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Rain-Scatter Interference in Terrestrial Microwave Systems

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The potential interference caused by scattering from raindrops is viewed as foreground reflections that "spoil" the off-beam antenna pattern. This approach greatly simplifies interference calculations and is sufficiently accurate for most engineering purposes.

The principal uncertainty that limits not only this simplified approach but also the "exact" solution is the inability to characterize the variable rainfall distribution along any given radio path by a single parameter, such as the rainfall rate, either at a point or as an average along the path.

I. INTRODUCTION

The potential cochannel interference between two stations, illustrated on Fig. 1a, is usually assumed to occur along the shortest path (dotted line) between stations. The transmission loss along the dotted path includes not only the free-space loss, but also any additional loss caused by obstructions or by site shielding. Transmission is further modified by the antenna patterns in the direction toward the other site. In addition, reflections from nearby trees and terrain may reduce the expected suppression of the antenna in directions away from the main beam.

If the two antenna beams intercept, any reflection from rain, aircraft, or birds in or near their common volume can provide another path by which interference can occur. While the magnitude of such reflections is normally small, this "sneak" path has free-space transmission with full antenna gain at one end and, thus, bypasses the suppression that is normally provided by the antenna patterns and shielding losses.

II. APPROACH

A simplified procedure for estimating the transmission loss by way of reflection from rain can be obtained, in most cases, by replacing Fig. 1a with Fig. 1b. In this case, antenna G_1 is replaced by a phantom

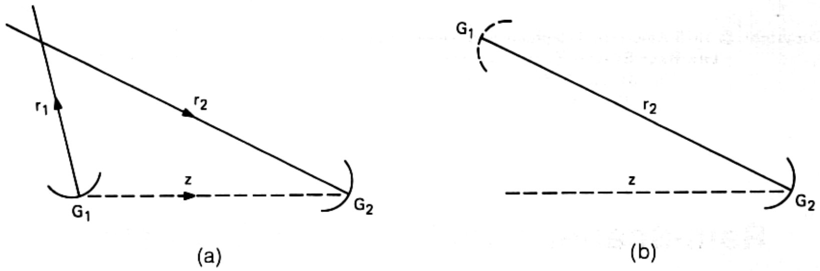

Fig. 1—(a) Plan view of two cochannel stations. (b) Interference path from common volume.

antenna, G_1', whose equivalent gain relative to an isotropic antenna is given on Fig. 2 for various frequencies and rates of rainfall. The phantom antenna, G_1', can be considered to be located at the beam intersection, which is at a distance r_2 from G_2.

In other words, the effect of rain is to fill in the nulls and reduce the normal side- and back-lobe suppression of the antenna, and then to raise it to within line of sight of the distant station. Even when all of the energy from the transmitter is scattered, G_1' cannot exceed the unity "gain" of an isotropic antenna. This means that for comparable antennas, distances, and transmitter power, the unwanted scattered signals will be less than the desired signal by the gain of the antenna nearest the heavy rainstorm.

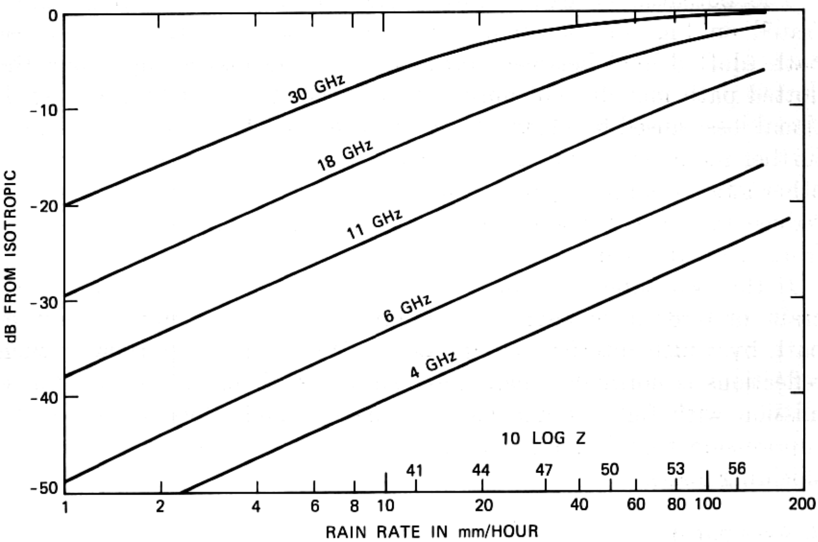

Fig. 2—Effective antenna gain G_1' during rain.

If the two antenna beams do not intersect, the antenna gain G_2 is reduced in accordance with its directivity pattern for the off-beam angle between its axis and the direction needed for full intersection.

The use of Fig. 2 requires an estimate of the rainfall rate R to be expected for a given percentage of the time. Typical values in current use that are expected to be exceeded for no more than 10^{-4} of the time are given in the following table.[1] For a probability of 10^{-5}, the above values of 10 log Z should be increased by about 6 dB.

Section of USA	R	10 log Z*
Southeastern and Gulf	80 mm/hr	53 dB
East and Midwest	56	51
Pacific region	33	47.5
Rocky Mountain region	13	41

* Reflectivity factor $Z \approx 10^5 (R/50)^{1.5}$.

While Fig. 2 provides a first-order solution that is sufficiently accurate for most engineering purposes, the complete rain-scattering problem is, of course, much more complex. It is necessary to consider the more general problem so that the quantitative significance of the many secondary factors can be evaluated and included whenever it is necessary to do so. Several solutions to the more complete scattering are given in Refs. 2 through 4. In spite of the variety of methods, symbols, and units, all of the papers referenced lead to essentially the same quantitative results for the same set of assumptions, because all use the same meteorological premises. The seeming complexity and diversity result in part from the three-dimensional geometry and in part from the inclusion of several factors that have been neglected in the first-order solution in Fig. 2 because their quantitative effects are small.

Atmospheric absorption is omitted from the first-order engineering solution because it is an unnecessary complication at 4 and 6 GHz; this loss can be included, if needed, at the end of the computation. To some extent, the omission of attenuation caused by rain is compensated for by the asymptotic behavior of the curves shown on Fig. 2. These curves include a factor to ensure that $G_1' \leqq 1$. More complicated methods (that theoretically are more accurate) add the expected absorption loss and then offset it in part by a gain due to "forward scatter." The difference between the two procedures is expected to be less than the effect of the uncertainty in the effective rain rate for a specified location and percentage of the time.

The polarization-mismatch factor is an additional loss of about $20 \log \cos \zeta$ dB, where ζ is the angle between the polarization of the desired and undesired signal. It is a safety factor that can ordinarily be neglected, because the accuracy of the other factors in the overall computation seldom warrants this precision. Some methods add 3- or 4-dB loss for this factor, but any assumption of a significantly larger amount (say, 20 to 25 dB because the interfering polarizations are expected to be orthogonal) seems unrealistic because the normal cross-polarization discrimination can be significantly reduced during rain.

In addition, some references include a dielectric factor $[(\epsilon - 1)/(\epsilon + 2)]^2$. This factor is negligible for rain but is automatically included in the factor Z used in Fig. 2.

Example

When the two stations illustrated in Fig. 1a are separated by $z = 30$ miles (50 km), the free-space loss between isotropic antennas is 138 dB at 4 GHz. The net transmission loss along the z direction is $138 - G_{1z} - G_{2z} + S$, where S is the shadow and shielding loss in dB and the antenna gains G_{1z} and G_{2z} (along the z direction) are also expressed in decibels. On the other hand, the coupling loss from intersecting beams illustrated in Fig. 1 is $138 - G_1' - G_2 + 20 \log r_2/z$, where G_1' is the equivalent antenna gain shown on Fig. 2 for the station nearest the rainstorm and G_2 is the antenna gain along r_2 of the more distant station. With 40-dB antennas at 4 GHz, $z = 50$ km, $r_2 = 62$ km, and a rainfall rate of $R = 100$ mm/hour, the coupling loss by way of rain scatter is $138 - (-25) - 40 + 2 = 125$ dB.

In calculating the data in Fig. 2 it was assumed that the area of maximum rainfall has an effective diameter $D = 1$ km; if the user prefers some other value, say, D' in km, the rain rate to be used in entering Fig. 2 should be $R' = RD'$. The model also assumes $(G_1/r_1^2) \geqq (G_2/r_2^2)$, but this is no restriction because of reciprocity.

III. DISCUSSION OF THE SCATTER PROBLEM

The remaining sections review the scattering equation that leads to the results given in Fig. 2. In addition to the rain-scatter problem, the general method given below also derives the radar equation, the expression for passive repeaters, and the equation for rough or smooth ground reflections. The free-space transmission loss for a path length r and a wavelength λ is given by

$$\frac{P_R}{P_T} = \frac{G_1 A_2}{4\pi r^2} = \frac{G_1 G_2 \lambda^2}{(4\pi r)^2} = \frac{A_1 A_2}{(\lambda r)^2}, \tag{1}$$

where G is the antenna gain and A is the area of the antenna aperture. The transmission loss P_R/P_T by way of reflection from a large plane surface, illustrated in Fig. 3, is the product of two free-space paths.

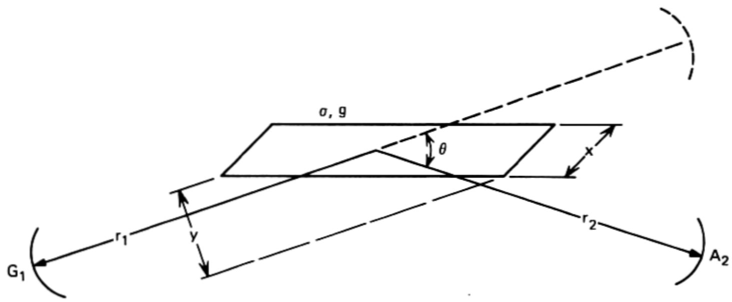

Fig. 3—Reflection from passive repeater.

The first section has an effective receiving area (cross section) denoted by σ and the second section has an effective transmitting antenna gain g that is related to both the cross section σ and the degree of both smoothness and reflectivity of the plane reflector. By algebraic rearrangement, the expression is separated into the free-space loss to the image of the distant antenna and a power reflection coefficient:

$$\frac{P_R}{P_T} = \left(\frac{G_1\sigma}{4\pi r_1^2}\right)\left(\frac{gA_2}{4\pi r_2^2}\right) = \underbrace{\left(\frac{G_1A_2}{4\pi(r_1+r_2)^2}\right)}_{\text{Free space to image antenna}}\underbrace{\left(\frac{\sigma g}{4\pi d_0^2}\right)}_{\text{Reflected power coefficient}}, \tag{2}$$

where

$$\sigma = \frac{xy}{\left(1+\dfrac{x}{\sqrt{\lambda d_0}}\right)\left(1+\dfrac{y}{\sqrt{\lambda d_0}}\right)}; \qquad d_0 = \frac{r_1 r_2}{r_1+r_2},$$

$$g = \sqrt{1+\left(\frac{4\pi\sigma K}{\lambda^2}\right)^2},$$

$K = 1$ for perfectly smooth surface (mirror)
$\approx \exp(-\phi)$ for irregular surfaces
$\rightarrow 0$ for very rough surface,

and

ϕ = standard deviation of phase variations caused by irregular surface.

By the simplifying assumptions that ordinarily apply to radar ($G_1 = G_2 = 4\pi A_2/\lambda^2$; $r_1 = r_2 = r$; $g = 1$; $K \ll 1$), eq. (2) reduces to the radar equation

$$\frac{P_R}{P_T} = \frac{G^2\lambda^2\sigma}{(4\pi)^3r^4}. \tag{3}$$

When both dimensions of the plane reflector in Fig. 3 are large compared with the first Fresnel zone ($\sqrt{\lambda d_0}$), the cross section $\sigma = \lambda d_0$. For this case, the transmission loss is the free-space loss along the image path multiplied by a power-reflection coefficient:

$$\frac{P_R}{P_T} = \left(\frac{G_1 A_2}{4\pi(r_1 + r_2)^2}\right)\left(\frac{\sigma^2 K}{\lambda^2 d_0^2}\right) = \left(\frac{G_1 A_2}{4\pi(r_1 + r_2)^2}\right)(K). \tag{4a}$$

When the y dimension of the plane reflector is less than the first Fresnel zone, $\sigma = y\sqrt{\lambda d_0}$, the corresponding transmission loss is

$$\frac{P_R}{P_T} = \left(\frac{G_1 A_2}{4\pi(r_1 + r_2)^2}\right)\left(\frac{y^2}{\lambda d_0}\right)(K). \tag{4b}$$

Finally, when both dimensions of the reflector are small compared with the first Fresnel zone, σ approaches the geometrical cross section of the reflector, and the corresponding transmission loss is

$$\frac{P_R}{P_T} = \left(\frac{G_1 A_2}{4\pi(r_1 + r_2)^2}\right)\left(\frac{xy}{\lambda d_0}\right)^2(K). \tag{4c}$$

In all of these cases, the first bracketed term is the free-space loss along the image path.

When the cross-sectional area σ of a single reflector is small compared with $\lambda^2/4\pi$ (or its surface is so rough that $K \to 0$), the energy is scattered in all directions. The antenna gain of the scatterer [see (2)] becomes $g = 1$, and this leads to

$$\frac{P_R}{P_T} = \left(\frac{A_2}{4\pi r_2^2}\right)\left(\frac{G_1\sigma}{4\pi r_1^2}\right) = \left(\frac{A_2}{4\pi r_2^2}\right)\left(\frac{\sigma}{A_1'}\right), \tag{5}$$

where

$$\frac{G_1}{4\pi} = \frac{1}{\beta_{1V}\beta_{1H}}$$

A_1' = cross-sectional area of beam from antenna 1 at distance r_1

and

$\beta_{V,H}$ = antenna beam widths in radians (assumed uniform).

The assumption of isotropic scattering is not strictly correct, but Setzer[5] has shown that, for frequencies below 30 GHz, the difference between isotropic scattering and the exact value is less than 3 dB at all angles.

The next step toward solution of the rain-scatter problem is to assume a large number (NV) of small particles, each with a cross section $\sigma \ll \lambda^2/4\pi$, randomly spaced with an approximately uniform

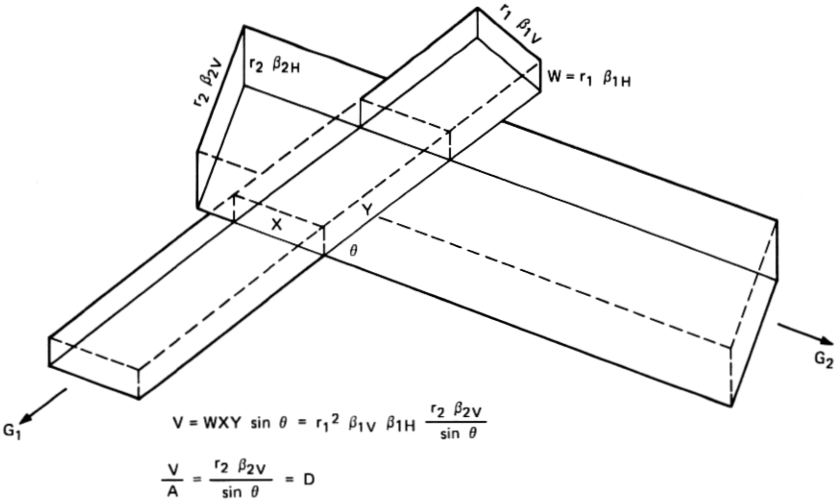

Fig. 4—Geometry of common volume.

density throughout the common volume illustrated in Fig. 4. The resulting transmission loss is

$$\frac{P_R}{P_T} = \left(\frac{A_2}{4\pi r_2^2}\right)\left(\frac{\sigma}{A'}\right)(VN) = \left(\frac{A_2}{4\pi r_2^2}\right)(\sigma N)\left(\frac{D}{1+\sigma ND}\right)$$
$$= \left(\frac{A_2}{4\pi r_2^2}\right)(G_1'), \quad (6)$$

where

N = number of particles per cubic meter,
V = common volume $A'D$,

and

$$D \leqq \frac{r_2\beta_2}{\sin\theta}$$
$$\leqq \left(\begin{array}{c}\text{size of rain cell with}\\ \text{maximum “uniform” rain rate}\end{array}\right).$$

The first bracket in (6) is the free-space transmission loss for a distance r_2 between an isotropic antenna and an antenna whose gain is $G_2 = 4\pi A_2/\lambda^2$. The term σN is a meteorological parameter (total cross section per cubic meter) that depends on the rate of rainfall and the wavelength. Essentially, the same assumptions on the distribution of raindrop diameters and on their velocity of fall are used by all the references with the result that

$$\sigma N = \frac{\pi^5 10^{-18} Z}{\lambda^4} \text{ meters}^{-1}, \tag{7}$$

Z = reflectivity factor,
= 180 $R^{1.6}$ from Ref. 2,
= 400 $R^{1.4}$ from table in Ref. 3,
= 200 $R^{1.6}$ from Ref. 4,

and

R = rainfall rate in mm/hour.

Since the primary interest is in heavy rains ($R > 50$ mm/hr), this work uses

$$Z = 10^5 \left(\frac{R}{50}\right)^{1.5}, \tag{8}$$

which is in good agreement with all references in the vicinity of $R = 50$ mm/hour, and splits the difference for much smaller and much larger values of R.

The equivalent antenna gain given in Fig. 2 is based on $G' = \sigma ND/(1 + \sigma ND)$ as defined in (6). The parameter D represents a somewhat uncertain (or "rubbery") length which, perhaps, can best be defined as the smaller of either V/A' or the diameter of the rain cell associated with the peak rain rate. One of the references uses $D = 4$ km in calculating its principal quantitative result, another uses $D = 1$ km, and a third uses 0.71 km. In this work, a value of $D = 1$ km is assumed in the results shown on Fig. 2.

Finally, the factor $1/(1 + \sigma ND)$ has been included to prevent a calculated value of G' greater than unity, because such a result would be inconsistent with the basic assumption of isotropic scattering.

IV. RAIN ATTENUATION

As previously mentioned, the first-order solution (using Fig. 2) considers only the scattering and omits the attenuation caused by rain. The attenuation computed by Setzer is shown by the solid lines on Fig. 5 and agrees with values given previously by Medhurst.[6,7] The dotted lines are calculated from the following empirical formula, which may be useful for engineering purposes:

$$\alpha = \frac{0.3R}{\left[1 + \left(\frac{\lambda}{2}\right)^2\right]^2} \text{ dB/km}, \tag{9}$$

where

λ = wavelength in cm.

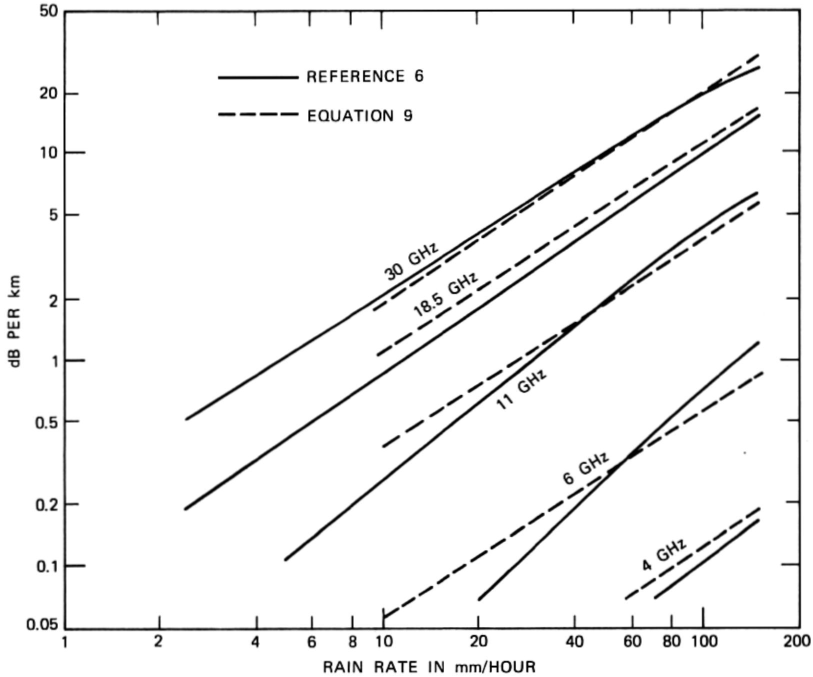

Fig. 5—Rain attenuation.

As in the scattering problem, there is some uncertainty regarding the effective distance to be used with the data in Fig. 5. The distance to be used in computing attenuation is greater than the diameter of a rain cell associated with the maximum rate of rainfall but cannot exceed the path length. Many authors prefer to think in terms of the *average* rate of rainfall over the actual path length, but this merely transfers the uncertainty from an effective distance to the determination of the average R. Based primarily on measurements in New Jersey, Hogg has related the average rainfall rate along a path to the rainfall rate at a point in the same climate.[8] Typical values to be exceeded for less than 10^{-5} of the time are given for path lengths in the 1- to 10-km range and extended to other climates. The results can be represented approximately by the following empirical formula:

$$\frac{\bar{R}}{R} = \left(1 + \frac{Rd}{750}\right)^{-\frac{1}{2}}, \tag{10}$$

where

$\bar{R}$ = average rainfall rate along path d (in km).

In principle, the rain attenuation can be computed if the number and size of drops are known at every point in the space between the two antennas at every instant in time. In practice, significant variations with time and location are likely because the distribution of the rainstorm is completely independent of the radio path.

Rain attenuation estimates are necessary in calculating the fade margin against set noise, but only the difference in attenuation between signal and interference affects the carrier-to-interference ratio. When the desired signal (C) and the interfering signal (I) are attenuated equally, the C/I ratio is unchanged and the interference-to-noise (I/N) ratio is less than in the absence of attenuation. When the desired signal is attenuated more than the interfering signal (as, for example, by heavy rain at a location to the left of the intersection illustrated on Fig. 1a) both the C/I and C/N are decreased and either one may be controlling. Estimates of outage time caused by rain attenuation (low C/N) can be obtained without reference to rain scatter or other types of interfering signals. Conversely, estimates of the probability that a rain-scatter interference will exceed the set noise (or some lower limit required by the difference in interfering effects between thermal noise and a single frequency tone) can be made independent of the rain attenuation. For an efficient microwave transmission system, it is essential that the probability of service degradation caused by interference should be less than for set noise alone. Any more sophisticated attempt to combine the two separate probabilities of outage requires more information on the distribution of rainfall with geography and time than is available.

V. CONCLUSION

The interference caused by scatter from raindrops is a complex problem, as indicated in the references. Some complexity is unavoidable when all pertinent factors are to be included regardless of their magnitude. Fortunately, a relatively simple engineering method for interference calculation can be obtained by neglecting several factors whose magnitudes are small compared with the fundamental uncertainty in the exact rainfall distribution along the path.

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