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Effect of Misalignments on Coupling Efficiency of Single-Mode Optical Fiber Butt Joints

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Analysis and computations made here, corroborated by experiment, determine the effects of axial displacement and angular misalignment on the power coupled between butt-joined, single-mode optical fibers. The absolute accuracy with which fibers must be joined on-centers is reduced for fibers with relatively smaller core; the angular accuracy is increased.

I. INTRODUCTION

The lowest-order mode in a clad optical fiber, the hybrid HE_{11} mode, is the only propagating mode for core sizes less than a few wavelengths in diameter ($v \leqq 2.4$).[1] Hence, small-core, single-mode glass fibers are attractive for transmitting optical signals because of their potentially low dispersive effects. A possible disadvantage lies in the difficulty of joining small-core fibers end-to-end.[2,3] It has been suggested[4] that butt joining may be made less critical by reducing the size of the fiber in the vicinity of the joint. The computation and experimental measurements disclosed here evaluate the advantage to be gained by such a procedure. It is found that as the fiber gets smaller, the accuracy with which the ends of the fibers to be joined must meet on-centers is indeed reduced. At the same time, however, as might be expected, the required angular alignment of the fibers becomes more critical.

Both calculations and experiments have been made under the assumption that the cladding is of sufficient extent that the role played by possible conversion of the zero-order mode to cladding modes need not be considered.[4] The calculation was made by matching the fields of the zero-order modes at the joint.

II. ANALYSIS

The power loss caused by displacement at a joint is readily found by first determining the ratio of power accepted by the displaced fiber to that presented by the sending fiber, that is, the power-coupling ratio. This is determined by assuming that the incident field E_i separates at the plane interface (which is perpendicular to the axis of the receiving fiber) into the desired zero-order mode that propagates in the receiving fiber, and into modes orthogonal to this propagating mode. The field of the propagating mode is represented by $B \cdot E_p$, since the form E_p is known but not the amplitude B. The field of the orthogonal modes is represented by E_o.

$$E_i = B \cdot E_p + E_o.$$

Multiplication by E_p and integration over the entire interface gives

$$\int E_i E_p dA = B \int E_p^2 dA + 0.$$

The desired power ratio, c, is then represented by B^2.

$$c = \left[\int E_i E_p dA \Big/ \int E_p^2 dA \right]^2.$$

The power-coupling ratio between fiber ends with parallel but translated axes is c_1. In order to show the effect of axial displacement of a given magnitude, independent of core size, the ratio c_1 is computed

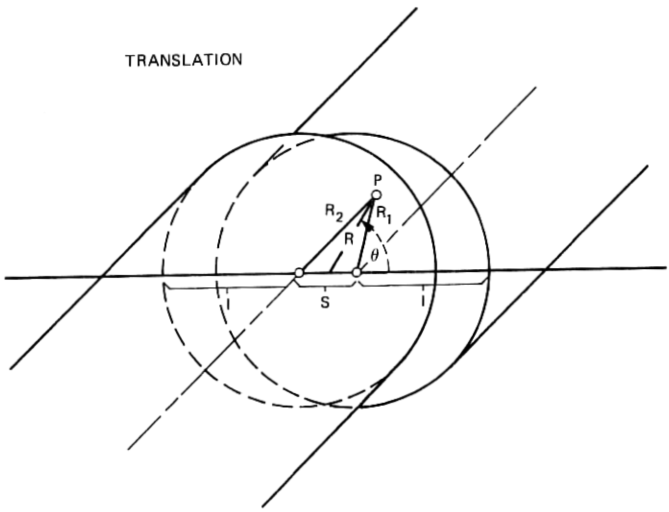

Fig. 1—Butt-joined fiber cores displaced by axial translation.

as a function of v, the normalized core size,[1] for several values of the parameter d, the normalized displacement.

$$v = \frac{2\pi a}{\lambda}(n_c^2 - n_o^2)^{\frac{1}{2}}, \tag{1}$$

$$d = \frac{2\pi s}{\lambda}(n_c^2 - n_o^2)^{\frac{1}{2}}, \tag{2}$$

where a is the core radius, s is the axial displacement distance, λ is the wavelength, and n_c and n_o are the core and cladding refractive indexes, respectively.

Figure 1 shows the cross section of a fiber displaced by axial translation. R and S are normalized to radius a. The origin is defined as the midpoint between core centers. R and θ are the polar coordinates of an arbitrary point P in the interface. R_1 is the distance from P to the center of the first fiber, and R_2 is the distance to the center of the second fiber.

$$R_1^2 = R^2 + \left(\frac{S}{2}\right)^2 - RS\cos\theta,$$

$$R_2^2 = R^2 + \left(\frac{S}{2}\right)^2 + RS\cos\theta.$$

Let A_{12} be the area of the interface of the displaced fibers, and A be the area of the cross section of the sending fiber; then

$$c_2(v) = \left[\int_{A_{12}} E(R_1)E(R_2)dA_{12} \bigg/ \int_A E^2(R)dA\right]^2.$$

The function E is defined[1] by

$$E(R) = \begin{cases} \dfrac{J_o(uR)}{J_o(u)}, & R \leqq 1, \\ \dfrac{K_o(wR)}{K_o(w)}, & R > 1, \end{cases}$$

where J_o and K_o are the regular and modified Bessel functions of order zero.

The values of u and w are determined from the eigenvalue equations,

$$v = (u^2 + w^2)^{\frac{1}{2}},$$

$$\frac{uJ_1(u)}{J_o(u)} = \frac{wK_1(w)}{K_o(w)}.$$

The integral in the denominator of the expression c_1 can be integrated analytically. For an infinite cladding,

$$\int_A E^2(R)dA = \pi\left[\frac{vJ_1(u)}{wJ_o(u)}\right]^2.$$

For a fiber of radius R_c, $(R_c > 1)$,

$$\int_A E^2(R)dA = \pi\left(\frac{v}{w}\frac{J_1(u)}{J_o(u)}\right)^2 - \frac{\pi R_c^2}{K_o^2(w)}\left[K_1^2(wR_c) - K_o^2(wR_c)\right].$$

The integral in the numerator can be divided so that portions of the integration can be done analytically.

The power-coupling ratio between fibers with angular displacement of the fiber axes is c_2. In order to show the effect of a fixed angular displacement of the axes for different core sizes, the ratio c_2 is computed as a function of v for several values of the parameter b, the normalized displacement angle,

$$b = \frac{\sin\phi}{\sqrt{1 - n_o^2/n_c^2}} \approx \frac{\phi}{\sqrt{2\Delta}}, \tag{3}$$

where ϕ is the angle between the fiber axes, and Δ is the ratio of core-to-cladding index difference to the core index.

Figure 2 shows a cross section of the fiber joint in the plane of the axes of the fiber and auxiliary cross sections perpendicular to the axes of the fibers. R', θ', z' and x', y', z' are coordinate systems oriented with respect to the sending fiber while R, θ, z and x, y, z are coordinate systems oriented with respect to the receiving fiber; ϕ is the angle between the axes of the two fibers (and therefore between the planes perpendicular to the axes). From Fig. 2 it can be seen that

$$R' = R(1 - \sin^2\theta \sin^2\phi)^{\frac{1}{2}}.$$

The field of the sending fiber at the point P of the interface L_1 is

$$E_i(R) = E(R') \cos \beta z',$$

where β is the normalized propagation constant.

$$\beta \approx \frac{v}{\sqrt{2\Delta}}.$$

For the small angles under consideration the maximum deviation of R'/R from 1 is at most $2 \cdot 10^{-2}$, so it is feasible to approximate R' by R. Therefore, $\beta z' = bvR \sin\theta$ and

$$c_2(v) = \left[\int_{A_{12}} E^2(R) \cos(bvR\sin\theta)dA \bigg/ \int_A E^2(R)dA\right]^2.$$

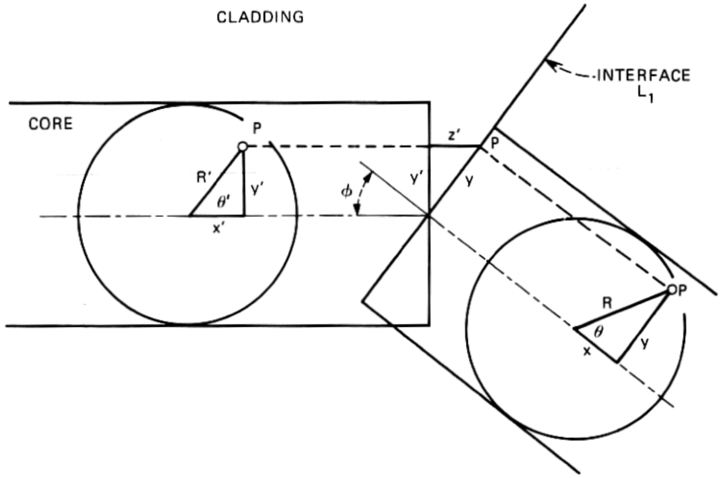

Fig. 2—Analytical model of butt-joined fiber cores displaced by angular misalignment, ϕ.

The integral in the numerator, since we have approximated R' by R, can be converted to a single integral.

$$\int_0^{R_c}\int_0^{2\pi} E^2(R)\cos(bvR\sin\theta)Rd\theta dR = 2\pi\int_0^{R_c} E^2(R)J_o(bvR)RdR.$$

The integral in the denominator is the same as before and can be integrated analytically.

III. EXPERIMENT

Experimental verification was carried out at microwave frequencies because equipment was readily available and dimensional control more certain than at optical frequencies. Polyfoam served as the core, and air as the cladding. The experimental arrangement is shown in Fig. 3. The polyfoam rod had dielectric constant of 1.06; hence, $(n_c^2 - n_o^2)^{\frac{1}{2}} = 0.245$. Results of computations from the analysis and experimentally measured points are shown overlaid in Figs. 4 and 5. The vertical point dimension indicates the disparity between two systematic measurements. Disparity between the analytical and experimental results for very small core dimensions and larger coupling loss is not understood; but it is not critical to the conclusions.

Figure 6 shows an overlay of calculated power coupling as a function of v due to both angular and lateral misalignments. It can be argued that the total power lost at the junction is roughly equal to the sum

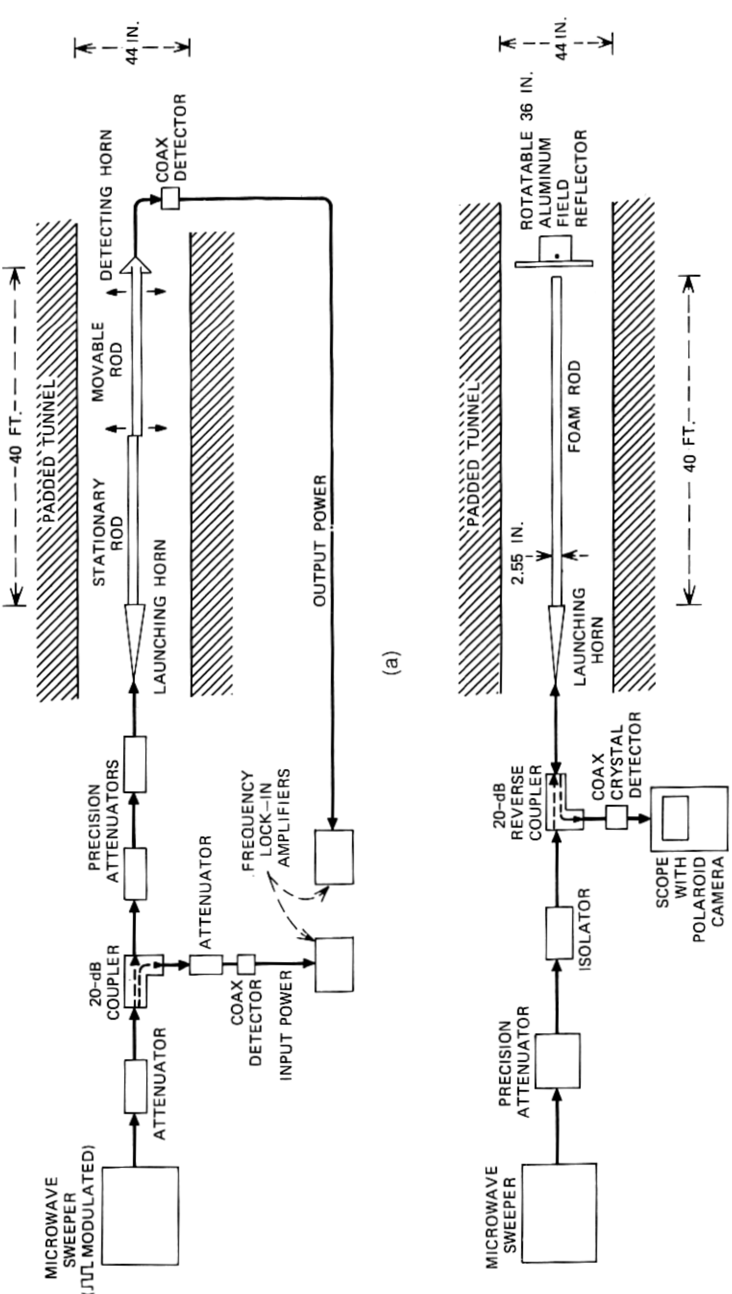

Fig. 3—Experimental arrangement used to corroborate analysis using microwave-guiding polyfoam rods. (a) Coupling vs axial displacement measurement configuration. (b) Coupling vs angular misalignment measurement configuration.

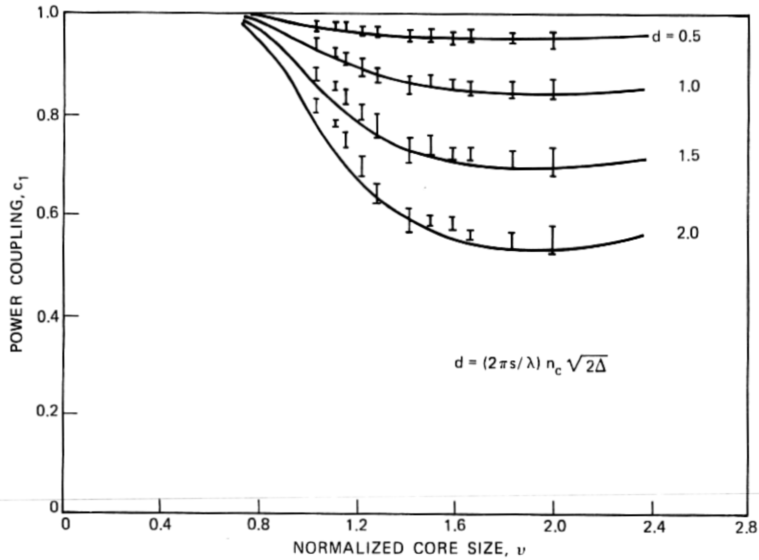

Fig. 4—Power coupling through translationally displaced joint as a function of normalized core size and axial translation, d.

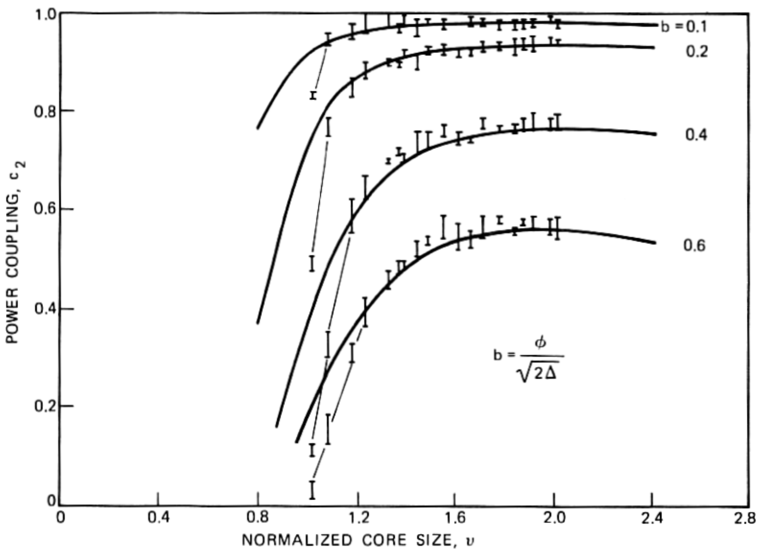

Fig. 5—Power coupling through angularly misaligned joint as a function of normalized core size and misalignment, b.

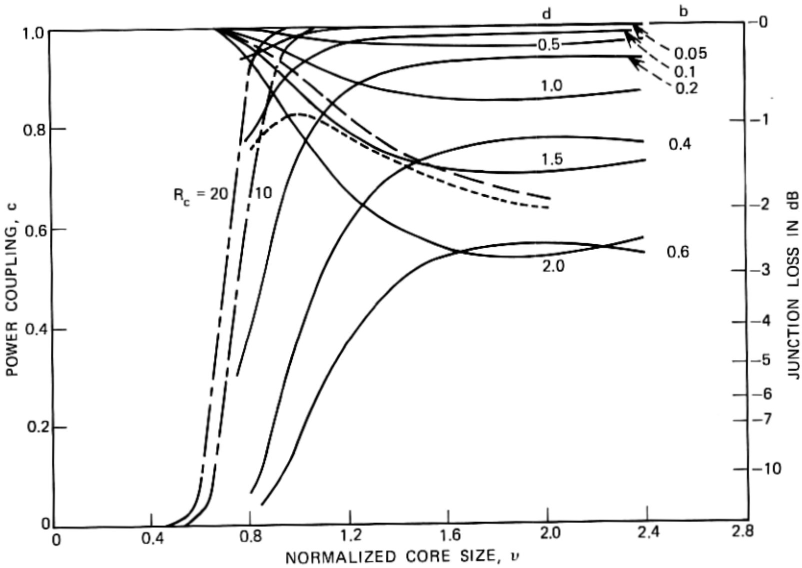

Fig. 6—Minimum power coupling through example joint as a function of normalized core size. Dashed line shows power coupled at maximum axial translation, dotted line shows composite minimum coupling.

of the lost power due to the two kinds of misalignment. Since many modes are involved in the scattered energy, and the displacement coordinates are different, the scattered modes must be essentially orthogonal, hence, power-additive. The dash-dot curves in Fig. 6 show the fraction of power lying within 10 and 20 radii of the core as labeled.

IV. CONCLUSIONS

In general, considering the nature of field spreading that accompanies the decrease in single-mode fiber core diameter, one concludes what one would expect. As the core size parameter, v, decreases below about 2, the fields spread, and axial displacement at the joints becomes less and less critical since more and more overlap of fields results from a given offset. At the same time, the effective "aperture" at the fiber end increases, the "antenna" becomes more directive, and the angular alignment becomes more critical.

More detailed conclusions must be drawn from the particular joining problem at hand. Suppose, for example, one wants to join single-mode fibers of 3-mil diameter, having a core loaded to produce an index difference at the interface of $\frac{1}{2}$ percent. If the refractive index of the fiber material is about 1.5, then

$$(n_c^2 - n_o^2)^{\frac{1}{2}} \approx n_c\sqrt{2\Delta} = 0.15.$$

If the fiber is designed to have $v = 2.0$ and the optical wavelength, λ, is 1 μm, the core radius may be found from eq. (1) to be

$$a \approx 2.1 \ \mu\text{m}.$$

Since 1.5 mils $\approx$ 38 μm, the cladding radius is about 18 times the core radius, so

$$R_c = 18.$$

Suppose the centering accuracy of the tool or fixture that will be used in aligning the fiber ends at a joint is about ± 1 μm. (It presumably will center the fiber with reference to its o.d.) And suppose the core is centered in the fiber with $\pm$1-percent accuracy, that is, the core center is never displaced from the fiber center by more than 1 percent of the fiber diameter. The net maximum displacement, then, will be

$$s_m \approx 1.75 \ \mu\text{m}$$

and, from (2),

$$d_m \approx 1.65.$$

If the fiber is drawn down to a smaller size at the end to decrease the lateral displacement sensitivity at the joint, it is reasonable to assume that the alignment tool maintains the same 1-μm accuracy and the core maintains the same $\pm$1-percent centering accuracy; the latter of which produces a net decrease in d_m. The dashed curve in Fig. 6 shows what happens to the minimum coupling, c_1, as v scales down with fiber size.

If at the same time one assumes that the net angular misalignment, ϕ, is less than about 0.01 radian, then from (3),

$$b \approx 0.1,$$

and the combined effect of angular and lateral displacement varies with v as shown by the dotted line in Fig. 6.

The total worst-case joint loss in this example, then, may be reduced by about a factor of two by drawing the fiber (core and cladding together) down to half its normal size at the ends for joining. At $v = 1$ the field extending outside the cladding is still negligible.

V. ACKNOWLEDGMENT

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