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An Analysis of the Effect of Lossy Coatings on the Transmission Energy in a Multimode Optical Fiber

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Lossy plastic coatings are used as a means of providing mechanical protection for optical fibers during the optical-cable manufacturing process. A model utilizing a quasi-ray analysis has been developed in this paper to determine the effects of lossy coatings on the transmission energy in a multimoded step-index optical fiber. Cladding thickness is the dominant fiber parameter that plays a critical role in preventing transmission loss due to a lossy coating. Other parameters that significantly affect transmission loss are transmitting wavelength, the real and imaginary part of the refractive index of the lossy coating, and the fiber core diameter.

I. INTRODUCTION

A thin lossy plastic coating applied to individual optical fibers is being considered as a means of decreasing crosstalk[1] between the optical fibers and as a mechanism for protecting the fibers during the cable-manufacturing process. The effect of the lossy coating on the transmission energy in a fiber is the subject of this paper.[2,3]

A quasi-ray tracing approach is used to describe energy propagation in a multimoded step-index optical fiber with a lossy plastic coating.[4,5] An integral expression for the power transmitted in the fiber is developed in terms of the geometry of the round fiber, the intrinsic loss of the fiber core, the reflection coefficient at the core-cladding interface, and the energy distribution at the launching end of the fiber. To calculate the reflection coefficient at the core-cladding boundary, the strategy followed is to replace the round fiber by a lossy multilayered semi-infinite slab model. A computer evaluation of the integral expressions for the transmitted and input power has been made. Included in this paper are the results of a study showing the functional relationship between power loss due to a lossy coating and cladding thickness, mode energy distribution, numerical aperture, and wave length.

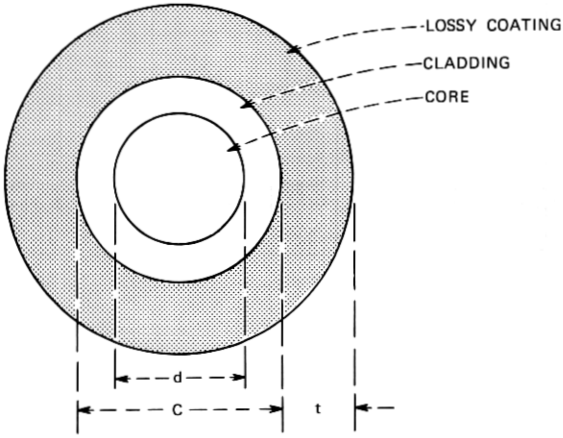

Fig. 1—Optical fiber with lossy coating.

II. DERIVATION OF TRANSMISSION AND REFLECTION LOSS FORMULAS

For the configuration shown in Fig. 1, we follow a technique developed in an earlier paper[1] and present an expression for the transmission of energy in the core of a multimode optical fiber whose cladding is surrounded by a lossy material. We assume that an optical source focuses its power on the center of the entrance end of a fiber, exciting meridional rays as shown in Fig. 2. We also assumed that:

(*i*) The input angular power distribution of the fiber is a gaussian function of the form

$$F(\theta) = F_e \epsilon^{-(\theta/\kappa\theta_c)^2}, \tag{1}$$

where κ is a parameter that is a measure of the width of the input beam and also an indication of how the power is distributed among the modes of the fiber, θ_c is the critical angle of the fiber, and F_e is a constant amplitude.

(*ii*) The propagating modes within the fiber are uncoupled, with the absorption coefficient α equal for all modes.

Under these assumptions, the fiber input and output powers are:

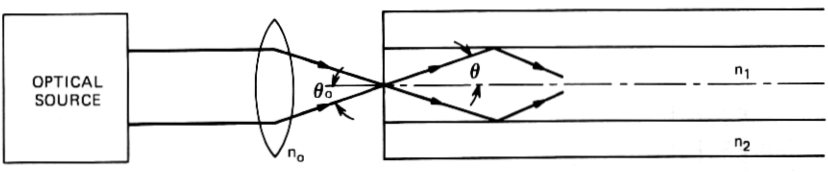

Fig. 2—Meridional ray fiber excitation.

$$P_{\text{input}} = 2\pi F_e \int_0^{\theta_{\max}} \sin\theta\epsilon^{-(\theta/\kappa\theta_c)^2} d\theta \quad (2)$$

and

$$P_{\text{output}} = 2\pi F_e \int_0^{\theta_{\max}} \sin\theta\epsilon^{-(\theta/\kappa\theta_c)^2} R^M(\theta)\,\epsilon^{-\alpha L \sec\theta} d\theta, \quad (3)$$

where

L is the fiber length,
M is the total number of bounces a ray makes while propagating down the fiber and is the largest integer smaller than

$$M = \left| \frac{L}{d} \tan\theta + \frac{1}{2} \right|, \quad (4)$$

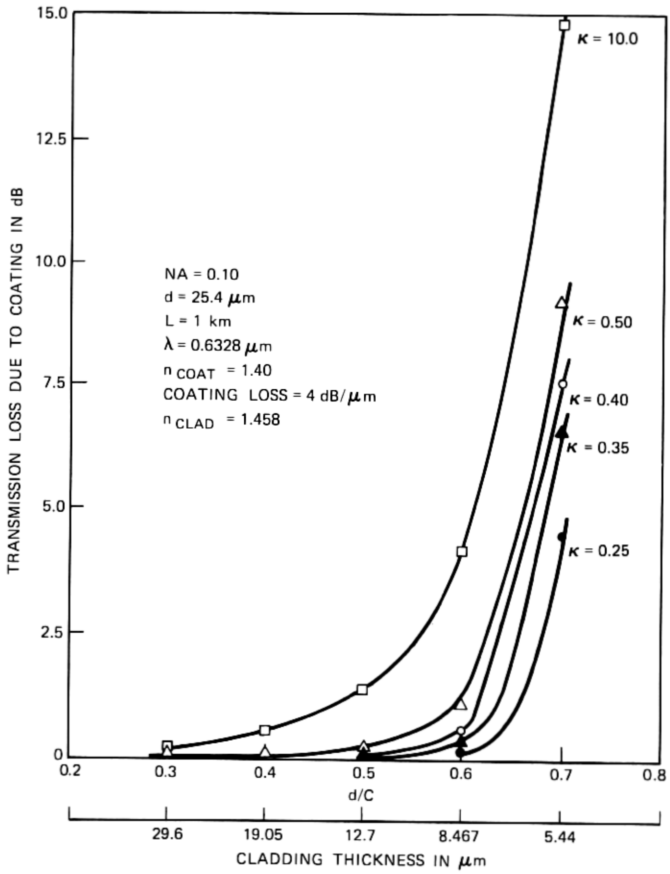

Fig. 3—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.10, d = 25.4 μm.

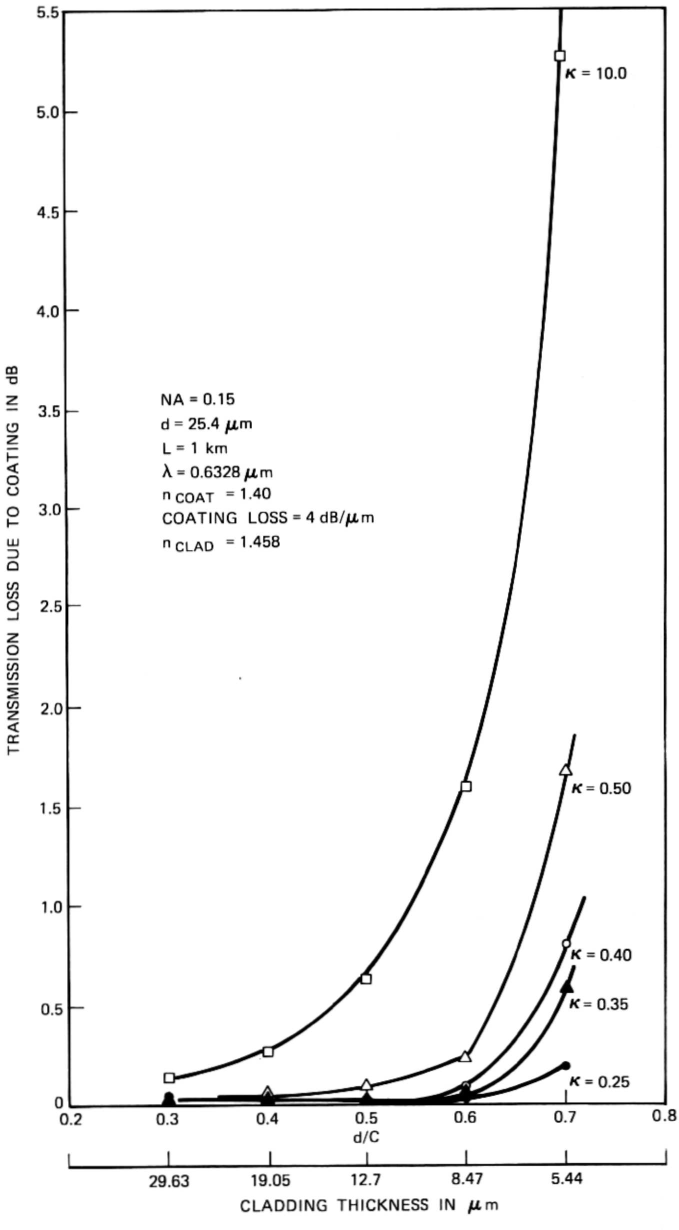

Fig. 4—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.15, d = 25.4 μm.

d is the fiber core diameter,

α is an absorption coefficient per unit length that takes into account both the fiber bulk absorption loss and scattering loss,

θ_{max} is the maximum input angle corresponding to the critical angle within the fiber, and

$R(\theta)$ is the reflection coefficient at the core-cladding boundary.

To calculate $R(\theta)$, the strategy followed is to replace the round fiber by a lossy multilayered semi-infinite slab model. The derivation of $R(\theta)$ for the slab model is shown in the appendix. The refractive indices and, in turn, the impedances of the media are complex to account for the lossy coating.

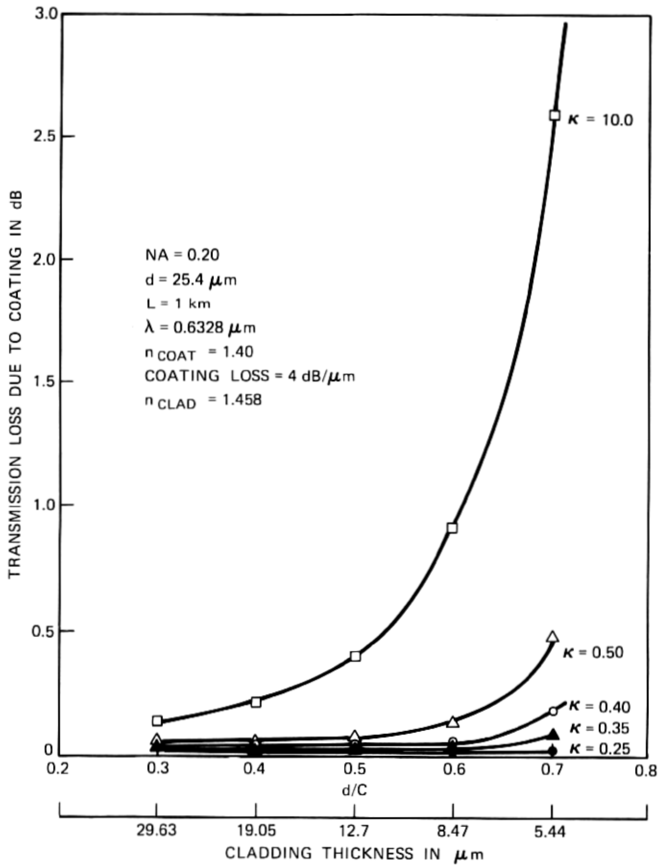

Fig. 5—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.20, d = 25.4 μm.

III. SUMMARY OF RESULTS OF THE COMPUTER STUDY

A computer program was written and the integrals (2) and (3) were evaluated for typical fiber parameters.[6] A number of studies were done to determine how transmission loss due to the coating varies as a function of cladding thickness, wavelength, core diameter, and the real and imaginary part of the coating refractive index. Figures 3 through 5 show, respectively, for fiber numerical apertures of 0.10, 0.15, and 0.20, the relationship between transmission loss due to the lossy coating and

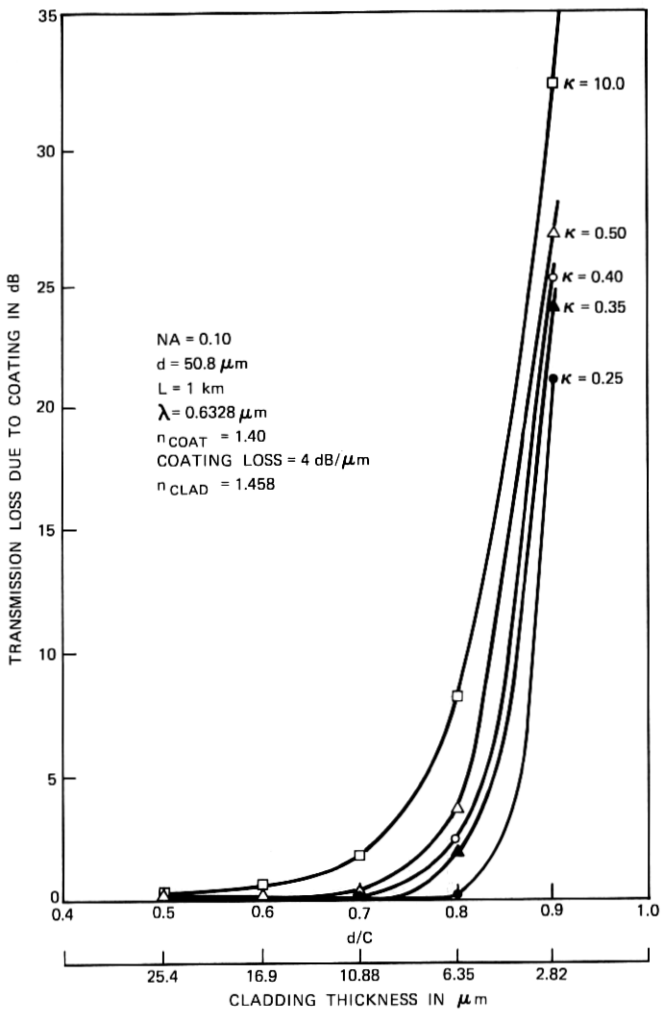

Fig. 6—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.10, d = 50.8 μm.

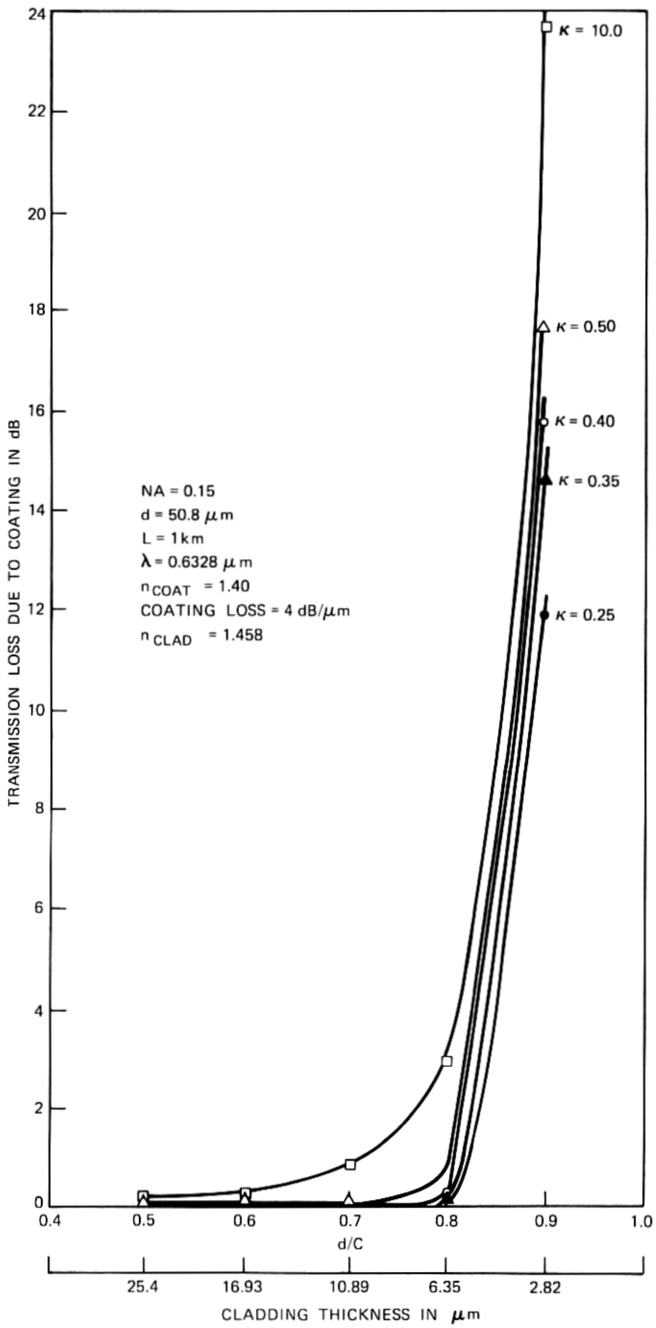

Fig. 7—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.15, d = 50.8 μm.

cladding thickness for a 25.4-μm fiber core diameter. For the same numerical apertures, Figs. 6 through 8 and Figs. 9 through 11 show this relationship for 50.8- and 75.2-μm core diameters.

For practical cladding thickness greater than 15 μm, increasing the cladding thickness will decrease the transmission loss due to the lossy coatings by approximately 0.04 dB/km per micrometer of cladding thickness for the higher-order modes ($\kappa = 10.0$). For the lower-order modes ($\kappa \leqq 0.5$), a cladding thickness of 15 μm should provide sufficient isolation to prevent transmission loss due to the presence of the lossy coating.

Calculations were made to determine the relationships between transmission loss due to a lossy coating and wavelength (λ), core

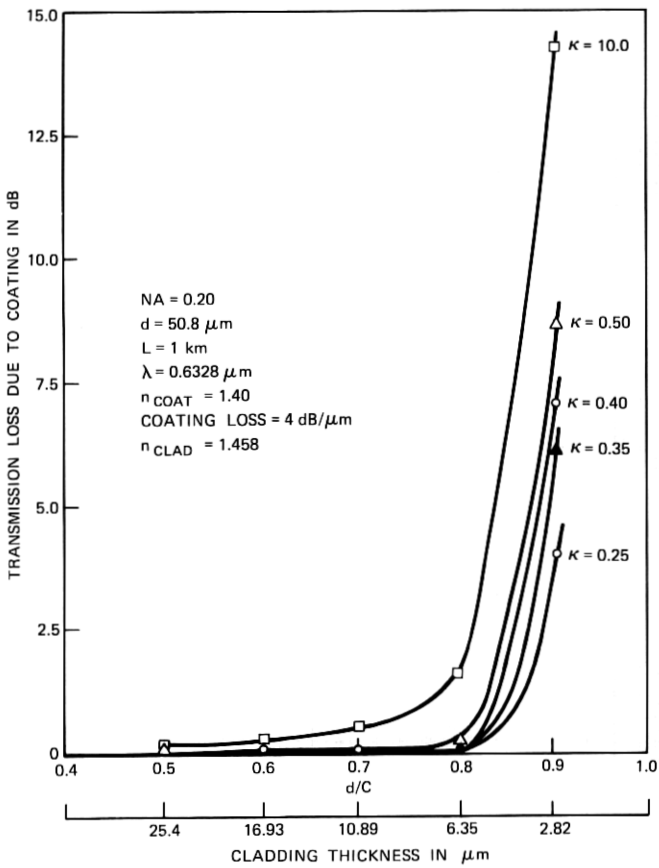

Fig. 8—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.20, d = 50.8 μm.

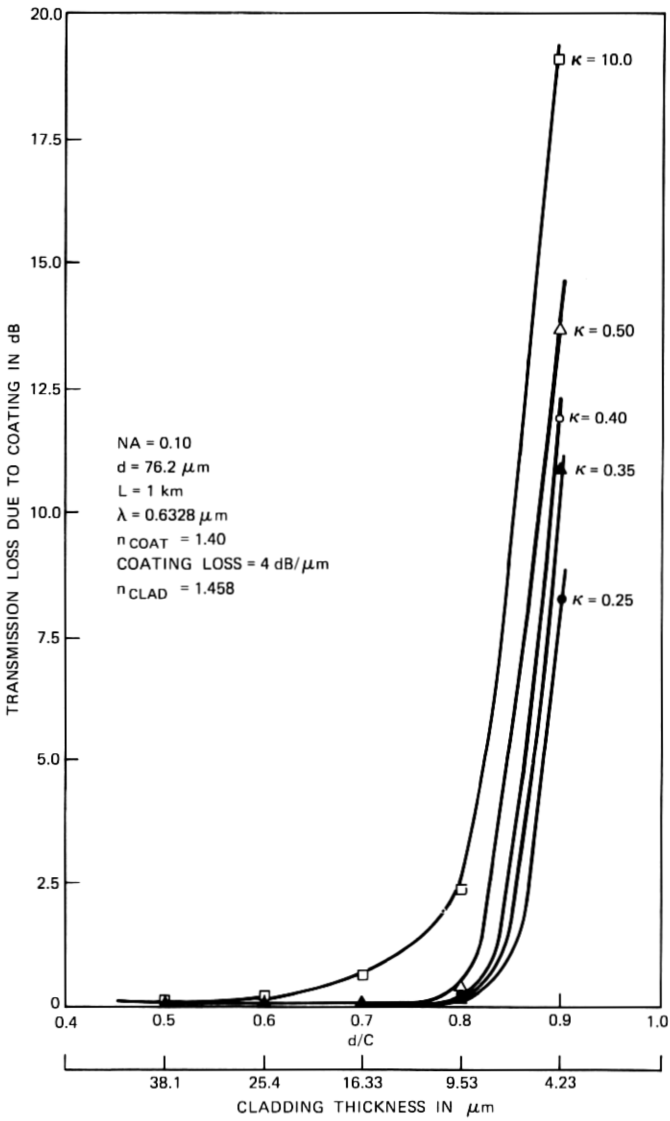

Fig. 9—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.10, d = 76.2 μm.

diameter (d), and the real and imaginary parts of the refractive index of the coating. A thin cladding of 8 μm was chosen in these calculations to easily illustrate the trends due to these parameters. This thin cladding was not intended to be a practical choice for a cladding thickness in an optical fiber. Figure 12 shows the transmission loss due to a

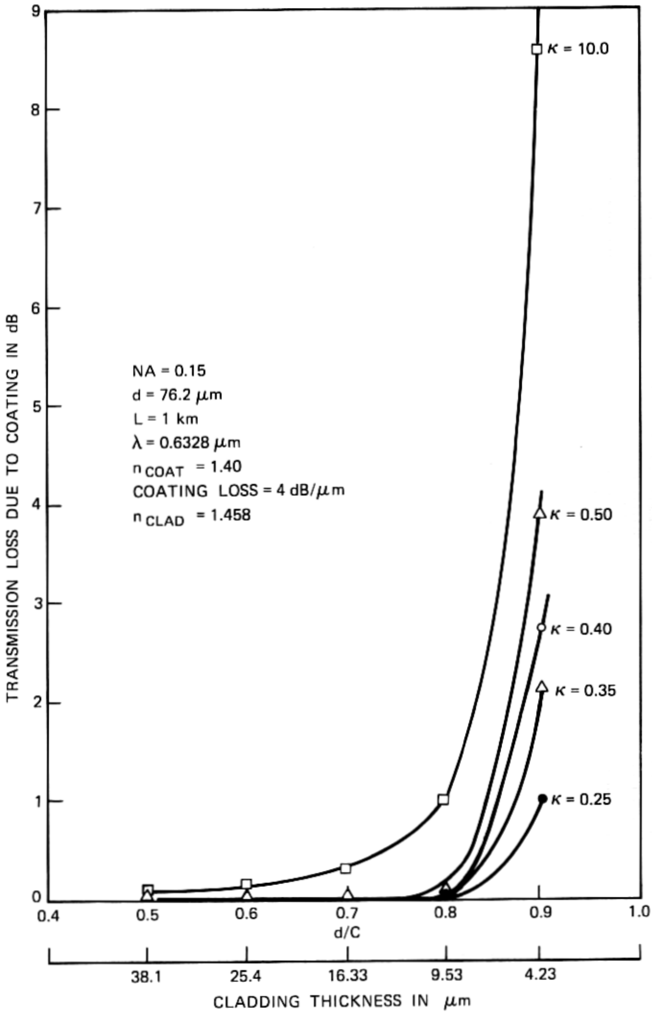

Fig. 10—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.15, d = 76.2 μm.

lossy coating of 4 dB/μm on a fiber whose numerical aperture was 0.15 and core diameter 50.8 μm. As expected, for a fixed cladding thickness, the transmission loss will increase as the wavelength increases. The increase in loss for the parameters chosen was, for the longer wavelengths (>0.8 μm), approximately 1.5 dB/km per micrometer of wavelength. Figure 13 shows the weak dependence of transmission loss on core diameter. Figures 14 and 15 show respectively the

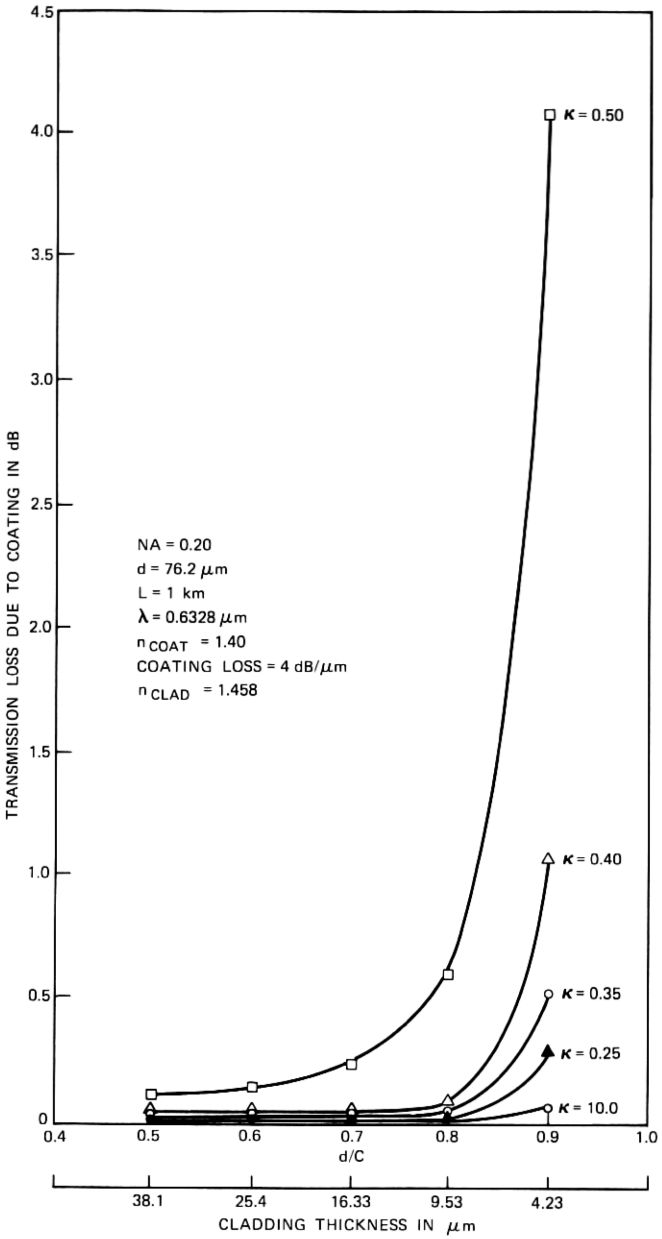

Fig. 11—Transmission loss due to coating vs d/C and vs cladding thickness; NA = 0.20, d = 76.2 μm.

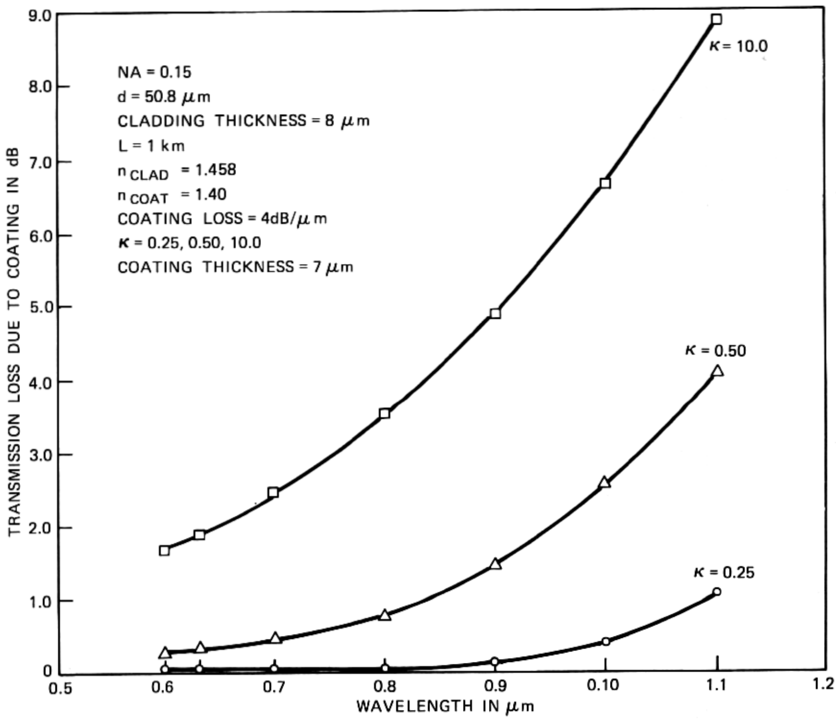

Fig. 12—Transmission loss due to coating vs wavelength.

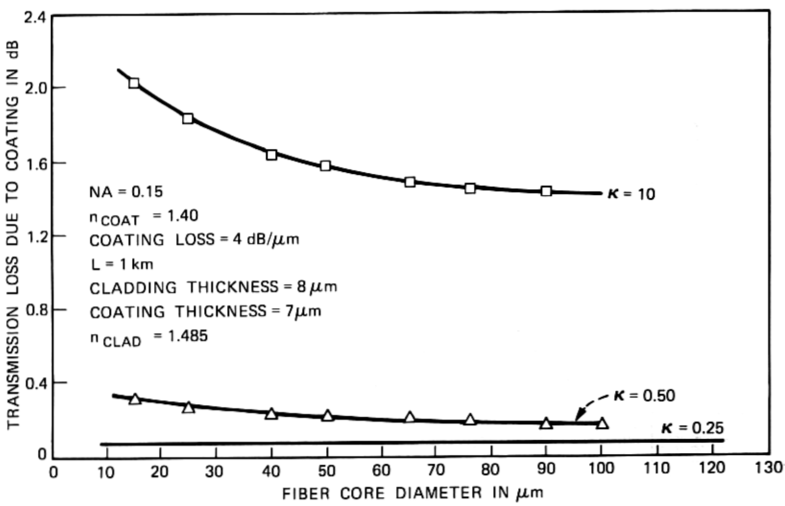

Fig. 13—Transmission loss due to coating vs core diameter.

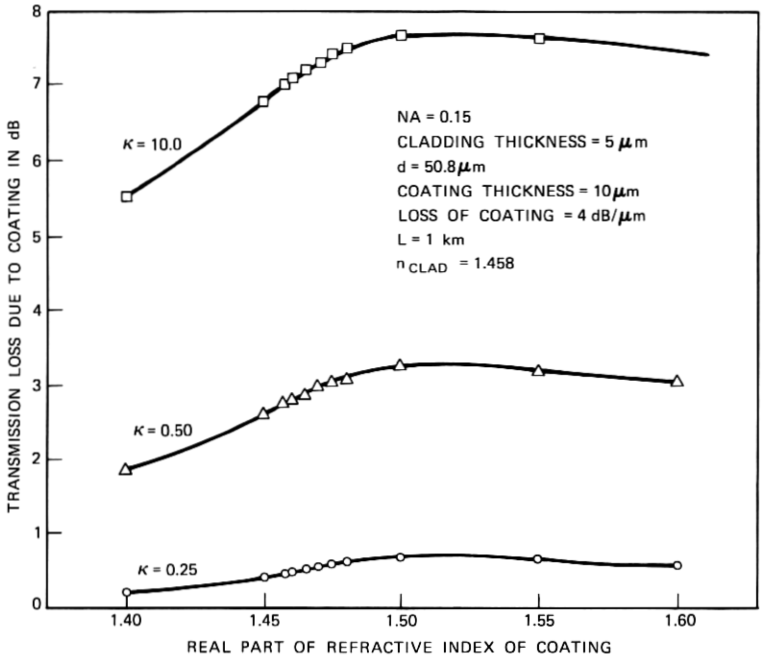

Fig. 14—Transmission loss due to coating vs real part of refractive index of coating.

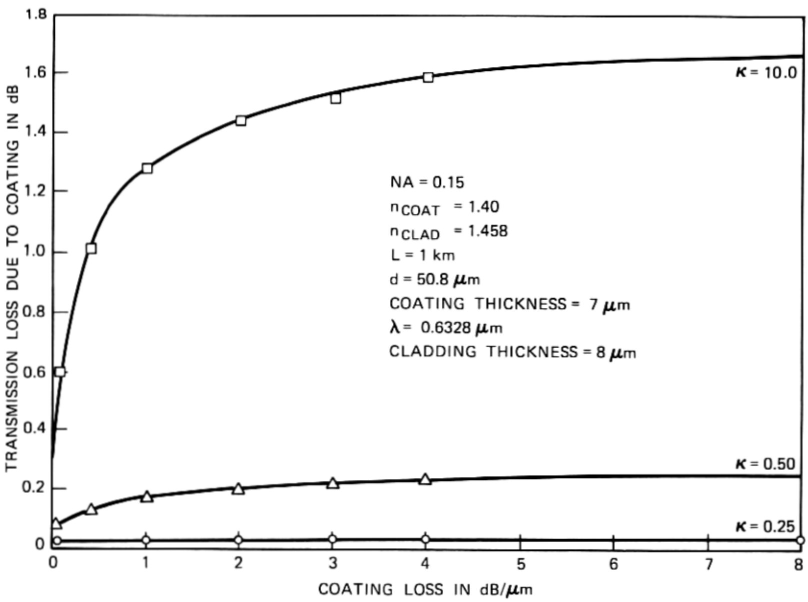

Fig. 15—Transmission loss due to coating vs imaginary part of refractive index of coating.

transmission loss as a function of the real and imaginary parts of the refractive index of the coating. Using Fig. 14, note that for a fiber with a cladding refractive index of 1.458 and coating loss of 4 dB/μm, the transmission loss increases, as the real part of the refractive index of the coating increases, up to a maximum at a value of the refractive index of approximately 1.51. As one increases the refractive index of the coating beyond 1.51, the transmission loss levels off to a constant value.

In Fig. 15, for a fixed real part of the refractive index of the coating equal to 1.40, the transmission loss was observed as a function of the coating loss (the imaginary part of the refractive index of the coating). The transmission loss increases rapidly with coating loss and reaches 80 percent of its final value for a coating loss of approximately 1 dB/μm. Further increase in the loss of the coating does not substantially affect the fiber transmission loss.

IV. CONCLUSIONS

The model described, along with an experimentally determined knowledge of the energy distribution (value of κ), can be used to choose fiber parameters that will prevent transmission loss caused by the lossy coating. Because of the meridional-ray assumption made in the analysis, a conservative estimate of the transmission loss is predicted by the model. The dominant fiber parameter that plays a critical role in preventing the transmission loss is cladding thickness. A cladding thickness of at least 20 μm is necessary to provide adequate isolation. The model also calculates the significant effect on transmission loss of transmitting wavelength, the real and imaginary part of the refractive index of the lossy coating, and the fiber core diameter.

V. ACKNOWLEDGMENT

The authors wish to thank Philip Rich for his assistance in processing the numerical data.

APPENDIX

Calculation of Input Impedance and Reflection Coefficient for a Multilayered Dielectric Medium

In this appendix, the elementary concepts of input impedance and reflection coefficient are developed for a multilayered dielectric medium. Consider the geometry shown in Fig. 16.

Let us suppose that between two semi-infinite media, denoted by 1 and $n + 1$, there are $n - 1$ layers of dielectric material denoted by 2, 3, $\cdots$, n. Let a plane wave be incident on the last layer at an angle of incidence θ_{n+1} and let the plane of incidence be the $X - Z$ plane.

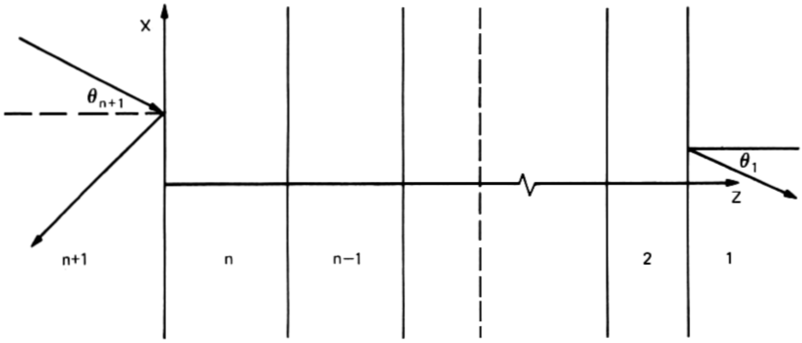

Fig. 16—Geometry used for the calculation of the reflection coefficients in a lossy multilayered dielectric medium.

As a result of multiple reflections at the boundaries of the layers, two waves will exist in each of the media with the exception of medium 1. Our problem is to determine the amplitude of the reflected wave in medium $n + 1$ and, hence, the reflection coefficient.

The following notation is used:

$d_j = z_j - z_{j-1} \equiv$ the thickness of the jth medium,

$k_j = \frac{2\pi}{\lambda} n_j \equiv$ the wave number in jth medium,

$\alpha_j = k_j \cos\theta_j \equiv z$ component of the wave vector in the jth medium,

$\phi_j = \alpha_j d_j =$ the phase change in the jth medium,

$Z_j =$ the self impedance of the jth medium,

$Z^j_{\text{in}} =$ the input impedance looking into the jth medium from the $j + 1$ medium.

The electric and magnetic fields in the jth medium are written as:

$$E_{jy} = A_j \exp\left[-i\alpha_j(z - z_{j-1})\right] + B_j \exp\left[i\alpha_j(z - z_{j-1})\right], \tag{5}$$

$$H_{jx} = \frac{1}{Z_j}\left\{A_j \exp\left[-i\alpha_j(z - z_{j-1})\right] - B_j \exp\left[i\alpha_j(z - z_{j-1})\right]\right\}. \tag{6}$$

The x and t dependency in this case is omitted for the sake of brevity but assumes the general form

$$\exp\left[i(k_{n+1}x \sin\theta_{n+1} - \omega t)\right].$$

A_j and B_j are the amplitudes of the incident and reflected waves in the jth medium ($B_1 = 0$). The amplitude A_{n+1} of the incident wave is assumed to be known. To obtain the reflection coefficient of interest:

$$\Gamma_{n+1} = B_{n+1}/A_{n+1}. \tag{7}$$

One can utilize formula (34) of Ref. 1 to express the input impedance looking into the nth medium from the $n+1$st medium:

$$Z_{\text{in}}^{(n)} = \frac{Z_{\text{in}}^{(n-1)} - iZ_n \tan \phi_n}{Z_n - iZ_{\text{in}}^{(n-1)} \tan \phi_n} \cdot Z_n. \tag{8}$$

The reflection coefficient of the incident wave can then be written in terms of input impedances as follows:

$$\Gamma = \frac{B_{n+1}}{A_{n+1}} = \frac{Z_{\text{in}}^{(n)} - Z_{n+1}}{Z_{\text{in}}^{(n)} + Z_{n+1}}. \tag{9}$$

In eq. (3), the power reflection coefficient R is used. In terms of Γ, R is defined as

$$R = |\Gamma|^2 = \left|\frac{B_{n+1}}{A_{n+1}}\right|^2. \tag{10}$$

For the examples in Section III, a four-layered medium composed of the fiber core, cladding, lossy coating, and surrounding air was used when calculating the reflection coefficient and the transmission loss due to the lossy coating. The refractive index of the lossy coating was defined as

$$N = n_{\text{coating}} - i\delta, \tag{11}$$

where δ is the imaginary or lossy part of the refractive index of the coating.

The relationship between δ and the operationally useful term, coating opacity, is[4]

$$\delta = 29.9\,\lambda\,(\text{opacity}), \tag{12}$$

where

opacity = the transmission loss of the coating in dB/μm
λ = wave length of the transmitted light in micrometers.

The term opacity is introduced here since it is an easily measurable indicator of the loss of the coating and a convenient input variable to the computer program.

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