the monotonicity of Var $L_n^{\text{I}}(T)$, as n varies, does not follow in an obvious way from general principles.

When all the parameters but n are prescribed, the ratios and, hence, the inequalities between the variances of either type I and/or type II measurements are independent of σ^2. These inequalities can then be expressed in terms of the correlation function $\rho(\cdot) \equiv R(\cdot)/\sigma^2$, and it turns out that, for any n and all $k \geqq 0$,

$$\text{Var } L_{n+k}^{\text{II}}(T) < \text{Var } L_\infty(T),$$

provided $\rho(\tau)$ is large enough. Some properties of Var $L_n^{\text{II}}(T)$, regarded as a function of n, are expressed next in terms of ρ.

Except for the offered load, a, which is allowed to vary, let all parameters be prescribed and let $\rho(t)$ be the correlation between two observations made t apart. As sketched in Fig. 16, $\rho(t)$ has the following properties valid for all t's (see also Figs. 17 and 18 for closely related results):

(*i*) In loss systems, $\rho(t)$ decreases monotonically as a increases and tends to 0 as a tends to infinity.
(*ii*) In delay systems, $\rho(t)$ increases monotonically as a increases and tends to 1 as a tends to the critical load c.

Let **n** be the smallest n for which $L_n^{\text{II}}(T) < L_\infty(T)$. As stated above, this inequality is satisfied whenever $\rho(\tau)$ is large enough. Hence, by means of (*i*) and (*ii*), we may conclude that

(1) In loss systems, **n** cannot increase as a increases (see Table I).
(2) In delay systems, **n** cannot decrease as a increases (see Table II).

Furthermore, since for all systems considered here, $\rho(t)$ decreases monotonically as t increases we have

(3) In delay-and-loss systems with arbitrary defection rates, **n** cannot decrease as the length of the observation period increases (all other parameters being kept fixed). This is illustrated in Tables III and IV.

As the number of waiting positions increases, so does $\rho(t)$ and we therefore also have

(4) In delay-and-loss systems with fixed a, c, and j, **n** cannot increase as d increases. (This assertion can be partially checked by comparing the data of Table I with those of Table II.)

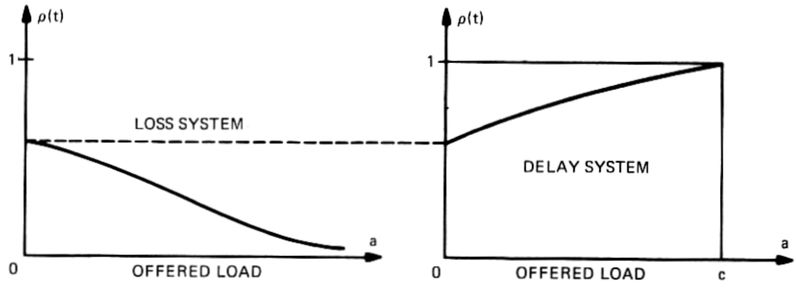

Fig. 16—Correlation between two observations made t apart vs the offered load.

[Note that for the values assigned to the parameters, the delay-and-loss system of Table II actually behaves like a (pure) delay system and, practically speaking, this table pertains to such a system. In the title of this table, however, the term "delay-and-loss" and the statement that $d = 80$ are retained so as not to obscure the conditions under which the computations were made.]

As can be seen from Tables I to IV, the difference between $\text{Var } L_n^{\text{II}}(T)$ and $\text{Var } L_\infty(T)$ is quite small and is certainly negligible in practical situations. The phenomenon studied in this section is of interest, however, because it contradicts a well-rooted feeling that a greater amount of information cannot entail a loss of accuracy; more important, however, is its implication that the common notion that the variance of the switch-count load can be regarded as the sum of the variances of the "source load" and of the switch-count error[2] is not unconditionally valid.

VIII. THE AUTOCOVARIANCE FUNCTION FOR SEQUENCES OF LOAD MEASUREMENTS

For the purpose of forecasting and/or controlling traffic volumes on trunk groups and switching devices, carried-load measurements are frequently performed over successive (nonoverlapping) intervals. The statistical analysis of such sequences of observations depends essentially on a knowledge of the autocovariance function (defined below). We shall therefore show how it can be computed by means of the variance formula derived in Ref. 1 and then describe some properties of the corresponding autocorrelation function.

From here on, we assume that all measurements are of type I and designate by $L_n(t, T)$ the switch-count load over $(t, t + T]$ $[L_n(0, T) \equiv L_n(T)]$. Then the autocovariance function, $\mathcal{R}_{nk}(T)$, for a sequence of observations performed over the nonoverlapping intervals $(0, T]$, $(T, 2T]$, $(2T, 3T]$, $\cdots$, is defined by

$$\mathcal{R}_{nk}(T) \equiv \text{Cov } \{L_n(0, T), L_n[kT, (k + 1)T]\}, \qquad k = 0, 1, \cdots.$$

For given n and T, this covariance is easily calculated for any value of k as soon as $\text{Var } L_{mn}(mT)$ is known for $m = 1, 2, \ldots, k + 1$. Indeed, for $k = 1$ we have (by the formula for the variance of sums of correlated variables)

$$4 \text{ Var } L_{2n}(2T) = 2 \text{ Var } L_n(T) + 2 \text{ Cov } [L_n(0, T), L_n(T, 2T)],$$

so that

$$\mathcal{R}_{n1}(T) \equiv \text{Cov } [L_n(0, T), L_n(T, 2T)] = 2 \text{ Var } L_{2n}(2T) - \text{Var } L_n(T).$$

Similarly, for $k = 2$, we have

$$9 \text{ Var } L_{3n}(3T) = 3 \text{ Var } L_n(T) + 4 \text{ Cov } [L_n(0, T), L_n(T, 2T)] + 2 \text{ Cov } [L_n(0, T), L_n(2T, 3T)],$$

so that

$$\mathcal{R}_{n2}(T) = \tfrac{9}{2}\,\mathrm{Var}\,L_{3n}(3T) - \tfrac{3}{2}\,\mathrm{Var}\,L_n(T) - 2\mathcal{R}_{n1}(T).$$

Hence, by a simple inductive process, we obtain the following expression:

$$\mathcal{R}_{nk}(T) = \frac{k^2}{2}\,\mathrm{Var}\,L_{kn}(kT) - \frac{k}{2}\,L_n(T) - (k-2)\mathcal{R}_{n2}(T) \\ - (k-3)\mathcal{R}_{n3}(T) - \cdots - 2\mathcal{R}_{n,k-1}(T).$$

By means of these formulas the (auto)correlation function, $\Gamma(k) \equiv \mathcal{R}_{nk}(T)/\sigma^2$, was computed for some loss, delay, and delay-and-loss systems. These results, which are presented in Figs. 17 and 18, suggest the following properties. In loss systems, the coefficient of correlation $\Gamma(k)$ for fixed $k(\geqq 1)$ increases monotonically as the load decreases and satisfies the following inequalities:

$$0 \leqq \Gamma(k) \leqq \Gamma_0(k) < 1,$$

where

$$\Gamma_0(k) \equiv \lim_{a \to 0} \Gamma(k).$$

Furthermore,

$$\Gamma_\infty(k) \equiv \lim_{a \to \infty} \Gamma(k) = 0.$$

In (pure) delay systems, the behavior of $\Gamma(k)$ is quite different. In this case, $\Gamma(k)$, for fixed $k \geqq 1$, increases monotonically as a increases and

$$\Gamma_0(k) \leqq \Gamma(k) \leqq \Gamma_\infty(k) = 1,$$

where $\Gamma_0(k)$ and $\Gamma_\infty(k)$ are defined as above. (Note that $\Gamma_0(k)$ is independent of the number of waiting positions.)

The dependence of $\Gamma(k)$ on a is somewhat more complicated in delay-and-loss systems. In this last instance, $\Gamma(k)$, for fixed $k(\geqq 1)$, first increases as a increases, reaches a maximum $\mathbf{\Gamma}(k)$, and then decreases monotonically as a further increases. $\Gamma(k)$ now satisfies the following inequalities:

$$\Gamma_\infty(k) = 0 \leqq \Gamma(k) \leqq \mathbf{\Gamma}(k),$$

where

$$\mathbf{\Gamma}(k) \equiv \max_a \Gamma(x) > \Gamma_0(k).$$

It is easy to show that $\mathcal{R}_{nk} = \sigma^2\Gamma(k)$ is asymptotically exponential for large values of k. But, as can be inferred from the behavior of $\Gamma(k)$ (see Figs. 17 and 18), deviation from exponentiality is rather pronounced for small k's. Hence, the assumption sometimes made in practice that $\mathcal{R}_{nk}$, regarded as a function of k, is exponential requires further investigation. In this connection, it seems that, at the very least, any fitting covariance function, $\mathcal{R}^*_{nk}$, should not be subjected to the requirement that $\mathcal{R}^*_{n0}(T) = \text{Var } L_n(T)$.

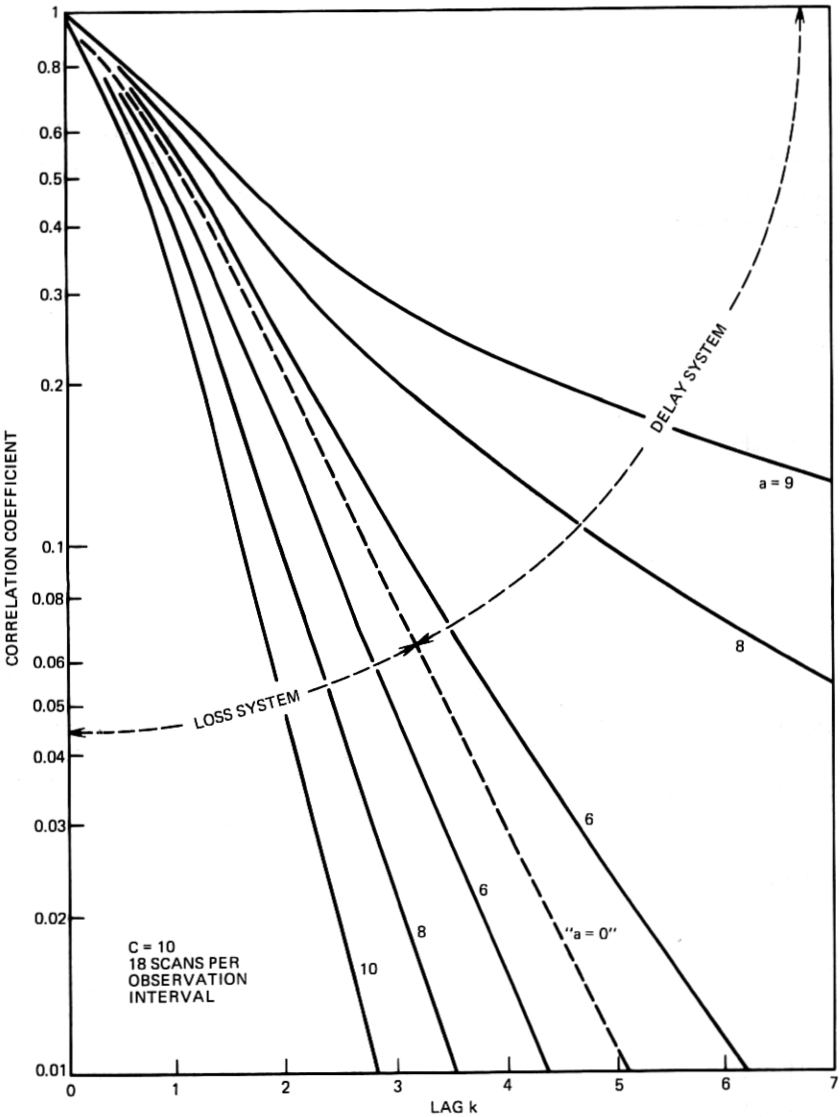

Fig. 17—Correlation between load measurements made over nonoverlapping intervals of unit length (Poisson input).

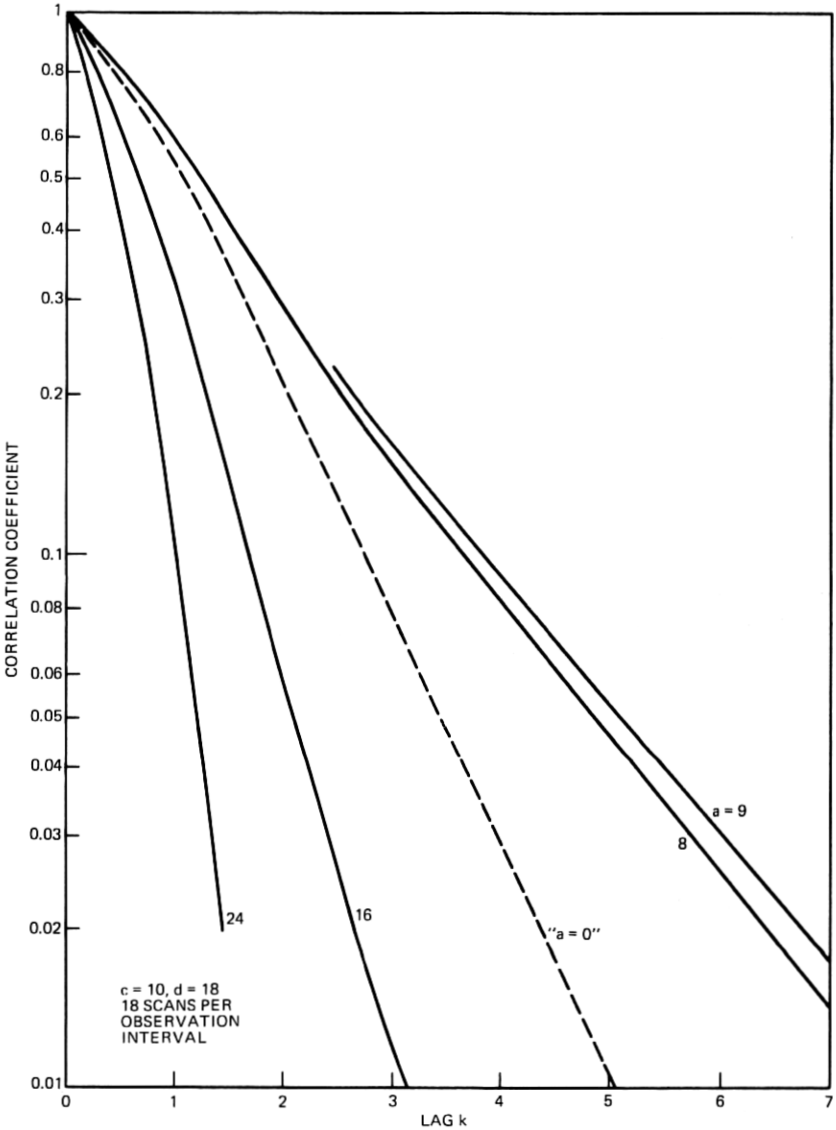

Fig. 18—Correlation between load measurements made over nonoverlapping intervals of unit length (Poisson input, delay-and-loss system).

IX. CONCLUSIONS

In this paper, we have presented numerical examples that shed considerable light on the behavior of the variance of the switch-count load. In particular, they show that, for relatively low offered loads, the variance in question increases as more waiting is allowed by the system while the converse holds at sufficiently high offered loads. But when the same variance is studied in terms of the carried load, a much simpler picture emerges: for a fixed value of this parameter the variance of the switch-count load always increases when either the number of waiting positions and/or the number of sources increase. And a decrease in the defection rates has a similar effect on the variance of the switch-count load as an increase in the number of waiting positions. As we have shown, these properties can be explained by a combination of simple probability and traffic considerations.

But the results of this paper are not exclusively qualitative. On the contrary, the charts illustrate that waiting, in general, affects the magnitude of the switch-count load variance to a degree that cannot be ignored in practice.

The reasonings by which we have explained the qualitative behavior of the switch-count load variance can be expected to hold for more general inputs and service-time distributions. The basis for this statement is that the "hole-filling" argument of Section VI remains meaningful since, to maintain the carried load at a given level, more calls must be offered when fewer of them are allowed to wait, and this fact, of course, is not affected by the shapes of the interarrival and holding-time distributions.

An unexpected result of our investigation is that the continuous load measurements are not minimum variance estimates of the carried load and that (discrete) scanning does not necessarily entail a loss of accuracy.

Finally, as we have shown, the variance formulas derived in Ref. 1 make it possible to compute exactly the autocovariance function for sequences of switch-count load measurements which are thus brought within the purview of time-series analysis. This, in turn, should help evaluate the performance of traffic-control methods based on load measurements.

REFERENCES

1. A. Descloux, "Variance of Load Measurements in Markovian Service Systems," B.S.T.J., *54*, No. 7 (September 1975), pp. 1277–1300.
2. W. S. Hayward, Jr., "The Reliability of Telephone Traffic Load Measurements by Switch Counts," B.S.T.J., *31*, No. 2 (March 1952), pp. 357–377.
3. V. E. Beneš, "The Covariance Function of a Simple Trunk Group, with Applications to Traffic Measurements," B.S.T.J., *40*, No. 1 (January 1961), pp. 117–148.
4. W. Feller, *An Introduction to Probability Theory and Its Applications*, vol. 1, 3rd ed. New York: John Wiley, 1967.