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Operational Limitations of Charge Transfer Devices

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The incomplete transfer of charge and the existence of random noise lead to the primary operational limitations of charge transfer devices. Owing to the signal dependence of the residual charge, which accumulates as a result of the incomplete transfer, signal detection with static detection levels becomes seriously impaired before the onset of significant signal attenuation or noise degradation. A scheme using dynamic detection levels is found to greatly extend the operational range of CTD's and achieves the minimum possible error rate for detecting uncorrelated charge packet sizes. By contrast, simple coding procedures are found to be ineffective in overcoming signal degradation due to incomplete transfer. Shannon's expression for maximum information transmission capacity is transformed into an expression for maximum information storage capacity. It is found that significantly larger storage capacities are possible with CTD's than have been achieved.

PRELIMINARY REMARKS

Proposition: Devices Function at Their Limits of Operation

One has only to design or fabricate a device which "exceeds specs," and then, because of his success, receive a set of revised (and more demanding) specifications from the systems people to appreciate this very basic principle. Or, if one fixed his goals too low, some clever fellow will come along with a new twist which makes full use of the device and revolutionizes the industry. On the other hand, devices function at their limits of operation *and no better*. This lies behind the widely recognized importance of ascertaining fundamental operational limitations in the early stages of device development.[1]

It is, therefore, the primary objective of this article to discuss the operational limitations of charge transfer devices (CTD's). At a time when the basic device structure to be developed is still uncertain, and with relatively little analysis of the charge transfer process, noise, error rates, storage capacities, etc. (and with even less experimental verification of these analyses), the results of such an article may seem very preliminary at best. In some respects this will be the case. However, sufficient progress has been made in understanding the operational features of CTD's, especially with respect to incomplete transfer and noise, that definite limitations can be placed on how large information storage capacity and how small (digital) error rates can be made.

It is my intention to outline CTD operational limitations in rather general (even perhaps philosophical) terms and to refer to appendixes, existing articles, work in preparation, etc., for mathematical details. The results are optimistic to the extent that they indicate how much better one can hope to do with CTD's than is commonly envisioned. However, at the same time, the results are pessimistic in that it is not obvious how one is to achieve this optimum use. Implementing our dynamic detection scheme, which results in the minimum possible error rate for digital signals, represents, nonetheless, a major step towards operating CTD's near full capacity.

I. INTRODUCTION

According to a most significant theorem due to Shannon,[2] the maximum information transmission capacity C_T of any channel is determined by the bandwidth and the signal/noise ratio of the channel. With a slight modification this theorem can be transformed into a theorem on the maximum information storage capacity C_s of any channel, in particular of a CTD. This C_s then places an upper limit on the number of bits of information which can be stored in an unregenerated section of a CTD. If we restrict consideration to codes in which the size of each charge packet is independent of preceding or subsequent packets, we can calculate a minimum error rate and specify an optimum detection scheme for digital signals. To obtain this result, some knowledge of the signal/noise ratio and the accumulation of residual charge owing to the incomplete transfer of portions of every charge packet is necessary.

In this paper, in order to establish the minimum possible error rate and maximum possible storage capacities attainable with CTD's, it will be necessary to first review the way incomplete charge transfer determines the attenuation-versus-frequency characteristic and the total bandwidth of a CTD. The role of incomplete charge transfer in signal degradation will then be discussed. Once the dependence of the accumulated residual charge on the preceding signal is understood, it is possible to devise a dynamic detection scheme for simple digital signals and to compare this with the basic method of absolute-amplitude or fixed-threshold detection (static levels). With any form of detection, noise will introduce errors. Following a review of CTD noise, error rates are discussed. It is found that in the presence of noise this dynamic detection scheme attains the theoretical minimum error rate. Finally, maximum information storage capacities are calculated using the bandwidth and signal/noise ratio characteristic of a CTD. The methods used to obtain specific results are sufficiently general that they can be used, for example, to calculate error rates for nonoptimal detection schemes.

II. REVIEW OF INCOMPLETE CHARGE TRANSFER, SIGNAL ATTENUATION, AND BANDWIDTH

As with other electronic devices, signal attenuation and device bandwidth are extremely useful concepts with which to discuss the maximum storage capacity and minimum error rates characterizing the operational limitations of charge transfer devices. In a CTD, signal attenuation arises primarily from the incomplete transfer of charge[3–9] and only secondarily from charge losses, for example, through thermal (or other) leakage currents. Let us first consider incomplete transfer, then signal attenuation, and finally bandwidth.

2.1 *Incomplete Transfer*

It is clear that the incomplete transfer of charge from one elemental cell to the next will lead to signal degradation.[7,9] The character of this degradation can be ascertained as follows. The charge $Q_{i,t}$ in the ith elemental cell at time t will be the charge in the previous cell during the previous transfer cycle (of period τ_o) diminished by the charge $Q^b_{i-1,t-\tau_o}$ left behind in the $(i-1)$th cell but increased by the charge left behind in the ith cell during the immediately preceding transfer $Q^b_{i,t-\tau_o}$, and also less any charge lost (or gained) during the previous storage, $Q^l_{i-1,t-\tau_o}$. Thus

$$Q_{i,t} = Q_{i-1,t-\tau_o} - Q^b_{i-1,t-\tau_o} + Q^b_{i,t-\tau_o} - Q^l_{i-1,t-\tau_o}. \tag{1}$$

In general, this equation is very difficult to solve since the Q^b_i and Q^l_i are nonlinear functions of Q_i. Nonlinearities, however, are common in electronic devices. Taking the usual approach, one makes a small-signal analysis in order to linearize the equations. Thus we write $Q_{i,t} = q_{i,o} + q_{i,t}$, where $q_{i,o}$ is the time-independent (dc) component (bias) and $q_{i,t}$ is the small (ac)-signal component. Similarly, Q^b and Q^l can be decomposed into dc and ac components. Substituting these into eq. (1) we can obtain two equations, one for the dc terms and one for the ac terms. The dc equation can give us the time-independent (dc) charge bias level at the output. While knowledge of this may be important in some applications, it does not lead to any significant operational limitations. Of greater importance is the solution of the equation for the ac terms.

The equation for the ac terms is from eq. (1)

$$q_{i+1,t+\tau_o} = q_{i,t} - q^b_{i,t} + q^b_{i+1,t} - q^l_{i,t}. \tag{2}$$

As part of the linearization, one takes $q^b_i = p\alpha q_i$ and $q^l_i = p\beta q_i$. (α and β can be calculated from the coefficients of a Taylor series expansion of Q^b and Q^l in terms of q: $p\alpha = dQ^b/dQ_o$ and $p\beta = dQ^l/dQ_o$, where p is the number of individual charge transfers within an elemental cell and Q_o is the charge to be transferred.) Substituting into (2) one obtains the basic equation for $q_{i,t}$:

$$q_{i+1,t+\tau_o} = q_{i,t} - p\alpha q_{i,t} + p\alpha q_{i+1,t} - p\beta q_{i,t} \tag{3}$$

which becomes upon taking the Fourier transform

$$q_{i+1}(\omega)e^{i\omega\tau_o} = q_i(\omega)(1 - p\alpha - p\beta) + p\alpha q_{i+1}(\omega). \tag{4}$$

Up to this point we have linearized eq. (1) and passed to the frequency domain, typical procedures in electrical engineering. We proceed to solve eq. (4) by first calculating

$$q_{i+1}(\omega)/q_i(\omega) = \frac{1 - p\alpha - p\beta}{1 - p\alpha e^{-i\omega\tau_o}} e^{-i\omega\tau_o}.$$

Then we note that since $q_{i+1}(\omega)/q_i(\omega)$ is independent of i, it follows that $q_N(\omega)/q_o(\omega) = [q_{i+1}(\omega)/q_i(\omega)]^N$, where N is the number of elementary cells in the shift register. Recognizing that $q_N(\omega)/q_o(\omega)$ is the transfer function of the shift register, $H(\omega)$, one finds that[7,9]

$$H(\omega) = e^{-i\omega N\tau_o}\left(\frac{1 - pa - p\beta}{1 - p\alpha e^{-i\omega\tau_o}}\right)^N. \tag{5}$$

As discussed in Section 2.2, $H(\omega)$ can in principle be used to determine $q_{N,t}$ for any given sequence of input charge packets $q_{t-N\tau_o}$, $q_{t-(N+1)\tau_o}$.

The first factor in $H(\omega)$ is just the phase delay in the signal present even in the limit of perfect transfer ($\alpha = \beta = 0$). The second factor contains a frequency-dependent attenuation and a further phase shift. To a good approximation we may write

$$H(\omega) = A(\omega)e^{-i\phi(\omega)}, \tag{6}$$

where the attenuation factor $A(\omega)$ is given by

$$A(\omega) = e^{-n\beta}e^{-n\alpha(1-\cos\omega\tau_o)} \tag{7}$$

and where the phase factor $\phi(\omega)$ is given by

$$\varphi(\omega) = N\omega\tau_o + n\alpha\sin\omega\tau_o. \tag{8}$$

($n = Np$, the total number of charge transfers in the shift register from input to output.) With knowledge of the device transfer function $H(\omega)$, we can discuss the attenuation $A(\omega)$ and then compute the device bandwidth.

2.2 *Attenuation*

The attenuation factor in eq. (7) can be interpreted as follows. The first factor results from charge loss. If a fraction β of charge is lost with each charge transfer, after n transfers the fraction remaining is just $(1-\beta)^n \approx \exp(-n\beta)$ (if $n\beta^2 \ll 1$). Charge loss is clearly frequency independent. The second factor results from the incomplete transfer of charge. For $\omega\tau_o \approx 0$, very-low-frequency components, the size of adjacent charge packets is approximately the same. Thus the charge incompletely transferred at site i is nearly compensated by the charge incompletely transferred at $i+1$. [$-p\alpha q_i + p\alpha q_{i+1} \approx 0$ in eq. (3).] Thus, apart from charge losses, $q_{i+1} \approx q_i$ and, hence, low-frequency components are expected to be attenuated very little. Equation (7) bears this out. By contrast, if $\omega\tau_o = 2\pi f/f_o \approx \pi$ ($f \approx f_o/2$ where $f_o = 1/\tau_o$ is the clock frequency), the attenuation is relatively large, $\exp(-2n\alpha)$. Again referring to eq. (3), $f \approx f_o/2$ implies that $q_{i,t}$ and $q_{i+1,t}$ are ~180 degrees out of phase and $q_{i,t} \approx -q_{i+1,t}$. Thus contributions to incomplete transfer add (rather than compensate as for low frequencies) and, again ignoring charge loss, eq. (3) predicts an attenuation of $(1-2\alpha)^n \approx \exp(-2n\alpha)$. Again, eq. (7) bears this out. For $\omega\tau_o = \pi/2$ ($f = f_o/4$) the attenuation is $\exp(-n\alpha)$, an intermediate case in which the phases of each successive packet differ by 90 degrees.

One further point concerning incomplete transfer should be emphasized. A charge packet which "loses" a fraction α of its charge in each of n transfers might be expected to be attenuated by a factor of $(1 - \alpha)^n \approx \exp(-n\alpha)$. However, eq. (7) for $A(\omega)$ shows how sensitive the actual degradation of a packet is to the presence and nature of the other charge packets composing the signal. Thus considering one "isolated" charge packet can be very misleading. In Appendixes A and B we discuss examples of attenuation in the time domain, and in

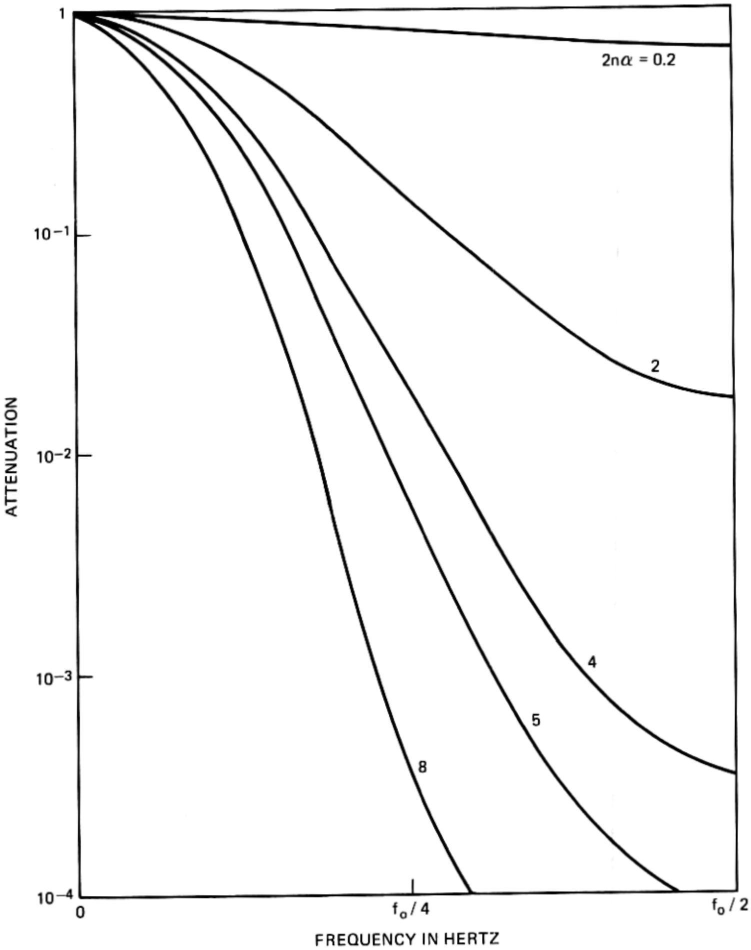

Fig. 1—Attenuation versus frequency for CTD's with $2n\alpha$ of 0.2, 2, 4, 5, 8.

Appendix C we consider $A(\omega)$ in more detail. $A(\omega)$ is plotted in Fig. 1 as a function of ω for several values of $n\alpha$. ($\beta = 0$.)

2.3 *Bandwidth*

In Fig. 1 we have plotted $A(\omega)$ for $0 \leqq f \leqq f_o/2$, although eq. (7) would seem to apply for $0 \leqq f$. The reason for this is basic. According to Shannon's sampling theorem,[2] $f_o/2$, one-half the clock frequency (one-half the sampling frequency), is the maximum frequency of the signal which can be transmitted. Thus given the clock frequency f_o, the maximum bandwidth a CTD can have is $f_o/2$.

Incomplete charge transfer clearly reduces the effective bandwidth of a CTD. This is evident from the attenuation plotted in Fig. 1. Normally one defines bandwidth by the size of the range of frequencies for which the attenuation A exceeds some fraction $\delta < 1$. A more convenient definition from the point of view of information transmission and storage capacity is that the bandwidth B be given by the following expression:

$$B = \int_0^{f_o/2} \frac{|A(f)|^2}{|A(0)|^2} df = \frac{f_o}{2} e^{-2n\alpha} I_o(2n\alpha) , \tag{9}$$

where I_o is a modified Bessel function.[10] In Fig. 2, B is plotted as a function of $n\alpha$. A slowly varying function, B decreases as $(f_o/2)\cdot(4\pi n\alpha)^{-\frac{1}{2}}$ for $n\alpha \gg 1$. Thus despite the rapid attenuation associated with $n\alpha \approx 10$ ($e^{-10} \approx 0.5 \times 10^{-4}$ for $f = f_o/4$ and $e^{-20} \approx 0.2 \times 10^{-8}$ for $f = f_o/2$), the bandwidth is still approximately $0.09 \times (f_o/2)$, 9 percent of its maximum value. The relative insensitivity of β to $n\alpha$

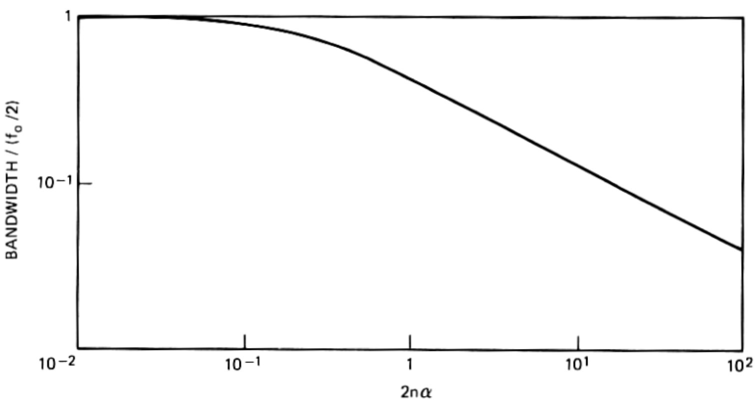

Fig. 2—Bandwidth versus $n\alpha$ for a CTD.

has an important effect on the maximum achievable CTD transmission and storage capacities as we shall see.

III. THE ROLE OF INCOMPLETE CHARGE TRANSFER IN SIGNAL DEGRADATION

Even though attenuation and bandwidth are frequency-domain (and hence analog) concepts, we shall see in Section VII that it is these quantities, along with the signal-power-to-noise-power ratio, which are needed in order to calculate the maximum information transmission and storage capacity for digital as well as analog signals in CTD's. Nonetheless, it is still helpful to discuss certain features of digital signals in the time domain in order to better appreciate certain operational limitations of CTD's. For the next few sections we shall ignore charge losses and consider only the much more important effects of incomplete charge transfer on the signal.

Suppose a charge packet of initial size Q_1 (representing a digital "one") follows some sequence of charge packets either of initial size Q_1 or of initial size Q_0 (representing a digital "zero"). If α is the coefficient of incomplete transfer, then after a single transfer the original charge packet has been reduced by a factor of $(1 - \alpha)$, and, after n similar transfers, by a factor of $(1 - \alpha)^n$. In addition, the original packet picks up some charge left behind by the preceding packets. This residual charge we shall refer to as Q_R, which is in general a function of the preceding signal. Thus the size of the charge packet representing the "one" at the output is given by

$$Q = (1 - \alpha)^n Q_1 + Q_R. \tag{10}$$

A brief analysis of eq. (10) reveals that the primary operational limitation imposed by incomplete transfer is the dependence of the residual charge Q_R on the preceding signal—the preceding sequence of zeroes and ones. Neither the attenuation of the size of the charge packet *per se* nor the accumulation of the incompletely transferred charge *per se* play a primary role. For example, suppose that we are using a very simple form of absolute-amplitude detection in which any charge packet of size $Q > \bar{Q} = (Q_1 + Q_0)/2$ is detected as a "one" and any packet of size $Q < \bar{Q}$ is detected as a "zero." Then a one preceded by a long string of zeroes will be detected as a zero if $n\alpha > 0.7$. Thus, under noiseless conditions we shall have a *non*zero error rate in cases where the signal attenuation is only one-half (see Appendix A). On the other hand, the accumulation of incompletely transferred charge can be reduced using zero-net-charge coding. As each bit of signal is coded into the same amount of total charge (distributed between two packets), the total amount of incompletely transferred charge from each bit is the same. Nonetheless, no significant improvement is obtained. Indeed, the maximum $n\alpha$ for zero errors under noiseless conditions remains a strong function of the preceding sequence of charge packets (see Appendix B). This result is true for analog as well as digital signals, as is clear from the discussion in Appendix C. The dependence of the residual charge on the previous signal suggests that more attention should be given the detection of the signal rather than its coding. This we discuss in the next section.

IV. THE OPTIMUM DETECTION OF SIMPLE DIGITAL SIGNALS

Let us suppose that some arbitrary sequence of charge packets of size Q_1 for a "one" and Q_0 for a "zero" have preceded the charge packet which we now wish to detect. The residue or residual charge added to the charge packet of interest can be designated Q_R as before, where Q_R is a function (given in Appendix A) of the preceding signal. If the charge packet which we are detecting is in fact a one, then the size $Q(1)$ of the packet will be

$$Q(1) = (1 - \alpha)^n Q_1 + Q_R. \tag{11}$$

If, however, the charge packet is zero, then the packet's size $Q(0)$ will be given by

$$Q(0) = (1 - \alpha)^n Q_0 + Q_R. \tag{12}$$

One will clearly optimize the detection (even in the presence of noise) if one choses for the detection level Q_d of the mean of $Q(1)$ and $Q(0)$:

$$Q_d \equiv (1 - \alpha)^n \tfrac{1}{2}(Q_1 + Q_0) + Q_R. \tag{13}$$

If $Q > Q_d$, we say that we have detected a Q_1 packet, and if $Q < Q_d$ we say that we have detected a Q_0 packet. Because Q_d is a function of Q_R which in turn depends on the entire preceding signal, we shall refer to this as a dynamic detection procedure. In contrast to Q_d given in (13), the static detection procedure mentioned in Section III has $Q_s = \bar{Q}/(1 - \alpha)$, and $Q > Q_s$ implies Q_1 and $Q < Q_s$ implies Q_0.

It is shown in Appendix A that under noiseless conditions this scheme of dynamic coding is errorfree regardless of the size of $n\alpha$ or of the nature of the preceding sequence of zeroes and ones. This again illustrates the role of the dependence of Q_R on the preceding signal, which we noted at the end of Section III. It is shown in Appendix A that

$$Q(1) - Q_d = (1 - \alpha)^n \frac{Q_1 - Q_0}{2} = Q_d - Q(0) \tag{14}$$

as is clear from eqs. (11), (12), and (13) as well. The quantity $(Q_1 - Q_0)$ may be referred to as the dynamic range of the device. Thus relative to the dynamic detection level, Q_d, the signal, $Q(1)$ or $Q(0)$, is attenuated as $(1 - \alpha)^n$. [Note that $[Q(1) - Q_d]$ and $[Q_d - Q(0)]$ are independent of the residual charge Q_R.] In the presence of noise, errors will clearly be introduced if $(1 - \alpha)^n(Q_1 - Q_0)/2$ approaches the noise level. This also shows that, having eliminated the signal-dependent residual charge, attenuation now plays an important role in limiting device operation. As n increases, this signal attenuation coupled with the compounding of noise both reduce the signal-to-noise ratio and lead to a reduction in the information transmission and storage capacities of the device. However, now it will be for $n\alpha \approx 4$ rather than for $n\alpha \approx 0.7$ that attenuation becomes limiting.

To set the dynamic detection level Q_d, Q_R must be realizable. In the absence of noise, this is always possible in principle since Q_R is an explicit function of the known, preceding signal. In the presence of noise, Q_R determined by eq. (31) also yields, for most cases of interest, nearly optimal detection in spite of the possibility that some preceding packets may have been incorrectly detected.[11]

In Section V, we briefly review noise in CTD's and then in Section VI we shall see how this dynamic detection scheme minimizes the error rate in the presence of noise. This further stresses the importance of detection in optimizing the operation of "simply coded" CTD's.

V. REVIEW OF NOISE

Noise in charge transfer devices is a fascinating subject which, unfortunately, can be only highlighted in this section.[12–17] Owing to the dramatic time dependence of the current during a single charge transfer, the noise generated during a single transfer is quite nonstationary. Since nearly all theories of noise in solid-state devices assume that the noise is stationary,[18] it is necessary to redo much of the theory taking into account the nonstationary aspect. A time-domain analysis has been found to be most convenient, whereas standard treatments are carried out in the frequency domain.

In Fig. 3 the most common sources of noise in CTD's are categorized. At the input, one has full shot noise only if the electrons enter the source independently (e.g., if generated by the random arrival of phonons in an imaging device or if injected by an emission-limited diode). At the output the nonrandom coupling to the clock line is the worst source of distortion in some cases. A distinction[14] is made between noise generated from transfer processes, typically thermal and trap-

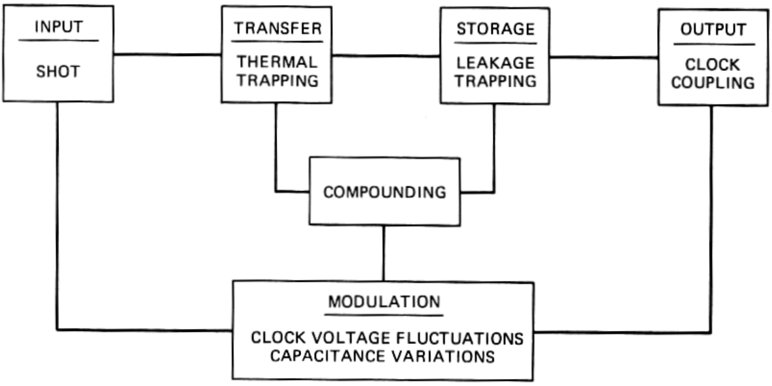

Fig. 3—Sources of noise for a CTD.

ping noise, and that from storage processes, typically leakage and again trapping noise. We shall return to this distinction shortly. Since a charge packet acquires some noise with each transfer-storage period, the noise component increases as the packet is transferred from input to output. This we refer to as compounding. Finally, the occurrence of clock voltage fluctuations in the presence of fabrication variations in the individual capacitances leads to a form of modulation noise. This type of noise is also compounded. For simplicity we have left out of Fig. 3 a number of less important noise sources.

Let us now return to the important distinction between storage process and transfer process noise.[14] It should be recalled that a CTD shift-register performs two functions simultaneously, the transfer of charge and the storage of charge. In Fig. 4 we indicate the basic distinction between the noise generated from these two processes. In the case of storage process (SP) noise, the charge fluctuation generated during each transfer cycle in each cell is essentially independent of that in any other cell. For transfer process (TP) noise this is not the case. Conservation of charge implies that if an excess of ΔQ is transferred from one storage region to the next, $-\Delta Q$ is left behind for the subsequent charge packet. This introduces a correlation in the noise in adjacent charge packets which leads to a suppression at low frequencies of the spectral density of TP noise. SP noise, by contrast, is uncorrelated and, therefore, the spectral density is flat (white). This difference between TP and SP noise is important for analog applications of CTD's and is discussed in more detail elsewhere.[14]

For digital applications we shall need the ratio S/N of the square of the signal charge to the mean-square noise charge at the detector.

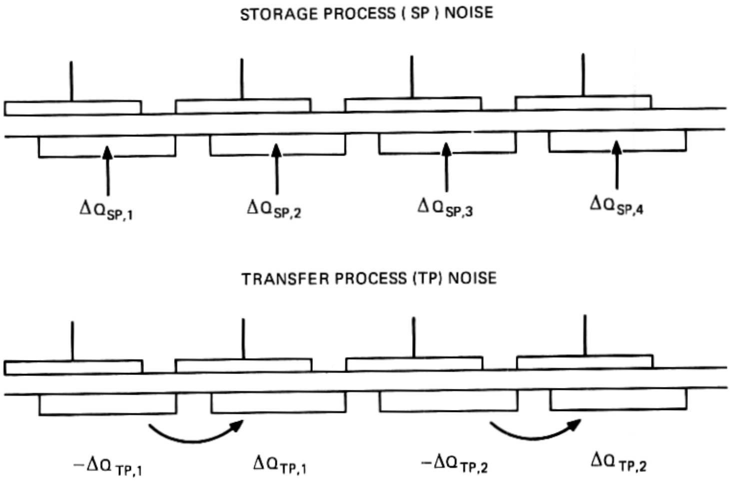

Fig. 4—The distinction between storage process and transfer process noise.

From the discussion of Section IV [see eq. (13)] the square of the effective signal charge at the detector is $(1 - \alpha)^{2n}[(Q_1 - Q_0)/2]^2$. The mean-square noise charge ΔQ^2 can be written

$$\Delta Q^2 = \Delta Q^2_{\text{Input}}(1 - \alpha)^{2n} + \Delta Q^2_{\text{SP}} H_{\text{SP}}(n) + 2\Delta Q^2_{\text{TP}} H_{\text{TP}}(n), \quad (15)$$

where $\Delta Q^2_{\text{Input}}$ is the input noise contribution, ΔQ^2_{SP} the storage process noise acquired by a single packet during a single clock period, ΔQ^2_{TP} the transfer process noise acquired by a single packet during a single charge transfer, $(1 - \alpha)^{2n}$ the attenuation from input to output, $H_{\text{SP}}(n)$ the compounding factor[12] for storage process noise, and $H_{\text{TP}}(n)$ the compounding factor[12] for transfer process noise. These compounding factors are approximately equal to n for $n\alpha \ll 1$; however, for $n\alpha \gtrsim 1$ they are suppressed[12] by incomplete transfer effects. For $n\alpha \gg 1$, $H_{\text{SP}}(\text{n}) \approx (n/\pi\alpha)^{\frac{1}{2}}$ and $H_{\text{TP}} \approx (2\alpha)^{-1}$, both of which are much less than n. Essentially the effect of incomplete transfer is to attenuate the accumulated noise as well as the signal. Owing to the correlation between the transfer-process noise components,[14] H_{TP} saturates. For shot noise, $\Delta Q^2_{\text{Input}} = qQ$, where Q is the (mean) total signal charge $(Q_1 - Q_0)$. For thermal noise, $\Delta Q^2_{\text{TP}} = \frac{2}{3}\text{k}TC$, where T is the temperature of the charge carriers and C is the storage capacitance. For our purposes here we shall ignore other noise contributions. Thus we find

$$S/N = (1 - \alpha)^{2n}[(Q_1 - Q_0)/2]^2/\Delta Q^2, \quad (16)$$

where ΔQ^2 is given by eq. (15). Equation (16) is plotted in Figs. 5 and 6 for $\alpha = 10^{-3}$ and $\alpha = 10^{-4}$ for $C = 1$, 0.1, 0.01, and 0.001 pF, for thermal noise only and for thermal and shot noise. We shall use eq. (16) in Section VI on error rates and in Section VII on storage capacity.

VI. MINIMUM DIGITAL ERROR RATES

No one would operate a CTD under conditions where errors in detection could occur even under noiseless conditions. However, in the presence of noise it is possible for a "one" to acquire sufficient net "negative" noise charge to be detected as a "zero" even under optimum conditions. It is the purpose of this section to calculate the probability of making a detection error, and to see to what degree the error rate (error probability times clock frequency) is minimized for the simple, two-level digital coding scheme by using dynamic detection.

Suppose that an arbitrary charge packet following an arbitrary sequence of charge packets would, under noiseless conditions, be of size Q_s at the output of the shift register. In the presence of noise the probability $P(Q)$ that the observed size of the packet is Q within dQ is given by

$$P(Q)dQ = \exp[-(Q - Q_s)^2/2\Delta Q^2]/(2\pi\Delta Q^2)^{\frac{1}{2}}dQ. \tag{17}$$

If we are using only zeroes and ones, then the probability P of detecting a certain "one" as a zero is given by

$$P_1 = \int_{-\infty}^{Q_d} P(Q)dQ\,, \tag{18}$$

where Q_d is the detection level (see Fig. 7). In eq. (18), P_1 depends upon $Q_s = Q(1)$ which in turn is a function of the sequence of signal charge packets preceding the one [see eqs. (11) and (12)]. To determine the average error probability, P_1 must now be averaged over all possible sequences of signals, in general a very difficult task.

Let us write eq. (18) in a slightly different form by changing variables.

$$P_1 = \int_{-\infty}^{Q_d - Q(1)} \frac{\exp(-Q^2/2\Delta Q^2)}{(2\pi\Delta Q^2)^{\frac{1}{2}}}\,dQ$$

or

$$P_1 = \int_{-\infty}^{[Q_d - Q(1)]/(\Delta Q^2)^{\frac{1}{2}}} e^{-x^2/2}dx/(2\pi)^{\frac{1}{2}}$$

or

$$P_1 = f_1\{[Q_d - Q(1)]/(\Delta Q^2)^{\frac{1}{2}}\}, \tag{19}$$

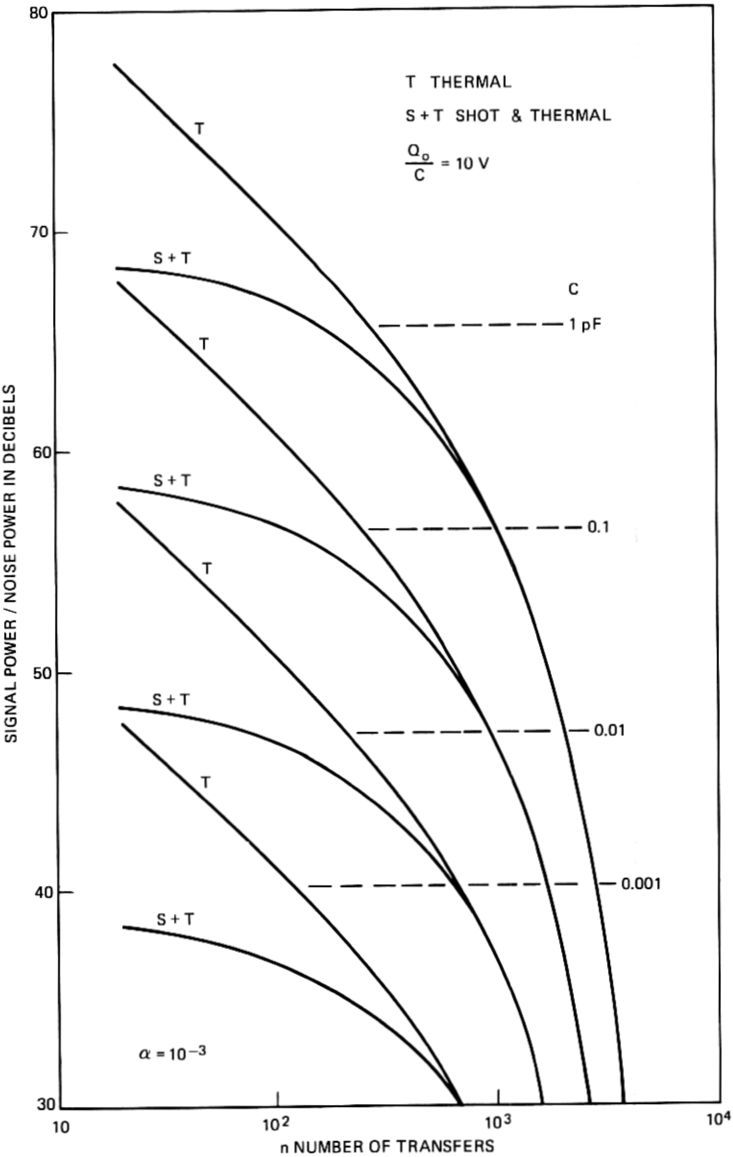

Fig. 5—Signal-to-noise ratio for CTD ($\alpha = 10^{-3}$) with storage capacitance C of 1 pF, 0.1 pF, 0.01 pF, 0.001 pF.

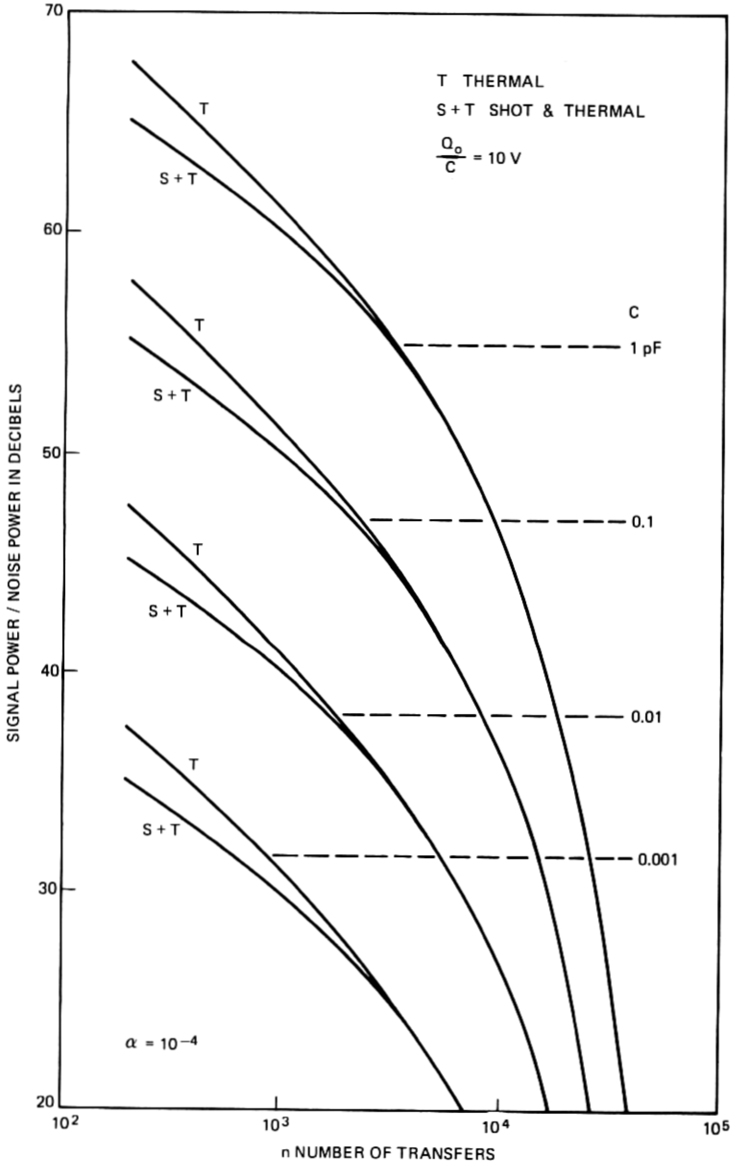

Fig. 6—Signal-to-noise ratio for CTD ($\alpha = 10^{-4}$) with storage capacitance C of 1 pF, 0.1 pF, 0.01 pF, 0.001 pF.

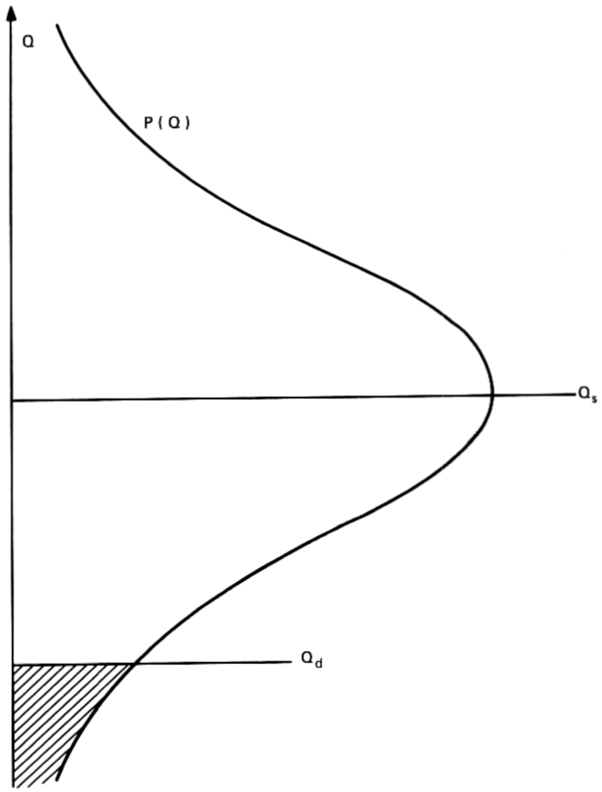

Fig 7.—A charge packet of size Q_s under noiseless conditions has a probability distribution of $P(Q)$ in the presence of noise. The probability that $Q < Q_d$ (that an error is made in detection) is the area "under" $P(Q)$ for $Q < Q_d$.

where, of course,

$$f_1(y) \equiv \int_{-\infty}^{y} e^{-x^2/2} dx/(2\pi)^{\frac{1}{2}}. \tag{20}$$

We desire $\langle P_1 \rangle = \langle f_1 \rangle$, where the brackets "$\langle\ \rangle$" denote averaging $Q(1)$ (and possibly Q_d) over all possible sequences of ones and zeroes $[Q(1) = Q(1-\alpha)^n Q_1 + Q_R$ from eq. (11)]. If $Q_s > Q_d$ for all possible Q_R (as it must if errors are to be avoided under noiseless conditions), then we show in Appendix D that $\langle P \rangle \geqq f\{\langle [Q_d - Q(1)]/(\Delta Q^2)^{\frac{1}{2}} \rangle\}$. In other words, the function f evaluated at the average of its argument is a lower bound to the average of the function, the average error probability. This permits putting a *lower* bound on the error rate.

If $[Q_d - Q(1)]$ is independent of Q_R, as is the case for the dynamic detection scheme discussed in Section IV, then

$$\langle P_1 \rangle = f_1\{[Q_d - Q(1)]/(\Delta Q^2)^{\frac{1}{2}}\} \tag{21}$$

and the average error probability is at the lower bound. For the dynamic detection scheme of Section IV, $[Q_d - Q(1)] = -(1-\alpha)^n \times (Q_1 - Q_0)/2$, which implies that

$$\langle P_1 \rangle = \int_{-\infty}^{(S/N)^{\frac{1}{2}}} e^{-x^2/2} dx/(2\pi)^{\frac{1}{2}}, \tag{22}$$

where (S/N) is given in eq. (18). If static detection had been used $[Q_d = \bar{Q}/(1-\alpha)]$, then the average error probability would always exceed the $\langle P_1 \rangle$ given in (22).

It remains to prove that the optimum (dynamic) detection scheme given in Section IV gives the lowest possible error probability for simple digital coding. The proof is as follows. We found above that for detecting a one the average error probability was at least

$$\langle P_1 \rangle_{\text{lb}} = f_1\{[\langle Q_d \rangle - \langle Q(1) \rangle]/(\Delta Q^2)^{\frac{1}{2}}\}. \tag{23}$$

(The "lb" stands for *l*ower *b*ound.) Noting the definition of f_1 [eq. (20)], we note that we can make $\langle P_1 \rangle_{\text{lb}}$ smaller by reducing $\langle Q_d \rangle$, $\langle Q(1) \rangle$ being already determined. However, we must also consider the error probability in detecting a "zero." Proceeding as for a "one," we obtain for P_0 the error probability for detecting a certain "zero,"

$$P_0 = f_0\{[Q_d - Q(0)]/(\Delta Q^2)^{\frac{1}{2}}\}, \tag{24}$$

where now

$$f_0(y) \equiv \int_y^{\infty} e^{-x^2/2} dx/(2\pi)^{\frac{1}{2}}. \tag{25}$$

Also

$$\langle P_0 \rangle_{\text{lb}} = f_0\{[\langle Q_d \rangle - \langle Q(0) \rangle]/(\Delta Q^2)^{\frac{1}{2}}\}. \tag{26}$$

From the definition of f_0, we note that we can made $\langle P \rangle_{\text{lb}}$ smaller by increasing $\langle Q_d \rangle$, $\langle Q(0) \rangle$ being already determined. Assuming that an equal number of "zeroes" and "ones" are used in the simple digital coding, then (by symmetry) choosing $\langle Q_d \rangle$ so that $\langle P_1 \rangle_{\text{lb}} = \langle P_0 \rangle_{\text{lb}}$, we shall achieve the minimum lower bounds. $\langle P_1 \rangle = \langle P_0 \rangle_{\text{lb}}$ for $\langle Q_d \rangle = [\langle Q(1) \rangle + \langle Q(0) \rangle]/2$. But for our dynamic detection scheme

$$\langle Q_d \rangle = \bar{Q}(1-\alpha)^{-1} = [\langle Q(1) \rangle + \langle Q(0) \rangle]/2 \tag{27}$$

and for our dynamic detection scheme $\langle P_1 \rangle = \langle P_1 \rangle_{\text{lb}}$ and $\langle P_0 \rangle = \langle P_0 \rangle_{\text{lb}}$. Therefore, since $\langle Q_d \rangle$ for the dynamic detection scheme produces the lowest possible lower bounds for error probabilities, and since the error probabilities are in fact equal to these lower bounds, no other detection scheme can detect with lower error probability. (It is possible that another scheme can do just as well, however, since it is only $\langle Q_d \rangle$ and not Q_d itself which is the determining factor.)

It is clear that this theorem places an operational limitation (a minimum error rate in detection) and CTD's using simple digital two-level coding. The theorem can be extended[11] to the dynamic detection of multilevel digital codes.

VII. MAXIMUM STORAGE CAPACITY

One use of the CTD is as a memory or storage element. In other applications the CTD can be used to shift or to transfer information from one location to another. To properly access the operations of CTD's in these applications one must calculate the maximum information transmission capacity and the maximum information storage capacity of the CTD. As a result of the work of Shannon, our labors are greatly diminished.

Shannon[2] proved a most profound theorem. Let B be the bandwidth of a transmission channel, and let S/N be the signal-power-to-noise-power ratio. Then the maximum transmission capacity of the channel C_T in bits per second is given by

$$C_T = B \log_2(1 + S/N). \tag{28}$$

This result can be understood for the CTD in the $S/N \gg 1$ range as follows. The number of levels into which a digital signal can be divided and still be detected with reasonably small error is $(S/N)^{\frac{1}{2}}$. $\text{Log}_2(S/N)^{\frac{1}{2}}$ is the maximum amount of information in bits detected with each charge packet. f_o is the rate at which charge packets are detected. Thus $f_o \log_2(S/N)^{\frac{1}{2}} = \frac{1}{2} f_o \log_2(S/N) \approx B \log_2(1 + S/N)$ is the number of bits of information transmitted per second. (In Section 2.3 we noted that for $n\alpha \ll 1$, $B \approx f_o/2$.) Shannon was, of course, much more interested in the $S/N \ll 1$ range. For this case his theorem implies that no matter how noisy the transmission channel may be, it is always possible to pass information along it. We shall not make use of Shannon's result in this latter range.

A more interesting quantity from the standpoint of the CTD is the maximum information storage capacity. This can be calculated from Shannon's Theorem[2] as follows. If C_T is the number of bits per second transmitted, then if one waits a time T_o equal to the time it takes the information to be transferred from the input to the output of the linear medium, the maximum storage capacity in bits C_s must be given by

$$C_s = T_o C_T = T_o B \log_2(1 + S/N). \tag{29}$$

For a transmission line, T_o is given by the length of the line divided by the propagation velocity. For a CTD, $T_o = N_o/f_o$ where $N_o = n/p$ and p is the number of charge transfers per clock period $T_o = 1/f_o$. Thus for a CTD we find for the maximum information storage capacity C_s in bits:

$$C_s = N_o(B/f_o) \log_2(1 + S/N). \tag{30}$$

[Strictly speaking, the maximum information storage capacity will actually be less than or equal to the C_s given in eq. (30). This is because as S/N decreases, the length of the code word increases.[2] However, for a CTD with N_o storage units, the maximum length of a code word is restricted to N_o. Thus for small S/N, the prediction of eq. (30)

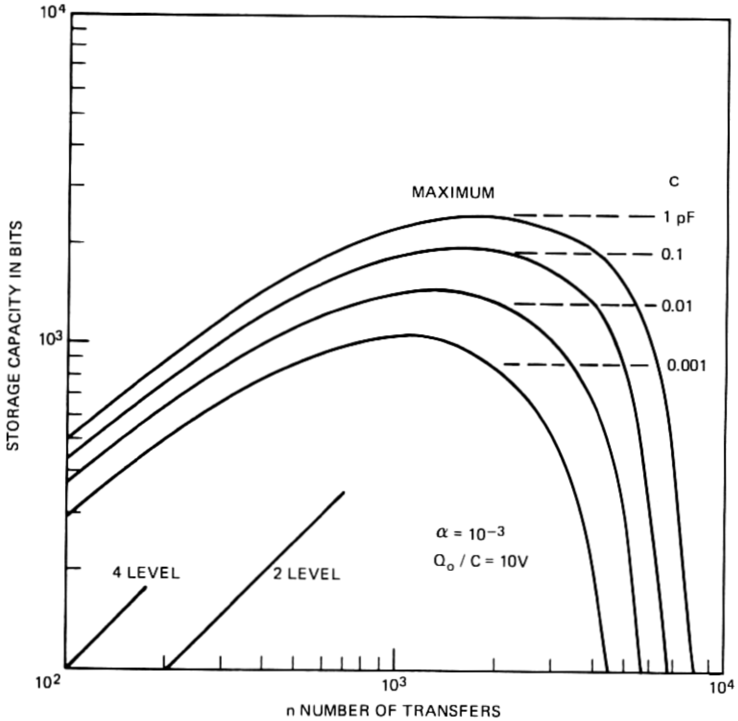

Fig. 8—Storage capacity C_s versus n, the number of charge transfers, for $\alpha = 10^{-3}$, $Q/C = 10$ volts, $C = 1$, 0.1, 0.01, 0.001 pF. Shown for comparison is C_s for 2- and 4-level digital codes.

may be in fact unrealizable. As our primary concern is for $S/N \gg 1$, the upper limit of C_s should be sufficiently accurate.]

Knowing N_o, (B/f_o), and (S/N) as functions of n, the number of charge transfers, we can calculate C_s versus n to determine the maximum C_s possible under various circumstances and for what n C_s is maximum. This has been done in Fig. 8 for $\alpha = 10^{-3}$ and in Fig. 9 for $\alpha = 10^{-4}$. In both figures $Q/C = 10$ volts, $Q = Q_1 = 2Q_0$, and storage capacitance $C = 1$, 0.1, 0.01, 0.001 pF. Also shown is C_s for two-level and four-level codes. Here n is limited by an n(maximum) for each code at the number of transfers beyond which signal degradation due to incomplete transfer would lead to errors in absolute-amplitude detection in the absence of noise. We note (*i*) that the maximum C_s occurs for n about a factor of three larger than for n(maximum) from the examples of simple coding and detection, and (*ii*) that the maximum value of C_s is about a factor of four to five

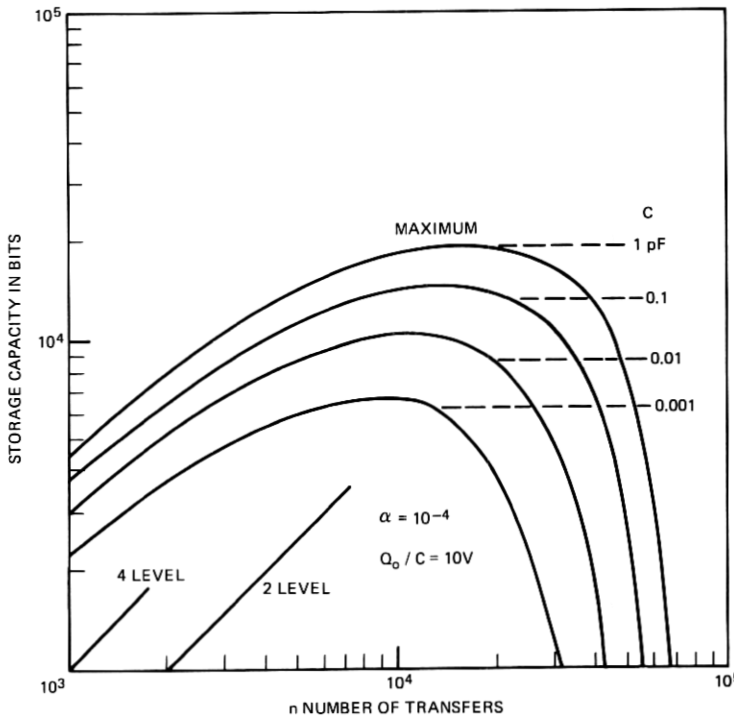

Fig. 9—Storage capacity C_s versus n, the number of charge transfers, for $\alpha = 10^{-4}$, $Q/C = 10$ volts, $C = 1$, 0.1, 0.01, 0.001 pF. Shown for comparison is C_s for 2- and 4-level digital codes.

larger than C_s at n(maximum) for these common digital codes. [It should also be noted that C_s at n(maximum) for two-level digital coding exceeds C_s at n(maximum) for multilevel coding. This is discussed in more detail elsewhere.[19]] It is encouraging to note that there exists such a margin between what is theoretically possible and what can be simply accomplished. From Section VI, however, it appears that it will be rather difficult to achieve optimum performance if this be desired (or essential).

VIII. CONCLUSIONS AND RECOMMENDATIONS

It is not surprising to find that incomplete charge transfer and random noise (especially shot and thermal) limit the bandwidth, storage capacity, and error rate of CTD's. What is surprising, however, is that the residual charge level Q_R resulting from the portions of charge (preceding the packet of interest) incompletely transferred is so strongly signal dependent that signal detection with static detection levels becomes seriously impaired prior to the onset of significant signal attenuation or noise degradation. Coding to offset the signal dependence of Q_R is found to be ineffective for the simple examples considered. On the other hand, by employing our dynamic detection scheme, which adjusts the detection levels to null out the signal dependence of the incompletely transferred charge, the operational range is significantly extended, limited only by the physically unavoidable effects of attenuation and noise. It is also shown that no detection scheme can be devised with a lower error rate than this dynamic detection scheme.

It might be concluded on the basis of the above result that more attention should be focused on detection rather than coding as a means of offsetting the worst effects of incomplete charge transfer. Noting the results shown in Figs. 8 and 9, however, it is apparent that substantial increases in storage capacity are possible with more sophisticated coding-decoding schemes.

IX. ACKNOWLEDGMENTS

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APPENDIX A

In this appendix the degradation of digital signals is discussed in general in some detail. In Appendix B these results are applied to certain specific cases using several simple coding procedures. In particular, the signal-dependent residual charge Q_R is discussed. These mathematical details should be of some assistance in understanding several equations presented in Sections III, IV, and VI of the text. The analysis will be in the time domain. In Appendix C a frequency domain analysis is given.

We shall denote by Q_N the size of the Nth charge packet at the input which precedes the packet of interest by N clock cycles. As in the text we shall denote by Q_R the size of the accumulated residual charge originating from the incompletely transferred portions of the preceding packets, Q_N. Mathematically Q_R is given by[7,19]

$$Q_R = (1-\alpha)^n \sum_{N=1}^{\infty} \binom{n+N}{N} \alpha^N Q_N \,, \tag{31}$$

where n is the total number of transfers from input to output and α is the coefficient of incomplete transfer for each transfer. [Equation (31), as well as eq. (3), are somewhat approximate. To obtain a "pure" binomial factor in (31), or equivalently to be able to write a single-transfer equation like eq. (3), one must assume that the actual transfer of charge can be approximated by simplified single transfers either on a per-cell basis as in (31) or on a storage-region basis as in eq. (3). The error involved in this approximation will be of the order of α or $n\alpha^2$, whichever is larger.]

The physical significance of (31) is the following. The portion of Q_N which will show up in Q_R are electrons incompletely transferred N times, each time introducing a factor α. The binomial factor gives the number of distinct alternative sequences of "transfer" or "no transfer" which can lead to a portion of Q_N contributing to Q_R.

Suppose now, as in the first example in Section III, we have a packet of size Q_1 preceded by an infinite string of packets of size Q_0. Then for Q_R one has

$$\begin{aligned} Q_R(000\cdots) &= (1-\alpha)^n \sum_{N=1}^{\infty} \binom{n+N}{N} \alpha^N Q_0 \\ &= -(1-\alpha)^n [1-(1-\alpha)^{-(n+1)}] Q_0 \,. \end{aligned} \tag{32}$$

Similarly,

$$Q_R(111\cdots) = -(1-\alpha)^n [1-(1-\alpha)^{-(n+1)}] Q_1 . \tag{33}$$

For a Q_1 following the string of Q_0's, the size of the charge packet Q[eq. (10)] at the output will equal $\bar{Q} = (Q_1+Q_0)/2$ if n is such that

$$(1-\alpha)^n Q_1 - (1-\alpha)^n [1-(1-\alpha)^{-(n+1)}] Q_0 = (Q_1+Q_0)/2$$

or if $(1-\alpha)^n = \frac{1}{2}$ [to within a factor of $(1-\alpha) \approx 1$]. Should $(1-\alpha)^n > \frac{1}{2}$, $Q < \bar{Q}$ and the one would be detected as a zero.

In Appendix B we shall discuss more complicated coding schemes to see whether Q_B (31) can be reduced or at least made less sensitive to the signal preceding the charge packet of interest. For the present let us continue to derive some of the other results stated in the text.

To cast eq. (13) into a simpler form we proceed as follows:

$$\begin{aligned} Q_d &= (1-\alpha)^n \frac{Q_1+Q_0}{2} + Q_R \\ &= (1-\alpha)^n\bar{Q} + (1-\alpha)^n \sum_{N=1}^{\infty} \binom{n+N}{N} \alpha^N (Q_N - \bar{Q} + \bar{Q}) \\ &= \bar{Q}\left[(1-\alpha)^n - (1-\alpha)^n \left(1 - \frac{1}{(1-\alpha)^{n+1}}\right)\right] + Q_R' \\ &= \bar{Q}(1-\alpha)^{-1} + Q_R', \end{aligned} \tag{13'}$$

where

$$Q_R' \equiv (1-\alpha)^n \sum_{N=1}^{\infty} \binom{n+N}{N} \alpha^N (Q_N - \bar{Q}) . \tag{34}$$

The static detection level, $Q_s = \bar{Q}/(1-\alpha)$, actually differs by a factor of $(1-\alpha)^{-1}$ from the $\bar{Q}$ used in Section III and in the discussion following eq. (33). The difference, while insignificant, arises from whether one takes Q_d to be the average of Q_1 and Q_0, the sizes of the charge packets at the input, or whether one takes Q_d to be the size at the output of an average charge packet [of size $\bar{Q} = (Q_1 + Q_0)/2$] following a string of similar packets. Thus for such a case

$$\begin{aligned} Q_s = Q &= \bar{Q}(1-\alpha)^n + (1-\alpha)^n \sum_{N=1} \binom{n+N}{N} \alpha^N \bar{Q} \\ &= \bar{Q}(1-\alpha)^n - (1-\alpha)^n[1 - (1-\alpha)^{-(n+1)}]\bar{Q} \end{aligned}$$

or

$$Q_s = \bar{Q}/(1-\alpha). \tag{35}$$

To show that by using the dynamic level Q_d given by eq. (13) one can have zero detection errors in the absence of noise we proceed as follows. Using eq. (11) one has at once that

$$Q(1) - Q_d = (1-a)^n(Q_1 - \bar{Q}) \tag{36}$$

independent of Q_R. As $Q_1 > Q_0$, $Q_1 > \bar{Q}$ and, therefore, $Q(1) - Q_d > 0$. Similarly using eq. (12) one finds $Q_d - Q(0) > 0$. Thus, in the absence of noise, $Q(1)$ and $Q(0)$ are always separated by Q_d, and hence no error need be made in distinguishing them.

APPENDIX B

In this appendix we use the results of Appendix A to investigate what improvement if any is possible in CTD operation by using several simple coding procedures. We shall assume noiseless absolute-amplitude detection using a static detection level at the average output charge level. Equation (31) for Q_R can be used to calculate the result of other coding procedures.

In Table I, I have enumerated four simple means of representing or coding a digital zero (0) and a digital one (1) using charge packets. The first is just to represent a 0 by a Q_0 packet and a 1 by a Q_1 packet. As calculated in Appendix A [see following (33)], a Q_1 following a long string of Q_0's [example "(a)"] will be detected as a Q_0 if $n\alpha > 0.7$. This is the "$n\alpha$ Limit" entry in the table. Finally, the size of the Q_1 packet is attenuated as $\exp(-n\alpha)$ as stated. For this coding a second example, "(b)," is given—a $\cdots Q_1Q_0Q_1Q_0 \cdots$ sequence. In this case Q_R is always sufficiently large for a Q_1 and sufficiently small for a Q_0 that under noiseless conditions $Q(1) > \bar{Q}$ and $\bar{Q} > Q(0)$ for any $n\alpha$. However, as noted in Section 2.2, such a signal is attenuated as $\exp(-n\alpha)$ attenuation.

One might hope that by preventing Q_R from becoming much differ-

TABLE I—FOUR SIMPLE MEANS OF REPRESENTING DIGITAL ZEROES AND ONES USING CHARGE PACKETS

Example	Representation		$n\alpha$ Limit*	Attenuation
	0	1		
	Q_0	Q_1		
(a) 1000 ···			0.7	$e^{-n\alpha}$
(b) 1010 ···			∞	$e^{-2n\alpha}$
	Q_0Q_0	Q_1Q_1		
(a) 1000 ···			(1) 0.7; (2) 1.67	$e^{-n\alpha}$
(b) 1010 ···			(1) 0.79; (2) 2.4	$e^{-n\alpha}$
	Q_1Q_0	Q_0Q_1		
(a) 1000 ···			(1) ∞; (2) 0.8	$e^{-2n\alpha}$, $e^{-n\alpha}$
(b) 1010 ···			(1) 2.36; (2) 0.785	$e^{-n\alpha}$
	$\bar{Q}\,Q_0$	$\bar{Q}\,Q_1$		
(a) 1000 ···			(1) 0.4; (2) 0.5	$e^{-n\alpha}$
(b) 1010 ···			(1) 1.6; (2) 3.1	$e^{-n\alpha}$

* The notation (1) refers to the first of the two packets forming a bit, and (2) refers to the second.

ent than $\bar{Q}$, one could increase the $n\alpha$ limit. In Table I three possibilities are given. The first consists merely of coding 0 into two adjacent Q_0 packets and 1 into two adjacent Q_1's. If one detects the second of the two Q_1 packets in sequence (a), the $n\alpha$ limit is increased to 1.67. For sequence (b) detecting the second Q_1 now has an $n\alpha$ limit of 2.4 while the signal is attenuated as $\exp(-n\alpha)$. How much of an improvement this offers, however, is questionable. To store the same amount of information n must be doubled reducing the 1.67 to an effective 0.835. To maintain the same information rate, f_o, the clock frequency must be doubled. This will increase α: if α is doubled, then the 1.67 limit, already reduced to 0.835, will be reduced further to about 0.42. Compared with the 0.7 limit of the simplest code, this is rather unfavorable. One compensation is that having two packets to detect rather than just one can be used to reduce the error rate induced by noise. However, one can do better, as the following example illustrates.

The third example in Table I is the zero-net-charge code. Here a 0 is coded as a Q_0 packet followed (in time) by a Q_1 packet. (In the register shifting from charge left to right this is represented as Q_1Q_0.) A 1 is coded as Q_0Q_1. The advantage of this procedure is that each pair, whether coding a 0 or a 1, contains the same amount of charge, $2\bar{Q}$. This prevents a buildup of charge in Q_R. The most demanding test is sequence (b) in which the $n\alpha$ limit is 2.36. This is a significant improvement over the 1.67 limit in the previous example. However, if one takes into account that to contain the same amount of information n must be doubled (as now each bit requires two charge packets) and that the clock frequency must be doubled to maintain the same data rate (which will increase α), one realizes that really very little has been achieved by increasing the upper limit on $n\alpha$ from $n\alpha < 0.7$ to $n\alpha < 2.4$ $(2.4/4 = 0.6)$. Other straightforward modifications of the basic 0, 1 code, of course, suffer from the same fault. Thus to achieve any improvement it is necessary that one still must be able to take advantage of the possibility of detecting *both* charge packets to do better than the simplest code. The reason for the failure of the zero-net-charge code in terms of frequency-domain concepts is given in Appendix C.

One final example is to follow the Q_0 or the Q_1 with an intermediate packet of size $\bar{Q}$. As seen in Table I, sequence (a) puts an $n\alpha$-limit of 0.5, which is inferior to the other codes. This attempt to reduce $|Q_B - \bar{Q}|$ by following Q_0 or Q_1 with a $\bar{Q}$ packet to "average" out the incompletely transferred charge is thus seen to be ineffective.

APPENDIX C

It is quite informative to briefly discuss in the frequency domain the effects of various digital coding schemes on the character of the signal.[7,9]

In Section 2.1 we noted that incomplete charge transfer leads to a frequency-dependent attenuation $A(\omega)$ given by

$$A(\omega) = \exp[-n\alpha(1 - \cos \omega\tau_o)] \tag{37}$$

for $\beta = 0$ in eq. (7). $A(\omega)$ is plotted in Fig. 1 for various values of $n\alpha$. As discussed in Section 2.2, low-frequency components ($f \ll f_o/2$) suffer very little attenuation, whereas components with frequency near half the clock frequency ($f \approx f_o/2$) are attenuated by $\exp(-2n\alpha)$, a large attenuation for $n\alpha \gtrsim 3$.

One can offset this high-frequency attenuation by the following scheme. If one takes every other charge packet and replaces it by a Q_1 if it originally was a Q_0, and by a Q_0 if it originally was a Q_1, then relative to $\bar{Q}$ one essentially multiplies each packet in turn by $+1, -1, +1, -1, +1, -1, \cdots$. This has the effect of converting the spectrum of the signal from $F(f)$ to $F(f_o/2 - f)$: the $f = 0$ component is attenuated as $A(f_o/2)$ and the $f = f_o/2$ component as $A(0)$. To better preserve the entire signal, one can sum the outputs of a register with attenuation $A(f)$ and a register with attenuation $A(f_o/2 - f)$. The ratio of maximum attenuation to minimum attenuation is thus improved from $\exp(-2n\alpha)$ to $2\exp(-n\alpha)/[1 + \exp(-2n\alpha)]$. However, distortion near $f = f_o/4$ is still significant for $n\alpha > 2$.

To see the effect of the zero-net-charge coding scheme on the signal, consider this example. If the clock frequency is f_o, the maximum frequency the CTD can carry is $f_o/2$. However, if two charge packets are devoted to each 0 or 1 as in the second through fourth examples in Table I, then the bandwidth is reduced to $f_o/4$. If the second example is chosen, then the band extends from $f = 0$ to $f = f_o/4$; if the third example (zero-net-charge coding) is chosen, then the band extends from $f = f_o/4$ to $f_o/2$, the lower-frequency components of the signal being carried at the higher frequencies and vice versa. If amplified by $\exp(+n\alpha)$, the ultimate effect of incomplete transfer on a signal coded using zero-net-charge coding is seen to be essentially the same as that on a signal coded using the second example. What is most striking, however, is that by reducing the clock frequency by a factor of two and using simple coding, one reduces $n\alpha$ by a factor of four, greatly reducing the attenuation.

By examining the effect of other coding schemes on the spectrum of the signal, and by taking into account the frequency-dependent attenuation accompanying charge transfer in CTD's it is possible to ascertain whether an improvement in (noiseless) detection will be in fact real or only apparent.

APPENDIX D

In noise, detection, and communication theory one often encounters an integral of the form

$$I(A) = \int_{-\infty}^{-A} e^{-x^2/2} dx/(2\pi)^{\frac{1}{2}}, \tag{38}$$

where $A > 0$. This expression, while extensively tabulated numerically, is difficult to work with analytically. In this appendix we shall (*i*) bound $I(A)$ between two simple analytic functions of A which differ by only a factor of 2, and (*ii*) prove that $\langle I(A)\rangle \geqq I(\langle A\rangle)$ for $A \geqq 0$.

(*i*) Bounds on $I(A)$:

In Fig. 10 we illustrate the motivation for our approximations. $I(A)$ is the area under the Gaussian for $x = -\infty$ to $x = -A$. If we draw a line tangent to the Gaussian at $x = -A$ and extend the line from $x = -A$ to the x-axis as shown, the area of the triangle formed by this tangent, the x-axis, and the vertical line $x = -A$ is clearly less than $I(A)$. Similarly, if an exponential curve $[B \exp(+Cx)]$ also tangent to the Gaussian at $x = -A$ and decaying to the left is drawn, then the area between this curve and the x-axis for $x \leqq -A$ is clearly greater than $I(A)$. Thus, if we calculate these two areas, we will have an upper and lower bound on $I(A)$. (These curves will clearly not cross the Gaussian if $A \geqq 1$, the inflection point of the Gaussian.)

To calculate the areas we proceed as follows. The slope of $\exp(-x^2/2)$ at $x = -A$ is $A \exp(-A^2/2)$, and of course its value at $x = -A$ is $\exp(-A^2/2)$. Thus the equation of the tangent is

$$y(x) = \exp(-A^2/2) + A \exp(-A^2/2)(x + A) \tag{39}$$

(which is zero for $x = -A - 1/A$, $f(-A - 1/A) = 0$) and of the exponential is

$$y(x) = \exp[-A^2/2 + A(x + A)]. \tag{40}$$

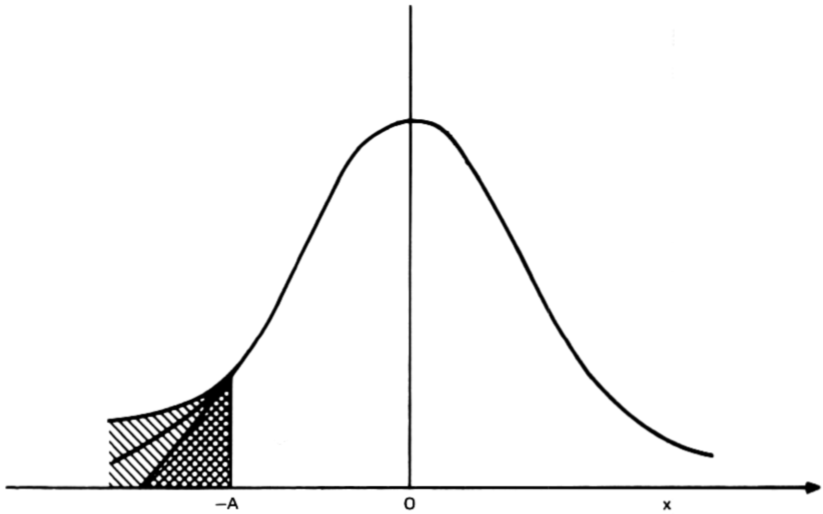

Fig. 10—Approximating the area under a portion of Gaussian curve by bounding the area between that of a right triangle (whose hypotenuse is tangent to the Gaussian at $x = -A$) and by the area under an exponential (also tangent to the Gaussian at $x = -A$).

Thus computing the appropriate areas we find that if $A \geqq 1$ then

$$D/2 < (2\pi)^{\frac{1}{2}} I(A) < D, \tag{41}$$

where

$$D = \exp(-A^2/2)/A. \tag{42}$$

Such bounds are very useful in calculating error rates, where one is seldom interested in accuracy better than a factor of two, and where upper and lower bounds are often very useful.

(*ii*) $\langle I(A) \rangle \geqq I(\langle A \rangle)$, $A \geqq 0$:

To evaluate $\langle I(A) \rangle$ is clearly very difficult even under the simplest of probability distributions of A, whereas $I(\langle A \rangle)$ is generally very easy to compute if $\langle A \rangle$ is known. $I(\langle A \rangle)$ can then be used as a lower bound for the more interesting $\langle I(A) \rangle$. We shall now prove the above inequality. (This result is reasonably well known.[20] The proof is given here for completeness.)

$I(A)$ is a function of A. According to the mean value theorem we may write

$$I(A) = I(\langle A \rangle) + \left.\frac{dI}{dA}\right|_{\langle A \rangle} (A - \langle A \rangle) + \frac{1}{2} \left.\frac{d^2 I}{dA^2}\right|_{A'} (A - \langle A \rangle)^2, \tag{43}$$

where $A'(A)$ lies between A and $\langle A \rangle$ and depends on A. Thus we may write

$$\langle I(A) \rangle = I(\langle A \rangle) + \frac{1}{2}\left\langle \frac{d^2I}{dA^2}\bigg|_{A'(A)} (A - \langle A \rangle)^2 \right\rangle. \tag{44}$$

Now then

$$\frac{d^2I}{dA^2} = A \exp(-A^2/2)/(2\pi)^{\frac{1}{2}} \tag{45}$$

which is zero or larger for $A \geqq 0$. Thus if we are averaging A over a probability distribution $P(A)$ for which $P(A < 0) = 0$, then $A'(A) \geqq 0$, and, consequently, the second term on the right-hand side will be zero or greater. Hence it follows that

$$\langle I(A) \rangle \geqq I(\langle A \rangle). \tag{46}$$

In Section VI this inequality is used to put a lower bound on the error rate for detecting digital signals.

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