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Receiver Design for Digital Fiber Optic Communication Systems, II

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This paper applies the results of Part I to specific receivers in order to obtain numerical results. The general explicit formulas for the required optical average power to achieve a desired error rate are summarized. A specific receiver is considered and the optical power requirements solved for. The parameters defining this receiver (e.g., bit rate, bias resistance, dark current, etc.) are then varied, and the effects on the required optical power are plotted.

I. INTRODUCTION

This paper will apply the theory of Part I to illustrate in detail how the required received optical power in a digital fiber optic repeater varies with the parameters such as the desired error rate, the thermal noise sources, the bit rate, detector dark current, imperfect modulation, etc. We shall begin by first applying the formulas of Part I to a specific realistic example to obtain reference point. We shall then derive curves of how the required power varies around this point as we vary the system parameters.

II. REVIEW OF RESULTS OF PART I

In Part I we derived explicit formulas for the required optical power at the input of a digital fiber optic communication system repeater to achieve a desired error rate. One formula was applicable when little or no internal (avalanche) detector gain was used, so that thermal noise from the amplifier dominated. The other formula was applicable when optimal gain was being used. These formulas are repeated below:

$$p_{\text{required}} = \frac{QZ^{\frac{1}{2}}}{GT}\frac{\hbar\Omega}{\eta}, \qquad \text{(Thermal Noise Dominates)} \qquad (1)$$

where

$$Z = \left\{ \frac{T}{e^2} \left[S_I + \frac{2k\theta}{R_b} + \frac{S_E}{R_T^2} \right] I_2 + \frac{(2\pi C_T)^2 S_E I_3}{Te^2} \right\}; \tag{1a}$$

$$p_{\text{required}} = \frac{1}{2T} (Q)^{(2+x)/(1+x)} \left[(Z)^{x/(2+2x)} (\gamma_1)^{x/(2+2x)} (\gamma_2)^{(2+x)/(1+x)} \right] \frac{\hbar\Omega}{\eta},$$

(Optimal Avalanche Gain) (2)

where

$$G_{\text{optimal}} = (Q)^{-1/(1+x)} \left[(Z)^{1/(2+2x)} (\gamma_1)^{1/(2+2x)} (\gamma_2)^{-1/(1+x)} \right],$$

where (referring to Fig. 1)

$\eta/\hbar\Omega$ = detector quantum efficiency/energy in a photon
T = interval between bits = 1/bit rate
G = average detector internal gain
G^x = detector random internal gain excess noise factor
Q = number of noise standard deviations between signal and threshold at receiver output. $Q = 6$ for an error rate of 10^{-9}. (See Fig. 21 in Part I for a graph of error rate vs Q.)
e = electron charge
$k\theta$ = Boltzman's constant·the absolute temperature
R_T = total parallel resistance in shunt with the detector including the physical biasing resistor and the amplifier input resistance
R_b = value of physical detector biasing resistor
C_T = total shunt capacitance across the detector including the shunt capacitance of the detector and that of the amplifier
S_I = amplifier shunt noise source spectral height (two-sided) in amperes^2/Hz
S_E = amplifier series noise source spectral height (two-sided) in volts^2/Hz.

I_2, I_3, γ_1, and γ_2 are functions only of the shapes of the input optical pulses and the equalized repeater output pulses, where the length of a time slot has been scaled out. These functions are defined in eqs. (23) and (34) of Part I.

Formulas (1) and (2) neglect dark current and assume perfect modulation (received optical pulses completely on or off). We shall investigate deviations from these idealizations later in the paper. For silicon detectors and bit rates above a few megabits per second, these idealizations are reasonable approximations.

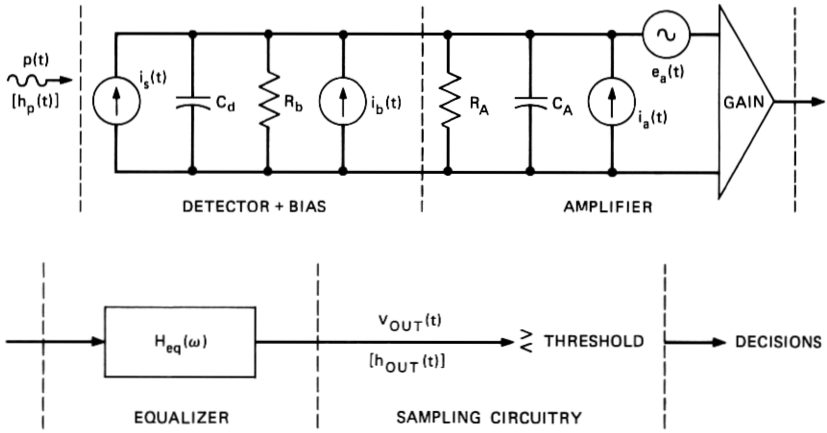

Fig. 1—Receiver.

III. A TYPICAL OPTICAL REPEATER

Consider the following practical optical repeater, operating at a bit rate of 2.5×10^7 bits per second and an error rate of 10^{-9}. The detector is a silicon device with excess noise exponent $x = 0.5$, quantum efficiency 75 percent, dark current before avalanche gain of 100 picoamperes, and an operating wavelength of 8500 angstroms. The front-end amplifier is a field-effect transistor in a common-source configuration. The total shunt capacitance across the detector is 10 pF. The detector biasing resistor is 1 megohm. The amplifier input resistance is 1 megohm. The amplifier shunt-current noise-source spectral height is equal to the thermal noise of a 1-megohm resistor. The amplifier series-voltage noise-source spectral height is equal to the thermal noise of a conductance with a value equal to the transistor transconductance, g_m, which is 5000 micromhos. The received optical pulses are half-duty-cycle rectangular pulses. The desired equalized output pulse is a raised cosine pulse [see Part I, eq. (25)] with parameter $\beta = 1$.

We must first calculate the value of Q which depends only upon the desired error rate. From Part I, Fig. 21, we see that for an error rate of 10^{-9}, $Q = 6$.

Next we must obtain the constants I_2, I_3, γ_1, and γ_2. These depend only upon the input optical pulse shape and the equalized output pulse shape. From (23) and (34) of Part I we obtain

$$I_2 = 0.804046, \qquad I_3 = 0.071966, \qquad \gamma_1 = 21.4106, \qquad \gamma_2 = 1.25424. \tag{3}$$

Using the above data we obtain the thermal noise parameter Z as follows:

$$Z = \left\{ \frac{\overset{T}{4 \times 10^{-8}}}{(1.6 \times \underset{e}{10^{-19}})^2} \left[8.28\, \overset{2k\theta}{10^{-21}} \left(10^{-6} + 10^{-6} + \frac{4 \times 10^{-12}}{5 \times 10^{-3}} \right) \overset{I_2}{0.804046} \right] + \frac{(2\pi \times 10^{-11})^2 \left(\frac{8.28 \times 10^{-21}}{5 \times 10^{-3}} \right) (\overset{I_3}{0.071966})}{(1.6 \times 10^{-19})^2 (4 \times 10^{-8})} \right\} = 4.8027 \times 10^5. \quad (4)$$

From the data we have $\hbar\Omega/\eta = 3.117 \times 10^{-19}$ joules.

We obtain from (1), at unity internal gain (no avalanche), $p_{\text{required}} = 3.25 \times 10^{-8}$ watts $= -44.89$ dBm (no gain).

We obtain from (2), at optimal avalanche gain, $p_{\text{required}} = 1.6409 \times 10^{-9}$ watts $= -57.85$ dBm, $G_{\text{optimal}} = 56.89$.

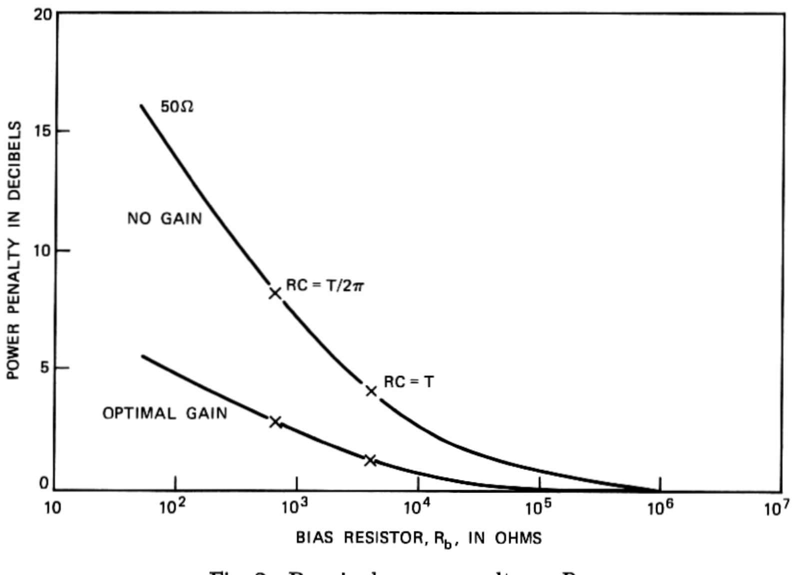

Fig. 2—Required power penalty vs R_b.

We therefore observe that optimal avalanche gain buys a 13-dB reduction in required optical power in this example. Before proceeding, we can check the validity of neglecting dark current. The average number of primary photoelectrons produced by the signal per pulse interval T is the required optical power multiplied by $\eta T/\hbar\Omega$. When shot noise is important (with avalanche gain) this number is 210 primary signal counts per interval T. The number of dark current counts per interval T is the dark current in amperes multiplied by T/e, which in this example is 25 primary dark current counts. Thus, the shot noise due to the dark current is about 10 percent of the signal shot noise. It is therefore a reasonable approximation to neglect this dark current noise. In Section VI we shall calculate precisely the effect of dark current upon the required optical power.

IV. VARYING THE PARAMETERS

In this section, we shall calculate the effect of varying parameter values used in the example of Section III.

4.1 *Biasing Resistor Value*

It was pointed out in Part I that the biasing resistor R_b should be sufficiently large so that the amplifier series noise source S_E dominates in the expression for Z of (1a). This was in fact the case in the example of Section III. We can calculate the penalty in required optical power for using a smaller biasing resistor. This penalty is plotted in Fig. 2 in dB with zero dB being the penalty associated with an infinitely large biasing resistor. The exact penalty of Fig. 2 is applicable with the other relevant parameters (which make up Z) given in the example above. However, the qualitative conclusions are that significantly more optical power is needed if one adheres to the "$RC = T$" design rather than the "large R" (high-impedance) design, in the absence of avalanche gain. Figure 3 shows how the optimal avalanche gain varies when R_b is changed. The qualitative conclusion is that the "$RC = T$" design requires significantly more avalanche gain that the "large R" (high-impedance) design. It should be pointed out that, for lower bit rates and/or a smaller capacitance C_T, the improvement associated with use of a large R_b rather than a value to keep $R_bC_T = T$ is more pronounced.

4.2 *Desired Error Rate*

As mentioned before, the error rate is coupled to the parameter Q in (1) and (2). Figures 4a and 4b show plots of the variation in the re-

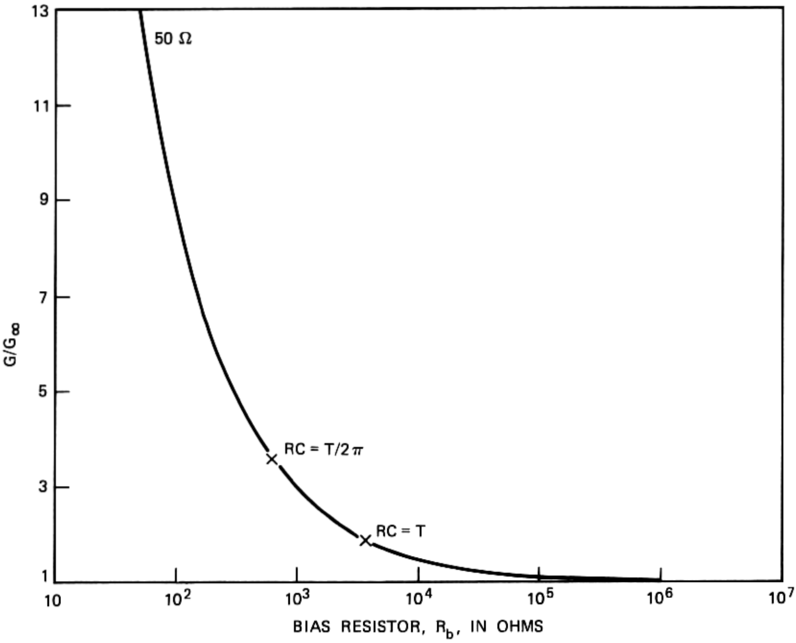

Fig. 3—Optimal gain penalty vs R_b, G_∞ = optimal gain at $R_b = \infty$.

quired power in dB with the desired error rate without gain and with optimal gain. The absolute power in dBm is only applicable to the example of Section III above. However, the difference in required power in dB between any two error rates is applicable in general, as should be apparent from (1) and (2), provided a silicon detector ($x = 0.5$) is being used.

4.3 *Bit Rate (1/T)*

As mentioned before, the pulse spacing T is scaled out of I_2, I_3, γ_1, and γ_2. These numbers depend only upon the input and output pulse *shapes* (e.g., half-duty-cycle rectangular input pulse, raised-cosine equalized output pulse). Therefore, the effect of the parameter T is explicitly given in (1) and (2) without any hidden dependencies. (This of course assumes that the input pulse shape is not limited by dispersion in the transmission medium and can therefore be held to a half-duty-cycle rectangle.) If we assume that the high-impedance design is being used and that this dominance of the term proportional to $1/T$ in Z of (1a) can be maintained as the bit rate is varied (becomes difficult

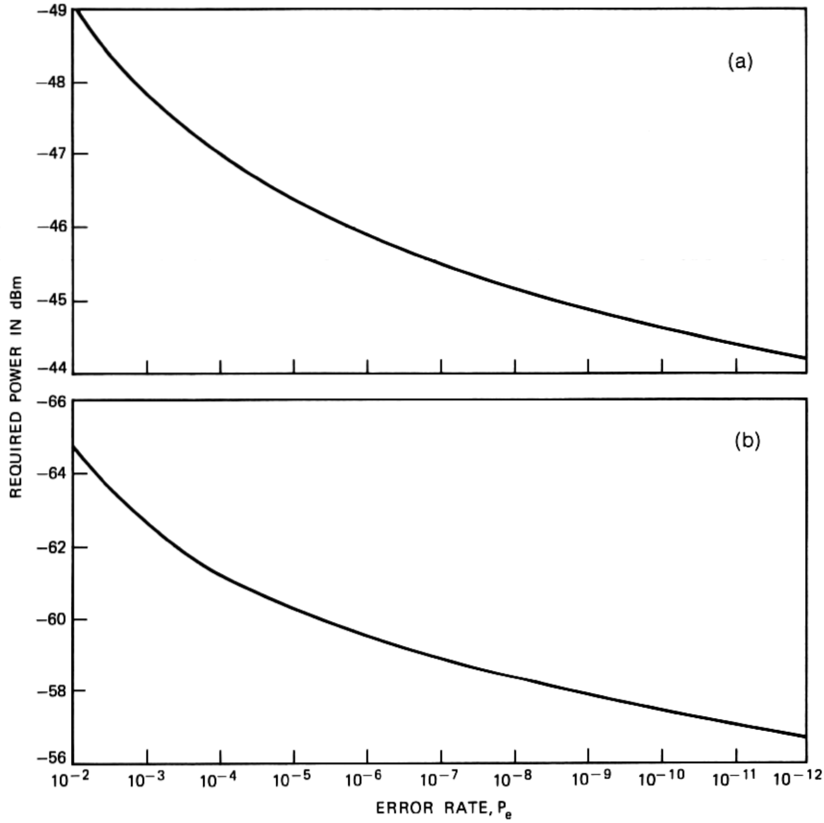

Fig. 4—(a) Required power vs error rate (no avalanche gain). (b) Required power vs error rate (optimal gain).

at low bit rates), then we have the following dependence of the required optical power upon the bit rate $1/T$ without gain and with optimal gain:

$$p_{\text{required}} \propto T^{-3/2} \quad \text{(no gain)} \quad \text{(4.5 dB/octave of bit rate)} \tag{5a}$$

$$p_{\text{required}} \propto T^{-7/6} \quad \text{(optimal silicon gain)} \quad \text{(3.5 dB/octave of bit rate)} \tag{5b}$$

$$G_{\text{optimal}} \propto T^{-1/3} \quad \text{(1 dB/octave of bit rate).}$$

One should be careful extrapolating (5a) and (5b) to very low bit rates. First, the shot noise is no longer negligible compared to the thermal noise at bit rates where the optimal gain is low. Thus (5a) loses validity at very low bit rates. Further, (5b) is only valid for optimal gains greater than unity. Near unity optimal gain, the silicon excess noise factor departs from $G^{.5}$. In addition, at low bit rates, dark current may not be negligible. It is reasonable to use (5a) and (5b) to extrapolate the results of the example in Section III to bit rates between 5 and 300 Mb/s.

V. THE EFFECT OF IMPERFECT MODULATION

The above formulas (1) and (2) assume that there is perfect modulation. That is, it was assumed that each optical pulse is either completely on or completely off. In this section we shall investigate two versions of imperfect modulation.

Case 1: Pulses Not Completely Extinguished

This case is illustrated in Fig. 5. In each time slot the optical pulse is either completely or partly on. The partly on pulse has the same shape as a completely on pulse, but has area EXT times the area of a completely on pulse. This may correspond to an externally modulated mode-locked laser source. Thus the ratio of the power received when a sequence of all "off" pulses is transmitted to the power received when a sequence of all "on" pulses is transmitted is EXT. Using the results of Part I eqs. (23) and (34), we obtain the following power requirements which are modifications of (1) and (2) above:

$$p_{\text{required}} = \left(\frac{1 + EXT}{1 - EXT}\right) \frac{QZ^{\frac{1}{2}}}{GT} \frac{\hbar\Omega}{\eta}, \qquad \text{(Thermal Noise Dominates)} \quad (6)$$

$$p_{\text{required}} = \frac{1 + EXT}{2T} \left(\frac{Q}{1 - EXT}\right)^{(2+x)/(1+x)} \times \left[(Z)^{x/(2+2x)} (\gamma_1')^{x/(2+2x)} (\gamma_2')^{(2+x)/(1+x)}\right] \frac{\hbar\Omega}{\eta}, \qquad \text{(Optimal Gain)} \quad (7)$$

where defining from Part I (23) and (34)

$$I_6 = \textstyle\sum_1 - (1 - EXT) I_1$$

we have

$$\gamma_1' = \frac{-\left(\sum_1 + I_6\right) + \sqrt{\left(\sum_1 + I_6\right)^2 + \dfrac{16(1+x)}{x^2} \sum_1 I_6}}{2 \sum_1 I_6}$$

$$\gamma_2' = \sqrt{1/\gamma_1' + \textstyle\sum_1} + \sqrt{1/\gamma_1' + I_6}.$$

[Compare (6) and (7) to (1) and (2).]

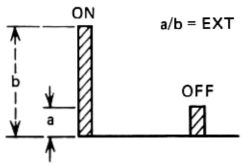

Fig. 5—Imperfect modulation, pulses not completely extinguished.

Using (1), (2), (7), and (8), we can calculate the extra required optical power due to a nonzero value of EXT with and without avalanche gain. When avalanche gain is being used, this power penalty depends upon the input and output pulse shapes. We plot in Fig. 6 the power penalty vs EXT, assuming the pulse shapes of the example in Section III above for the avalanche gain case.

Case 2: Pulses on a Pedestal

This case is illustrated in Fig. 7. The received optical pulses arrive on a pedestal, which may correspond to inability to completely extinguish the light from a modulated source which is not in a pulsing (mode-locked) condition. We set the ratio of average received optical power when all pulses are "off" to average received optical power when all

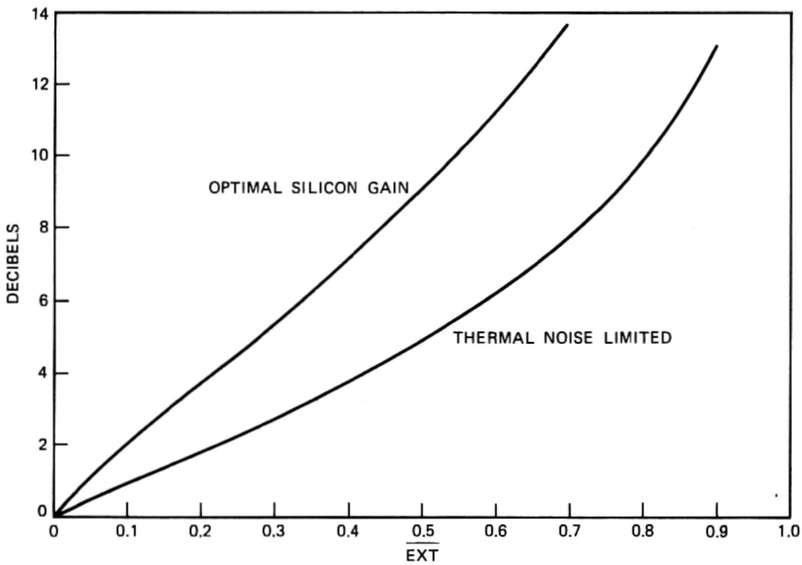

Fig. 6—EXT penalty (dB) vs EXT.

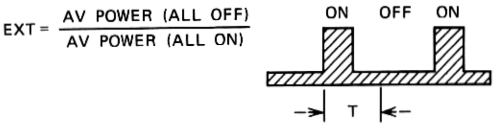

Fig. 7—Imperfect modulation, pulses on pedestal.

pulses are "on" to be EXT in analogy to Case 1 above. This ratio will remain fixed if the pulse changes in propagation from transmitter to receiver. Using the results of Part I we obtain the following formulas for the required optical power:

$$p_{\text{required}} = \frac{1 + EXT}{1 - EXT} \frac{QZ^{\frac{1}{2}}}{GT} \frac{\hbar\Omega}{\eta}, \qquad \text{(Thermal Noise Dominates)} \qquad (8)$$

$$p_{\text{required}} = \frac{1 + EXT}{1 - EXT} \frac{(Q)^{(2+x)/(1+x)}}{2T}$$
$$\times \left[(Z)^{x/(2+2x)}(\gamma_1'')^{x/(2+2x)}(\gamma_2'')^{(2+x)/(1+x)}\right] \frac{\hbar\Omega}{\eta}, \qquad \text{(Optimal Gain)} \qquad (9)$$

where defining from Part I (23) and (34)

$$\Sigma_1' = \Sigma_1 + \left(\frac{EXT}{1 - EXT}\right) I_2$$

$$I_7 = \Sigma_1 - I_1 + \left(\frac{EXT}{1 - EXT}\right) I_2$$

we have

$$\gamma_1'' = \frac{-(\Sigma_1' + I_7) + \sqrt{(\Sigma_1' + I_7)^2 + \dfrac{16(1+x)}{x^2}(\Sigma_1')I_7}}{2(\Sigma_1')I_7}$$

$$\gamma_2'' = \sqrt{1/\gamma_1'' + \Sigma_1'} + \sqrt{1/\gamma_1'' + I_7}.$$

Once again we can use (1), (2), (8), and (9) to calculate the penalty for nonzero extinction. This penalty is plotted in Fig. 8 vs EXT where we assume the input and output pulse shapes of the example in Section III when there is optimal avalanche gain.

VI. THE EFFECT OF DARK CURRENT

In order to allow for dark current, we must solve the following set of simultaneous equations which treat the dark current as an equivalent pedestal-type nonzero extinction. (When thermal noise dominates, dark current is either negligible or its shot noise can be added trivially to the amplifier parallel current noise source S_I.)

$$p_{\text{required}} = \frac{Q^{(2+x)/(1+x)}}{2T} (Z)^{x/(2+2x)} (\gamma_1''')^{x/(2+2x)} (\gamma_2''')^{(2+x)/(1+x)} \frac{\hbar\Omega}{\eta}, \quad \text{(At optimal avalanche gain)} \tag{10}$$

where defining from Part I (23) and (34)

$$\Sigma_1'' = \Sigma_1 + \delta I_2$$
$$I_8 = \Sigma_1 - I_1 + \delta I_2$$

we have

$$\gamma_1''' = \frac{-(\Sigma_1'' + I_8) + \sqrt{(\Sigma_1'' + I_8)^2 + \frac{16(1+x)}{x^2} \Sigma_1'' I_8}}{2 \Sigma_1'' I_8}$$

$$\gamma_2''' = \sqrt{1/\gamma_1''' + \Sigma_1''} + \sqrt{1/\gamma_1''' + I_8}$$

$$i_d = (2 p_{\text{required}}) \frac{\eta e \delta}{\hbar\Omega}, \tag{11}$$

where i_d = primary dark current in amperes.

There are various ways to solve (10) and (11) simultaneously. One way is to solve (10) first with $\delta = 0$ for p_{required}. Then one can solve

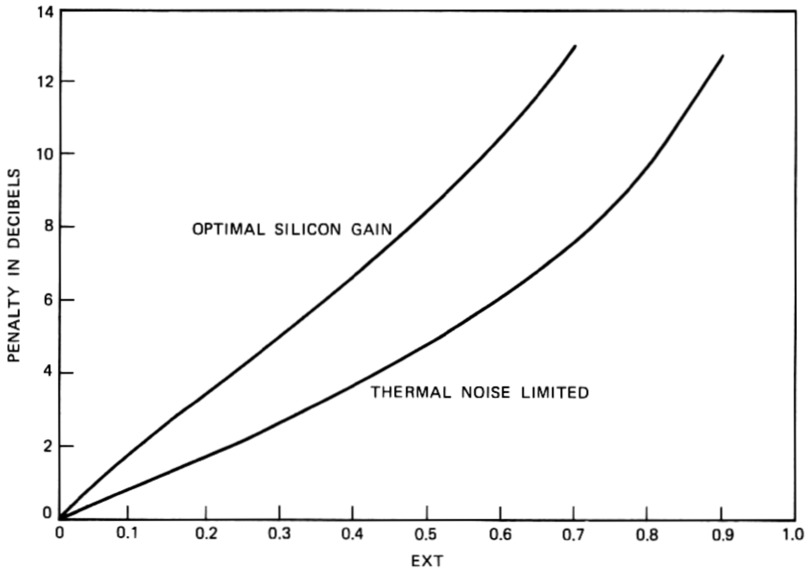

Fig. 8—Power penalty vs EXT.

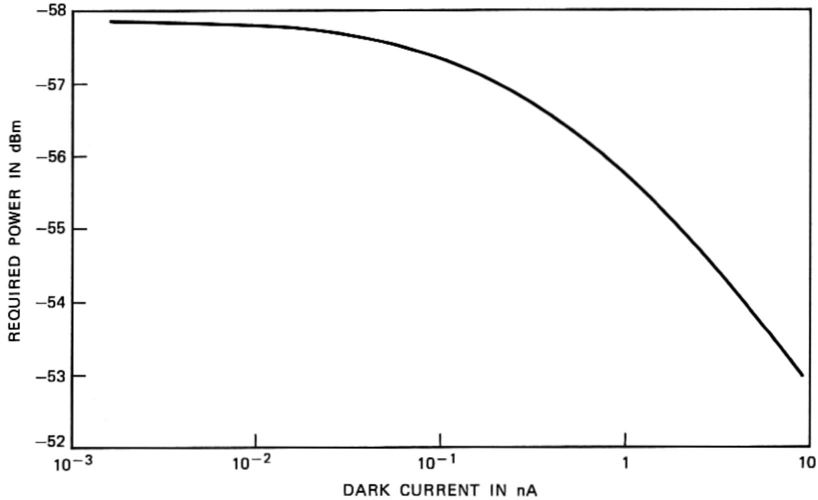

Fig. 9—Required power vs dark current.

(11) for a new value of δ. One then resolves (10) and then (11), etc.–repeating the iterations until satisfactory convergence is obtained. Figure 9 shows a plot of the required power in dBm for the example in Section III vs dark current in nanoamperes. We see that a dark current of 100 picoamperes results in an optical power requirement which is about 0.5 dB more than that which would be required with zero dark current. Thus it was reasonable to neglect dark current when calculating the required power in Section III. Dark current will result in even less of a penalty at higher bit rates. Although the curve of Fig. 9 is applied to the specific example of Section III, it is apparent in general that, at bit rates above a few megabits per second and with primary dark currents less than 0.1 nanoampere, dark current will have a small effect upon the required optical power.