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Scattering Losses Caused by the Support Structure of an Uncladded Fiber

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We consider an uncladded dielectric waveguide core that is held by dielectric supports. The radiation losses caused by the support structure are being considered. The analysis is simplified by using a slab waveguide model held by slab-shaped supports. Only order-of-magnitude estimates are attempted. The radiation losses can be reduced by reducing the refractive index contrast between the supports and the surrounding medium with the help of an index matching liquid. The radiation losses remain large unless the index match is sufficiently close.

I. INTRODUCTION

A typical optical fiber consists of a core, the refractive index of which is larger than that of the cladding material surrounding the core.[1] The cladding serves the purpose of keeping any outside influence, such as dust, at a safe distance from the core. The requirement of a lower refractive index for the cladding makes it difficult to find suitable cladding materials for one of the most promising core materials–fused silica. Fused silica is particularly useful as fiber core material because of its inherently low absorption loss. However, the refractive index of fused silica $n = 1.46$, is lower than that of most other glasses. In particular, there are as yet no low-loss materials suitable for fiber claddings, the refractive indices of which are lower than that of fused silica. The few available materials have absorption losses that rule out their use as claddings for low-loss optical fiber waveguides.

It appears natural to ask whether a fiber without cladding could not be made. In principle a dielectric fiber waveguide works just as well without a cladding if it could be suspended in air or in vacuum. But since no method of levitating the fused silica fiber without mechanical supports has yet been devised, the necessity exists of holding the uncladded core by some kind of supporting structure. If the supports of the fiber core have a refractive index lower than that of the naked core, the amount of light scattered out of the core by contact with the supports may be tolerable. Since the supports touch the naked core only occasionally, their dielectric losses need not be exceptionally small. Solid materials of lower index than fused silica, but with much higher losses, do exist.

The scattering losses of the supports could be reduced by submerging the core and the support structure in a low-loss index matching liquid.[2] If it were possible to match the refractive index of the supports perfectly, the submersion technique could eliminate all scattering losses from the supports so that only the losses of the index matching liquid and the heat loss of the fiber supports would remain. The heat losses of the supports would equal their average value averaged over the entire length of the guide. Because of the low filling factor, this average loss could be tolerably small. However, even this index matching technique encounters certain problems. It is unlikely that a low-loss liquid could be found that matches the refractive index of the support structure perfectly. But even if this were possible, the index match would work only at one fixed temperature so that scattering losses would still occur if the ambient temperature drifts from the design value.

In order to explore the requirements for low scattering losses of a partially index-matched support structure, a model calculation is carried out in this paper. For simplicity we limit the discussion to the TE modes of the slab waveguide in the hope of obtaining order-of-magnitude estimates. The model to be investigated is shown in Fig. 1.

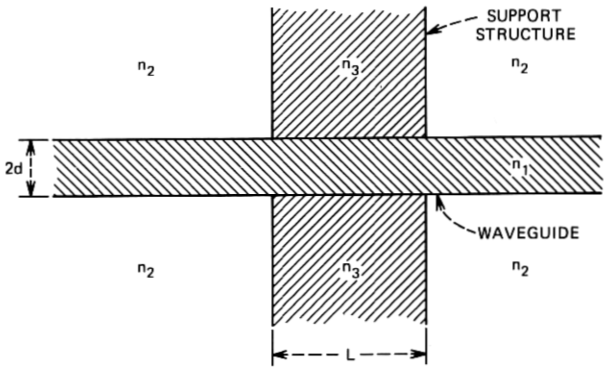

Fig. 1—Dielectric slab waveguide core of refractive index n_1 supported by two slabs of index n_3.

The dielectric slab of refractive index n_1 is embedded in a medium of refractive index n_2. This surrounding medium can be thought of either as air or as a suitable index matching liquid. The support structure is simulated by two dielectric slabs attached at right angles to the waveguide core. The refractive index of these support pieces is n_3. The shape of the model supports does not resemble the shape to be expected of actual supports. However, it is hoped that this model will provide insight into the scattering losses to be expected from an actual support structure. Since only order-of-magnitude estimates are expected from this model, we can use rather crude approximations of the mathematical expressions. The idea for using a naked fiber core held by a dielectric support structure originated with R. Kompfner.

II. SCATTERING LOSS THEORY

Our calculation of the scattering losses caused by the waveguide supports is based on the usual expansion of the field of the dielectric waveguide in terms of normal modes.[1] We restrict ourselves to TE modes of the slab waveguide whose only electric field component E_y is tangential to the surface of the slab. The field is expanded in terms of normal guided and radiation modes of the slab waveguide

$$E_y = \sum_\nu C_\nu \mathcal{E}_\nu + \int_0^\infty q(\rho)\mathcal{E}(\rho)d\rho. \tag{1}$$

The expansion coefficients in (1) are obtained from[1]

$$C_\mu = C_\nu \int_0^L K_{\mu\nu}(z)e^{-i(\beta_\nu-\beta_\mu)z}dz. \tag{2}$$

An analogous expression holds for $q(\rho)$ with $K_{\rho\nu}$ instead of $K_{\mu\nu}$. It can be shown [see eqs. (9.2–21) and (9.2–30) of Ref. 1] that the coupling coefficient $K_{\mu\nu}$ is given by

$$K_{\mu\nu} = \frac{\omega\epsilon_0}{4P}\int_{-\infty}^{\infty}[n^2(x) - n_0^2(x)]\mathcal{E}_\nu\mathcal{E}_\mu^* dx. \tag{3}$$

$n_0(x)$ is the refractive index distribution of the ideal waveguide, $n_0(x) = n_1$ inside of the waveguide core and $n_0(x) = n_2$ outside of the core region. The index distribution $n(x)$ is defined as being $n(x) = n_1$ in the waveguide core and $n(x) = n_3$ in the region of the waveguide support. L is the width of the support and P is the power carried by the incident mode. ω and ϵ_0 are, respectively, the radian frequency of the field and the permittivity of vacuum.

The relative power loss from mode ν to mode μ caused by the support is given by[1]

$$\frac{\Delta P_{\mu\nu}}{P_\nu} = \frac{|C_\mu|^2}{|C_\nu|^2} = |K_{\mu\nu}|^2 \frac{4\sin^2(\beta_\nu - \beta_\mu)\frac{L}{2}}{(\beta_\nu - \beta_\mu)^2}. \tag{4}$$

In particular, this formula applies to the reflection loss R of the mode ν which is obtained by setting $\beta_\mu = -\beta_\nu$. The radiation loss of mode ν is similarly obtained from the formula[1]

$$\frac{\Delta P_s}{P_\nu} = \int_0^\infty \frac{|q(\rho)|^2}{|C_\nu|^2} d\rho = 4\int_{-n_2k}^{n_2k} |K_{\rho\nu}|^2 \frac{\sin^2(\beta_\nu - \beta)\frac{L}{2}}{(\beta_\nu - \beta)^2} \frac{|\beta|}{\rho} d\beta. \tag{5}$$

The integration variable in the integral on the extreme right-hand side was changed from ρ to β.

In order to evaluate the coupling coefficients, we need the field expressions for the guided and radiation modes only outside of the core region, because the integrand in (3) vanishes inside of the waveguide core. We have for $|x| > d$

$$\mathcal{E}_\nu = \left\{\frac{2\gamma_\nu P}{(n_1^2 - n_2^2)\beta_\nu(1 + \gamma_\nu d)\omega\epsilon_0}\right\}^{\frac{1}{2}} k\kappa_\nu e^{-\gamma_\nu(|x|-d)}. \tag{6}$$

Equation (6) for the guided TE modes follows from eqs. (8.3–12) and (8.3–18) of Ref. 1, with the help of (8.6–16). The parameters γ_ν and κ_ν are related to the free-space propagation constant $k = 2\pi/\lambda$ in the following way

$$\gamma_\nu^2 = \beta_\nu^2 - n_2^2k^2 \tag{7}$$

and

$$\kappa_\nu^2 = n_1^2k^2 - \beta_\nu^2. \tag{8}$$

The field of the TE radiation modes is obtained from eqs. (8.4–4), (8.4–9), and (8.4–18) of Ref. 1.

$$\mathcal{E}_\rho = \left\{\frac{\rho^2k^2P}{2\pi\omega\epsilon_0|\beta|(\rho^2\cos^2\sigma d + \sigma^2\sin^2\sigma d)}\right\}^{\frac{1}{2}} \cdot \left\{\left(\cos\sigma d - i\frac{\sigma}{\rho}\sin\sigma d\right)e^{-i\rho(|x|-d)} + \left(\cos\sigma d + i\frac{\sigma}{\rho}\sin\sigma d\right)e^{i\rho(|x|-d)}\right\}. \tag{9}$$

The coefficient for coupling between the guided modes ν and μ follows from (3) and (6)

$$K_{\mu\nu} = \frac{n_3^2 - n_2^2}{n_1^2 - n_2^2} \frac{\kappa_\nu \kappa_\mu (\gamma_\nu \gamma_\mu)^{\frac{1}{2}}}{(\gamma_\nu + \gamma_\mu)[|\beta_\nu \beta_\mu|(1 + \gamma_\nu d)(1 + \gamma_\mu d)]^{\frac{1}{2}}}. \tag{10}$$

The power reflection coefficient R_ν for the νth mode is obtained from (4) and (10) by setting $\beta_\mu = -\beta_\nu$, $\kappa_\mu = \kappa_\nu$, and $\gamma_\mu = \gamma_\nu$

$$R_\nu = \frac{(n_3^2 - n_2^2)^2 \kappa_\nu^4 \sin^2 \beta_\nu L}{4(n_1^2 - n_2^2)^2 \beta_\nu^4 (1 + \gamma_\nu d)^2}. \tag{11}$$

It is interesting to consider the reflection coefficient in the limit $\gamma_\nu = 0$ at the cutoff point of the guided mode. We obtain from (11) with $\beta_\nu = n_2 k$ and $\kappa_\nu^2 = (n_1^2 - n_2^2)k^2$

$$R_\nu = \frac{(n_3^2 - n_2^2)^2}{16 n_2^4} (4 \sin^2 n_2 k L) \cdot \tag{12}$$

The sine factor (multiplied by 4) on the right-hand side of this equation describes the interference between the reflection from the front and back surface of the support structure. If we omit this factor we obtain the reflection from only one of the two interfaces in the form

$$\bar{R}_\nu = \frac{(n_3^2 - n_2^2)^2}{16 n_2^4} = \frac{(n_3 + n_2)^2}{(2n_2)^2} \frac{(n_3 - n_2)^2}{(2n_2)^2}. \tag{13}$$

Comparison of eq. (13) with the correct expression for the power reflection coefficient from a dielectric interface [see eq. (1.6–36) of Ref. 1]

$$R = \left(\frac{n_3 - n_2}{n_3 + n_2}\right)^2 \tag{14}$$

provides an indication of the accuracy of our approximation. Instead of solving the infinite system of coupled equations, we obtained the coefficient of the reflected wave and hence the power reflection coefficient by considering only the incident and the reflected wave alone. The resulting equation (13) agrees with the correct equation (14) in the limit $(n_3 - n_2) \to 0$. The approximate solution of the infinite equation system is thus a good approximation only for small index differences. However, it is certainly quite satisfactory as an order-of-magnitude estimate.

The coupling coefficient for mode ν and a radiation mode characterized by the parameter ρ follow from (3), (6), and (9)

$$K_{\rho\nu} = \frac{n_3^2 - n_2^2}{(n_1^2 - n_2^2)^{\frac{1}{2}}} \cdot \frac{k\kappa_\nu(\gamma_\nu)^{\frac{1}{2}}\rho(\gamma \cos \sigma d - \sigma \sin \sigma d)}{(\beta_\nu^2 - \beta^2)[\pi|\beta\beta_\nu|(1 + \gamma_\nu d)(\rho^2 \cos^2 \sigma d + \sigma^2 \sin^2 \sigma d)]^{\frac{1}{2}}}. \quad (15)$$

The radiative power loss caused by one support follows from (5) and (15). The integral cannot be solved exactly. Instead of resorting to numerical integration, I worked out an approximate solution which holds only as an order-of-magnitude estimate.

$$\frac{\Delta P}{P} = \frac{2}{3\pi} \frac{(n_3^2 - n_2^2)^2}{(n_1^2 - n_2^2)^{\frac{3}{2}}} \frac{k\kappa_\nu^2\gamma_\nu^3}{\beta_\nu(1 + \gamma_\nu d)(\beta_\nu + n_2 k)^2(\beta_\nu - n_2 k)^3}. \quad (16)$$

The radiation loss depends on the width L of the support structure. For large values of L/λ, the width dependence disappears. The approximation (16) holds in this limit. Comparison of (16) with a few sample values of the numerical integration has shown that this approximation can depart from a few percent to as much as 50 percent from the value obtained by integrating (5).

III. DISCUSSION AND NUMERICAL RESULTS

The radiation losses caused by the waveguide supports are particularly high for single-mode operation. Our discussion is thus directed towards multimode applications. However, Fig. 2 shows the dependence of the radiation loss on the width L of the supports for a single-mode case. It is apparent that the interference of radiation originating at the front and back surface of the support slab causes the radiation loss curve to oscillate. These oscillations die out with increasing width of the supports. The parameter V, that is used to label the curve in Fig. 1 and all remaining figures, is defined by

$$V = (n_1^2 - n_2^2)^{\frac{1}{2}}kd. \quad (17)$$

V is a parameter that combines frequency, slab width, and refractive index difference. Its values determine the number of modes that can

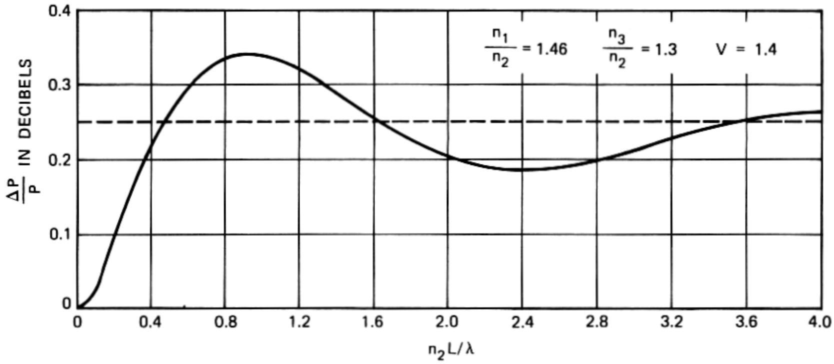

Fig. 2—Relative scattered power as a function of support thickness L. The dotted line represents eq. (16), which holds approximately for $L \to \infty$.

propagate on the waveguide. The lowest-order even TE mode has no cutoff, the cutoff value of V for this mode is thus $V_c = 0$. The cutoff of the next mode, the first odd mode, is given by $V_c = \pi/2 = 1.57$. In general, we obtain the cutoff condition for all the even and odd slab modes from the formula[1]

$$V_c = \nu \frac{\pi}{2}. \tag{18}$$

For even modes, ν assumes the values 0, 2, 4, etc.; for odd modes we have $\nu = 1, 3, 5$, etc.

The solid curve of Fig. 2 was obtained by numerical integration of (5) and (15). The dotted line also shown in the figure results from the approximation (16). This approximation holds for $L \to \infty$ and is thus independent of the variable n_2L/λ.

The following figures apply to multimode operation and show the radiation losses caused by one set of waveguide supports as a function of the mode angle θ. Each guided mode can be decomposed into two plane waves, the propagation vectors of which form angles $+\theta$ and $-\theta$ with the waveguide axis. The angle θ is defined by

$$\cos \theta_\nu = \frac{\beta_\nu}{n_1 k}. \tag{19}$$

The mode angles assume discrete values θ_ν corresponding to the discrete values β_ν of the propagation constant of the guided modes. Figures 3 through 8 show θ as a continuous variable. It is important to remember that only certain discrete values of θ are allowed. The number of modes that exist below a certain value of θ is approximately proportional to

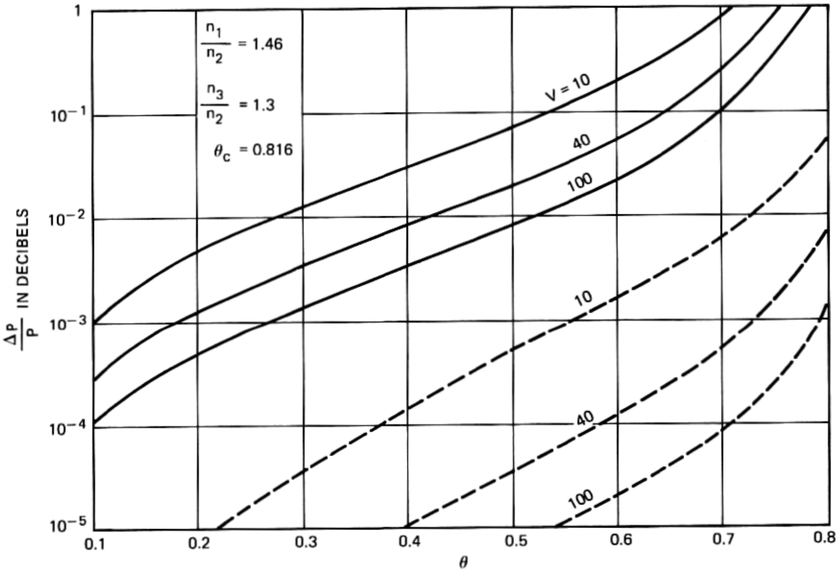

Fig. 3—Relative scattered power (solid lines) as a function of mode angle θ. The waveguide core and the supports are in an air environment. The dotted lines represent the reflection loss of each mode.

this angle in case of the slab waveguide but it is proportional to the square of θ in case of the round optical fiber. For the slab, the number of TE and TM modes with angles smaller than a given value θ is approximately given by ($\theta \ll 1$ is assumed)

$$N_\theta = \frac{4}{\pi}\,\theta n_1 kd. \tag{20}$$

The total number of guided TE and TM modes that can be supported is approximately given by

$$N_{\max} = \frac{4}{\pi} V. \tag{21}$$

For the round fiber we have[3] (d = fiber radius)

$$N_\theta = \tfrac{1}{2}(\theta n_1 kd)^2 \tag{22}$$

modes with angles less than θ. The maximum number of modes that the fiber can support is[3]

$$N_{\max} = \tfrac{1}{2}V^2. \tag{23}$$

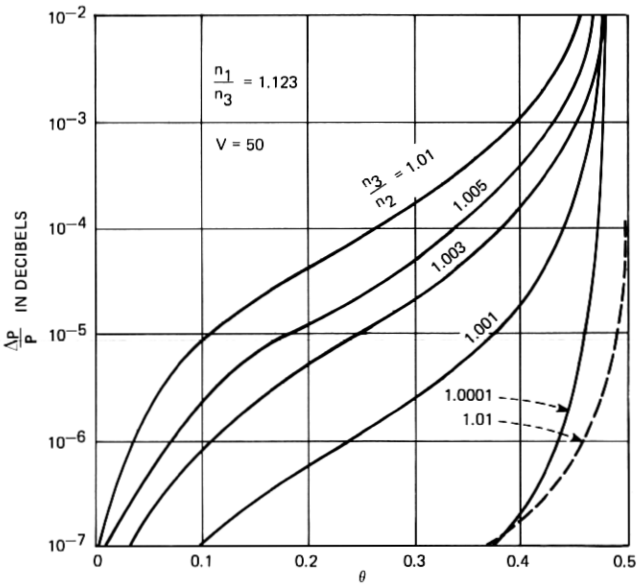

Fig. 4—Scattering losses as a function of mode angle θ. The core index is assumed to be $n_1 = 1.46$, the support index is $n_3 = 1.3$. Core and supports are immersed in an index matching liquid. The dotted line is the reflection loss for $n_3/n_2 = 1.01$.

Figure 3 shows the relative power loss as a function of mode angle θ for three different values of V. The refractive indices are chosen to represent a quartz fiber in air, $n_1/n_2 = 1.46$. It was assumed that the supports consist of a material with refractive index $n_3 = 1.3$. The solid curves represent the radiation losses while the dotted curves show the reflection losses for the modes with mode angle θ. It is apparent that the reflection losses are much smaller than the radiation losses. Mode conversion from each guided mode to all the other guided modes has not been considered. The maximum mode angle θ in this and all the following figures (with the exception of Fig. 4) coincides approximately with the right-hand edge of the graph.

Figure 4 shows the radiation losses for a fixed value $n_1/n_3 = 1.123$ of the ratio of core index to the index of the supports. This figure was drawn for the case that the core index is again $n_1 = 1.46$ and $n_3 = 1.3$, but allows for the fact that an index matching liquid is used in an attempt to reduce the scattering losses. The ratio n_3/n_2, that indicates the degree of index matching, is used as a curve parameter. The dotted curve is the reflection loss for $n_3/n_2 = 1.01$. The reflection losses for all the other index ratios are much smaller. The critical angle is different for each curve of this figure.

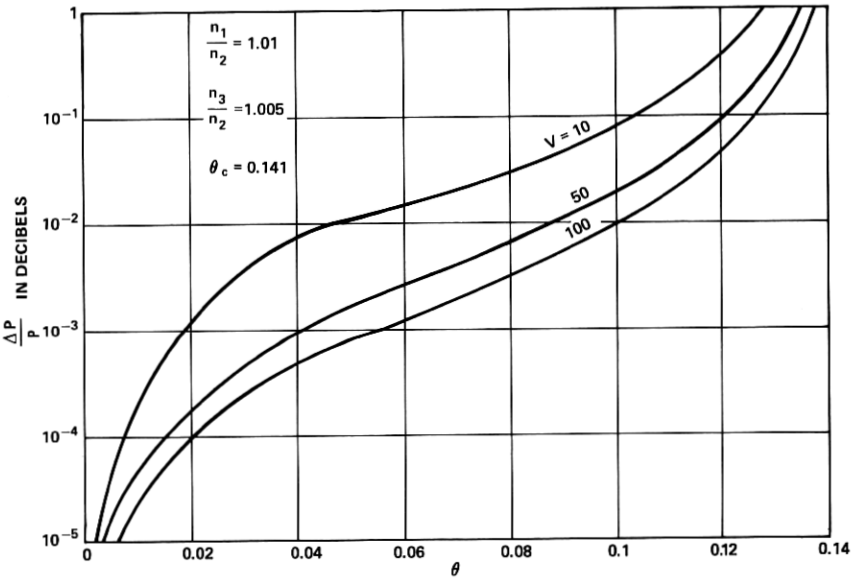

Fig. 5—Relative scattering losses as a function of mode angle θ, $n_1/n_2 = 1.01$, $n_3/n_2 = 1.005$.

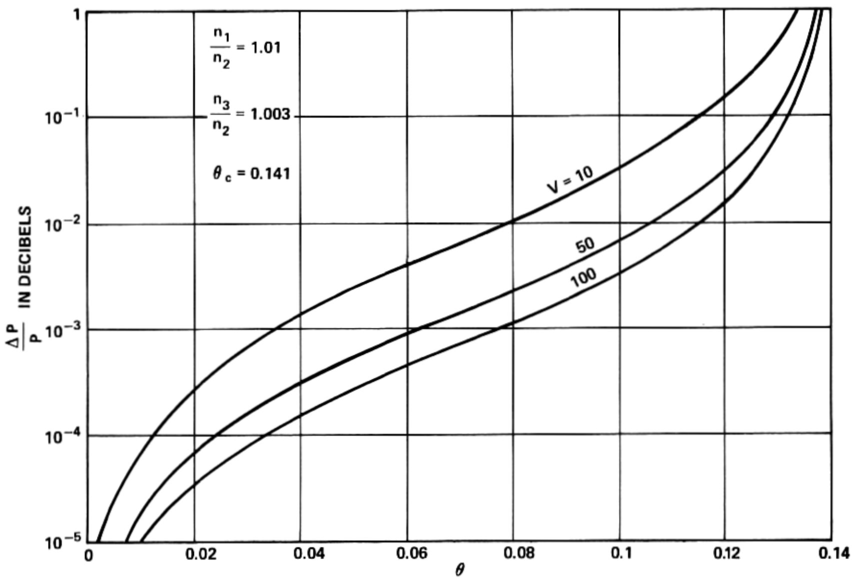

Fig. 6—Same as Fig. 5, $n_3/n_2 = 1.003$.

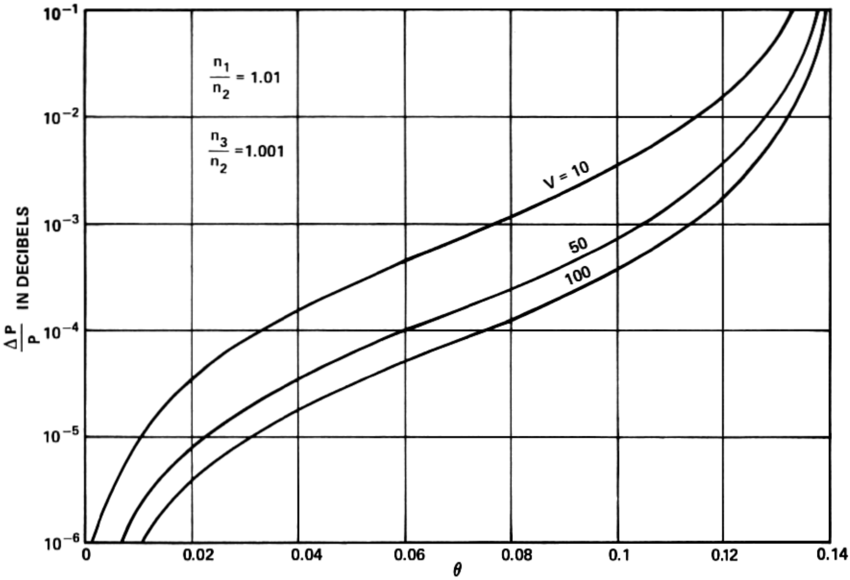

Fig. 7—Same as Fig. 5, $n_3/n_2 = 1.001$.

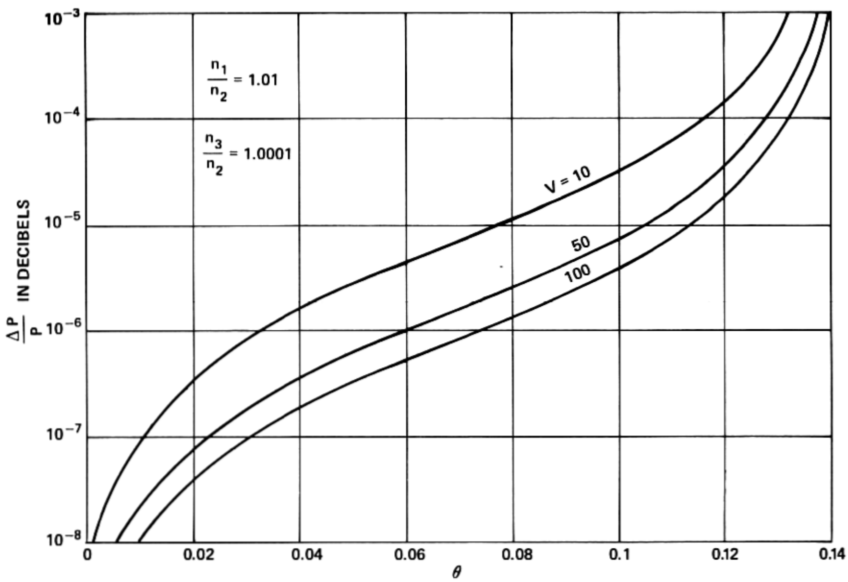

Fig. 8—Same as Fig. 5, $n_3/n_2 = 1.0001$.

The remaining four figures are all similar to each other. All show the radiation losses as functions of mode angle for three different values of V and for different values of the ratio n_3/n_2, but fixed value $n_1/n_2 = 1.01$. It is thus assumed that the index of the matching fluid remains the same, while the index of the supports is changed slightly. The reflection losses remain below the scale of all these figures.

For an evaluation of the total radiation losses to be expected from a fiber held by many supports, it is necessary to know the total number of supports. Let us assume for simplicity that we have one support per centimeter. A fiber of 1-km length is thus held by a total of 10^5 supports. In order to stay below a radiation loss of 10 dB/km, we must remain below the $\Delta P/P = 10^{-4}$ dB line of the figures. As a first crude approximation, we can assume that all modes, the loss of which remains below this line, are received at the end of the fiber while the modes that exceed this loss are lost. For a slab waveguide, the number of modes that can travel through the fiber is directly proportional to the mode angle θ, so that the ratio of transmitted to dissipated modes can be read off the horizontal axis. For the $n_3/n_2 = 1.01$ curve of Fig. 4, modes with angles less than 0.26 radian stay below the 10^{-4} line and can be considered as being transmitted through the fiber of 1-km length. This means that roughly half the modes are lost while half of them are being transmitted. However, if the waveguide is a round fiber (and if we accept the applicability of the loss curves for this case) we find that only one-quarter of all the modes can be transmitted while three-quarters of the modes and hence three-quarters of the power (for uniform initial power distribution) is lost. These estimates ignore mode conversion between the guided modes. The small values of the reflection loss suggest that conversion between guided modes is reasonably small. However, some power is converted from modes with small angles to large angle modes so that the loss estimate made on the basis of the individual mode losses alone must be optimistic.

A comparison of Fig. 4 and Fig. 5 shows clearly that it is advantageous to try to match the refractive index of the supports but keep the core index as large, compared to the surrounding liquid, as possible. The curves of Fig. 5 show much larger loss values because the difference in refractive index of core and index matching liquid is small, so that the fields extend farther into the liquid and thus are more effectively scattered by the supports. In order to achieve losses as low as those of the $n_3/n_2 = 1.01$ curve of Fig. 4, with a guide whose core-to-liquid index ratio is only 1.01, requires that the index ratio of support and matching liquid be better than 1.001.

IV. CONCLUSIONS

The problem of radiative power loss caused by light scattering from the supports of an unclad fiber has been investigated. The study was based on a slab waveguide model with dielectric slabs used as supports. The study comes to the conclusion that tolerably low scattering losses are obtained only if the supports are made less visible to the wave by index matching with a suitable matching liquid. The matching liquid must itself have very low dielectric losses. It is important to make the index difference between core and supports as large as possible (with the core index being larger than that of the supports) and match the support index as closely as possible. For a core-to-support index ratio of $n_1/n_3 = 1.123$, an index ratio of $n_3/n_2 = 1.01$ between supports and matching liquid is sufficient to allow one-quarter of all the modes of a round fiber to travel with acceptably low radiation losses. If the core index is more nearly equal to the refractive index of the supports, a much better index match for the supports is required. To achieve conditions comparable to the last example requires an index match of the supports of better than one tenth of a percent if $n_1/n_2 = 1.01$. It may be difficult to maintain such a good index match over the whole range of expected operating temperatures.

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