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Heat Transfer in Electronic Systems With Emphasis on Asymmetric Heating

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The trend in electronic circuit design is toward increasing power dissipation density. The performance and reliability of increasing number of electronic systems are now seriously threatened by thermal effects, so that it is necessary to reappraise the relevant thermal design procedures. This paper examines natural convection cooling and concerns the prediction of maximum temperatures of electronic cabinets containing arrays of vertically oriented circuit cards with unequal power dissipation levels. By means of a vertical channel model, the effects of channel spacing, channel height, and power dissipation level are assessed with emphasis on asymmetric powering of the channel walls. Methods are indicated for rapid evaluation of maximum temperatures and optimum channel spacing with asymmetric heating. The present results show that asymmetry reduces the thermal performance of the channel. Consequently, the power dissipation on the channel walls should be made nearly equal.

I. INTRODUCTION

For the past several years, the trend in electronic equipment design has been toward ever-increasing circuit speeds. Now it is common for response times of telecommunication equipment to be specified in terms of nanoseconds while, in high-performance data processing systems, the required response times are given in the picosecond range.

The increase in circuit speed is facilitated by integration of circuit functions, circuit miniaturization, and higher-density packaging. Although the miniaturization of circuits results in decreased power dissipation per circuit, the power generation per unit volume, which is the important parameter in determining the circuit temperature, is actually increased due to the much higher packaging densities. The thermal problem is also compounded by the lower operating temperature requirements of integrated circuits. This gives rise to a challenging undertaking in thermal design. And yet a poor thermal design could possibly lead to complete failure or unreliable performance of equipment. Thus, it is imperative that a sound thermal design be initiated, and this at the earliest possible stage of system planning.

The thermal design of a modern electronic system is based on a rational selection of a cooling option followed by a thoughtful design consideration. A design should not only be practicable, but also economical, serviceable, reliable, and compatible with other system components. It is the purpose of this paper to present design data on natural convection cooling of modern electronic systems. The emphasis is on the prediction of maximum temperatures in equipment. Although natural convection is the oldest method used in electronics cooling, the method is still employed extensively in new generations of equipment both in the communications and the data processing fields. In many cases, this method is used in conjunction with one or more of the newer

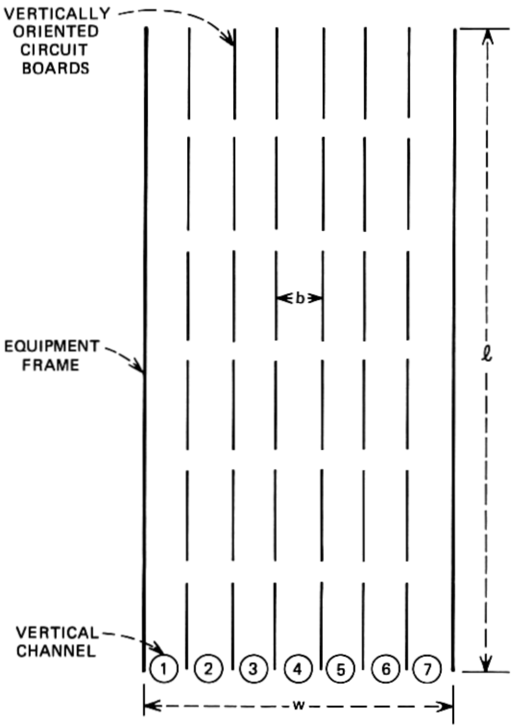

Fig. 1—Schematic diagram of electronic equipment.

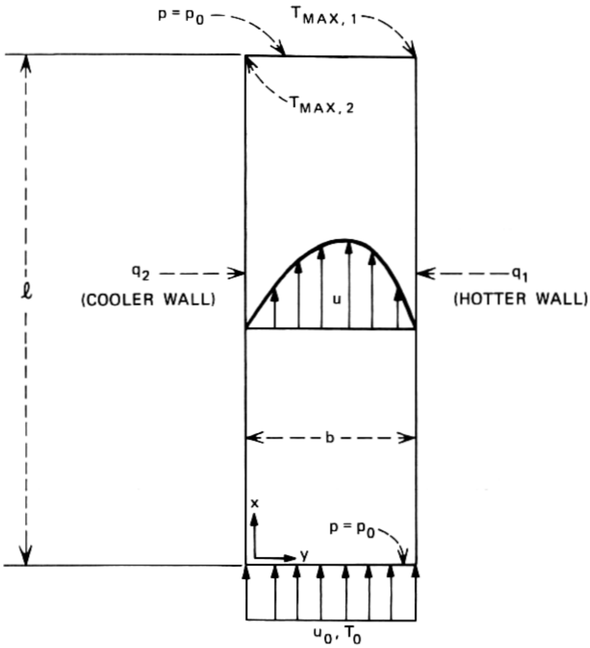

Fig. 2—Two-dimensional vertical channel model.

methods such as forced flow and the application of heat pipes and liquid cooling.

The present paper concerns equipment wherein circuit boards containing heat-dissipative components are mounted vertically in channel-like fashion as shown in Fig. 1. Since the circuit boards are not identically powered, the average heat fluxes from the columns are different, giving rise to asymmetrical air flow. In each channel, the heat transfer may be modeled by that in a vertical, two-dimensional, smooth-walled channel as shown in Fig. 2. The channel walls are treated as uniformly heated. However, the two wall heat fluxes need not be identical.

A number of investigations have been reported in the literature concerning free convection in vertical channels. Ostrach[1] solved the combined free (natural) and forced convection problem in a vertical channel in fully developed flow with symmetrical uniform heat flux and internal heat generation. By fully developed flow is meant a situation in which the fluid velocity is invariant in the direction of flow. Engel and Mueller[2] investigated the effect of nonisothermal channel walls by assuming constant heat fluxes. Their results, however, are limited to symmetrical wall heating. Lauber and Welch[3] considered the combined free and forced convection between vertical flat plates which are heated at uniform but different heat fluxes. Their work is confined to fully developed flow.

Two standard references widely applied to heat transfer in electronic equipment are the experimental work of Elenbaas[4] and the numerical solution of Bodoia and Osterle.[5] Inasmuch as the above references deal with a channel whose walls are at a constant temperature, two factors are therefore ignored in their results: the effect of nonisothermal surfaces and asymmetric heating. These effects are considered in the present paper.

It is to be noted that, as in the references cited above, radiative transfer of heat is ignored in this paper. In some electronic systems, radiation may be a significant factor in heat removal. Inasmuch as temperatures of heat sources are of interest, the neglect of radiation usually results in conservative (too high) estimates of the temperatures.

II. ANALYSIS

Referring to Fig. 2 which shows a two-dimensional, straight vertical channel, let the channel height be ℓ and width be b. The channel walls

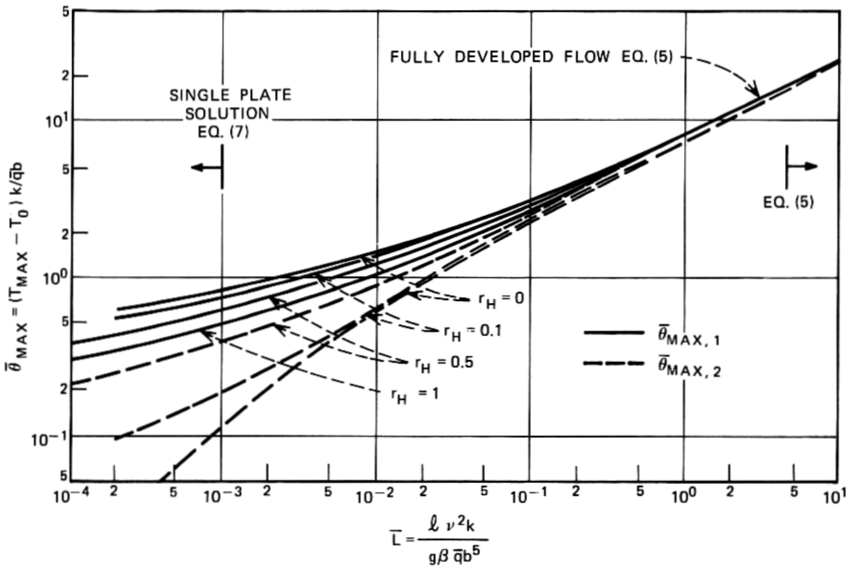

Fig. 3—Relation between dimensionless maximum wall temperature rise and the dimensionless parameter $\bar{L}$ (from Aung, et al.[6]).

are heated causing fluid to rise between them by free convection. The thermal conditions of the walls are characterized as uniform heat flux (UHF), but the individual values for the two walls need not be the same. The fluid that enters the channel at the ambient temperature T_0 is assumed to have a flat velocity profile u_0. The equations expressing conservation of mass, momentum, and energy for free convection in the constant-property (except for density in the buoyancy term) laminar flow may be written respectively as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{g_c}{\rho}\frac{dp'}{dx} + g\beta(T - T_0), \tag{2}$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

where p' is the pressure excess above the hydrostatic pressure.

The boundary conditions are:

$$\left.\begin{aligned}
&\text{For } x = 0 \text{ and } 0 < y < b: \\
&\qquad u = u_0, \qquad T = T_0; \\
&\text{For } y = 0 \text{ and } x \geqq 0: \\
&\qquad u = 0, \qquad v = 0, \qquad k\frac{\partial T}{\partial y} = -q_2; \\
&\text{For } y = b \text{ and } x \geqq 0: \\
&\qquad u = 0, \qquad v = 0, \qquad k\frac{\partial T}{\partial y} = q_1.
\end{aligned}\right\} \tag{4}$$

At $x = 0$ and $x = \ell$: $p = p_0$, where p_0 is the hydrostatic pressure.

The system of eqs. (1) to (4) may be cast in nondimensional form and then solved by numerical integration using a digital computer. Details are contained in Refs. 6 and 7. The latter references show that the numerical results agree well with experimental data. It may be pointed out that, for a channel whose height is large compared to the spacing so that the so-called fully developed flow condition exists, the governing eqs. (1) to (3) may be simplified and the solution is then obtainable in closed form. This is indicated in Ref. 8. In the present paper, these results will be applied to indicate the effect of operating parameters on maximum wall temperatures. The cooling fluid considered is exclusively air.

III. RESULTS

When the channel walls are individually heated at uniform heat fluxes, the quantity that is of design interest is generally the maximum temperature rise on the hotter wall. This quantity may be obtained from the general results shown in Fig. 3. It is clear from the latter that the maximum temperature increase depends on the parameter $\bar{L}$ which in turn depends on the quantity $\ell/\bar{q}b^5$. When the latter quantity is large, fully developed flow is approached[6] and the desired information on the maximum temperature increase is given by a rather simple expression which follows.

3.1 *Results at Large* $\ell/\bar{q}b^5$

At large values of the quantity $\ell/\bar{q}b^5$, the flow in the channel exhibits interesting characteristics whereby the velocity distribution across the channel remains unaltered with axial distance. All axial temperature variations are also given by linear relations. Typical velocity and temperature distributions in this situation are given in Fig. 4. This is the so-called fully developed flow situation. Clearly, it prevails most commonly in a channel whose height ℓ is large compared to its width. In a channel with developing flow, typical velocity and temperature dis-

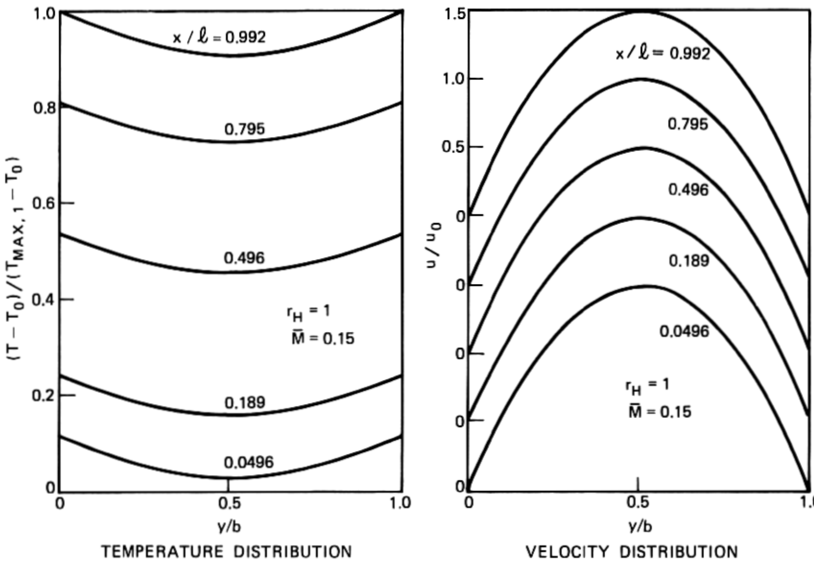

Fig. 4—Typical temperature and velocity distributions in a channel having nearly fully developed flow.

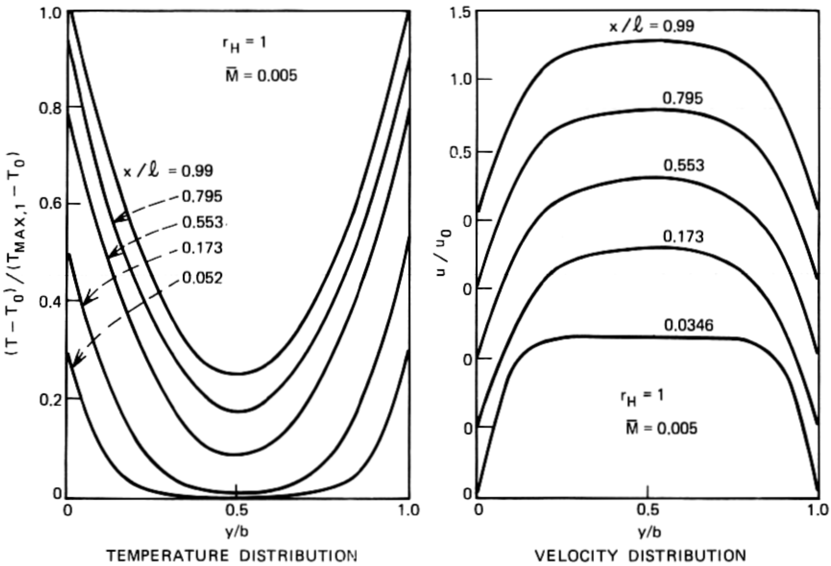

Fig. 5—Typical temperature and velocity distributions in a channel having developing flow.

tributions are as indicated in Fig. 5. In both Fig. 4 and Fig. 5, the quantity $\bar{M}$ is a dimensionless mass flow parameter which is related to $\bar{L}$ (see Refs. 6 and 8).

Figure 3 shows that, as fully developed flow is approached, all curves coalesce into one. Based on the derivations given in Ref. 8, it can be shown that, in fully developed uniform heat flux channels with asymmetric heating, the maximum temperatures on the hotter and cooler walls become practically identical and may be evaluated using the equation:

$$\left.\begin{array}{r}\bar{\theta}_{\max,1} = \bar{\theta}_{\max,2} = 6.9285(\mathrm{Pr})^{-1/2}(\bar{L})^{1/2}, \\ \bar{L} \geqq 5, \qquad \text{all } r_H\end{array}\right\}. \tag{5}$$

For air at 120°F, the Prandtl number (Pr) is about 0.7. Evaluating the thermal physical properties for air in eq. (5) at a temperature of 120°F, we may rewrite eq. (5) as:

$$\left.\begin{array}{r}T_{\max,1} - T_0 = T_{\max,2} - T_0 = 0.1610(\bar{q})^{0.5}(\ell)^{0.5}(b)^{-1.5} \\ \text{for } \dfrac{\ell}{\bar{q}b^5} \geqq 1.19 \times 10^9, \qquad \text{all } r_H\end{array}\right\}. \tag{6}$$

In eq. (6), ℓ is expressed in ft, $\bar{q}$ in W/ft^2, and b in ft. Note that, if $T_{\max,1}$ alone is to be evaluated, eq. (6) is applicable at $\ell/\bar{q}b^5 \geqq 4.76 \times 10^7$.

The reason is apparent from Fig. 3 where at $\bar{L} \geqq 0.2$ the deviations among curves of $\bar{\theta}_{max,1}$ at all values of r_H may be neglected.

3.2 *Results at Small $\ell/\bar{q}b^5$*

The counter-case to the situation just discussed is the one in which the channel height ℓ is relatively short compared to its width b. Again, in this situation the maximum temperature rise on the channel walls may be calculated by a simple relation. Note that, to use this relation, which will be given below, it is not necessary that ℓ be smaller than b. It is only necessary to fulfill the validity limit attached to the equation.

For relatively short channels, the results in Ref. 6 indicate that the maximum temperature in each wall is independent of the other wall, and may be calculated using the result for a single vertical flat plate.[9] The latter result is valid for the asymmetrically heated channel when $\bar{L} \leqq 10^{-3}$. Thus, we may write:

$$\theta_{max} = 2.05(L)^{1/5} \qquad \text{at } \bar{L} \leqq 10^{-3}. \tag{7}$$

Again, inserting thermal physical properties of air at 120°F, the above may be written:

$$\left.\begin{aligned} T_{max} - T_0 &= 8.66\ell^{1/5}q^{4/5} \\ \text{at } \frac{\ell}{\bar{q}b^5} &\leqq 2.35 \times 10^5, \qquad \text{all } r_H \end{aligned}\right\}. \tag{8}$$

In eq. (8), $(T_{max} - T_0)$ is in degrees Fahrenheit, ℓ in feet, and q in watts per square foot of surface area. Equation (8), which gives the maximum temperature on either the hotter or cooler wall when q is appropriately replaced by q_1 or q_2, shows that the maximum temperature on the hotter wall, subject to the attached condition, is independent of the channel spacing which is to be expected. It may also be noted for reference that

$$\frac{\ell}{q_1 b^5} = \left(\frac{1 + r_H}{2}\right)\frac{\ell}{\bar{q}b^5}. \tag{9}$$

3.3 *Results at Intermediate $\ell/\bar{q}b^5$*

If the proposed design is such that neither eq. (6) nor eq. (8) may be applied, then the maximum temperature increase in the channel can be evaluated using Fig. 3. Parametric curves can be generated from Fig. 3 to display the effect of various operating variables on the maximum temperature rise on the hotter wall. Since the value of $\bar{L}$ depends on the channel spacing, the average heat flux, and the channel height, any one of these may be varied while others remain constant at typical

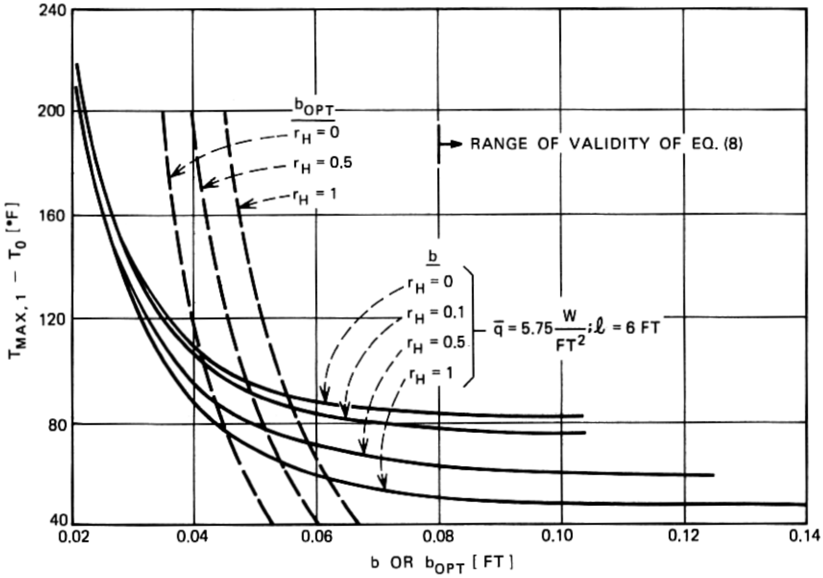

Fig. 6—Effect of channel spacing on the maximum wall temperature rise.

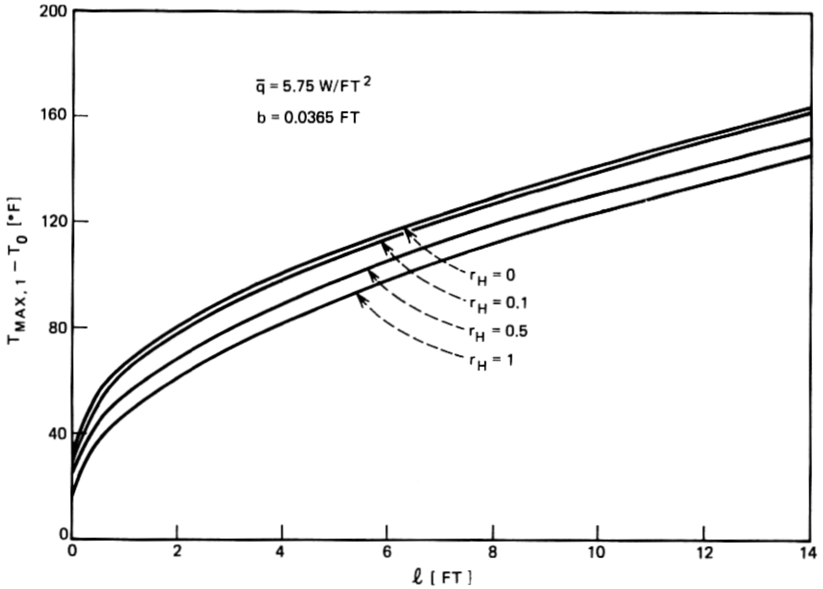

Fig. 7—Effect of channel height on the maximum wall temperature rise.

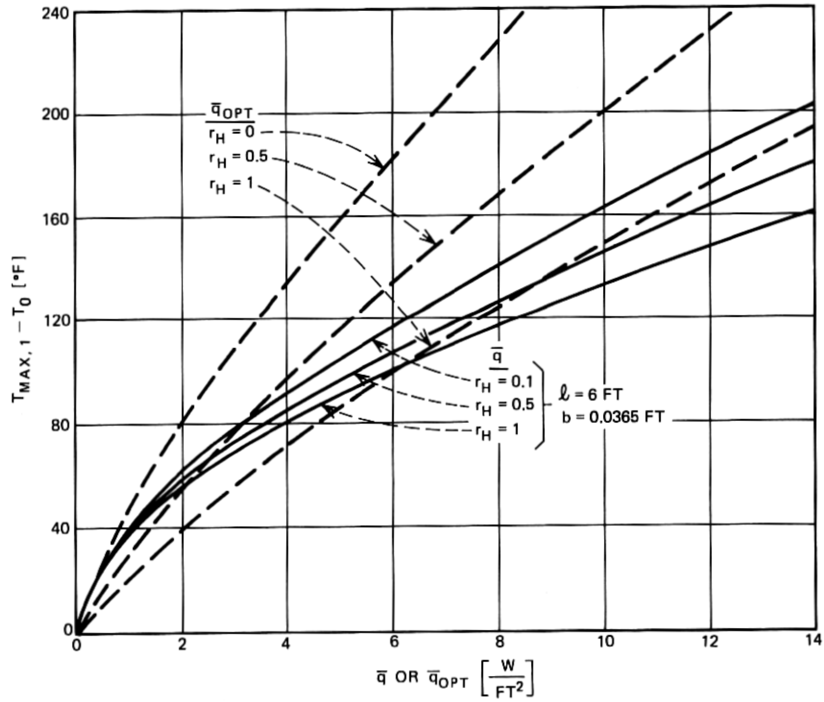

Fig. 8—Effect of average heat flux on the maximum wall temperature rise.

operating values. The following conditions are chosen as an example:

$$\begin{aligned} b &= 0.0365 \text{ foot } (0.4375 \text{ inch}) \\ \ell &= 6 \text{ feet} \\ \bar{q} &= 5.75 \text{ watts per square foot.} \end{aligned} \tag{10}$$

The resulting curves are shown in Figs. 6 through 8 which can be used by the designer. To obtain Fig. 6, ℓ and $\bar{q}$ are obtained from eqs. (10) and, corresponding to each pair of assigned values of b and r_H, $\bar{L}$ is calculated. A value of $\theta_{\max,1}$ is then obtained from Fig. 3. The maximum temperature rise can then be obtained. By varying r_H and b, the solid curves in Fig. 6 result. The range of validity of eq. (8) is also indicated. Figures 7 and 8 are obtained in the same manner. The dashed lines in Figs. 6 and 8 pertain to optimum spacing which will be discussed later. From Figs. 6 through 8, it can be seen that at any fixed maximum temperature rise the effect of asymmetry is to increase the necessary channel spacing, decrease the channel height, or decrease the heat flux when any one of these quantities is taken as the sole variable in the equipment.

The effect of asymmetric wall heating may be more conveniently examined by replotting Fig. 3 in the manner of Fig. 9. In the latter, the maximum temperature increase on each wall of an asymmetric channel is normalized by the maximum temperature increase on a similar channel wherein the same average heat flux is distributed evenly on the two walls, so that $r_H = 1$. Clearly, as the degree of asymmetry is increased so that r_H is decreased without affecting the average heat flux, the result is that the maximum temperature on the hotter wall is raised while that on the cooler wall is depressed. Therefore, in equipment design, it is desirable to obtain equal heating on the two channel walls. This is also desirable from the standpoint of optimum spacing, as will be seen.

With the help of Fig. 9, the maximum temperature increases in an asymmetric channel may be evaluated once the symmetric heating value is known. The latter may be obtained rapidly with the aid of the nomogram shown in Fig. 10 once ℓ, b, and $\bar{q}$ are known, r_H being 1 in Fig. 10. The use of the latter has been described in Ref. 7 but is repeated here for ease of reference. To use Fig. 10, first locate Points 1, 2, and 4.

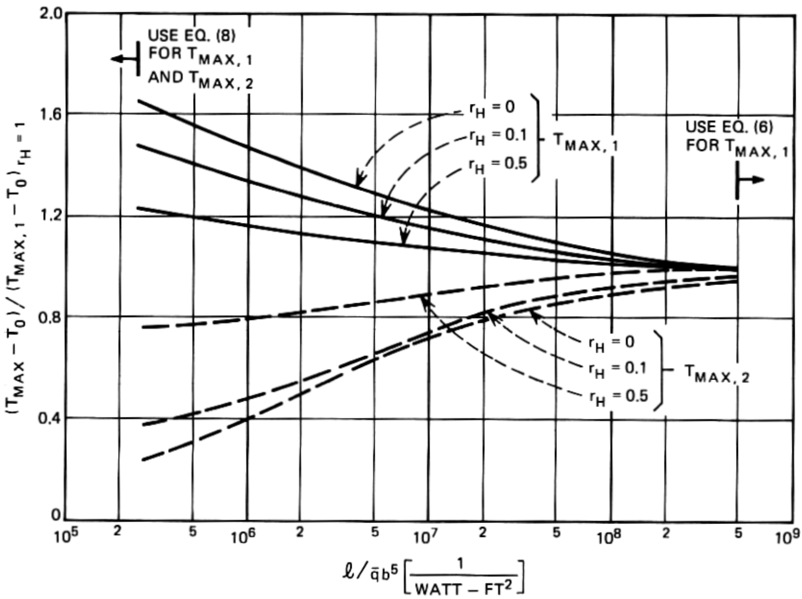

Fig. 9—Maximum wall temperature rise as a function of the quantity $\ell/\bar{q}b^5$.

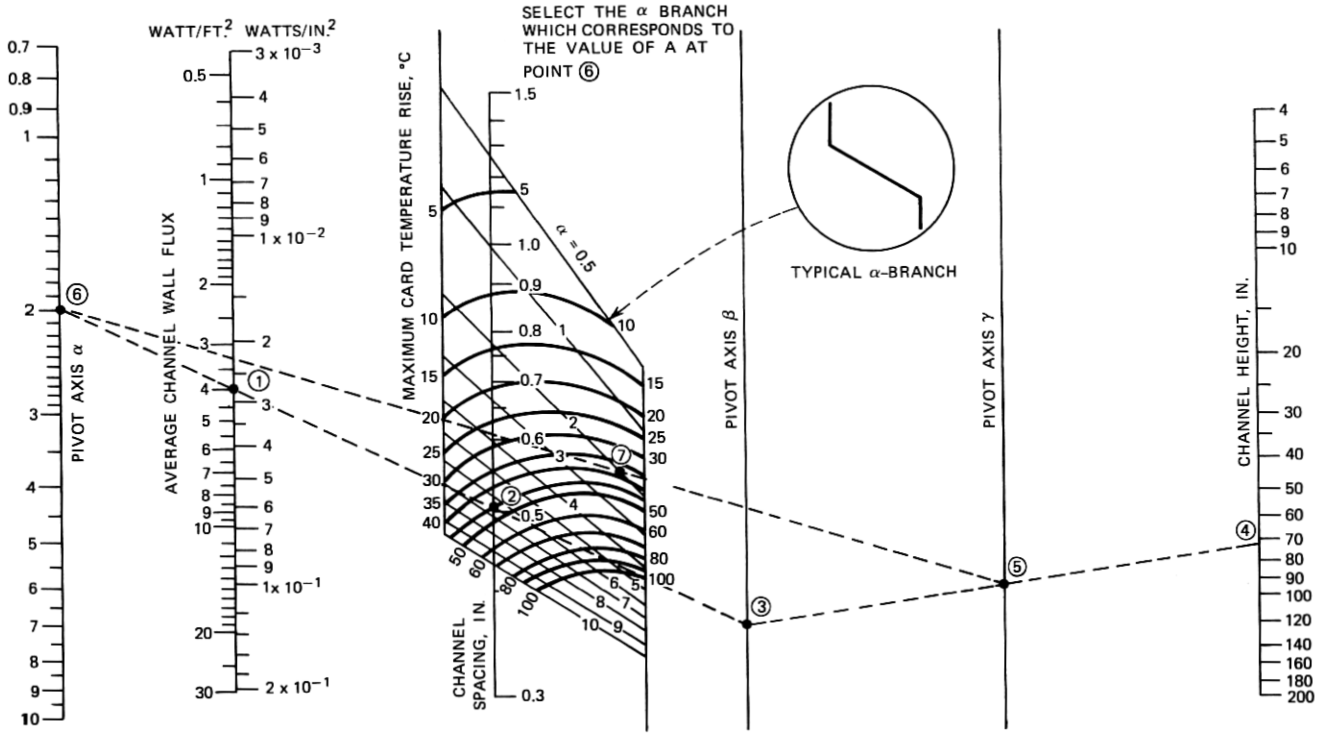

Fig. 10—Nomogram for evaluation of maximum wall temperature increase, $r_H = 1$ (from Aung, Kessler, and Beitin[7]).

Draw a line connecting Points 1 and 2. Extend this line to intersect with the Pivot Axis β to give Point 3, and with Pivot Axis α to give Point 6. Draw a line joining Points 3 and 4. The intersection of this line with Pivot Axis γ gives Point 5. Draw a line connecting Points 5 and 6. Read the α value at Point 6 which for the case illustrated is 2. The intersection of line 5–6 with the appropriate α-branch ($\alpha = 2$ in this case) in the center of the nomogram (Point 7) gives the maximum card temperature rise over room ambient, which in this example is 38°C or 68°F. It should be noted that each α-branch is made up of three segments: a left-side vertical line which corresponds to flow around a single flat plate, a right-side vertical line which corresponds to fully developed flow, and an inclined straight line joining the two vertical ones. Moreover, the asymptotes of the vertical lines coincide for all values of α. Hence, if the line joining Points 5 and 6 does not intersect the appropriate α-branch on its inclined segment, either the left or the right vertical segment should be chosen depending on whether the 5–6 line passes above the inclined segment or below it.

3.4 *Optimum Spacing Including the Effect of Asymmetry*

Consideration may now be given to the overall performance of the channel as a heat-removing device. In electronic equipment the maximum device junction temperature is usually specified. Since the junction temperature is related to the wall temperature, the maximum wall temperature is therefore implied. In modern equipment, it is desirable to increase the packing density and hence the total power dissipation of the entire equipment without increasing the maximum wall temperature beyond the allowable limit. As shown in the appendix, the task here is to select b so as to maximize the heat transfer. The latter value will be designated q_{opt} and the corresponding spacing is called b_{opt}. Design curves for finding b_{opt} and q_{opt} will be given below. The emphasis here is on asymmetrically heated channels. A more detailed discussion of optimum spacing in a symmetric channel may be found in Ref. 7. Note that maximum power dissipation is realized only when the equipment is strictly operating at b_{opt} and q_{opt}. If a different (smaller) spacing is used, then a different (smaller) heat flux must be employed to yield the same maximum temperature increase. For this purpose, design curves such as those in Figs. 6 through 8 can be consulted.

Following Bodoia,[10] it can be shown that the power dissipation is maximized if the channel parameters are so selected that the channel is operated at the point where the slope of a log-log plot of Nu versus Ra is one-half, where Nu is the Nusselt number and Ra the Rayleigh number defined as:

$$\mathrm{Nu} = \frac{\bar{q}b}{(T_{\max,1} - T_0)\mathrm{k}} = \frac{1}{\bar{\theta}_{\max,1}},$$

$$\mathrm{Ra} = \mathrm{Pr} \times \frac{\mathrm{g}\beta(T_{\max,1} - T_0)b^4}{\ell\nu^2} = 0.7 \times \bar{\theta}_{\max,1} \times \bar{L}.$$

From Fig. 3, a relation between Nu and Ra can be obtained. This is shown in Fig. 11. Calling the values of Nu and Ra for maximum power dissipation $\mathrm{Nu_{opt}}$ and $\mathrm{Ra_{opt}}$, respectively, Table I may be constructed from Fig. 11. In Table I, E is an efficiency of heat transfer defined as

$$\left.\begin{aligned} E &= \frac{\bar{p}}{(\bar{p})_{r_H=1}} \\ \text{where} \qquad \bar{p} &= \mathrm{Nu_{opt}}/(\mathrm{Ra_{opt}})^{1/2} \\ &\propto \frac{h_{\mathrm{opt}}}{b_{\mathrm{opt}}} \end{aligned}\right\}. \tag{11}$$

In the above, h_{opt} and b_{opt} are optimum values of $h_{\max,1}$ and b which give maximum power dissipation.

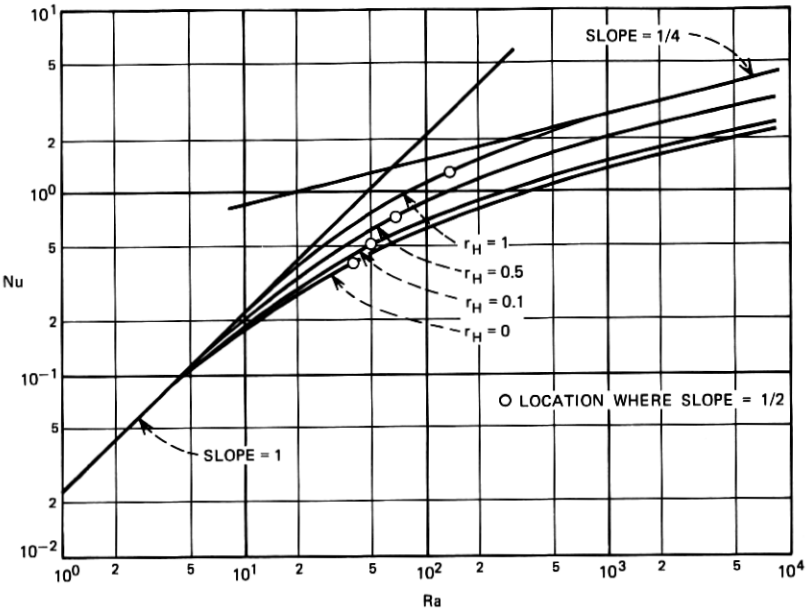

Fig. 11—Relation between Nusselt and Rayleigh numbers.

TABLE I—OPTIMUM RAYLEIGH AND NUSSELT NUMBERS

r_H	Ra_{opt}	Nu_{opt}	E
0	42	0.43	65%
0.1	51	0.51	70%
0.5	70	0.73	86%
1.0	135	1.18	100%

From Table I, it is seen that the efficiency E varies in the same manner as r_H. Asymmetry therefore decreases the efficiency of heat transfer. From the thermal standpoint all equipment should therefore be designed to yield symmetrical heating as closely as possible.

Using values of Ra_{opt} from Table I, it is possible to obtain optimum spacings for different maximum temperatures once the channel height is specified. In like manner, the necessary heat flux at optimum spacing corresponding to different prescribed maximum temperatures can also be obtained from Nu_{opt}. Results have been obtained for a 6-foot channel with $r_H = 0.0$ and $r_H = 0.5$ and compared with the case $r_H = 1$ in Figs. 6 and 8 (indicated by the dashed line). It may be ascertained from these figures that the effect of asymmetric heating is to decrease b_{opt} and q_{opt}. The net effect of asymmetry is to decrease the total power dissipation in the cabinet (see appendix).

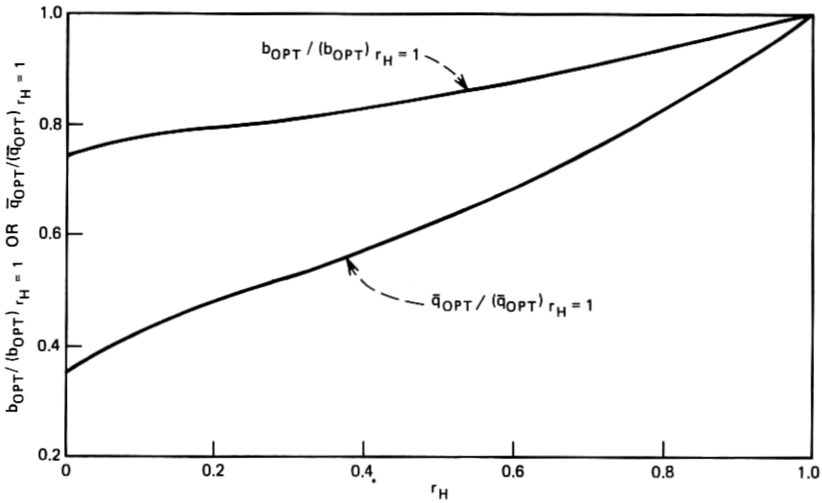

Fig. 12—Optimum spacing and optimum heat flux as a function of the heat flux ratio r_H.

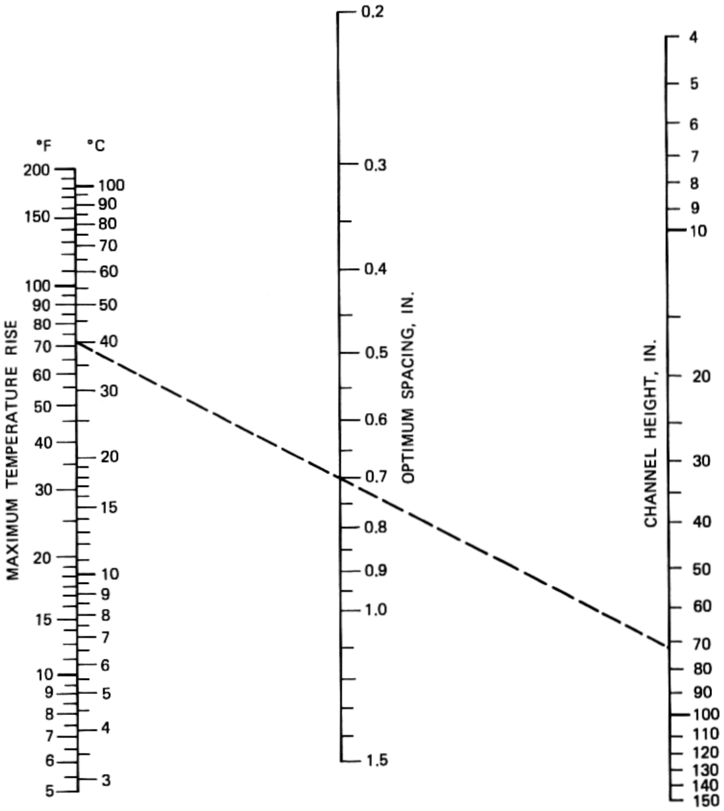

Fig. 13.—Nomogram for evaluation of optimum spacing, $r_H = 1$ (from Aung, Kessler, and Beitin[7]).

Using the values of $\mathrm{Ra_{opt}}$ and $\mathrm{Nu_{opt}}$ given in Table I, it is possible to obtain a general relation between optimum spacing b_{opt} or optimum heat flux q_{opt} and r_H, since

$$\frac{b_{\mathrm{opt}}}{(b_{\mathrm{opt}})_{r_H=1}} = \frac{(\mathrm{Ra_{opt}})^{1/4}}{(\mathrm{Ra_{opt}})^{1/4}_{r_H=1}}$$

and

$$\frac{\bar{q}_{\mathrm{opt}}}{(\bar{q}_{\mathrm{opt}})_{r_H=1}} = \frac{\mathrm{Nu_{opt}}}{(\mathrm{Nu_{opt}})_{r_H=1}}.$$

The results are indicated in Fig. 12. With the aid of this figure and the nomogram given in Fig. 13 which yields $(b_{\mathrm{opt}})_{r_H=1}$, the optimum spacing at specified r_H, ℓ, and maximum hotter wall temperature rise

$(T_{\max,1} - T_0)$ can be rapidly evaluated. Note that the maximum temperature rise to be entered in Fig. 13 is also the specified value of $(T_{\max,1} - T_0)$.

IV. CONCLUDING REMARKS

In the present paper, consideration has been given to the free or natural convection cooling of electronic cabinets containing arrays of vertical circuit cards with asymmetric heating. It was indicated that fully developed flow prevailed in a channel whose height is large compared to its width. In this case, the maximum temperature in the channel can be calculated using the simple relation presented here. In contrast, when the channel spacing is relatively large, the maximum temperature on each wall is independent of the conditions on the other wall but can again be calculated by a simple equation.

In situations where neither of the above two approaches can be applied, the maximum temperature in the channel can be evaluated rapidly using the graphical procedure outlined in this paper. The optimum channel spacing for maximizing the power dissipation is discussed and it is emphasized that to reduce adverse thermal effects channel walls should be powered as equally as possible.

APPENDIX

The total power dissipation in the cabinet per unit depth can be written as

$$\begin{aligned} \text{Power} &= 2\ell N \bar{q} \\ &= 2\ell N h (T_{\max,1} - T_0), \end{aligned}$$

where N is the number of channels (see Fig. 1). If the thickness of the channel wall is neglected, we have

$$N = \frac{W}{b}.$$

Hence,

$$\begin{aligned} \text{Power} &= 2\ell W (T_{\max,1} - T_0) \frac{h}{b} \\ &\propto \frac{h}{b} \end{aligned}$$

for fixed $(T_{\max,1} - T_0)$, W, and ℓ. Clearly, to obtain the largest possible power one needs to obtain the maximum h (that is $\bar{q}$) and the minimum b, the combination of which gives the specified $(T_{\max,1} - T_0)$.

NOTATIONS

b	clear channel or card spacing in feet (ft)
c_p	specific heat of air in W-s/lb_m-F
g	acceleration due to gravity in ft/s^2
h	$\bar{q}/(T_{max,1} - T_0)$, average heat transfer coefficient, in W/ft^2-F
k	thermal conductivity of air in W/ft-F
L_1	$(\ell\nu^2 k)/(g\beta q_1 b^5)$, dimensionless
$\bar{L}$	$(\ell\nu^2 k)/(g\beta\bar{q}b^5)$, dimensionless
ℓ	channel height in ft
Pr	Prandtl number defined by $\mu c_p/k$, dimensionless
p	pressure in lb_f/ft^2
p_0	hydrostatic pressure in lb_f/ft^2
p'	$(p - p_0)$ in lb_f/ft^2
q	heat flux on channel wall per unit surface area, in W/ft^2
$\bar{q}$	$(q_1 + q_2)/2$, W/ft^2
r_H	q_2/q_1, dimensionless
T	temperature in degrees Fahrenheit (F)
T_0	ambient temperature in degrees Fahrenheit
u, v	axial and transverse velocity in ft/s
W	width of cabinet
x, y	axial and transverse coordinates in ft, see Fig. 2
β	thermal expansion coefficient of air in 1/F
μ	dynamic viscosity in lb_m/ft-s
ν	kinematic viscosity of air in ft^2/s
ρ	density of air in lb_m/ft^3
$\bar{\theta}$	dimensionless temperature rise defined in Fig. 3

Subscripts

1	refers to hotter wall
2	refers to cooler wall
max	maximum value
max, 1	maximum value on hotter wall
max, 2	maximum value on cooler wall

REFERENCES

1. Ostrach, S., "Combined Natural and Forced-Convection Laminar Flow and Heat Transfer of Fluids with and without Heat Sources in Channels with Linearly Varying Wall Temperatures," NACA TN 3141 (1954).
2. Engel, R. K., and Mueller, W. K., "An Analytical Investigation of Natural Convection in Vertical Channels," ASME Paper No. 67-HT-16, 1967.
3. Lauber, T. S., and Welch, A. U., "Natural Convection Heat Transfer Between Vertical Flat Plates with Uniform Heat Flux," Proc. Third Int. Heat Transfer Conf., Chicago, Illinois (1966), pp. 126–131.

4. Elenbaas, W., "Heat Dissipation of Parallel Plates by Free Convection," Physica, *9*, No. 1 (1942), pp. 1–28.
5. Bodoia, J. R., and Osterle, J. F., "The Development of Free Convection Between Heated Vertical Plates," J. Heat Transfer, Trans. ASME, Series C, *84*, No. 1 (1962), pp. 40–44.
6. Aung, W., Fletcher, L. S., and Sernas, V., "Developing Laminar Free Convection Between Vertical Flat Plates with Asymmetric Heating," Int. J. Heat Mass Transfer, *15*, 1972, pp. 2293–2308.
7. Aung, W., Kessler, T. J., and Beitin, K. I., "Natural Convection Cooling of Electronic Cabinets Containing Arrays of Vertical Circuit Cards," ASME Paper No. 72-WA/HT-40. See also W. Aung, T. J. Kessler, and K. I. Beitin, "Free Convection Cooling of Electronic Systems," IEEE Trans. Parts, Hybrids, and Packaging, *9*, No. 2 (June 1973), pp. 75–86.
8. Aung, W., "Fully Developed Laminar Free Convection Between Vertical Plates Heated Asymmetrically," Int. J. Heat Mass Transfer, *15*, 1972, pp. 1577–1580.
9. Sparrow, E. M., and Gregg, J. L., "Laminar Free Convection from a Vertical Plate with Uniform Surface Heat Flux," Trans. ASME, February 1956, pp. 435–440.
10. Bodoia, J. R., Ph.D. Thesis, Carnegie Institute of Technology, 1959.