The Field Singularity at the Edge of an Electrode on a Semiconductor Surface

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Near the edge of a charged electrode on the surface of a semiconductor, the field in the semiconductor may become very large because of the accumulation of charge at the electrode edge. Such large local fields are undesirable, not only because they may cause local breakdown, but also because they make the behavior of a semiconductor device difficult to predict.

In the present paper we consider a simple mathematical model of an electrode edge-semiconductor-insulator configuration and derive conditions under which large local fields may be avoided. More accurately, since the electrode edge and semiconductor corner angles are assumed to be perfectly sharp, we derive conditions under which the local field in the semiconductor is nonsingular. It is necessary to include the effect of the surrounding insulator, because even for small insulator-semiconductor dielectric constant ratios, a field singularity in the insulator will be coupled back into the semiconductor.

I. INTRODUCTION

Beneath a charged electrode situated on the surface of a semiconductor, at points far from the electrode edge, the electrostatic field is regular and quasi-one-dimensional, with its maximum value at the electrode. Near the electrode edge, however, the field may become very large because of the accumulation of surface charge at the sharply curved electrode edge.[1] Also, the jump in dielectric constant between the semiconductor and the surrounding insulating material may produce a large local field intensity. Such a field may be so large as to cause avalanche breakdown near the edge, but, in any case, the presence of such an edge effect makes the behavior of a semiconductor device difficult to predict.

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In this paper we consider a simple mathematical model of an electrode edge-semiconductor-insulator configuration, namely a sharp-edged electrode on top of a semiconductor mesa, as in Fig. 1. We study the behavior of the potential, or rather its singular part, in the two wedge-shaped semiconductor and insulator regions shown in the inset circle of Fig. 1, assuming that the potential is locally planar and that its singular part satisfies Laplace's equation, both in the insulator and in the semiconductor. Since the treatment is local and the electrode edge and mesa corner are replaced by mathematically sharp wedges, our analysis can only predict the existence or nonexistence of a singular field at the edge and cannot produce an estimate of local field strength, which depends on conditions far from the edge.

We derive an estimate of the order of the singularity in the potential of the form $\varphi = 0(r^p)$, where r is the distance from the corner and $p > 0$. The local field is thus of order r^{p-1}, singular for $p < 1$. We consider p as a function of the semiconductor wedge angle α, the electrode wedge angle β, and the insulator-semiconductor permittivity ratio η, for α and β between zero and 180° and η between zero and one.

We find that, to avoid a field singularity, we must make β greater than 90° and α less than 90°. In particular, if we take $\beta = 180°$, any α less than 90° yields a nonsingular field. Such a configuration might be realized, for example, by using an overhanging electrode on an undercut semiconductor mesa, as in Fig. 2. The length of the overhang must be several Debye lengths for small electrode potential and several depletion layer thicknesses for large reverse bias, in order that the present theory be applicable.

Figure 3 summarizes our principal results. It gives the range of α, for $90° \leqq \beta \leqq 180°$, $0 \leqq \eta \leqq 1$, within which the field is nonsingular.

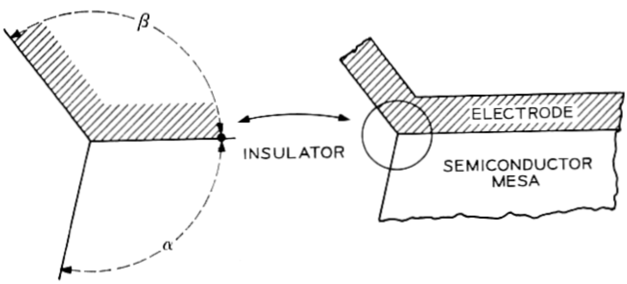

Fig. 1—Mesa with sharp-edged electrode.

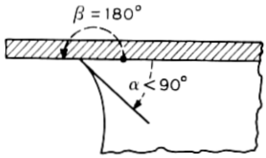

Fig. 2—Overhanging electrode on undercut mesa.

It should be noted that our results apply without modification to edge fields in capacitors and, with the appropriate interpretation, to steady temperature fields in conductors. In the latter case, it would be interesting to study the corresponding thermal stresses, as, for example, in a glass-to-metal seal.

II. A MODEL OF THE ELECTRODE EDGE

We consider the electric field in a current-free semiconductor, near an electrode edge whose cross-section is sketched in Fig. 4. The sketch shows a typical semiconductor mesa with corner angle α, surrounded by insulating material, and supporting an electrode with corner angle β. For given insulator-semiconductor permittivity ratio $\eta = \epsilon_0/\epsilon_1$ (≈ 0.1 for air-silicon, 0.3 for silica-silicon), we wish to choose α and β to avoid local field singularities.

In the semiconductor the dimensionless potential φ satisfies an equation of the general form

$$\nabla^2\varphi = f(\varphi), \tag{1}$$

where the Laplacian operator is made dimensionless with the Debye length for small potential and by the depletion layer thickness for large reverse bias (see, for example, Ref. 2). "Large" or "small" distance then means large or small with respect to one of these typical lengths. In particular, we shall assume that the electrode is so large in a direction perpendicular to the cross-section that the local field may be treated as planar.

We seek solutions of equation (1) which have field singularities at the vertex $r = 0$, that is, a bounded potential φ such that $| \nabla\varphi |$ is unbounded at $r = 0$. In the neighborhood of such a singularity the individual second derivatives which make up the Laplacian will be very large, although they must combine to make the Laplacian equal to the bounded function $f(\varphi)$. As far as the singular part of the solution is concerned then, the specific form of $f(\varphi)$ is unimportant and we can in fact set $f(\varphi)$ equal to zero. The singular solution then satisfies Laplace's

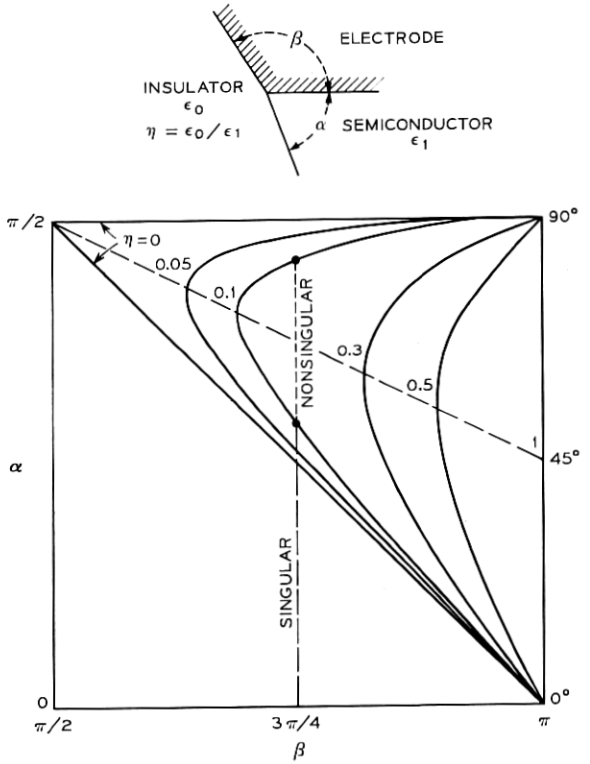

Fig. 3—Bounds on semiconductor angle α as a function of electrode angle β for a nonsingular field.

equation

$$\nabla^2\varphi = \partial^2\varphi/\partial r^2 + \partial\varphi/r\,\partial r + \partial^2\varphi/r^2\,\partial\theta^2 = 0, \tag{2}$$

in the neighborhood of $r = 0$, both in the semiconductor $(0 < \theta < \alpha)$ and in the insulator $(\alpha < \theta < 2\pi - \beta)$. At the electrode faces we have the boundary conditions

$$\varphi(r, 0) = \varphi(r, 2\pi - \beta) = 1, \tag{3}$$

while at the semiconductor-insulator interface, in the absence of surface charge, we have the continuity conditions*

* As we shall see, no matter how small η is, equation (5) couples any singularity in the insulator back into the semiconductor. Satisfaction of this condition is an essential feature of the problem.

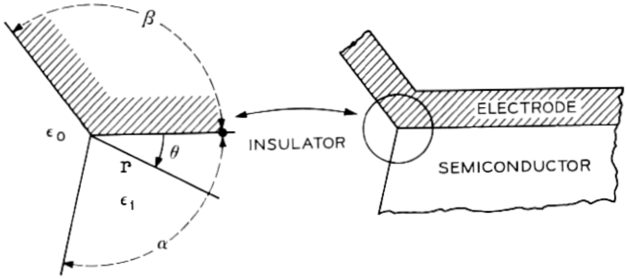

Fig. 4—Semiconductor-electrode edge configuration.

$$\varphi(r, \alpha^-) = \varphi(r, \alpha^+), \tag{4}$$

$$\partial\varphi(r, \alpha^-)/\partial\theta = \eta\partial\varphi(r, \alpha^+)/\partial\theta. \tag{5}$$

We now expand φ in positive powers of r, setting

$$\varphi(r, \theta) = \begin{cases} 1 + \sum_{k=1}^{\infty} A_k r^{p_k} \sin p_k\theta, & \text{for} \quad 0 \leqq \theta \leqq \alpha, \\ 1 + \sum_{k=1}^{\infty} B_k r^{p_k} \sin p_k(2\pi - \beta - \theta), & \text{for} \quad \alpha \leqq \theta \leqq 2\pi - \beta, \end{cases}$$

satisfying equation (2) and boundary condition (3). The A's, B's, and p's are chosen to satisfy the continuity conditions (4) and (5), which take the form

$$A \sin p\alpha - B \sin p(2\pi - \beta - \alpha) = 0,$$

$$A \cos p\alpha + \eta B \cos p(2\pi - \beta - \alpha) = 0.$$

These equations have a nontrivial solution only if the coefficient determinant vanishes, giving the characteristic equation for p

$$\eta \sin p\alpha \cos p(2\pi - \beta - \alpha) + \cos p\alpha \sin p(2\pi - \beta - \alpha) = 0. \tag{6}$$

We wish to find the smallest positive value of p which will satisfy this equation, as a function of α, β, and η. For this value of p we have

$$\varphi - 1 = 0(r^p),$$

$$|\nabla\varphi| = 0(r^{p-1}),$$

singular for $p < 1$, in the neighborhood of $r = 0$.

III. THE FLAT SURFACE AND THE MESA

Before treating the general problem, let us consider two special cases, in order to gain insight into the behavior of p as a function of α, β, and η. In the very common case of a thin electrode ($\beta = 0$) on a flat semiconductor surface ($\alpha = \pi$), as in Fig. 5, the characteristic equation (6) reduces to

$$\sin p\pi \cos p\pi = 0, \tag{7}$$

for all η. It is satisfied by $p = 1$, $p = \frac{1}{2}$, the latter being the smallest value of p. In this case, near the electrode edge the field is singular, like $r^{-\frac{1}{2}}$, no matter what insulating material is used. This is the same singularity as that obtained in the classical Weber problem of the disk electrode. The same local behavior was also found in Ref. 3, where a closed-form solution of the above problem was derived for the linearized semiconductor equation and $\eta = 0$, $\eta = 1$.

Now let us attempt to reduce the singularity by cutting away the semiconductor, placing the electrode on top of a mesa, as shown in Fig. 6 for $\alpha = \pi/2$. For $\eta = 0$ this configuration gives a one-dimensional, regular field, normal to the electrode, so that we might expect that it would be advantageous for small η. With $\beta = 0$, $\alpha = \pi/2$, equation (6) becomes

$$\eta \sin \frac{p\pi}{2} \cos \frac{3p\pi}{2} + \cos \frac{p\pi}{2} \sin \frac{3p\pi}{2} = 0, \tag{8}$$

satisfied by $p = 1$, for all η. However, note that, for $\eta = 0$, it also has the smaller root $p = \frac{2}{3}$. If we examine the equations for A and B, we find that, in this case, $A = 0$, so that the root $p = \frac{2}{3}$ gives a singularity only in the insulator for $\eta = 0$. However, a perturbation for small positive η shows that A is of order η, so that the singularity is coupled back into the semiconductor for any positive η, no matter how small.

On the other hand, for $\eta = 1$, the smallest root of equation (8) is $p = \frac{1}{2}$. In this case p is independent of α, for $\eta = 1$ corresponds to a single dielectric filling the whole space around the electrode. Now, if

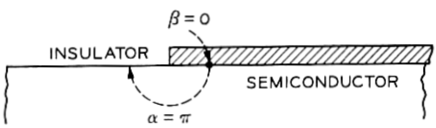

Fig. 5—Flat semiconductor surface.

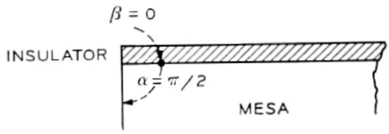

Fig. 6—Right-angled mesa corner.

p is a continuous function of η, its value for $0 < \eta < 1$ must lie between the value for $\eta = 0$ and the value for $\eta = 1$; that is, we must have

$$\tfrac{1}{2} < p < \tfrac{2}{3},$$

for $\beta = 0$, $\alpha = \pi/2$, and $0 < \eta < 1$. For small η, the singularity is weakened in some sense, but not removed, by the formation of a mesa.

These two special cases indicate that the calculation of $p = p(\alpha, \beta, \eta)$ is not completely straightforward. They also suggest that the simple cases $\eta = 0$, $\eta = 1$ can be used as a framework for the general calculation.

IV. LIMITING VALUES OF PERMITTIVITY RATIO

Let us first consider the case $\eta = 1$. In this case equation (6) becomes

$$\sin p(2\pi - \beta) = 0,$$

independent of α, as one would expect. Its smallest positive root is

$$p = p(\alpha, \beta, 1) = p_1(\beta) = \pi/(2\pi - \beta), \tag{9}$$

singular for $\beta < \pi$. Those familiar with potential theory will recognize this as the singularity at the tip of a wedge-shaped electrode, protruding into a uniform dielectric. It has been studied in detail by Wasow, Lehman, and Joyce.[4–6]

The other limiting case $\eta = 0$ gives two roots. Equation (6) becomes

$$\cos p\alpha \sin p(2\pi - \beta - \alpha) = 0,$$

with the roots

$$p = p(\alpha, \beta, 0) = p_0^-(\alpha) = \pi/2\alpha, \tag{10}$$

$$p = p(\alpha, \beta, 0) = p_0^+(\alpha, \beta) = \pi/(2\pi - \beta - \alpha). \tag{11}$$

The first of these roots gives a singular field in the semiconductor for $\alpha > \pi/2$; the second gives a singular field for $\alpha + \beta < \pi$ (in the insulator only, for $\eta = 0$) but weakly coupled back into the semiconductor for small positive η.

For $0 < \eta < 1$, p must lie between p_1 and the smaller of the two values p_0^-, p_0^+. Now $p_0^- > p_0^+$ for $\alpha < (2\pi - \beta)/3$, so that p lies between p_1 and p_0^+, for $0 \leqq \alpha \leqq (2\pi - \beta)/3$, and between p_1 and p_0^-, for $(2\pi - \beta)/3 \leqq \alpha \leqq \pi$. Also $p_0^- < p_1$, for $\alpha > (2\pi - \beta)/2$, so that we finally obtain the series of bounds listed below:

$$\frac{\pi}{2\pi - \beta} < p < \frac{\pi}{2\pi - \beta - \alpha}, \quad \text{for} \quad 0 < \alpha < \frac{2\pi - \beta}{3}, \tag{12}$$

$$\frac{\pi}{2\pi - \beta} < p < \frac{\pi}{2\alpha}, \quad \text{for} \quad \frac{2\pi - \beta}{3} < \alpha < \frac{2\pi - \beta}{2}, \tag{13}$$

$$\frac{\pi}{2\alpha} < p < \frac{\pi}{2\pi - \beta}, \quad \text{for} \quad \frac{2\pi - \beta}{2} < \alpha < \pi. \tag{14}$$

Now the largest value of the upper bound is attained at the point where the upper bound of equation (12), an ascending hyperbola as a function of α, meets the upper bound of equation (13), a descending hyperbola, that is at the point $\alpha = (2\pi - \beta)/3$, where $p = 3\pi/2(2\pi - \beta) = p_{\max}$. For $\beta < \pi/2$ this maximum is less than unity, so that a field singularity can be avoided only by choosing the electrode angle β greater than 90°. Similarly, the upper bound of equation (13) implies that the semiconductor angle α must be chosen less than 90° to avoid a field singularity.

A particularly simple way of satisfying these requirements is the combination of overhanging electrode ($\beta = \pi$) and slightly undercut semiconductor mesa ($\alpha \leqq \pi/2$) shown in Fig. 7. In order that the theory be applicable, the length of the overhang must be several characteristic lengths, i.e. several Debye lengths for small electrode potential and several depletion layer thicknesses for large electrode potential. The mesa corner needs to be undercut only enough to be certain that α is

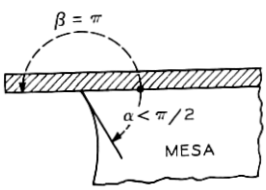

Fig. 7—Overhanging electrode on undercut mesa.

never greater than 90°, within fabrication tolerances. In Section V we document these preliminaries.

V. ARBITRARY PERMITTIVITY RATIO

For detailed calculations it is convenient to rewrite the characteristic equation (6) in the form

$$(1 + \eta) \sin p(2\pi - \beta) + (1 - \eta) \sin p(2\pi - \beta - 2\alpha) = 0 \qquad (15)$$

from which it is a simple matter to calculate the derivative

$$\frac{\partial p}{\partial \alpha} = \frac{2(1-\eta)p \cos p(2\pi-\beta-2\alpha)}{(1+\eta)(2\pi-\beta) \cos p(2\pi-\beta)+(1-\eta)(2\pi-\beta-2\alpha) \cos p(2\pi-\beta-2\alpha)}. \qquad (16)$$

Now, whereas it is difficult to solve equation (6) directly for p as a function of α, β, η, because of the existence of neighboring higher roots, it is a simple matter to integrate equation (16), starting from $\alpha = 0$, where $p = \pi/(2\pi - \beta)$ for all η. This calculation was carried out for $\eta = 0, 0.1, 0.3, 0.5, 0.7, 1.0$ and for $\beta = 0, \pi/2, 3\pi/4, \pi$. The results are shown in Figs. 8 through 11, for the two lowest branches of p.* As Section IV predicts, a nonsingular field ($p \geqq 1$) becomes possible only for $\beta \geqq \pi/2$, with the range of permissible values of α and η, increasing from $\alpha = \pi/2$, $\eta = 0$, at $\beta = \pi/2$, to $0 < \alpha < \pi/2$, $0 < \eta < 1$, at $\beta = \pi$.

This range of values of α for given β and η is easily found. If we set $p = 1$ in equation (15), we find that it is satisfied by two values of α, which bound the permissible range for a nonsingular field. Figure 3 shows the result of this elementary calculation. For example, for $\beta = 3\pi/4$, $\eta = 0.1$, the field is singular for $\alpha < 0.291\pi = 52.5°$, regular for $0.291\pi < \alpha < 0.459\pi = 82.5°$, and singular again for larger α. The slanting dashed line gives the minimum value of β, and the corresponding value of α, for which the field is nonsingular for given η.

VI. ACKNOWLEDGMENT

This problem was suggested originally by S. Sze. The authors profited from several illuminating discussions with J. McKenna. The calculations were carried out by Miss Judith B. Seery.

* Two branches are shown to indicate the topology, although of course only the lower branch is of interest.

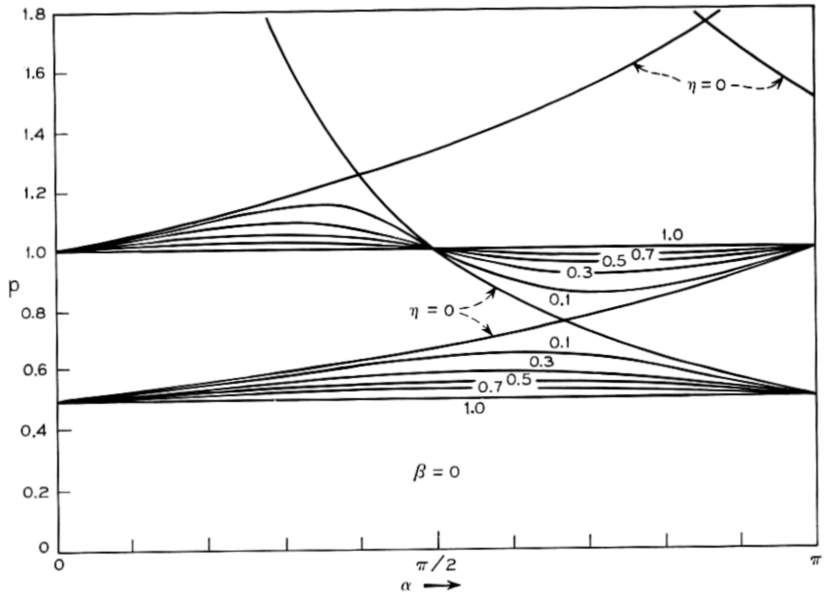

Fig. 8—$p(\alpha)$ for $\beta = 0$ and various η.

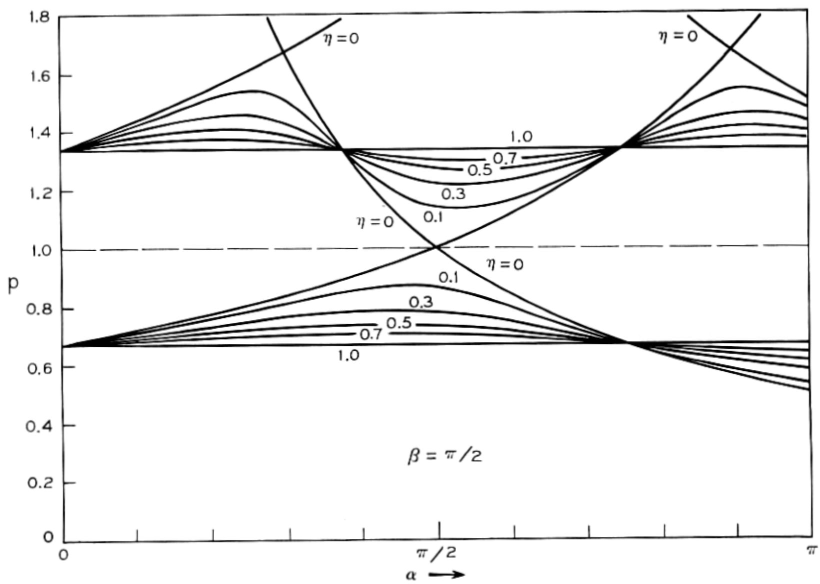

Fig. 9—$p(\alpha)$ for $\beta = \pi/2$ and various η.

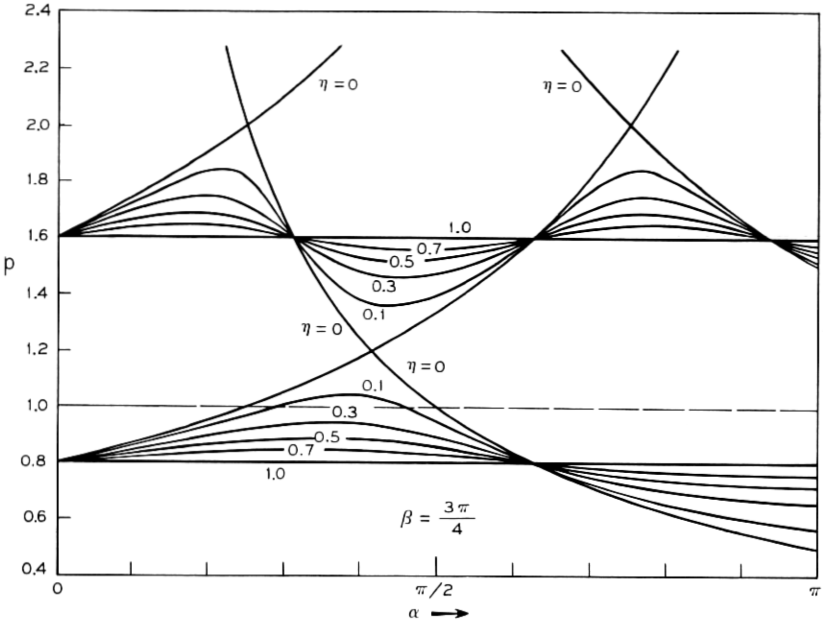

Fig. 10—$p(\alpha)$ for $\beta = 3\pi/4$ and various η.

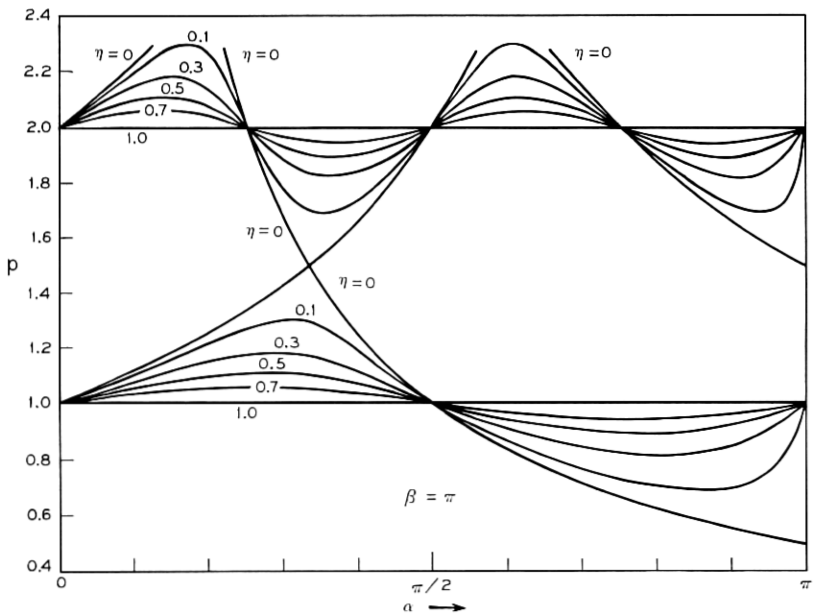

Fig. 11—$p(\alpha)$ for $\beta = \pi$ and various η.

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