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Perturbation Calculations of Rain-Induced Differential Attenuation and Differential Phase Shift at Microwave Frequencies

By J. A. MORRISON and T. S. CHU

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In a recent note[1] calculated results of differential attenuation and differential phase shift, as a function of rain rate, were given at frequencies of 4, 18.1, and 30 GHz. The calculations have since been done at 11 GHz also. These results are based on scattering of a plane electromagnetic wave by oblate spheroidal raindrops. The point matching procedure used to obtain nonperturbative solutions to the problem was briefly described, and full details will be presented later.[2] Somewhat similar calculations have been carried out by Oguchi[3] at 19.3 and 34.8 GHz.

The purpose of this note is to point out that a modification of Oguchi's earlier first-order perturbation approximation,[4] for spheroidal raindrops with small eccentricity, gives results which are quite close to those obtained by the point matching procedure. We also give these modified perturbation results at frequencies in the range up to 100 GHz, although they may be less reliable at the higher frequencies, particularly at the heavier rain rates.[4] We remark that the perturbation results are obtained quite inexpensively, whereas the point matching procedure is very costly.

The surface of an oblate spheroidal raindrop is given in spherical coordinates by

$$r = R(\theta) = a(1 - \nu \sin^2 \theta)^{-\frac{1}{2}} = a[1 + \tfrac{1}{2}\nu \sin^2 \theta + 0(\nu^2)], \quad (1)$$

for $0 \leqq \theta \leqq \pi$, independently of the azimuthal angle φ. It was assumed[1] that the ratio of minor to major axis depends linearly on the radius

$\bar{a}$ (in cm) of the equivolumic spherical drop; specifically $a/b = (1 - \bar{a})$. Thus, from (1), $a = \bar{a}(1 - \bar{a})^{\frac{1}{3}}$ and $\nu = \bar{a}(2 - \bar{a})$. We may rewrite (1) in the form

$$R(\theta) = \bar{a}[1 + 2\bar{a}(\tfrac{1}{2}\sin^2\theta - \tfrac{1}{3}) + 0(\bar{a}^2)], \tag{2a}$$

or

$$R(\theta) = \bar{a}[1 + \nu(\tfrac{1}{2}\sin^2\theta - \tfrac{1}{3}) + 0(\nu^2)]. \tag{2b}$$

Then, rather than perturbing about a spherical drop of radius a, with perturbation parameter ν, as did Oguchi,[4] we perturb about the equivolumic spherical drop of radius $\bar{a}$, and take either ν or $2\bar{a}$ as the perturbation parameter.

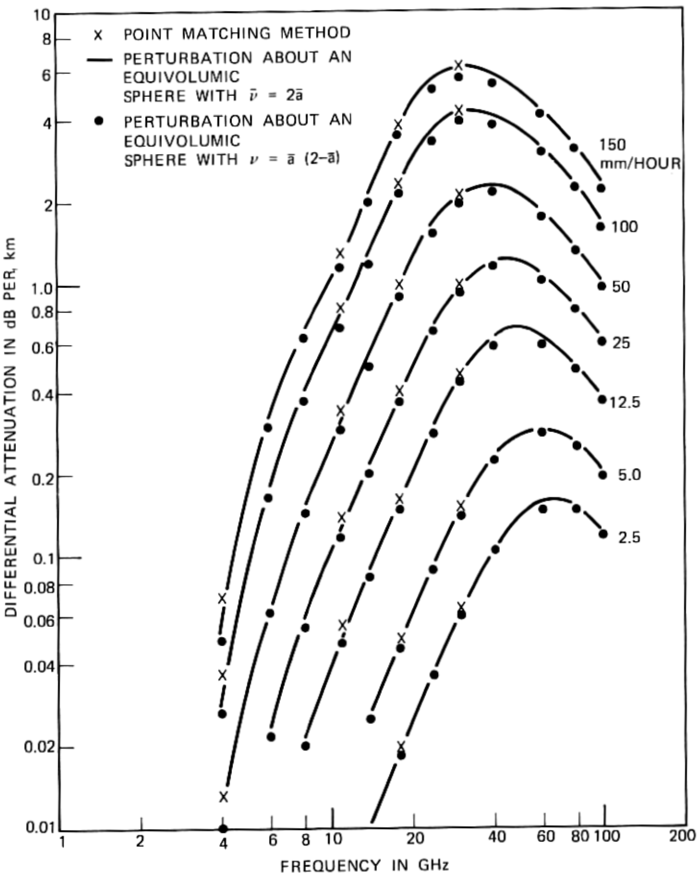

Fig. 1—Rain-induced differential attenuation.

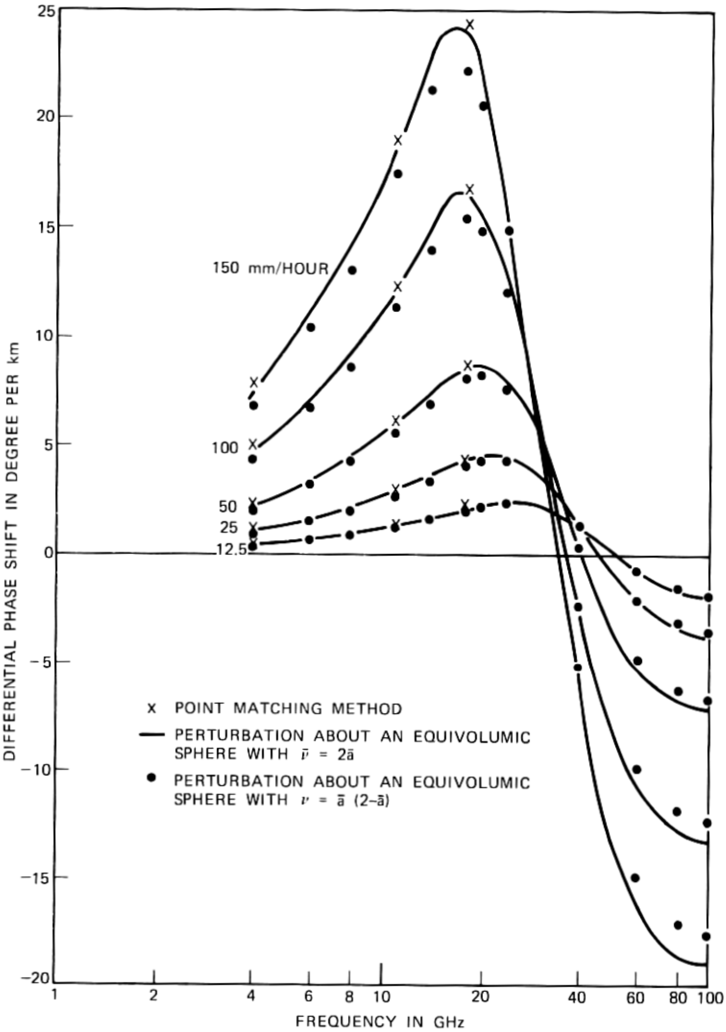

Fig. 2—Rain-induced differential phase shift.

Oguchi's first-order perturbation results have been generalized to axisymmetric raindrops which are nearly spherical, as will be discussed in the detailed paper.[2] There the first-order approximations to the forward scattering functions $S_{\text{I}}(0)$ and $S_{\text{II}}(0)$, for horizontally disposed oblate spheroidal raindrops, will be compared to the values obtained by the point matching method, for the 14 different drop sizes $\bar{a} = 0.025$

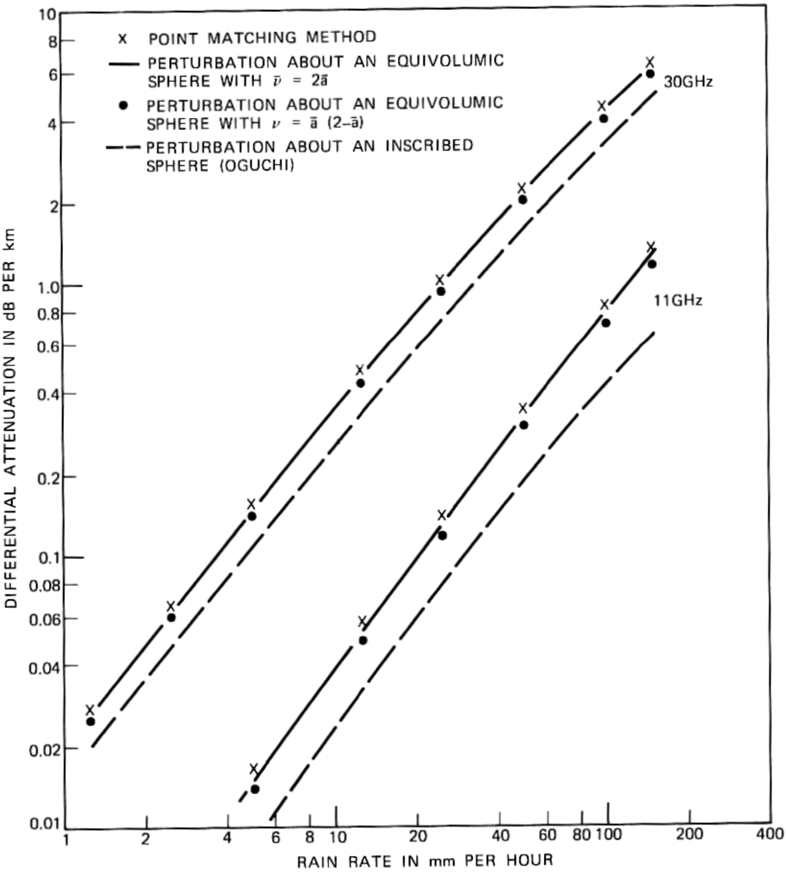

Fig. 3—Comparison between point matching and three perturbation methods for differential attenuation.

(0.025) 0.35. The subscripts I and II correspond to vertical and horizontal polarizations of the incident electric field, respectively. Here we consider only the differential attenuation and differential phase shift, which are obtained[1] by summing the real and imaginary parts of $S_{II}(0)-S_{I}(0)$ over the Laws and Parsons drop size distribution.[5] We comment that for the larger drop sizes the perturbation parameter is not small.

Although extra first-order correction terms arise in the expansions about the equivolumic spherical drop, given in (2), they correspond to a constant change in the radius of the drop. Hence the corresponding

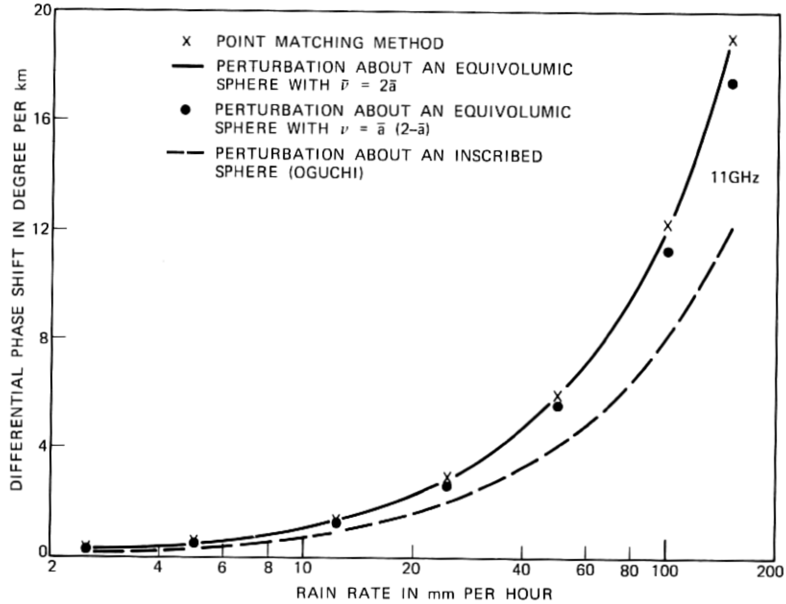

Fig. 4a—Comparison between point matching and three perturbation methods for differential phase shift at 11 GHz.

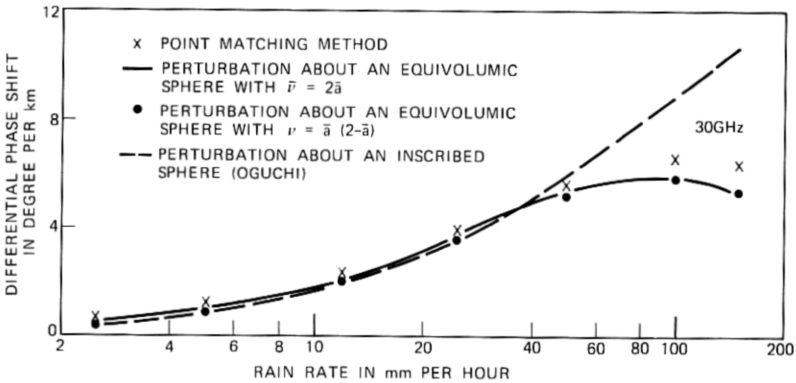

Fig. 4b—Comparison between point matching and three perturbation methods for differential phase shift at 30 GHz.

increments in the forward scattering functions are the same for both polarizations, and therefore do not affect the difference $S_{II}(0)–S_{I}(0)$. Thus Oguchi's formulas[4] may be applied directly to calculate this difference, by replacing a by $\bar{a}$, and using either ν, or $\bar{\nu} = 2\bar{a}$, as the perturbation parameter. We remark, however, that some simplifications may be made[2] in the four expressions given in equation (37) of Oguchi's 1960 paper.

Using the approximate forward scattering functions from the perturbations about an equivolumic sphere, we obtained the differential attenuation and differential phase shift versus frequency for various rain rates as shown in Figs. 1 and 2. The refractive indexes of water at 20°C were obtained as described in the previous note.[1] The curves are calculated with the perturbation parameter $\bar{\nu} = 2\bar{a}$, while the dots are calculated with the perturbation parameter ν. The point matching solutions are included as crosses; these show good agreement with the curves, whereas the dots deviate more from the crosses. (In order to avoid confusion, we have omitted the dots and crosses corresponding to the differential phase shift at 30 GHz.) On the other hand, we found much greater discrepancy between the point matching results and the approximate results from the perturbations about an inscribed sphere. This discrepancy is illustrated in Figs. 3, 4a, and 4b for 11 and 30 GHz. Discrepancies for 4 and 18.1 GHz are similar to those for 11 GHz. The above comparison between the point matching solution and three perturbation solutions is consistent with the order of geometrical errors in the three approximations to the oblate spheroid, i.e., the largest error corresponds to (1) and the smallest error corresponds to (2a).

The differential attenuation and differential phase shift recently presented by Watson and Arbabi[6] from 4 through 36 GHz are based upon Oguchi's perturbation solution. These numerical values are in general considerably lower than those of the point matching solution, except for the differential phase shift around 30 GHz, where the differential phase shift from the point matching method decreases sharply. The differential phase shift becomes negative at millimeter wavelengths, and hence remains a significant factor in depolarization.

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