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Gain-Induced Modes in Planar Structures

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The properties of modes in a slab structure with gain in the center region and loss in the surrounding medium are investigated. The propagation constant and field distribution of the lowest-order modes are determined. The cutoff frequencies and propagation constants of the next-higher mode are given. Furthermore, the effect of a refractive index depression or increase in the center region is determined. The depression does not destroy the mode, as may be expected, but causes it to have a cutoff frequency. Comparison of these results to the experimental data shows that gain-induced modes play an important part in the lateral confinement in stripe-geometry GaAs lasers.

I. INTRODUCTION

Modes in cylindrical structures with refractive index boundaries are well known. Their existence and basic properties are generally visualized by superposition of plane waves reflected at the index boundaries. Much less is known about modes in structures with spatially nonuniform gain or attenuation.[1,2] However, it seems intuitively possible that some kind of mode should also exist in that case; indeed, Kogelnik[3] showed that a cylindrical structure with a radial gain profile can support a Gaussian beam of constant diameter even if there are no refractive index differences present. Evidently, the nonuniform transparency of the medium counteracts the natural tendency of the beam to spread.

In this paper we will consider a planar structure with stepwise discontinuities of gain or absorption. This geometry is of considerable practical interest. As will be shown in this paper, the lateral confinement of the optical field in a stripe-geometry GaAs laser is due to the gain-loss interface at the edge of the stripe. Furthermore, it is easy to create nonuniform gain distributions in planar structures either by masking and optically pumping or by nonuniform injection current distribution. The modes induced by these gain distributions can be easily influenced from the outside by changing the pumping intensity or injection current.

II. GAIN-INDUCED MODES IN A THREE-LAYER STRUCTURE

The physical configuration with which we will concern ourselves in this paper is shown in Fig. 1. All material parameters are assumed to be constant throughout the regions and differ only at the boundaries $x = \pm a$. As is well known from the theory of refractive guiding,[4] the modes can be divided into four classes: TE even and odd in x direction and TM even and odd in x. The two lowest-order even modes have no cutoff frequency, whereas the odd modes can be guided only above a certain cutoff frequency. For our application, the lowest-order even modes are of greatest interest and we will deal with them first.

Before going into details, let us explain the notation which we will use subsequently.

(*i*) The complex relative dielectric constant ϵ is split into refractive index n and extinction coefficient k according to

$$\epsilon = (n + jk)^2. \tag{1}$$

(*ii*) u determines the x dependence of the fields inside the center layer $|x| \leqq a$, i.e., they have the functional form

$$\begin{matrix}\sin \\ \cos\end{matrix}\left(u\frac{x}{a}\right).$$

(*iii*) w gives the dependence of the fields in the cladding $|x| \geqq a$ which follow the function $e^{-w|x/a|}$. u and w are related by

$$u^2 + w^2 = (\epsilon_1 - \epsilon_2)(k_0 a)^2. \tag{2}$$

With these quantities we can derive the characteristic equations (the

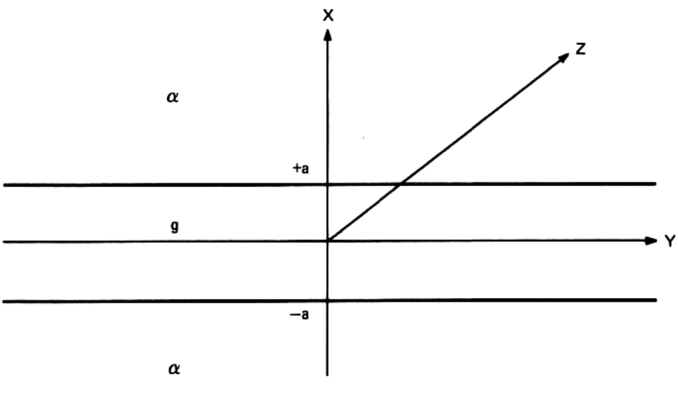


Fig. 1—Cross section of slab waveguide.

field components are listed in Appendix A) for the even modes.

$$\begin{array}{cc} \underline{\text{TE Mode}} & \underline{\text{TM Mode}} \\ w = u \tan u & \frac{\epsilon_1}{\epsilon_2} w = u \tan u \end{array} \tag{3}$$

They are formally the same as in the case of purely reactive modes but, since ϵ_1 and ϵ_2 are complex, the solutions will be complex.

We determine under which conditions eqs. (3) have a solution. We restrict ourselves to cases where the fields cannot increase exponentially with increasing distance from the interface nor can there be any movement of wavefronts from infinity toward the guiding structure. This limits w to the first quadrant. It is then easy to derive a necessary but not sufficient condition from eqs. (2) and (3): $\epsilon_1 - \epsilon_2$ has to stay in the first or second quadrant, i.e., Im $(\epsilon_1 - \epsilon_2) > 0$, which is equivalent to

$$n_1 k_1 - n_2 k_2 > 0. \tag{4}$$

Let us interpret this inequality for some special cases:

(*i*) $k_1 = k_2$. Condition (4) reduces to $n_1 > n_2$ which is the well-known requirement for refractive guiding.

(*ii*) $n_1 = n_2$. The condition (4) now reads $k_1 > k_2$, equivalent to saying that the center region should be more transparent than the sides, which agrees with physical intuition.

(*iii*) $n_1 = n_2 + \Delta n$, where Δn is small compared to n_2 but not necessarily small compared to k. In this case (4) can be expressed by

$$\frac{\Delta k}{k_1} + \frac{\Delta n}{n_2} > 0, \tag{4a}$$

where $\Delta k = k_1 - k_2$. This inequality shows that there can be modes even if Δn or Δk is negative, as long as the other mechanism is strong enough to generate the mode. However, since (4) is only a necessary condition, this case requires further examination.

The second problem we have to address ourselves to is that of the *cutoff frequency*. For refractive guidance the lowest-order even modes have no cutoff frequency. We will now establish the conditions under which gain-induced modes have no cutoff frequency either. For small values of $(\epsilon_1 - \epsilon_2)(k_0 a)^2$, w and u will be small and the tangent in eq. (3) can be replaced by its argument. The solution of eqs. (2) and (3) is thus $w = (\epsilon_1 - \epsilon_2)(ak_0)^2$ for TE modes and $w = (\epsilon_2/\epsilon_1) \times(\epsilon_1 - \epsilon_2)(ak_0)^2$ for TM modes with ak_0 very small. This result shows that the solution of (3) exists no matter how small (ak_0) is, independent of the guide parameters, as long as $\epsilon_1 - \epsilon_2$ is in the first quadrant. In other words, *a gain-induced mode will have no cutoff frequency as long as there is a refractive index difference to keep* $\epsilon_1 - \epsilon_2$ *in the first quadrant. If there exists a refractive index depression, i.e.,* $Re\ (\epsilon_1 - \epsilon_2) < 0$*, the modes do have a cutoff frequency.*

We will now consider the case of *gain-induced modes without a refractive index change.* We specialize the discussion to small extinction coefficients in the order of 10^{-4}, whereas n is typically greater than one. This covers typical laser applications reasonably well. The characteristic equations for TE and TM modes [eqs. (3)] differ only by a factor of $\epsilon_1/\epsilon_2 = 1 + j(2/n)(k_1 - k_2)$ which is sufficiently close to one to cause only a negligible perturbation. In the following, we will therefore neglect the difference between TE and TM modes.

To simplify the discussion we define two new quantities: A normalized frequency is given by

$$v = ak_0\sqrt{\epsilon_1 - \epsilon_2}. \tag{5}$$

For gain-induced modes, v reduces to the form

$$v = ak_0 n \sqrt{j2\,\frac{(k_1 - k_2)}{n}}\,, \tag{6}$$

which means that in this particular case the phase angle of v is independent of the material parameters. It is furthermore customary[5] to use a normalized propagation constant b

$$b = \frac{(\beta_z/k_0)^2 - \epsilon_2}{\epsilon_1 - \epsilon_2} \quad \text{or} \quad \left(\frac{\beta_z}{k_0}\right)^2 = \epsilon_2 + (\epsilon_1 - \epsilon_2)b. \tag{7}$$

Since in our application $\epsilon_1 - \epsilon_2$ is a small quantity, $(\beta_z/k_0)^2$ is always equal to ϵ_2 plus a small perturbation, whereas the variation of $|b|$ is in the order of one, thus alleviating some computational problems. The two parameters u and w are related to v and b by $u = v\sqrt{1 - b}$ and $w = v\sqrt{b}$.

With these new quantities, the two characteristic eqs. (3) are transformed into

$$\sqrt{b} = \sqrt{1 - b}\tan\left(v\sqrt{1 - b}\right). \tag{8}$$

It should be noted that the normalized propagation constant b is exclusively a function of v. It is, therefore, only necessary to solve the characteristic eq. (8) once to cover all material parameters and dimensions, assuming, of course, the validity of the initial assumptions.

The solution of the characteristic equation was done on the computer. Figure 2 shows the normalized propagation constant b for the lowest-order mode as a function of $|v|$. We can interpret b more easily

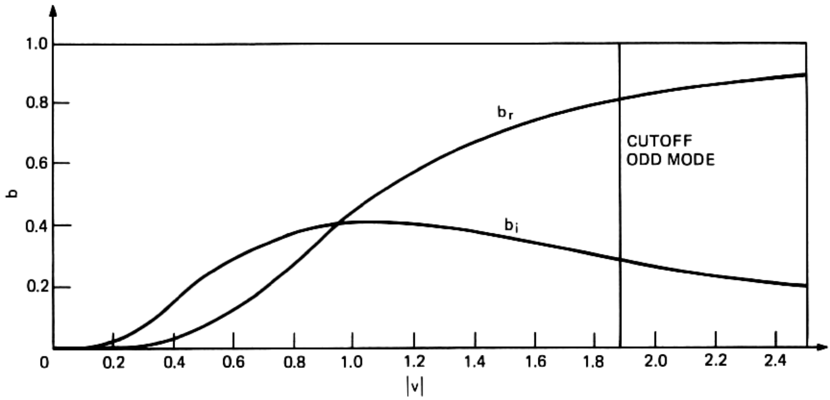


Fig. 2—$b(|v|)$ for lowest-order even mode ($n_1 = n_2$).

if we rewrite the propagation constant β_z [eq. (7)] to show its real and imaginary parts.

$$\frac{\beta_z}{k_0} \approx n - b_i(k_1 - k_2) + j(k_2(1 - b_r) + k_1 b_r). \tag{9}$$

The attenuation or gain of the mode as represented by the imaginary part of β_z/k_0 is solely dependent on b_r, which varies between 0 and 1. For small values of b_r, the attenuation will be determined by the outer medium and only for higher values of b_r will the gain in the center region be of significance. This situation corresponds very closely to the case of refractive guidance. For further reference we have included in Fig. 3 the field distribution with $|v|$ as parameter.

For any practical application, it is important to know up to which frequency or dimension the guide is single moded. We will therefore determine the cutoff frequency of the lowest-order odd mode and of the first-order even mode. We define the cutoff value of b as the one at which the radiation condition just ceases to be fulfilled. It turns out that this is the case when w is purely imaginary, i.e., only power is radiated away from the guiding structure, but the amplitude does not decrease with increasing $|x|$. Re $(w) = 0$ is equivalent with the condition Re $(b) = 0$ [eq. (7)]. The solution of this problem has to be found on the computer. The results are

$$|v_{\text{odd}}| = 1.877 \qquad |v_{\text{even}}| = 2.759.$$

For future reference we have included $b(|v|)$ for the first-order even and the lowest-order odd mode (Figs. 4 and 5).

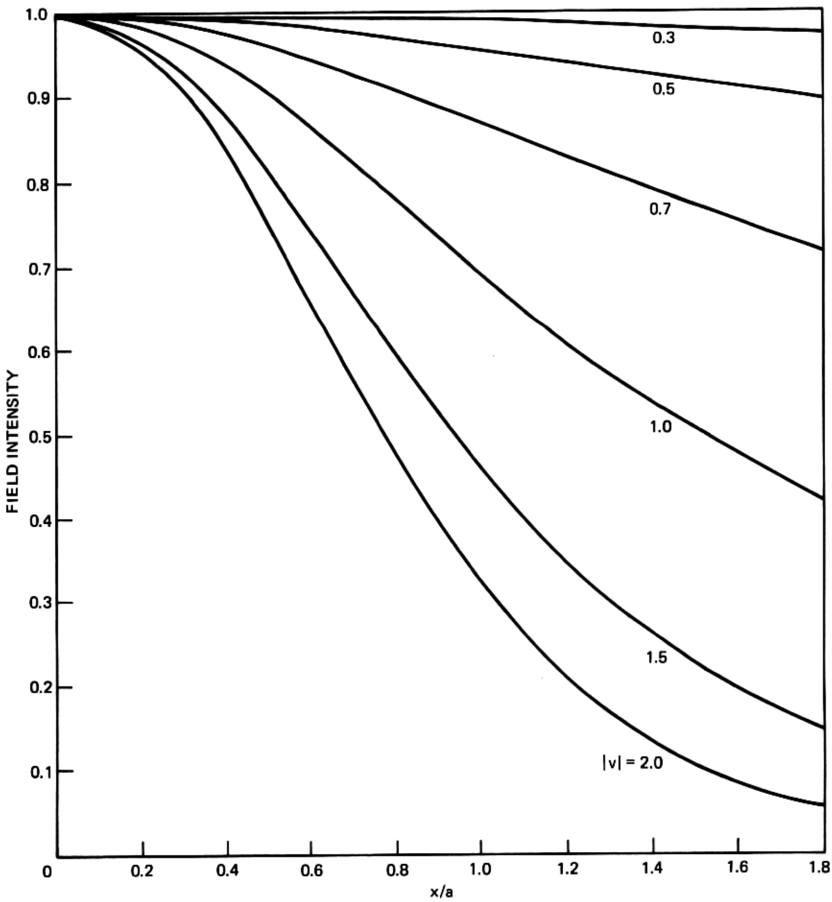


Fig. 3—Field intensity for lowest-order even mode.

III. THE INFLUENCE OF ADDITIONAL REFRACTIVE INDEX DIFFERENCES

In general, if there is a gain or loss difference between two regions, the refractive index will be different as well. Gain and refractive index are related by the Kramers-Kronig relations and thus the presence of gain can change the refractive index. Furthermore, in the case of the injection laser, the injected carriers change the refractive index due to the plasma resonance. In this particular case, the gain region can be expected to have a lower refractive index than the side regions. It is thus necessary to explore the effects of refractive index increases and depressions in the center region in conjunction with gain-loss differences.

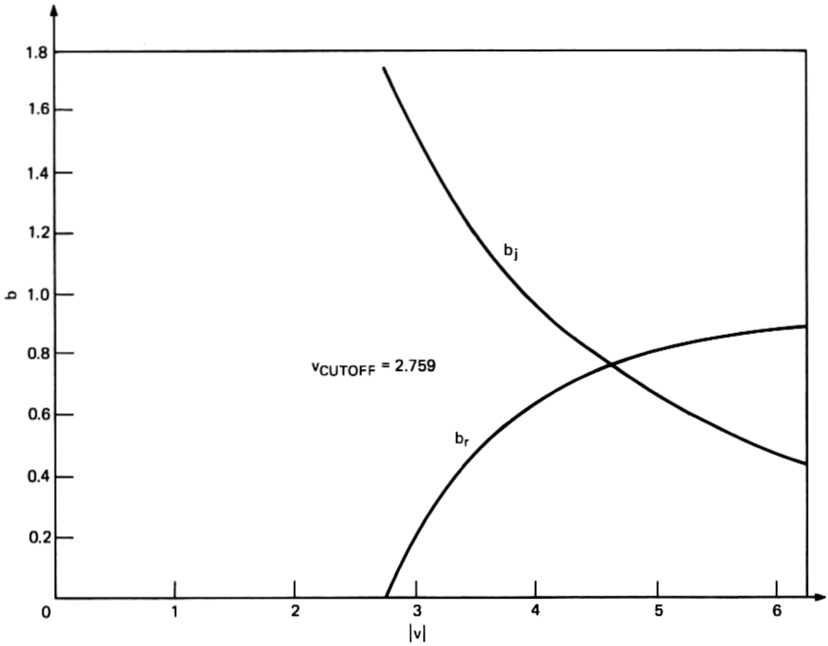


Fig. 4—$b(|v|)$ for first-order even mode.

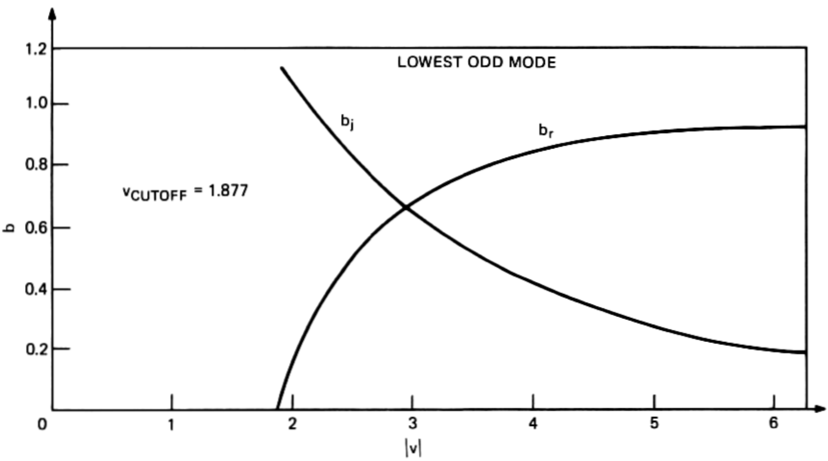


Fig. 5—$b(|v|)$ for lowest-order odd mode.

If there is an increase in refractive index in the center, we can expect the confinement of the energy to increase with increasing index difference. However, in the case of a refractive index depression in the center, the two confining mechanisms will counteract each other.

Let us consider the effect of *increased* refractive index in detail. In contrast to the case of gain-induced modes where the phase of v was $\pi/4$ [eq. (6)] independent of material parameters, it is now dependent on the complex difference $\epsilon_1 - \epsilon_2$. This precludes a representation of the guiding parameters independently of the waveguide parameters as it was possible in the previous case. We are thus forced to determine a solution of the characteristic equation for each set of parameters. It is therefore of importance to develop approximations for the propagation constant. We treat first the case, $\Delta k/\Delta n \ll 1$. We regard the waveguide as a perturbed refractive guide. The $b(v)$ characteristic is plotted in Fig. 6. Obviously, both v and b are real. A small perturbation Δk has two effects: (*i*) the characteristic eq. (8) becomes complex and yields an imaginary part b_i, whereas the real part is to a first order unperturbed; and (*ii*) the propagation constant (9) is now approximated by

$$\frac{\beta_z}{k_0} \approx n + \Delta n b_r - b_i \Delta k + j(k_2 + b_r \Delta k + b_i \Delta n), \tag{10}$$

i.e., there appears a second imaginary component $b_i \Delta n$, which acts like a gain since Δn and b_i are both positive. This effect is due to the improvement in guiding by the gain-loss mechanism. The calculations are straightforward and are listed in Appendix B. We note here only the result:

$$b_i = \frac{\Delta k}{\Delta n} \frac{(1 - b_0)v_0\sqrt{b_0}}{1 + v_0\sqrt{b_0}}, \tag{11}$$

where the quantities indexed with a zero denote the unperturbed state. In particular, $v_0 = ak_0n\sqrt{2\Delta n/n}$. We notice that b_i contains a factor $1 - b_0$, which decreases with b_0 approaching unity. Inserting (11) into eq. (10) yields an expression for the loss (or gain) of the mode:

$$\alpha \approx k_0 \left[k_2 + \Delta k \left(b_0 + \frac{(1 - b_0)v_0\sqrt{b_0}}{1 + v_0\sqrt{b_0}} \right) \right]. \tag{12}$$

We note that the function

$$f(v_0) = b_0 + \frac{(1 - b_0)v_0\sqrt{b_0}}{1 + v_0\sqrt{b_0}}$$

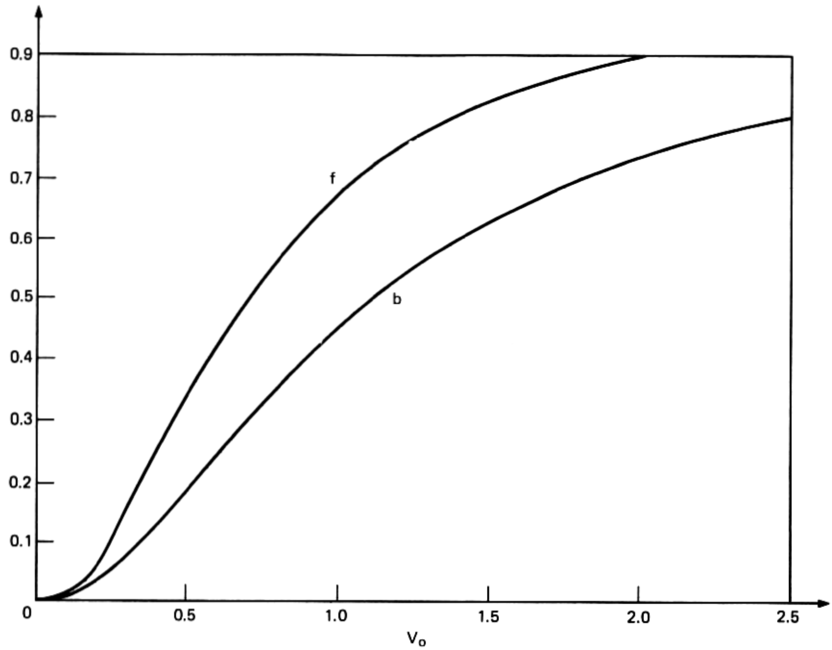


Fig. 6—$b(v)$ for lowest-order even mode in a refractive slab guide and loss-weighing function $f(v_0)$ for lowest-order even mode.

associated with Δk is only dependent on v_0 and we have thus again achieved a representation valid for all waveguide parameters. Figure 6 shows that $f(v_0)$ approaches unity much faster than $b_0(v_0)$; the loss is approaching the loss or gain in the center region indicating improved confinement due to the gain-loss difference.

We will now consider the refractive index *depression* in the center region. From our discussion of cutoff frequencies, we know that modes in this case do have a cutoff frequency if they exist at all. It can be determined from the condition that $w = v\sqrt{b}$ is purely imaginary.

In Fig. 7 real and imaginary parts of b are plotted as functions of s/λ with the parameters listed in the figure caption. Since b_r is larger than one, eq. (10) shows that the effective refractive index is smaller than that of either medium (Δn is negative). The phase velocity is therefore larger than that of a plane wave in either medium, as in metallic waveguides. Altogether, this is quite a different behavior from the ordinary dielectric waveguide. One would expect the index depression to counteract the confinement and, if the depression is strong enough, to destroy it completely. As discussed before, this is only the

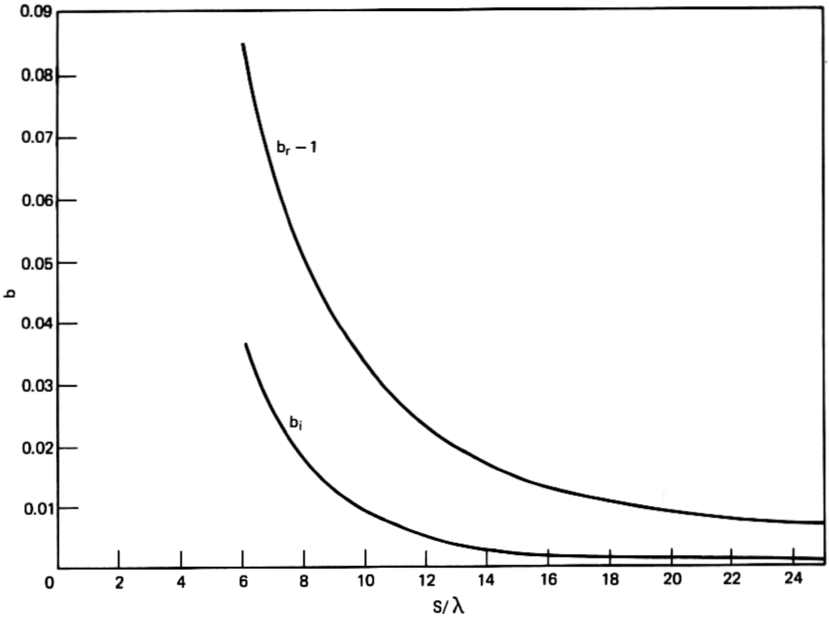


Fig. 7—$b(s/\lambda)$ for lowest-order mode for a refractive index depression $\Delta n = 10^{-2}$, $n = 3.56$, $k_1 = 2.10^{-4}$, $k_2 = 10^{-4}$.

case below the cutoff frequency. To ensure that the solution of the characteristic equation is real and not a freak of the computer program, we have derived an approximate solution for $\Delta k/\Delta n \ll 1$ in Appendix C. The normalized propagation constant is given by

$$b = 1 + \frac{\pi^2}{4v^2}\left\{1 + j\frac{2}{v}\left(1 + v\frac{\Delta k}{2|\Delta n|}\right)\right\}, \tag{13}$$

where

$$v = \frac{s}{\lambda}\pi n\sqrt{\frac{2|\Delta n|}{n}}.$$

IV. GAIN-INDUCED MODES IN STRIPE-GEOMETRY GaAs LASERS

The function of the stripe geometry has been viewed as selecting a filament and preventing others from forming.[6] We will now show that, in contrast to a laser with a wide area contact, the stripe geometry does provide a confining mechanism for the optical power and is not merely "selecting" the filament.

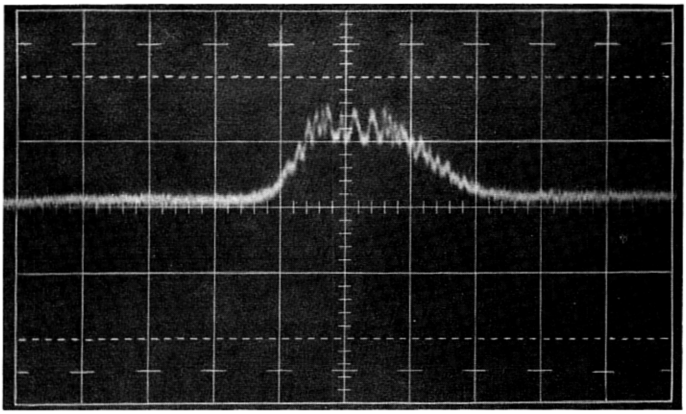
In stripe-geometry lasers, the flow of carriers is confined in a stripe region parallel to the junction (laterally). This is done by proton bombarding the adjacent regions.[7] The proton bombardment and subsequent annealing alter the free-carrier density but not the optical properties of the material.[8] There are no deliberately built-in refractive index changes between the active and the passive regions. The experiments show that with stripe widths of $\sim 12\ \mu m$ the lasers operate generally in a single lateral mode; the origin of this confinement has not yet been explained. Figure 8a gives a linear scan of a typical nearfield distribution. However, at high current levels or wider stripe widths, numerous deviations from such a single-mode field distribution occur. Figure 8b gives an example of the nearfield in such a case. The field still fills essentially the whole stripe width but is highly nonuniform. Frequently the intensity maximum changes its position with changing current. It is clear that the simple case we have been analyzing so far cannot explain these effects. However, not enough knowledge is available at this time about the numerous parameters involved. We therefore try only to isolate the common properties of the majority of stripe-geometry lasers and explain them in terms of gain-induced modes. We had three major facts to consider:

(*i*) For stripe widths in the order of $12\ \mu m$ and less, there is generally a single mode for current levels close to threshold.
(*ii*) At stripe widths above $\sim 18\ \mu m$, the field distribution is very often nonuniform even at threshold.
(*iii*) The threshold current increases steeply with decreasing stripe width.

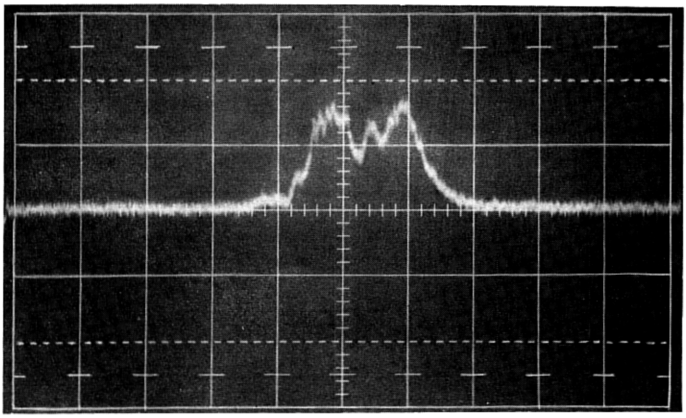
Since we know that there must be a gain-loss difference between the stripe and the surrounding regions, we will apply the previous results on gain-induced modes and investigate if the experimental results can be explained by this effect or if additional mechanisms have to be invoked. In doing so we must keep in mind that we can expect the agreement between experiment and this theory to be only qualitative since a number of effects are neglected. We mention the few most obvious:

(*i*) The loss in the layers above and below the active region contributes to the overall loss. This contribution is dependent on stripe width.
(*ii*) The gain depends on the field intensity and is therefore not constant.
(*iii*) The mirror loss may depend on the stripe width.

The mirror reflectivity R of the lowest-order mode was determined to



(a)



(b)

Fig. 8—Linear scan of nearfield distribution of stripe geometry lasers with stripe width of $\sim$12 μm. One division $\approx$3.7 μm. Current in both cases is $\sim$20 percent above threshold.

be $\sim$0.3 and hence the mirror loss, defined by $\alpha_M = 1/L \ln 1/R$, to be $\cong$10/cm for $L = 400$ μm sample length.[9] The attenuation of GaAs is $\sim$10/cm. We ask now how much gain would one need to reach threshold with an active region width of $\sim$12 μm. In the following, we will call gain g the excess gain over the intrinsic attenuation in the active region. To reach threshold, the effective gain of the lowest-order mode ($gb_r - \alpha(1 - b_r)$ from eq. (9)) must equal the mirror losses:

$$gb_r - \alpha(1 - b_r) = \alpha_M, \tag{14}$$

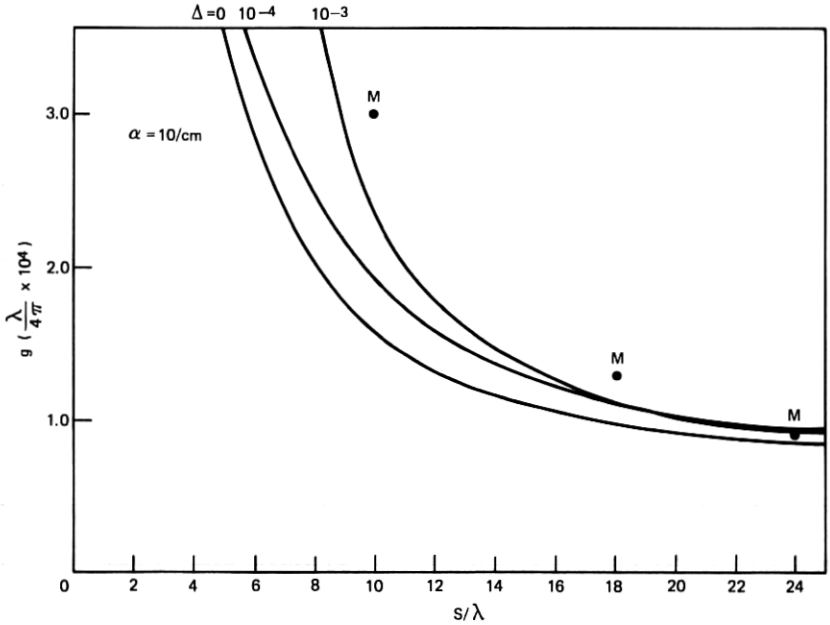


Fig. 9—Threshold gain as function of stripe width. Mirror loss $\alpha_M = 10$/cm, $n = 3.56$, $\alpha = 10$/cm, and refractive index depressions $\Delta n = 10^{-3}$, 10^{-4}.

where α is the loss in the side regions. b_r–as pointed out before–is exclusively a function of $|v|$ (Fig. 2). $g\lambda/4\pi$ is plotted as a function of stripe width in Fig. 9. We can see that the gain value corresponding to $\sim$12 μm stripe width is quite realistic and we conclude that the gain-induced guiding must play at least some role in the stripe-geometry confinement. As a further piece of experimental evidence, it has been shown by Dyment[10] that the threshold current in a stripe-geometry laser increases strongly with decreasing stripe width. There is, unfortunately, some uncertainty about the relationship between current and gain. Generally, an exponential dependency is assumed ($g \sim J^q$, where q varies between 1.5 and 3). We have used $q = 2$ to insert Dyment's measured results into Fig. 9. We have furthermore introduced $g(s/\lambda)$ curves for refractive index depressions of 10^{-4} and 10^{-3} which correspond to carrier densities of 10^{18} and 10^{19}/cm, respectively. The agreement is satisfactory considering the accuracy of the measurements. It therefore seems reasonable to conclude that the gain-induced modes provide the dominant confining mechanism in stripe-geometry lasers. In view of this conclusion we will now derive a few properties of the gain-induced modes which should be useful for predicting some properties of the stripe-geometry lasers. Wherever the ouput of the laser is to be coupled into an optical system, it is desirable to reduce the stripe width without increase in threshold current. Equation (14) was therefore evaluated for various loss values in the side region. Figure 10 shows that–at least for zero refractive index difference–α has very little influence on the necessary gain. Evidently the increased loss in the higher α case is compensated for by improved guidance.

A refractive index depression generates a gain-width relationship of the form [eq. (13)]

$$g = \alpha_M + \frac{k_0}{4\pi n^2} \frac{1}{(s/\lambda)^3} \frac{1}{\sqrt{\frac{2\Delta n}{n}}}. \tag{15}$$

It is quite evident that it is very difficult to compensate for the very strong s/λ dependence by proper choice of Δn. It is, however, possible to reduce the stripe width with a refractive index increase, in which case the effective gain is given by [eq. (12)]

$$g = \frac{\alpha_M + \alpha(1 - f(v))}{f(v)}, \tag{16}$$

where $f(v)$ is plotted in Fig. 6. However, at present, the technological difficulties of this approach have not been solved.

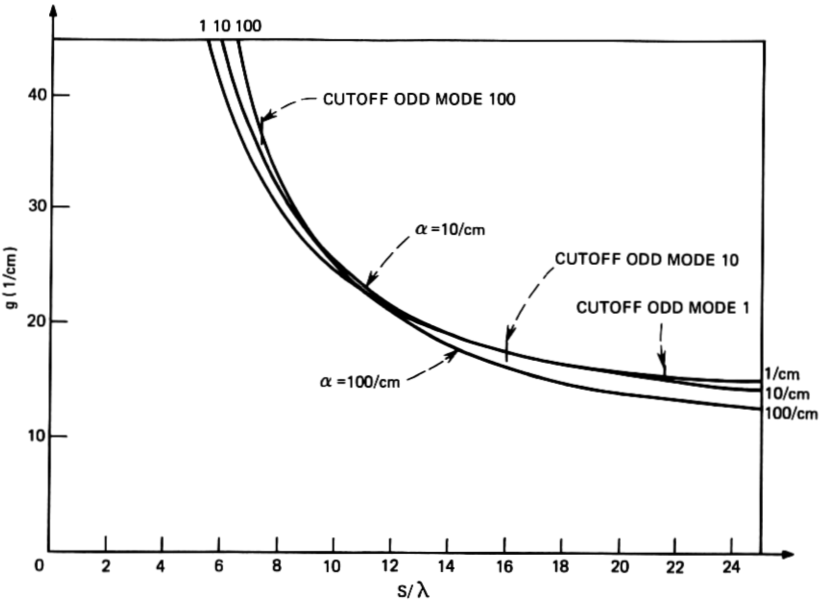


Fig. 10—Threshold gain as function of stripe widths for different attenuation α in the side regions. $\alpha_M = 12.5/\text{cm}$.

V. CONCLUSION

Modes caused by gain attenuation differences in a slab structure have been investigated. They can exist without refractive index differences, in which case the lowest-order even modes do not have a cutoff frequency. The cutoff frequencies for the next-higher modes are given. The influence of additional refractive index increase or decrease in the center region is studied. One particularly interesting result is that, in the presence of gain, a refractive index depression does not remove the mode, but it causes it to have a cutoff frequency.

An application of these results to the problem of lateral confinement in stripe-geometry lasers shows that gain-induced modes are mainly responsible for the optical confinement. Thus the stripe geometry does not just select a filament, but provides a confining mechanism for the optical power. (A similar conclusion was reached independently by F. R. Nash.[11]) The sharp increase of the threshold current with reduced stripe width is a direct consequence of this gain-induced confinement. It therefore appears impossible to reduce the stripe width significantly below ~10 μm without a substantial sacrifice in the threshold current. A deliberately built-in refractive index increase in the center region should alleviate this situation.

APPENDIX A

Field Components in Slab Guide

TE		TM
	$|x| \leqq a$	
$E_y = A\omega\mu_o \cos u \frac{x}{a} e^{-j\beta_z z}$		$H_y = A\omega\epsilon_0\epsilon_1 \cos u \frac{x}{a} e^{-j\beta_z z}$
$H_x = -A\beta_z \cos u \frac{x}{a} e^{-j\beta_z z}$		$E_x = A\beta_z \cos u \frac{x}{a} e^{-j\beta_z z}$
$H_z = -jA \frac{u}{a} \sin u \frac{x}{a} e^{-j\beta_z z}$		$E_z = jA \frac{u}{a} \sin u \frac{x}{a} e^{-j\beta_z z}$
	$|x| \geqq a$	
$E_y = A\omega\mu_0 \cos u e^{-[j\beta_z z + w(|x/a|-1)]}$		$H_y = A\omega\epsilon_0\epsilon_2 \cos u e^{-[j\beta/_z z + w(|x/a|-1)]}$
$H_x = -A\beta_z \cos u e^{-[j\beta_z z + w(|x/a|-1)]}$		$E_x = A\beta_z \frac{\epsilon_1}{\epsilon_2} \cos u e^{-[j\beta_z z + w(|x/a|-1)]}$
$H_z = \mp jA \frac{w}{a} \cos u e^{-[j\beta_z z + w(|x/a|-1)]}$		$E_z = \pm jA \frac{w}{a} \frac{\epsilon_1}{\epsilon_2} \times \cos u e^{-[j\beta_z z + w(|x/a|-1)]}$

The notations are explained in the list of notations at the end of the paper. The separation parameters are related to each other by

$$(\beta_z a)^2 + u^2 = \epsilon_1 (k_0 a)^2, \qquad (\beta_z a)^2 - w^2 = \epsilon_2 (k_0 a)^2,$$

and $u^2 + w^2 = (\epsilon_1 - \epsilon_2)(k_0 a)^2$.

APPENDIX B

Approximate Solution of Characteristic Equation for Refractive Index Increase in Outer Region

We assume that $\Delta k/\Delta n \ll 1$, i.e., the refractive index step is governing the guiding properties. The normalized frequency v can then be expressed by

$$v \approx \frac{s}{\lambda} \pi n \sqrt{2 \frac{\Delta n}{n}} \left(1 + j \frac{\Delta k}{2\Delta n}\right). \tag{17}$$

We treat the characteristic equation for TE modes first [eqs. (3)]. The regular solution for $\Delta k = 0$ will yield real values for u and w. For small $\Delta k/\Delta n$, both will be complex with a small imaginary part. We list the real and imaginary part of the characteristic equation separately, neglecting second-order terms:

$$u_r \sin u_r = w_r \cos u_r \tag{18a}$$

$$u_i[(1 + w_r) \sin u_r + u_r \cos u_r] = w_i \cos u_r. \tag{18b}$$

The real part (18a) is identical to the unperturbed equation and we assume, therefore, u_r and w_r to take the values of u and w, respectively, of the $\Delta k = 0$ case. Combining (18a) and (18b) yields the following relation between u_i and w_i:

$$u_i[u_r^2 + w_r(1 + w_r)] - w_i u_r = 0. \tag{19}$$

Now we have to relate u_i, w_i to b_i, the quantity we really want to determine.

$$w = v\sqrt{b} \qquad u = v\sqrt{1 - b}. \tag{20}$$

With a small imaginary part of b we get

$$w = v_r \sqrt{b_r} \left[1 + j\left(\frac{v_i}{v_r} + \frac{b_i}{2b_r}\right)\right] \tag{21a}$$

$$u = v_r \sqrt{1 - b_r} \left[1 + j v_r \left(\frac{v_i}{v_r} - \frac{b_i}{2(1 - b_r)}\right)\right]. \tag{21b}$$

These equations show that, if the real parts of w and u are unperturbed to the first order, the real part of b will be unperturbed also. If we introduce eqs. (21) into the characteristic eq. (19), b_i can be determined

$$\boxed{b_i = \frac{\Delta k}{\Delta n} \frac{(1 - b_r)v_r\sqrt{b_r}}{1 + v_r\sqrt{b_r}}}. \tag{22}$$

The characteristic equation for the TM mode has an additional factor ϵ_1/ϵ_2. It is easy to show that the real part of b is altered by Δb from the TE value b_0

$$\boxed{\frac{\Delta b_r}{b_0} = -\frac{\Delta n}{n} \frac{4(1 - b)}{1 + w}}. \tag{23}$$

The imaginary part is to a first approximation the same as for the TE mode [eq. (22)].

APPENDIX C

Approximate Solution of Characteristic Equation for Refractive Index Depression in Center Region

Again we assume $\Delta k/|\Delta n| \ll 1$. Δn will be negative. The normalized frequency v is now expressed by

$$v = j\frac{s}{\lambda} n\pi \sqrt{\frac{2|\Delta n|}{n}} \left(1 + j\frac{\Delta k}{2|\Delta n|}\right) = jv_i\left(1 - j\frac{\Delta k}{2|\Delta n|}\right). \tag{24}$$

Let us assume that u is very close to $\pi/2$,

$$u = \pi/2 + \delta + j\psi, \tag{25}$$

where δ and ψ are small quantities. Since

$$w = v\sqrt{1 - \left(\frac{u}{v}\right)^2},$$

we use

$$w = v\left[1 - \frac{1}{2}\left(\frac{u}{v}\right)^2\right], \tag{26}$$

where we note that u/v is small compared to one. The characteristic equation to a first-order approximation takes the form

$$\left(\frac{\pi}{2} + \delta + j\psi\right) + jv_i(\delta + j\psi) = 0. \tag{27}$$

The solution is

$$\delta \approx -\frac{\pi}{2}\frac{1}{1 + v_i^2} \qquad \psi = \frac{\pi}{2}\frac{v_i}{1 + v_i^2}. \tag{28}$$

It is now easy to calculate b, which is related to u by

$$b = 1 - \left(\frac{u}{v}\right)^2. \tag{29}$$

The result is

$$\boxed{b = 1 + \frac{\pi^2}{4v_i^2}\left\{1 + j\frac{2}{v_i}(1 + v_r)\right\}} \tag{30}$$

keeping in mind that $v_r/v_i \ll 1$ and $v_i^2 \gg 1$.

NOTATIONS

$\alpha = \dfrac{4\pi k}{\lambda}$	attenuation constant
a	halfwidth of slab
b	normalized propagation constant [defined in eq. (7)]
$s = 2a$	width of slab
$\Delta n = n_1 - n_2$	
$\Delta k = k_1 - k_2$	
ϵ_0	dielectric constant of vacuum
ϵ_1, ϵ_2	relative dielectric constants (subscript 1 refers to the center and 2 to the outer region)
g	gain in center region
$k_0 = \omega\sqrt{\mu_0\epsilon_0}$	free-space wave number
$k_{1,2}$	extinction coefficient (subscript 1 refers to the center and 2 to the outer region)
λ	free-space wavelength
μ_0	permeability of free space
$n_{1,2}$	refractive index (subscript 1 refers to the center and 2 to the outer region)
$\omega = \dfrac{2\pi c}{\lambda}$	angular frequency
u	separation parameter
$v = ak_0\sqrt{\epsilon_1 - \epsilon_2}$	normalized frequency
w	separation parameter

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