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A Novel Implementation of Digital Phase Shifters

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A novel technique is presented for implementing a variable digital phase shifter which is capable of realizing noninteger delays. The theory behind the technique is based on the idea of first interpolating the signal to a high sampling rate, then using an integer delay, and finally decimating the signal back to the original sampling rate. Efficient methods for performing these processes are discussed in this paper. In particular, it is shown that the digital phase shifter can be implemented by means of a simple convolution at the sampling rate of the original signal.

I. INTRODUCTION

In digital systems, linear phase shift or delay of a signal waveform by an integer multiple of the sampling period is a simple process that can be implemented as a cascade of unit delays in the network. If, however, it is desired to delay the signal waveform by an amount not equal to an integer multiple of the sampling period, then the process is considerably more difficult. In this case, the signal must be interpolated to obtain new samples of its waveform at noninteger sample times.

In this paper, we propose a novel implementation for achieving such noninteger delays. The theory is based on the application of the concepts of decimation and interpolation proposed by Schafer and Rabiner[1] and Crochiere and Rabiner.[2] It is shown that the actual implementation of the phase shifter or interpolator can be achieved by means of a simple convolution.

Applications in which such noninteger delays in the signal waveform are required often occur when digital systems must interface with analog systems. For example, in the cancellation of echoes, digital systems are often used to generate artificial echoes by means of a simulation of an echo model. These artificial echoes are then subtracted from the original analog signal to cancel its echo. For best cancellation, the digital simulated echo may have to be delayed by a noninteger multiple of the sampling period.

A second potential application occurs when multiple signals must be processed together such as in a phased-array antenna system (e.g., for seismic processing). In this case, the signal waveforms from the various elements must be shifted by noninteger multiples of the sampling period relative to each other.

A third application of noninteger delays is in pitch, synchronous synthesis of speech.[3] In this case, a parametric representation of speech is generated at a fixed sampling rate (usually 100 Hz); however, the synthesis parameters are required at time instances between the sampling intervals to avoid producing transients in the synthesized signal. Using the variable phase shifter proposed in this paper, the synthesis parameters can be readily interpolated to any point between sampling intervals.

II. BASIC CONCEPTS OF THE PHASE SHIFTER

Figure 1 illustrates the basic operation of the phase shifter. To implement a delay of l/D samples, where l and D are any integers, the sampling rate, f_r, of the input signal $x(n)$ is first increased by an integer factor D [by inserting $D - 1$ zero-valued samples between each sample of $x(n)$]. The resulting signal $v(n)$ is then filtered by a low-pass filter $h(n)$ (generally a linear-phase FIR filter is used here) to remove its periodic frequency components, which are centered about integer multiples of the original sampling frequency.[1,2] The output of the filter $u(n)$ is an interpolated version of the input signal $x(n)$. The signal $u(n)$ is then delayed by l samples at the high sampling rate to produce the signal $w(n) = u(n - l)$. It will be assumed that l satisfies the condition.

$$0 \leqq l \leqq D - 1. \tag{1}$$

Finally, the output $y(n)$ is obtained by desampling or decimating $w(n)$, i.e., by choosing every Dth sample of $w(n)$. The net effect is to delay the original signal $x(n)$ by a noninteger delay of $(l/D)T$ where $T = 1/f_r$ is the sampling period at the low rate. In addition, an integer delay is introduced in the signal due to the delay of the low-pass filter $h(n)$.

The structure in Fig. 1 can be analyzed in a straightforward manner. Let $X(e^{j\omega})$, $V(e^{j\omega})$, $W(e^{j\omega})$, $Y(e^{j\omega})$, and $H(e^{j\omega})$ be the Fourier transforms of $x(n)$, $v(n)$, $w(n)$, $y(n)$, and $h(n)$ respectively. Then, the relationships

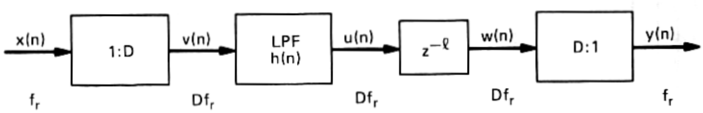

Fig. 1—Block diagram of the phase shifter.

between them can be given as in Refs. 1 and 2:

$$V(e^{j\omega}) = X(e^{j\omega D}), \tag{2}$$

$$W(e^{j\omega}) = H(e^{j\omega})e^{-j\omega l}V(e^{j\omega}), \tag{3}$$

and

$$Y(e^{j\omega}) = \frac{1}{D}\sum_{m=0}^{D-1} W(e^{-j2\pi m/D}e^{j\omega/D}). \tag{4}$$

In eq. (4), the terms in the summation for $m = 1, 2, \cdots, D-1$ correspond to high-frequency components of $W(e^{j\omega})$, which are aliased into the low-frequency band from 0 to $f_r/2$ due to the desampling process. We assume that the low-pass filter $H(e^{j\omega})$ attenuates these high-frequency components to a point where such aliasing can be considered negligible. That is, it has a stop-band cutoff frequency of $1/2D$ (normalized to the high sampling rate Df_r) and a stop-band ripple δ_s that is sufficiently small to prevent aliasing. With these assumptions, (4) becomes

$$Y(e^{j\omega}) \cong \frac{1}{D} W(e^{j\omega/D}) \tag{5}$$

and with the aid of (2) and (3) it can be written as

$$Y(e^{j\omega}) \cong \frac{1}{D} H(e^{j\omega/D})e^{-j\omega l/D}V(e^{j\omega/D})$$

$$\cong \frac{1}{D} H(e^{j\omega/D})e^{-j\omega l/D}X(e^{j\omega}). \tag{6}$$

We now assume that $H(e^{j\omega})$ is a FIR filter with exactly linear phase and has a unit sample response duration of N samples. Then, its delay will be $(N-1)/2$ samples at the high sampling rate. If it is desired that this delay be an integer delay at the low sampling rate, then N must be chosen such that $(N-1)/2$ is an integer multiple of D. That is,

$$\frac{N-1}{2} = ID, \tag{7}$$

where I is a positive integer and

$$N = 2ID + 1. \tag{8}$$

If the particular application does not require that the delay of the filter appear as an integer delay at the low sampling rate, then condition (8) is entirely optional and need not be used.

We now impose the constraint that the passband response of $H(e^{j\omega/D})$ have a gain of D and be essentially flat (i.e., have very small passband ripples). Then the filter response of $H(e^{j\omega/D})$ over the passband is approximately [assuming (8) applies]

$$H(e^{j\omega/D}) \cong De^{-(j\omega/D)[(N-1)/2]}$$
$$\cong De^{-j\omega I}. \tag{9}$$

Substituting (9) into (6) gives the final desired result:

$$\frac{Y(e^{j\omega})}{X(e^{j\omega})} \cong e^{-j\omega I}e^{-j\omega l/D} \tag{10}$$

or in terms of z-transforms

$$\frac{Y(z)}{X(z)} \cong z^{-I}z^{-l/D}. \tag{11}$$

Thus, the structure in Fig. 1 is essentially an all-pass network [over the passband of $H(e^{j\omega/D})$] with a fixed integer delay of I samples due to the processing delay of the low-pass filter $h(n)$ and a variable noninteger delay of l/D samples. If N does not satisfy condition (8), then I in eqs. (10) and (11) will not be an integer. In either case, the output, $y(n)$, in Fig. 1 is an approximation to $x(n - l/D - I)$.

III. IMPLEMENTATION OF THE PHASE SHIFTER

The design of the phase shifter in Fig. 1 suggests a structure which involves two different sampling rates. In this section, we show that the actual implementation of the phase shifter can be achieved considerably easier as a straightforward convolution at the low sampling rate.

Since the duration of $h(n)$ is N samples and $D - 1$ out of every D samples of $v(n)$ are zero valued, the filter $h(n)$ spans approximately N/D nonzero samples of $v(n)$. More precisely, because of the constraint imposed on N in (8), $h(n)$ spans Q nonzero samples of $v(n)$ for the computation of some output points and $Q - 1$ nonzero samples of $v(n)$ for the computation of other output points [Q is defined in eq. (13)]. To avoid this implementation difficulty, it is convenient to consider instead a new filter $h'(n)$ whose length N' is

$$N' = QD \geqq N, \tag{12}$$

where $h'(n)$ is obtained by extending $h(n)$ with $N' - N$ zero-valued coefficients. Obviously, the filter $h'(n)$ has the same exact frequency response and delay as $h(n)$, but it spans *exactly* Q nonzero samples of $v(n)$ [although one nonzero sample of $v(n)$ may be multiplied by a zero valued coefficient of $h'(n)$]. Since we wish to keep N' as small as possible, consistent with (12) we can choose Q to be

$$Q = \left[\frac{N}{D}\right], \tag{13}$$

where the brackets indicate that the number is rounded to the next largest integer.

With these assumptions, we can now relate the output $y(n)$ in Fig. 1 to $x(n)$ and $h'(n)$ by the expression[2]

$$y(n) = \sum_{k=0}^{Q-1} h'[kD + (-l) \oplus D]x(n-k), \tag{14}$$

where $\oplus$ corresponds to modulo addition. By letting

$$g_l(k) = h'[kD + (-l) \oplus D] \quad k = 0, 1, \cdots, Q-1, \tag{15}$$

(14) then becomes

$$y(n) = \sum_{k=0}^{Q-1} g_l(k)x(n-k), \tag{16}$$

which is the form of a simple convolution. Therefore, the phase shifter can be implemented by a Q point convolution of $x(n)$ with $g_l(n)$, where $g_l(n)$ is an appropriate subset of the coefficients of $h'(n)$. To obtain a zero incremental phase shift, we use the coefficients $\{g_0(0) = h'(0), g_0(1) = h'(D), \cdots, g_0(Q-1) = h'[(Q-1)D]\}$. To obtain a delay of $(l/D)T$ (or a phase shift of $\omega l/D$), we use the coefficients $\{g_l(0) = h'[(-l) \oplus D], g_l(1) = h'[D + (-l) \oplus D], \cdots, g_l(Q-1) = h'[(Q-1)D + (-l) \oplus D]\}$. If we want a variable phase shifter, we can store all D sets of coefficients and use the appropriate set as suggested in Fig. 2.

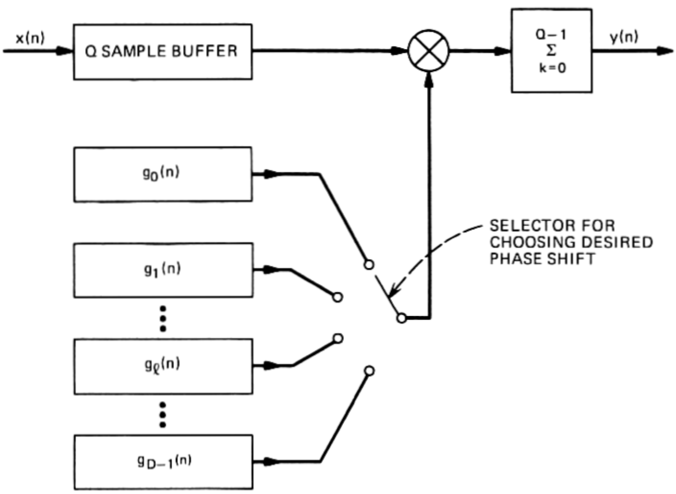

Fig. 2—A practical implementation of a variable phase shifter.

V. CONCLUSIONS

We have presented a method for designing an incremental digital phase shifter that can shift the phase of a waveform by a noninteger number of samples. Conceptually, the process can be thought of as a sample rate increase, a delay, and a sample rate decrease as indicated in Fig. 1. Practically, it can be implemented as a straightforward convolution as shown in Fig. 2. From the discussion of the theory, it is also clear that the design trade-offs of the phase shifter are directly related to the characteristics of the low-pass FIR filter. That is, the passband ripples of $H(e^{j\omega})$ determine how close the phase shifter is to an ideal all-pass network (over the passband), and the stop-band ripples determine the amount of distortion due to aliasing. Finally, the cutoff frequency of the filter determines the usable frequency range of the phase shifter.

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