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Statistical Behavior of Rain Attenuation*

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Thirty-one sets of experimental data on the statistics of microwave rain attenuation at frequencies above 10 GHz, in the U.S.A., England, Japan, Italy, and Canada, indicate that: (i) the distribution of rain attenuation α, *in dB, is approximately lognormal with a standard deviation* σ_a *of* $\log_{10}\alpha$ *ranging from 0.46 to 0.71 for earth-space paths, and from 0.33 to 0.86 for terrestrial paths; (ii) the distribution of the rain fade duration* τ *is also approximately lognormal with a standard deviation* σ_τ *of* $\log_{10}\tau$ *ranging from 0.44 to 0.76 for both earth-space paths and terrestrial paths. We propose a theory to explain this general behavior. A theoretical upper bound for the fade duration distribution in the tail region is also given.*

The findings in this paper simplify the determination of rain attenuation statistics needed for the design of earth-satellite radio links and terrestrial radio links.

I. INTRODUCTION

The statistics of rain attenuation are important for the design of both terrestrial and earth-satellite radio links using frequencies above 10 GHz. Many experiments[1–33] have been performed to obtain data on rain attenuation for different frequencies, path lengths, and geographical locations. This paper presents the behavior of rain attenuation statistics found in our study to be common to the available experimental data.†

II. DEFINITIONS

Let

$V(t)$ be the time-varying amplitude of the received signal voltage normalized to its nonfaded level,

* An excerpt of this paper has been presented at the 1972 IEEE International Conference on Communications at Philadelphia, and included in the Proceedings of the Conference.

† These data are actual measured microwave rain attenuations and not the approximate attenuations calculated from rain rate data.

$\alpha(t) = -20 \log_{10} V(t)$ be the time-varying rain attenuation in dB,

$P(\alpha \geqq A)$ be the expected fraction of time that $\alpha(t)$ exceeds any specified value A,

P_o be the expected fraction of time that rain falls at the location of the radio link,

$P_c(\alpha \geqq A)$ be the expected fraction of raining time that α exceeds A,

α_m be the median value of α during the raining time, i.e., $P_c(\alpha \geqq \alpha_m) = 0.5$,

$\tau(A)$ be the duration of rain attenuation fades with α exceeding any specified threshold A in dB,

$P[\tau(A) \geqq b]$ be the probability that the fade duration $\tau(A)$ exceeds any specified duration b, and

$\bar{\tau}(A)$ be the average duration.

Notice that

$$P(\alpha \geqq A) = P_o \cdot P_c(\alpha \geqq A). \tag{1}$$

Thus, P_o may also be called the probability of rainfall and $P_c(\alpha \geqq A)$ the conditional distribution of α under the condition that the rain is falling.

III. SUMMARY OF RESULTS

3.1 *Attenuation Distribution*

(*i*) The available experimental data on both earth-space paths and terrestrial paths in the U.S.A., England, Japan, Italy, and Canada show consistently that the conditional distribution, $P_c(\alpha \geqq A)$, is approximately lognormal within the attenuation range, $1 \text{ dB} \leqq \alpha \leqq 50 \text{ dB}$, of practical interest.*

(*ii*) We propose a theory to explain the lognormal behavior of attenuation $\alpha(t)$. In essence, the value of the rain attenuation, $\alpha(t)$, at any time instant can be multiplicatively affected by a large number of random time-varying parameters of the environment such as the present states and the past histories of the weather conditions at various locations all over the world. The large number of random multiplicative components and the central limit theorem lead to the lognormal distribution of $\alpha(t)$.

* This means the distribution of signal amplitude $V(t)$ is approximately log-lognormal.

(*iii*) These results indicate that three parameters, P_o, σ_a (the standard deviation), and α_m (the median), are sufficient to determine the rain attenuation distribution $P(\alpha \geqq A)$. These parameters depend on geographic locations.

(*iv*) α_m increases almost linearly with the path length because the median rain rate usually is small and is almost uniform over the entire path.

(*v*) The dependence of α_m on frequency follows the theoretical prediction of Sezter[20] for small rain rate because the median rain rate is usually small. The rain rate data in Refs. 13 and 34 to 37 indicate that the median rain rate ranges from 0.5 to 10 mm/h, depending on location.

(*vi*) σ_a decreases slightly as the path length increases because of the averaging effect of the propagation volume.[24,25]

(*vii*) σ_a decreases slightly as frequency increases.

3.2 *Fade Duration Distribution*

(*i*) The experimental data indicate that the fade duration distributions, $P[\tau(A) \geqq b]$, are also approximately lognormal. The physical reason for the lognormal duration is the same as that for the lognormal attenuation.

(*ii*) The probability of occurrence of long fade durations has an absolute upper bound given by

$$P\left(\frac{\tau}{\bar{\tau}} \geqq X\right) \leqq \frac{1}{2}\,\mathrm{erfc}\left(\frac{\log_{10} X}{M}\right)^{\frac{1}{2}} \qquad (2)^*$$

for any $X \geqq 1$. For example, $P(\tau/\bar{\tau} \geqq 10) \leqq 0.0161$ means that no more than 1.61 percent of the total number of fades will have durations longer than $10 \cdot \bar{\tau}$.

IV. EXPERIMENTAL DATA

4.1 *Rain Attenuation Distribution*

Thirty-one sets of experimental data on rain attenuation distribution, each with a time base of six months or longer, are summarized in Tables I, II, and III for earth-space paths, long terrestrial paths (> 10 km), and short terrestrial paths ($\leqq$ 10 km), respectively.†

* The constant M is defined in eq. (13).

† In the literature, there are many other sets of experimental data with a time base less than six months. Those data are not included because the short-term distributions of rain attenuation are fairly random.

TABLE I—RAIN ATTENUATION EXPERIMENTS ON EARTH–SPACE PATHS

Authors	Reference Number	Location	Frequency (GHz)	Path Length (km)	P_o	σ_a	α_m (dB)	Time Base
Wilson and Ruscio	4, 5, 17	Crawford Hill, N. J.	30	Earth-Space	5.23×10^{-2}	0.57	1.6	Dec. 8, 1967–Feb. 28, 1969
Wilson and Ruscio	4, 5, 17	Crawford Hill, N. J.	16	Earth-Space	5.23×10^{-2}	0.71	0.33	Dec. 8, 1967–Feb. 28, 1969
Wilson	15	Crawford Hill, N. J.	16	Earth-Space	5.9×10^{-2}	0.67	0.3	April 1, 1969–Aug. 7, 1969
Wilson	15	Sayreville, N. J.	16	Earth-Space	5.9×10^{-2}	0.60	0.48	April 1, 1969–Aug. 7, 1969
Wilson	15	Parkway, N. J.	16	Earth-Space	5.9×10^{-2}	0.70	0.38	April 1, 1969–Aug. 7, 1969
Davies	31	England	19	Earth-Space	8.5×10^{-2}	0.46	0.4	June 1968–May 1970

Note: This table does not include the results of an earth-space path at Point Reyes (30 miles north of San Francisco), California by K. O'Brien because the rain attenuation exceeded 8 dB only once (for 8 minutes) during the 9-month period of the experiment at that location.

TABLE II—RAIN ATTENUATION EXPERIMENTS ON LONG TERRESTRIAL PATHS

Authors	Reference Number	Location	Frequency (GHz)	Path Length (km)	P_o	σ_a	α_m (dB)	Time Base
Hathaway and Evans	7	Mobile, Ala.	11	43.5	3.7×10^{-2}	0.40	7.1	1956
Zimmerman	8	Mobile, Ala.	17	22.9	3.7×10^{-2}	0.46	6.4	Aug. 1, 1958–Aug. 31, 1959
Zimmerman	8	Mobile, Ala.	17	14	3.7×10^{-2}	0.49	4	Aug. 1, 1958–Aug. 31, 1959
Funakawa, et al.	11	Near Tokyo, Japan	12.62	80	10×10^{-2}	0.42	3.1	Nov. 1964–Oct. 1965
Turner, Easterbrock, et al.	12, 13, 14	Southern England	11	24	8.5×10^{-2}	0.51	0.21	Jan. 1967–Dec. 1968
Turner, Easterbrock, et al.	12, 13, 14	Southern England	18	24	8.5×10^{-2}	0.51	0.55	Jan. 1967–Dec. 1968
Turner, Easterbrock, et al.	12, 13, 14	Southern England	36	24	8.5×10^{-2}	0.52	2.3	Jan. 1967–Dec. 1968
Turner, Easterbrock, et al.	12, 13, 14	Southern England	11	24	6.4×10^{-2}	0.40	0.76	1964
Turner, Easterbrock, et al.	12, 13, 14	Southern England	11	30.6	6.4×10^{-2}	0.34	1.2	1963 and 1964
Turner, Easterbrock, et al.	12, 13, 14	Southern England	11	38.62	6.4×10^{-2}	0.38	1.0	1964
Turner, Easterbrock, et al.	12, 13, 14	Southern England	11	58	6.4×10^{-2}	0.37	1.4	1964
Blevis, et al.	6	Ottawa, Canada	15	15.78	5.8×10^{-2}	0.67	0.4	May–Oct. 1965
Stracca	33	Italy	18	20	6×10^{-2}	0.56	1.82	May–Oct. 1967
Stracca	33	Italy	11	20	6×10^{-2}	0.71	0.45	May–Oct. 1967

TABLE III—RAIN ATTENUATION EXPERIMENTS ON SHORT TERRESTRIAL PATHS

Authors	Reference Number	Frequency (GHz)	Path Length (km)	Location	Time Base	σ_a	α_m (dB)	P_o
Barnett and Bergmann	44	17.83	5.23	Palmetto, Georgia	Nov. 1970–June 1971	0.57	0.55	5.7×10^{-2}
Barnett and Bergmann	44	17.71	5.07	Palmetto, Georgia	Nov. 1970–June 1971	0.6	0.5	5.7×10^{-2}
Barnett and Bergmann	44	17.95	5.07	Palmetto, Georgia	Nov. 1970–June 1971	0.57	0.55	5.7×10^{-2}
Semplak	2	18.5	2.6	Crawford Hill, N. J.	Jan. 1–Sept. 25, 1970	0.86	0.074	4.03×10^{-2}
Semplak	2	30.9	2.6	Crawford Hill, N. J.	Jan. 1–Sept. 25, 1970	0.78	0.24	4.03×10^{-2}
Semplak	1	18.5	6.4	Crawford Hill, N. J.	1967 + 1968 + 1969	0.86	0.11	5.08×10^{-2}
Semplak	1	30.9	1.9	Crawford Hill, N. J.	1968 + 1969	0.85	0.11	5.55×10^{-2}
Delange and Dietrich	30	60	1.03	Crawford Hill, N. J.	1970	0.47	0.94	4.15×10^{-2}
Gray	3	100	0.61	Crawford Hill, N. J.	1970	0.55	0.46	4.15×10^{-2}
Kenny	16	18.4	4.3	Merrimack Valley, Mass.	April 1970–July 1971	0.44	1.1	4×10^{-2}
Hickin	9	18	10	England	March 1964–Feb. 1966	0.85	0.045	6.6×10^{-2}

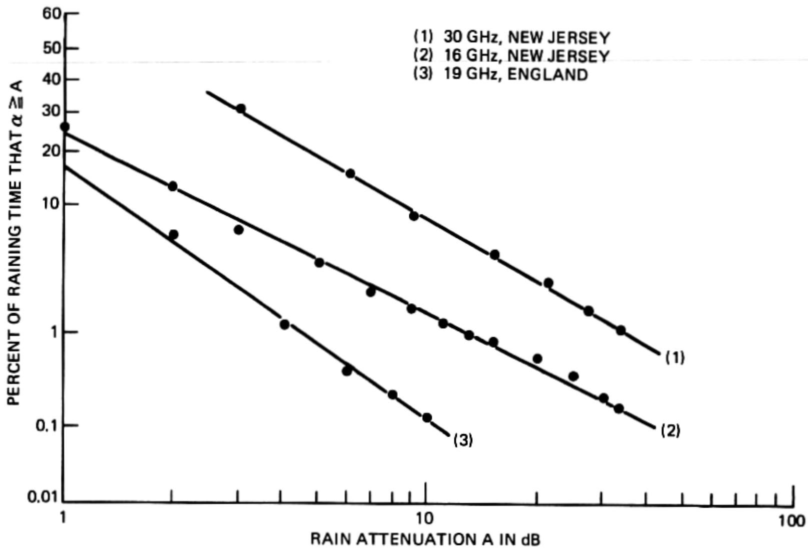

Fig. 1—Lognormal distributions of rain attenuation on earth-space paths.

The original data are all given in terms of the unconditional distribution $P(\alpha \geqq A)$. Based upon P_o (either available or estimated), we convert these unconditional distributions into the conditional distribution $P_c(\alpha \geqq A)$ by eq. (1). This conversion is done to exclude the dry periods in which the rain attenuation is identically zero.*

When plotted on a lognormal coordinate system, these conditional distributions, $P_c(\alpha \geqq A)$, are all approximately straight lines within the attenuation range, $1 \text{ dB} \leqq \alpha \leqq 50 \text{ dB}$, of practical interest. Figures 1 to 3 show nine examples. The equation describing the lognormal distribution is

$$P_c(\alpha \geqq A) = \tfrac{1}{2} \operatorname{erfc}\left[\frac{\log_{10} A - \mu_a}{\sqrt{2}\sigma_a}\right] \tag{3}$$

where erfc $(\sim)$ denotes the complementary error function; σ_a is the standard deviation of $\log_{10} \alpha$ during the raining time; and

$$\mu_a = \log_{10} \alpha_m \tag{4}$$

is the mean value of $\log_{10} \alpha$ during the raining time. The estimated values of σ_a and α_m are given in Tables I, II, and III.

However, the accuracy of these estimated values of σ_a and α_m is

* Absorption by the clear atmosphere is not considered in this paper.

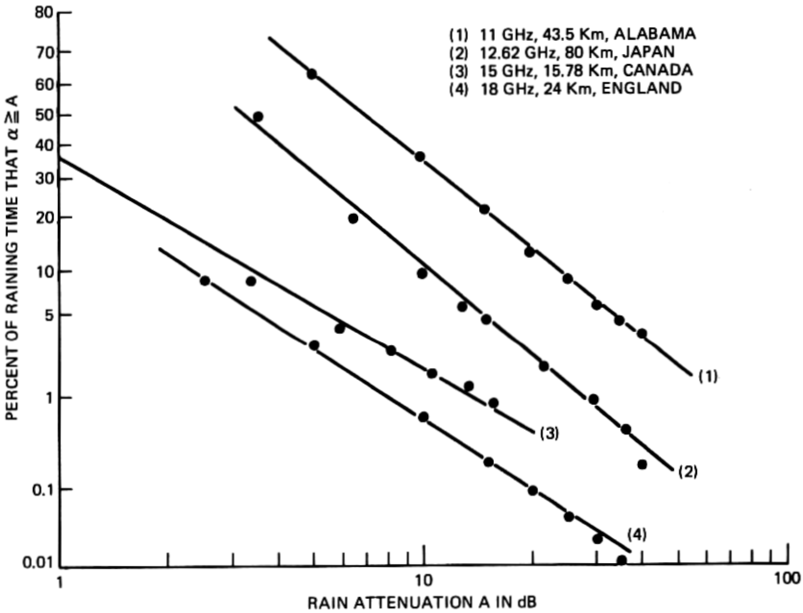

Fig. 2—Lognormal distributions of rain attenuation on long terrestrial paths.

limited by two problems:

(*i*) The time base of the experiment may not be sufficiently long to yield stable statistics, and

(*ii*) some of the published experimental data do not provide the probability P_o of rain during the experiment.*

Let

$$\beta(t) = \frac{[\log_{10} \alpha(t)] - \mu_a}{\sigma_a}. \tag{5}$$

If α is lognormally distributed, then β will be normally distributed with zero mean and unity standard deviation. This allows us to pool all the available data of β on the same graph paper for comparison. Figures 4, 5, and 6 show the pooled data of the earth-space paths, long terrestrial paths, and short terrestrial paths, respectively. It is seen that these experimental results of β are indeed normally distributed with zero mean and unity standard deviation.

* The P_o values at Crawford Hill, New Jersey, are provided by D. C. Hogg and R. A. Desmond from their rain gauge records. The P_o values in Alabama, England, Japan, and Canada are estimated from the information in Refs. 6, 8, 11, and 13. The P_o values in Italy and Massachusetts are assumed values using some judgment.

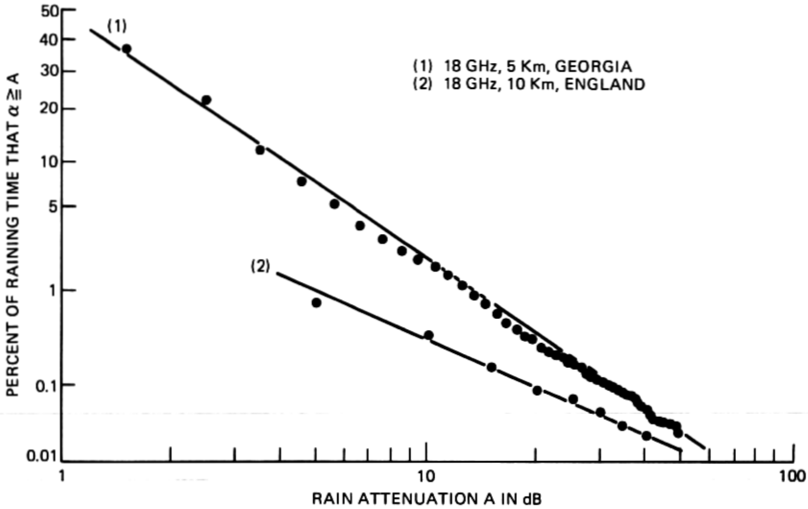

Fig. 3—Lognormal distributions of rain attenuation on short terrestrial paths.

However, Fig. 6 shows that the data of short paths have significant deviations from the normal distribution in the tail region ($\beta \geqq 2.8$). Furthermore, most of the deviations in the tail region are downward from the straight line approximation. The reason for this nonsymmetric deviation is discussed in Appendix B.

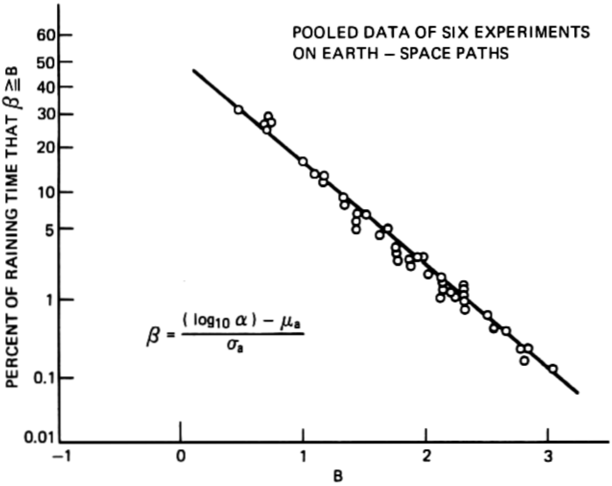

Fig. 4—Normal distribution of β of earth-space paths.

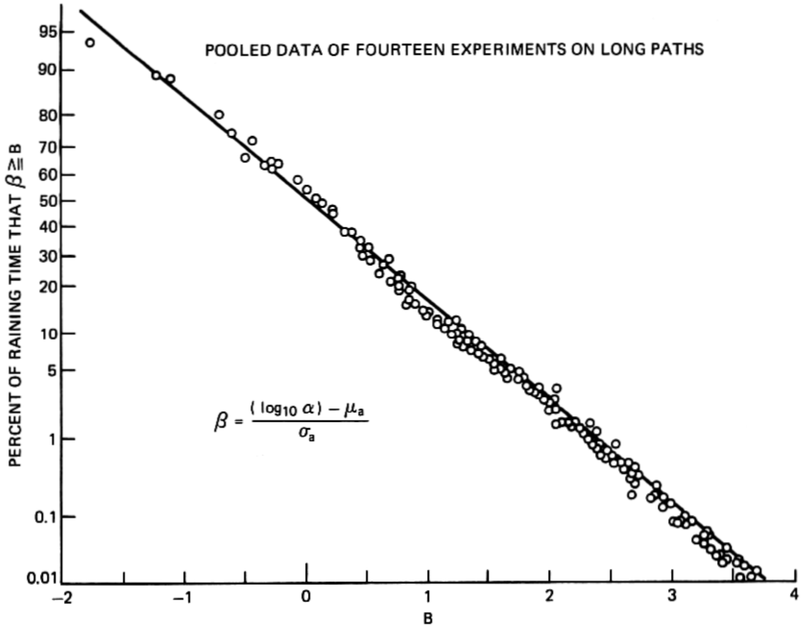

Fig. 5—Normal distribution of β of long terrestrial paths.

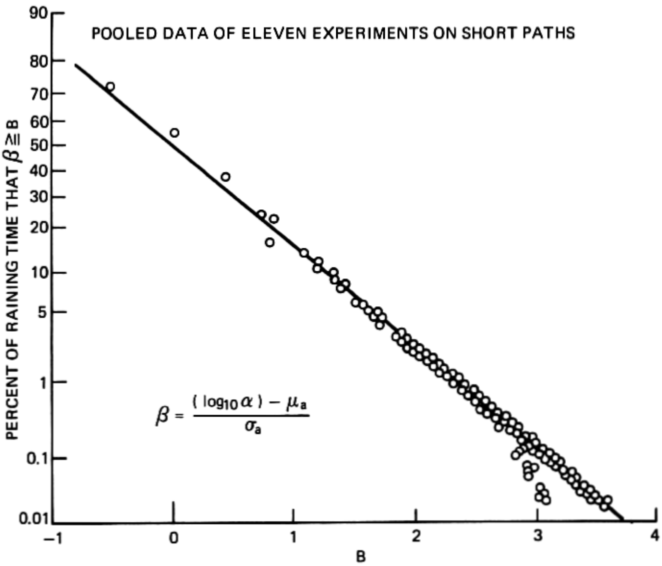

Fig. 6—Normal distribution of β of short terrestrial paths.

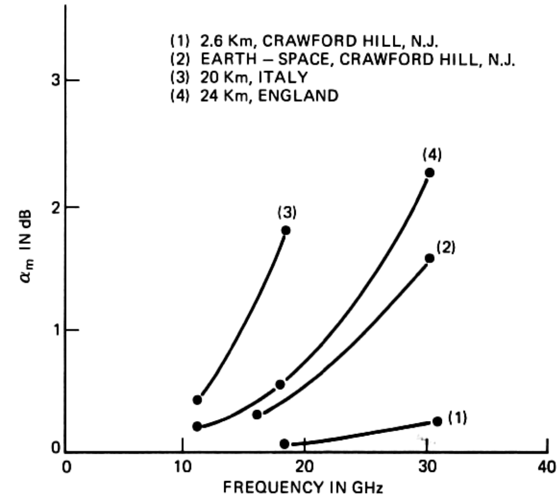

Fig. 7—Effect of frequency on median attenuation α_m.

4.2 *Dependence of σ_a and α_m on Path Length and Frequency*

The experimental results, discussed in Section 4.1, show that the three parameters, P_o, σ_a, and α_m are sufficient to determine the distribution $P(\alpha \geqq A)$. Therefore, it is important to study the dependence of σ_a and α_m on path length and frequency.

The effects of path length and frequency on σ_a and α_m are shown in Figs. 7 to 10. These experimental results indicate the effects of path

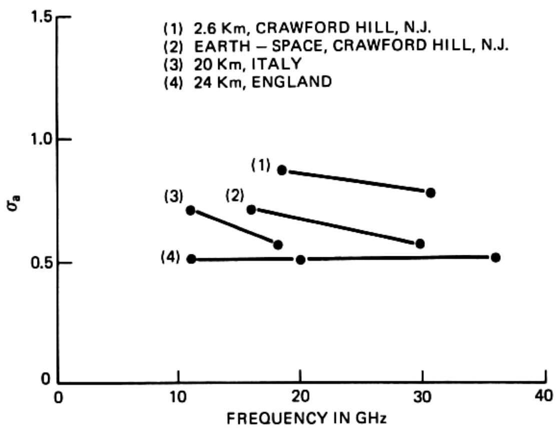

Fig. 8—Effect of frequency on standard deviation σ_a.

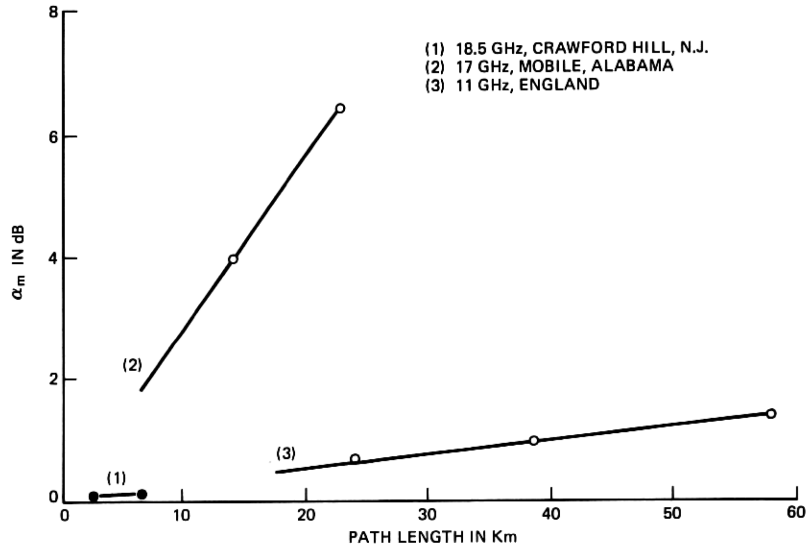

Fig. 9—Effect of path length on median attenuation α_m.

length and frequency on σ_a and α_m as stated in terms (*iv*) to (*vii*) of Section 3.1.*

The theoretical calculation[20] indicates that the increase of rain attenuation with frequency f is slightly faster than the square law in the range: 10 GHz $\leqq f \leqq$ 60 GHz, and 1 mm/h $\leqq$ rain rate $\leqq$ 15 mm/h. This is the basis of the dashed curves in Fig. 7. The slopes of straight lines in Fig. 9 are proportional to median rain rate and extinction coefficient. Curves (1) and (2) in Fig. 9 indicate that the median rain rate in Alabama is much larger than that in New Jersey.

4.3 *Fade Duration Distribution*

The available nine sets of experimental data on the histogram of the durations $\tau(A)$ of rain attenuation fades are summarized in Table IV.† We convert these histograms into the cumulative distribution $P[\tau(A) \geqq b]$. On a lognormal coordinate system, these fade duration distributions are all approximately straight lines. Figure 11 shows two examples.

* In Fig. 8, the σ in England seems to increase slightly with frequency in contrast to those in the U.S.A. and Italy. A possible reason for this inconsistency is that the time bases for the three sets of data for 11, 18, and 36 GHz measured in England are not concurrent.

† Some of the available data in the literature are not included because of short time base.

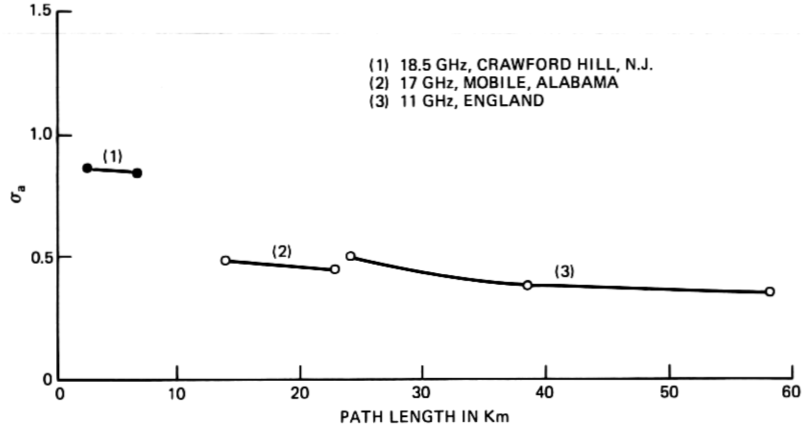

Fig. 10—Effect of path length on standard deviation σ_a.

The equation describing the lognormal distribution of $\tau(A)$ is

$$P[\tau(A) \geqq b] = \tfrac{1}{2}\,\mathrm{erfc}\left[\frac{\log_{10} b - \mu_\tau}{\sqrt{2}\sigma_\tau}\right] \tag{6}$$

where μ_τ and σ_τ are the mean and the standard deviation respectively of $\log_{10} \tau(A)$. The estimated values of σ_τ and the average fade duration $\bar{\tau}(A)$ are given in Table IV. Again, the unstable statistics, caused by insufficient time bases, limit the accuracy of these estimated values of σ_τ and $\bar{\tau}$.

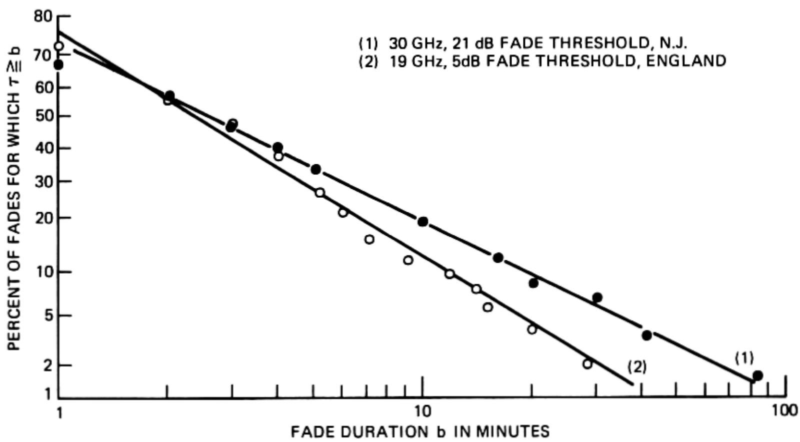

Fig. 11—Lognormal distributions of fade duration.

TABLE IV—EXPERIMENTS ON DISTRIBUTION OF FADE DURATION

Authors	Reference Number	Location	Frequency (GHz)	Path Length (km)	σ_τ	$\bar{\tau}(A)$ (min)	Fade Threshold (A dB)	Time Base
Barnett	44	Palmetto, Georgia	18	5	0.44	2.4	36	November 1970–June 1971
Wilson and Ruscio	4, 5, 17	Crawford Hill, N. J.	16	Earth-Space	0.76	8	9	Dec. 2, 1967–Feb. 28, 1969
Wilson and Ruscio	4, 5, 17	Crawford Hill, N. J.	30	Earth-Space	0.76	11.4	9	Dec. 2, 1967–Feb. 28, 1969
Wilson and Ruscio	4, 5, 17	Crawford Hill, N. J.	30	Earth-Space	0.67	7.7	21	Dec. 2, 1967–Feb. 28, 1969
Davies	31	England	19	Earth-Space	0.53	4.4	5	June 6, 1968–May 31, 1970
Semplak and Turin	18	Crawford Hill, N. J.	18.5	6.4	0.65	7.2	5	June 23, 1967–Oct. 31, 1967
Semplak and Turin	18	Crawford Hill, N. J.	18.5	6.4	0.57	5.8	10	June 23, 1967–Oct. 31, 1967
Semplak and Turin	18	Crawford Hill, N. J.	18.5	6.4	0.63	5.1	20	June 23, 1967–Oct. 31, 1967
Stracca	33	Italy	11	20	0.75	4.8	10	May–Oct 1967

V. THEORY

5.1 *Rain Attenuation Distribution*

Existing theory[19–28] and experimental data[1–19,29–33] indicate that rain attenuation is a complicated function of many parameters of the propagation medium: the total number of rain drops in the path, the drop size distribution, the fine grain spatial characteristics of the rain density along the path, wind velocity, the presence of up or down drafts, raindrop shape, raindrop cant angles, the storm cell shapes and sizes, raindrop temperature, etc. In other words, the attenuation $\alpha(t)$ is a function of many random time-varying parameters of the medium. Furthermore, through the coupling of the atmosphere, the above-mentioned parameters of the propagation medium depend on the present states and the past histories of the weather conditions at many near or faraway locations and altitudes.

We will assume that the rain attenuation can be affected by a large number of random time-varying multiplicative components:

$$\alpha(t) = S_1(t) \cdot S_2(t) \cdot S_3(t) \cdots S_n(t). \tag{7}$$

Each of $\{S_i(t)\}_{i=1}^{i=n}$ represents the random modification factor due to an environmental parameter.

Taking logarithms on both sides of eq. (7) yields

$$\log \alpha = \log S_1 + \log S_2 + \cdots + \log S_n \tag{8}$$

which shows that $\log \alpha$ is a summation of a large number of random variables. Then by the central limit theorem,[38] the distribution of $\log \alpha$ approaches a normal distribution for large n if there is no dominant component. Therefore, the distribution of α is approximately lognormal.*†

The basis for the multiplicative formulation (7) is that the environmental parameters affect the rain attenuation $\alpha(t)$ in a proportional fashion (i.e., in terms of percentage) rather than an additive fashion. For example, at 30 GHz frequency, the theoretical calculation shows that, when the rain temperature decreases from 20°C to 5°C, the rain attenuation increases by approximately 4 percent. This means the

* It is interesting to note that, in Fig. 12 of Ref. 10, the lognormal distribution appears to be a reasonable fit to the probability distribution of rainfall rate. The physical reason for this behavior probably is similar to that for rain attenuation α discussed in this section.

† A basic characteristic of a lognormal random variable y is that its value can vary in the entire semi-infinite range: $0 \leqq y < \infty$. For the rain attenuation problem, the attenuation α(dB) possesses this basic characteristic. On the other hand, the normalized signal amplitude $V(t)$ is strictly confined to the finite range: $0 \leqq V(t) \leqq 1$, which rules out the possibility of a lognormal distribution for $V(t)$.

variation of rain attenuation, due to 15°C variation of rain temperature, can be 4 dB, 0.4 dB, or 0.04 dB if the attenuation at 5°C is 100 dB, 10 dB, or 1 dB, respectively. Similar arguments apply to the effects of other parameters on the attenuation. A more general interpretation of formulation (7) is discussed in Appendix A.

We emphasize that some of the components $\{S_i(t)\}_{i=1}^{i=n}$ may be extremely slowly varying functions, which may take several months or even several years in order to show their effects. For example, the rainfall intensity-duration-frequency data of the Weather Bureau[32,39] show that in New Jersey the return period[40] for a rain rate exceeding 150 mm/h, which continues for a 5-minute duration, is about 5 years. Therefore, if the time base of a rain attenuation experiment is less than 5 years, the chance of missing these rare and extreme events is very high. The justification for considering such a long-term distribution of $\alpha(t)$ is that the microwave radio systems, which are designed based on these statistics, contains many repeaters, each sampling its own rain universe and contributing to total path outage.

5.2 *Fade Duration Distribution*

We also assume that the duration $\tau(A)$ of a rain attenuation fade with $\alpha \geqq A$ is affected by a large number of random time varying multiplicative components

$$\tau(A) = X_1 \cdot X_2 \cdot X_3 \cdots X_n. \tag{9}$$

Each of $\{X_i\}_{i=1}^{i=n}$ represents the random modification factor of an environmental parameter. Therefore, the long-term distribution of $\tau(A)$ is also approximately lognormal.

5.3 *Upper Bound for Fade Duration Distribution*

In radio system design, one is concerned with the occurrence probability of an unusually long continuous outage. It is desirable to have a "quick estimate" of the fade duration distribution, especially in the tail region of long duration. In our experimental and theoretical study[41–43] of the lognormal distributions of durations of rain and multipath fading, W. T. Barnett[41] has found an upper bound on the fade duration distribution as discussed in the following.

Let*

$$x = \frac{\tau}{\bar{\tau}}. \tag{10}$$

* The idea of normalizing fade duration τ to the average duration $\bar{\tau}$ originates from the work[48] of S. O. Rice on fade duration distributions.

The lognormal distribution of x can be written as

$$P(x \geqq X) = \tfrac{1}{2} \operatorname{erfc}\left[\frac{\log_{10} X - \mu}{\sqrt{2}\sigma}\right] \tag{11}$$

where μ and σ are the mean and the standard deviation, respectively, of $\log_{10} x$. For lognormally distributed x, it is easily shown that

$$\bar{x} = e^{\mu/M + \frac{1}{2}(\sigma^2/M^2)} \tag{12}$$

where $\bar{x}$ is the mean value of x, and

$$M = \log_{10} e \cong 0.434. \tag{13}$$

The definition (10) implies that

$$\bar{x} = \frac{\bar{\tau}}{\bar{\tau}} = 1. \tag{14}$$

Equations (12) and (14) show that

$$\mu = -\frac{1}{2}\frac{\sigma^2}{M}. \tag{15}$$

Substituting (15) into (11) yields

$$P(x \geqq X) = \tfrac{1}{2} \operatorname{erfc}\left[\frac{\log_{10} X + \sigma^2/2M}{\sqrt{2}\sigma}\right]. \tag{16}$$

Therefore, the lognormal distribution of x is completely determined by only one parameter σ.

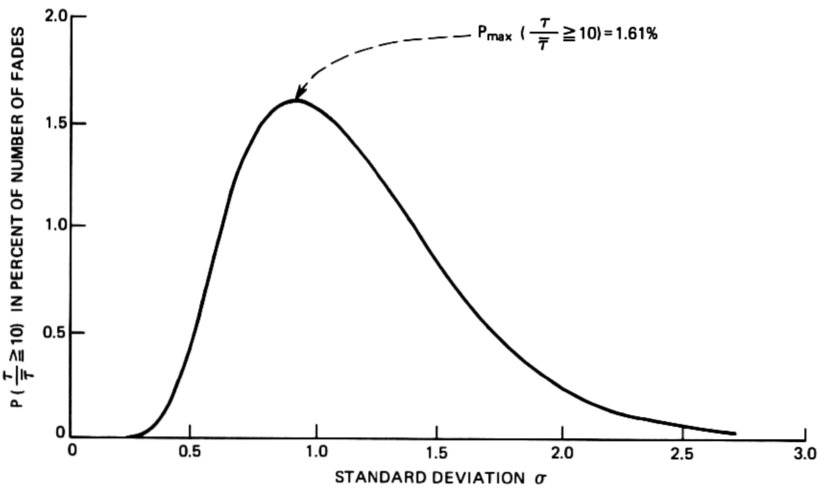

Fig. 12—Effect of standard deviation σ on the probability of long duration of fade.

By differentiating (16) with respect to σ, it is easily shown that

$$\left.\begin{aligned} \frac{dP}{d\sigma} &= 0 \\ \frac{d^2P}{d^2\sigma} &< 0 \end{aligned}\right\} \quad \text{at } \sigma = \sqrt{2M \log_{10} X} \tag{17}$$

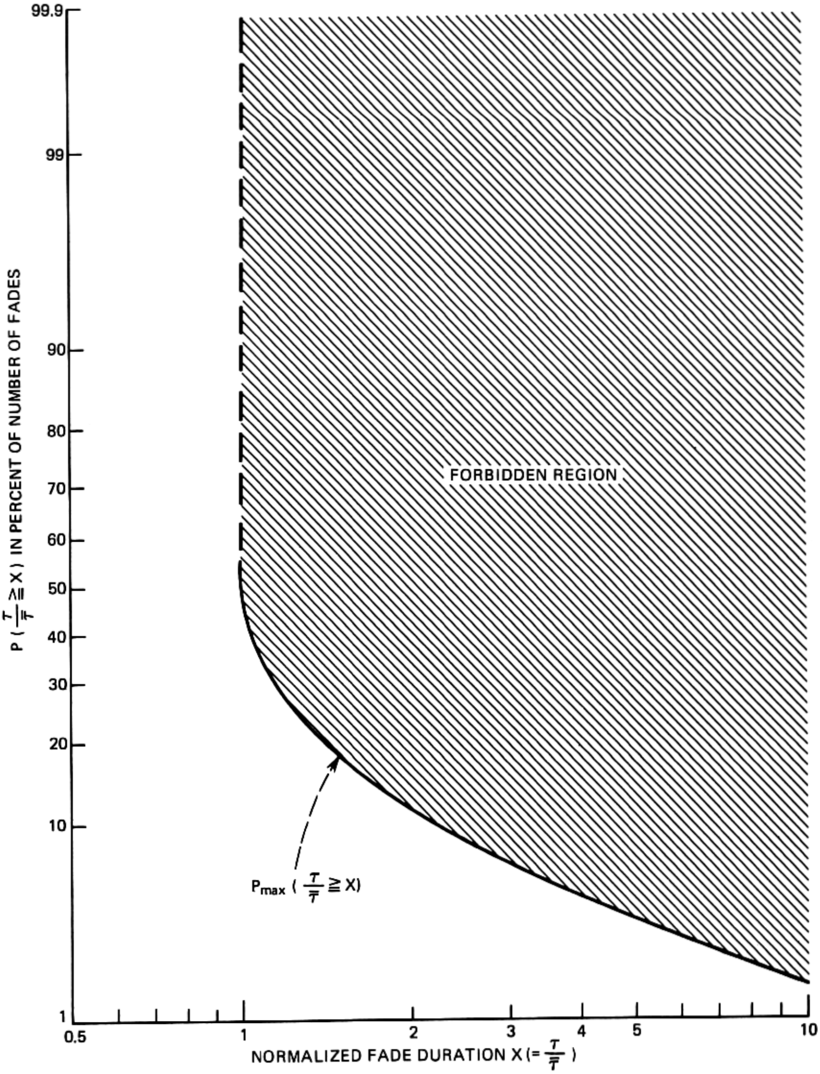

Fig. 13—Maximum probability for various normalized fade durations.

which implies that for any $X \geqq 1$ the probability $P(x \geqq X)$ as a function of σ has a maximum value:

$$P_{\max}(x \geqq X) = \tfrac{1}{2} \operatorname{erfc}\left[\sqrt{\frac{\log_{10} X}{M}}\right] \tag{18}$$

at

$$\sigma = \sqrt{2M \log_{10} X}. \tag{19}$$

For example, Fig. 12 shows $P(x \geqq 10)$ as a function of σ.

Figure 13 shows the maximum probability $P_{\max}(x \geqq X)$ as a function of X as given by eq. (18). By the use of "C-discriminant equa-

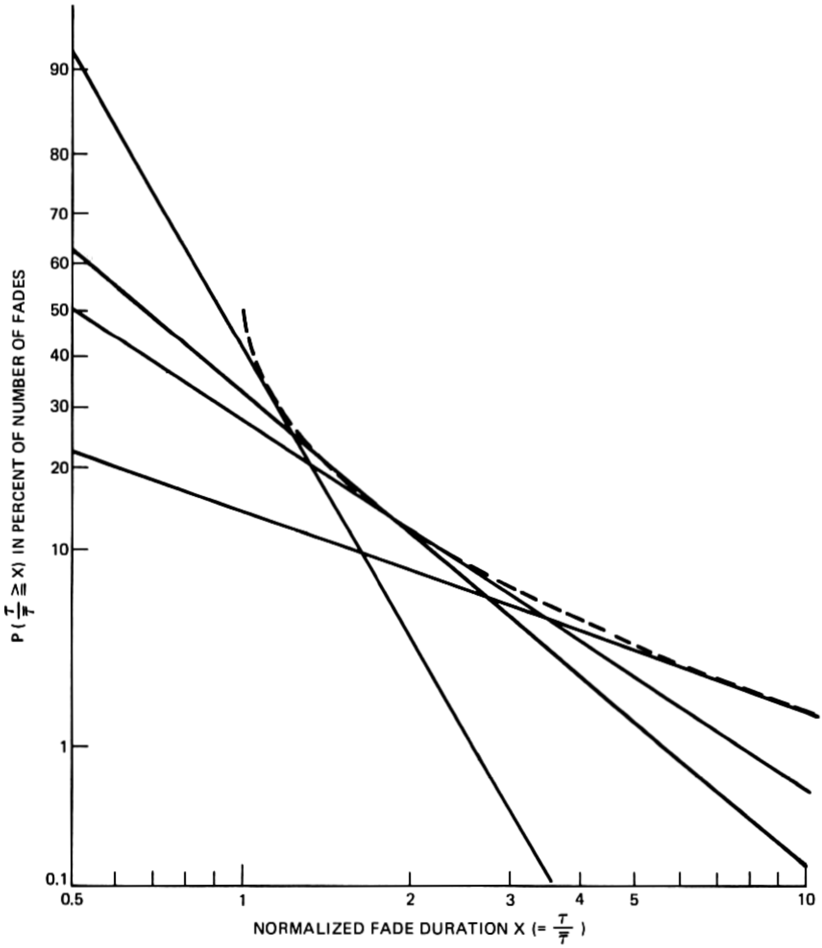

Fig. 14—The envelope of the family of lognormal duration distributions.

tion" in the theory of ordinary differential equation,[45] it can be shown that eq. (18) is, in fact, the *envelope* of the family of lognormal distributions (16) with σ as the family parameter. In other words, the family (16) are all tangent to (18) as shown in Fig. 14. The shaded area in Fig. 13 is a forbidden region where the lognormal duration distribution will never penetrate. Therefore, the upper bound for the fade duration distribution is given by eq. (2).

VI. CONCLUSION

Both the experimental data and theory indicate that both the rain attenuation distribution and the fade duration distribution are lognormal. The detailed results have already been given in Section III (Summary of Results).

VII. ACKNOWLEDGMENTS

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APPENDIX A

Generalized Interpretation of Formulation (7)

From a more general viewpoint, the effect of an environmental parameter $Z_i(t)$ on $\alpha(t)$ may be more complicated than the simple linear proportional relation

$$\alpha(t) \propto Z_i(t), \quad i = 1, 2, \cdots, n. \tag{20}$$

For example, $\alpha(t)$ may depend not only on the present value, but also on the past history of the environmental parameter $Z_i(t)$; then the relation between $\alpha(t)$ and $Z_i(t)$ becomes

$$\alpha(t) \propto \int_{-\infty}^{t} H_i(t, t')Z_i(t')dt' \qquad i = 1, 2, \cdots, n \tag{21}$$

where $H_i(t, t')$ is the impulse response of $\alpha(t)$ if the input $Z_i(t)$ is an impulse $\delta(t - t')$ applied at t'. In order to allow for the more general and complicated relation between $\alpha(t)$ and $Z_i(t)$, we shall use the mathematical operator notation:

$$\alpha(t) \propto G_i[Z_i(t)], \quad i = 1, 2, 3, \cdots, n \tag{22}$$

where G_i is a transformation from $Z_i(t)$ into $\alpha(t)$. Then the formulation becomes

$$\alpha(t) = G_1(Z_1) \cdot G_2(Z_2) \cdots G_n(Z_n). \tag{23}$$

Let

$$S_i(t) = G_i[Z_i(t)] \quad i = 1, 2, 3, \cdots, n. \tag{24}$$

Substituting (24) into (23) leads to (7).

The formulation can be further generalized to include the cases where the effects of various environmental parameters on α are not completely separable. Let

$$\gamma = \log_{10} \alpha, \tag{25}$$

$$c_i \xi_i = \log_{10} \quad S_i = \log_{10} G_i(Z_i)$$

$$i = 1, 2, 3, \cdots, n - 1, \text{ and} \tag{26}$$

$$R = \log_{10} S_n. \tag{27}$$

Substituting (25), (26), and (27) into (8) gives

$$\gamma(\xi_1, \xi_2, \cdots, \xi_{n-1}) = \bar{\gamma} + C_1(\xi_1 - \bar{\xi}_1) + C_2(\xi_2 - \bar{\xi}_2) + \cdots + C_{n-1}(\xi_{n-1} - \bar{\xi}_{n-1}) + (R - \bar{R}) \tag{28}$$

where

$$\bar{\gamma} = C_1\bar{\xi}_1 + C_2\bar{\xi}_2 + \cdots + C_{n-1}\bar{\xi}_{n-1} + \bar{R}. \tag{29}$$

Equation (28) can be interpreted as the first-order Taylor series expansion of $\gamma(\xi_1, \xi_2, \cdots, \xi_{n-1})$ with a remainder term,* $R - \bar{R}$. The main point in eq. (28) is that the effects of the $(n - 1)$ random variables $\{\xi_i\}_{i=1}^{i=n-1}$ on γ do not have to be completely separable because of the remainder term $R - \bar{R}$. If $R - \bar{R}$ does not dominate the sum in the right-hand side of eq. (28), then, by central limit theorem, the distribution of γ (i.e., $\log_{10} \alpha$) is approximately normal even if the effects of various environmental parameters are not completely separable. (This argument originates from Refs. 46 and 47.) Therefore, the formulation (7) includes very general and complicated relationships between the environmental parameters and rain attenuation $\alpha(t)$.

* If higher-order derivatives of $\gamma(\xi_1, \xi_2, \cdots, \xi_{n-1})$ exist, then $R - \bar{R}$ represents the sum of all the higher-order terms; otherwise, $R - \bar{R}$ represents the difference between $\gamma(\xi_1, \xi_2, \cdots, \xi_{n-1})$ and its first-order Taylor series expansion.

APPENDIX B

Deviations of Experimental Data From Lognormal Distribution

Aside from the experimental error, there are two major factors which contribute to the deviations of experimental data from the lognormal distribution.

B.1 *Effect of Finite Number of Components*

Davenport and Root[38] have indicated that when the number, n, of components is finite, the normal distribution may well give a *poor* approximation to the *tails* of the distribution of the sum (8) even though the limiting form of the sum distribution is, in fact, normal. Therefore, in practice, we believe that the deviation of the experimental data from the lognormal distribution may increase toward the tails.

B.2 *Effect of Time Base*

Since some of the components in eq. (7) are extremely slowly varying, reducing the time base will reduce the number of contributing components since the influences of slow components are approximately constant in a short experiment. Therefore, we expect the deviation of the experimental data from the lognormal distribution to increase as the time base decreases.

The minimum required time base for the convergence of the experimental data to the lognormal distribution increases as

(*i*) path length decreases, or
(*ii*) operating frequency decreases, or
(*iii*) attenuation range of interest increases.

Furthermore, when the time base of a rain attenuation experiment is not long enough, the deviations of the short-term distribution $P_s(\alpha \geqq A)$ from the long-term distribution $P(\alpha \geqq A)$ are usually nonsymmetric in the deep fade region, i.e., $P_s(\alpha \geqq A)$ is more likely to be less than $P(\alpha \geqq A)$. This nonsymmetry is caused by:

(*i*) The distribution of fade duration $\tau(A)$ is lognormal, which is nonsymmetric with respect to the average fade duration $\bar{\tau}(A)$. Typically, about 70 percent of fade durations are shorter than the average duration.
(*ii*) The probability distribution of the number $N(A, T)$ of observed deep fades, exceeding the margin A dB in the period T, is somewhat similar to a Poisson distribution which is also

nonsymmetric with respect to the average number $\bar{N}(A, T)$ of deep fades in a period T.

Since, when the time base is too short, both $N(A, T)$ and $\tau(A)$ have a higher chance of being less than their average values $\bar{N}(A, T)$ and $\bar{\tau}(A)$, respectively, then the short-term distribution $P_s(\alpha \geqq A)$ tends to deviate downward from the long-term distribution in the tail region.*

B.3 *Effect of Time Base on Standard Deviation* σ_a

Since it takes a long time base to include appreciable effects of slow components, we expect that σ_a increases slightly as the time base increases, and reaches an asymptotic value only after the time base is long enough to include the effects of all the possible slow components.

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* This idea originates from the work of A. Vigants on the deviations of experimental data from the theoretical distribution for diversity signals subject to multipath interference fading.

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