Transit-Time Variations in Line-of-Sight Tropospheric Propagation Paths

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We present in this paper transit-time variations in line-of-sight propagation paths and systems operating at frequencies up to 30 GHz. We discuss variations due to both atmospheric changes (no precipitation) and rain and point out some relationships to PCM systems.

I. INTRODUCTION

In a recent paper[1], J. R. Pierce considered stable synchronization of large digital transmission networks, and pointed out that the realization of such a synchronized network calls for, among other things, more information concerning network transit-time variations. In this paper, we seek extreme values for these variations in line-of-sight propagation paths in order to provide some of this information. Estimates are given for (*i*) the maximum variation in transit-time, $\Delta\tau_{\max}$, which one might expect over the period of a year, and (*ii*) the maximum time derivative, $\dot{\tau}_{\max}$, which one might encounter. The estimated values are related to digital systems, with most specific examples given for a 500 megabit transmission rate. Variations due to changes in the atmosphere (no precipitation), and those due to rain are discussed separately. In the longer line-of-sight paths achieved in tandem systems, repeaters are assumed to be stable, that is, the concern herein is with atmospheric variations only. Delays associated with selective fading are not discussed, but they are believed not to exceed the given estimates of the maximum variations.

The transit-time τ is given by the familiar relationship

$$\tau = \frac{1}{c} \int_{P_1}^{P_2} n \, ds \tag{1}$$

where c is the velocity of light, n the medium refractive index, ds the differential path length, and the limits P_1 and P_2 represent the end points of the path. If one assumes Δn_{max}, the maximum index of refraction change which is expected over a given period of time, and if one assumes that the changes in n over the entire path S are perfectly correlated and equal to Δn_{max}, then the corresponding maximum transit-time variation, $\Delta\tau_{max}$, is given by

$$\Delta\tau_{max} = \frac{1}{c}(\Delta n_{max})S. \tag{2}$$

In a subsequent section, $\Delta\tau_{max}$ will be computed after estimating Δn_{max}. A similar set of assumptions concerning the time derivative $\dot{n}$ leads to an estimate of $\dot{\tau}_{max}$, namely,

$$\dot{\tau}_{max} = \frac{1}{c}\dot{n}_{max}S. \tag{3}$$

II. MAGNITUDE OF TRANSIT-TIME VARIATIONS IN THE ATMOSPHERE

Letting the superscript a denote the atmosphere, the estimate of Δn^a_{max} to be used in this section is based on data found in Bean and Dutton.[2] The data comprise eight years of point observations at six locations in the United States. The locations are listed in Table I where the maximum, minimum, and range Δn of the index of refraction are given in N units, with $N = (n - 1) \times 10^6$. A value which approximately strikes an average for the six locations listed will be used, namely, $\Delta n^a_{max} = 1.0 \times 10^{-4}$. Substituting in equation (2),

$$\Delta\tau^a_{max} = 0.333 \times 10^{-9}S \tag{4}$$

where S is in kilometers. Equation (4) is plotted in Fig. 1. Since the atmospheric index of refraction, n^a, may be regarded as independent of frequency up to 30 GHz[2], the curve in Fig. 1 may be considered applicable up to this frequency.

TABLE I—EXTREMES IN INDEX OF REFRACTION FOR SIX LOCATIONS IN THE UNITED STATES

Location	max N	min N	ΔN
Washington, D. C.	393	277	116
San Antonio, Texas	387	258	129
Bismarck, N. D.	368	260	108
Colorado Springs, Col.	307	214	93
Salt Lake City, Utah	323	230	93
Tatoosh Island, Wash.	354	292	62

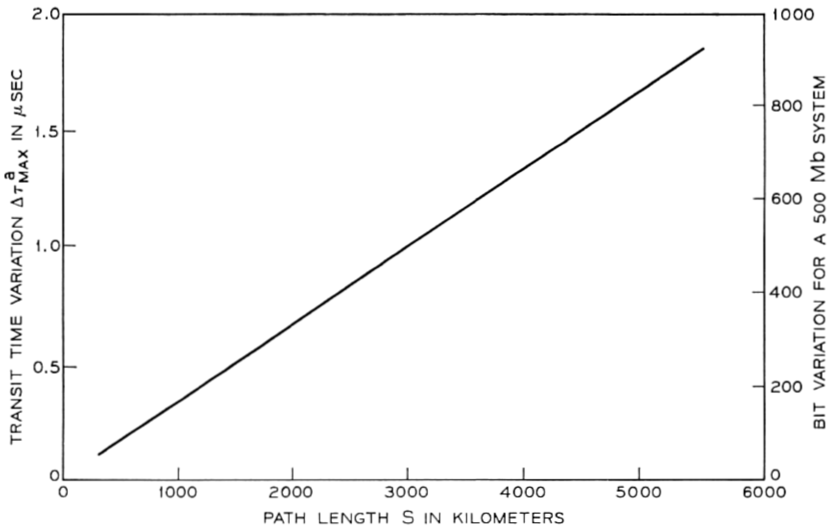

Fig. 1—Estimated maximum atmospheric (no precipitation) transit-time variations for long path length line-of-sight tropospheric communications systems.

For long paths, $\Delta\tau^a_{\max}$ is overestimated because the assumption that Δn is perfectly correlated and equal over the entire path becomes unrealistic as S increases. However, the overestimation is not excessive because those components in Δn which primarily are due to seasonal and diurnal variations are highly correlated and combine to be of the order of 50 N units (approximately 30 N units for seasonal and 20 N units for diurnal). Thus, $\Delta\tau^a_{\max}$ is conservative to within a factor of two for the continental United States. As an example, for a 3000-mile path in Fig. 1, $\Delta\tau^a_{\max} = 1.6\ \mu s$; certainly, it would be no less than 0.8 μs. In terms of a 500 Mb system, $\Delta\tau^a_{\max} = 1.6\ \mu s$ is equivalent to an 800 bit variation.

For short path lengths, more detailed data are needed and these are in Fig. 2. One notes that, for short paths, a Δn^a which typifies a given region should be used instead of the $\Delta n^a_{\max}$ of 100 N units. Therefore, Fig. 2 shows $\Delta\tau_{\max}$ versus path length for each of the six locations listed in Table I.

Now consider the problem of synchronizing two clocks separated by a distance S. For a digital system of pulse spacing T, two clocks may be considered synchronized if they are in phase to within a factor f of a pulse spacing. For the sake of argument in this discussion, we choose $f = 0.1$, and we draw on Fig. 2 the lines $f \cdot T$ for pulse rates of 50, 100,

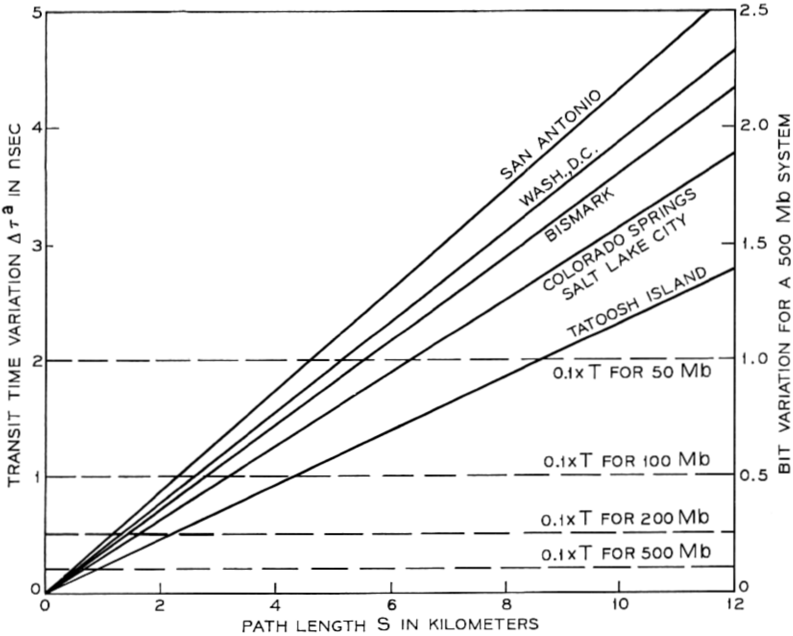

Fig. 2—Maximum atmospheric (no precipitation) transit-time variations for short paths. Variations are shown for six locations in the United States.

200, and 500 Mb. The intersections of the lines $f \cdot T$ with the curves for $\Delta\tau_{\max}$ determine the maximum path lengths for which atmospheric transit-time variations may be neglected. For instance, these lengths are respectively 5, 2.5, 1.25 and 0.5 km for the 50, 100, 200, and 500 Mb rates when the region of concern is Washington, D. C. For some other criterion of synchronization, that is for some other f, these path lengths, of course, will be different.

From Fig. 2, it is evident that for any but the shortest path lengths, some compensation for refractive index changes must be incorporated in a digital system running on a universal clock. Figures 1 and 2, however, do not provide information on how rapidly one must compensate. This question is answered by first considering the time derivative of transit-time variations which are due to turbulence, and then by considering those due to the motion of synoptic scale air masses.

III. TIME DERIVATIVE OF TRANSIT-TIME VARIATIONS

Table II shows the rms values of the hourly transit-time variations[3,4] which are due to atmospheric turbulence. Because of the randomness

TABLE II—RMS HOURLY TRANSIT-TIME VARIATIONS IN THE TURBULENT ATMOSPHERE

RMS Delay (seconds)	Path Length (miles)	Frequency (GHz)
1.5×10^{-12}	2.25	10
4×10^{-12}	3.5	1
15×10^{-12}	10	1
22×10^{-12}	60	1

of turbulent motion, one would expect the rms values to be proportional to the square root of the path length S. Thus, on an hourly basis, the variations for a transcontinental path work out to be 0.2 ns, which is one tenth of the pulse spacing in a 500 Mb system. On the basis of a five minute interval, measurements show that rms variations are two orders of magnitude smaller than the hourly changes.[4] Thus it appears that compensation for turbulence-induced atmospheric transit-time variations can be made on the order of tens of minutes for transcontinental links, and even more slowly for shorter paths.

In contrast with the turbulence-related phenomena discussed above, the motion of synoptic-scale air masses brings about changes in index-of-refraction which are correlated over large regions, that is, for regions covering hundreds of kilometers, changes in n will be proportional to path length, S, not to $(S)^{\frac{1}{2}}$. For instance, an advancing cold front brings with it a decrease in temperature θ, a decrease in water vapor pressure e, an increase in total pressure P, and an accompanying change ΔN_F which is well correlated over the entire frontal advance.

For the purpose of discussion, a model front is shown in Fig. 3. The model front passes transversely across a microwave transmission path. The index of refraction change ΔN_F occurs over a distance S_Δ, and the

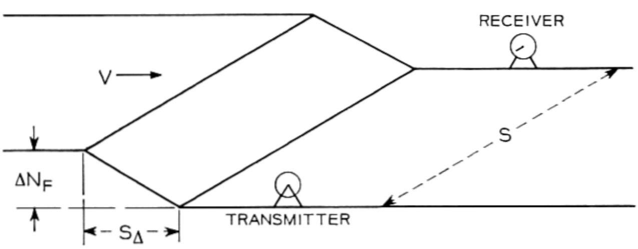

Fig. 3—Model of a frontal system moving across a line-of-sight transmission path.

front moves at velocity v. This model permits one to estimate the maximum rate of change of transit-time $\dot{\tau}_{\max}$.

Proceeding with the estimate of $\dot{\tau}_{\max}$, from Fig. 3,

$$\dot{n}_{\max} = 10^{-6} \frac{\Delta N_F}{S_\Delta} \cdot v \Big|_{\max} = 10^{-6} \frac{\Delta N_F}{t_\Delta} \Big|_{\max}$$

where $t_\Delta = S_\Delta / v$. Substituting in equation (3),

$$\dot{\tau}_{\max} = 0.33 \times 10^{-11} \frac{\Delta N_F}{t_\Delta} \Big|_{\max} S. \tag{5}$$

Estimates of the quantity $\Delta N_F / t_\Delta \mid_{\max}$ have been obtained from temperature, pressure, and relative humidity data continuously recorded at Crawford Hill since January 1, 1967. The records were examined for events of rapidly changing atmospheric conditions. N was computed for conditions just prior to and just after these events using the formula[2]

$$N = 77.6 \frac{P}{\theta} + 3.73 \times 10^5 \frac{e}{\theta^2} \tag{6}$$

where θ is expressed in kelvins, and P and e in millibars. ΔN_F was taken as the difference between the two computed values for each event. Table III gives the date, time, ΔN_F, t_Δ, and $\Delta N_F / t_\Delta$ for the largest events recorded. It is noted from the Table that $\Delta N_F / t_\Delta \mid_{\max} = .0788$. Rounding off to .08, and substituting in equation (5),

Table III—Index of Refraction Changes Accompanying Rapidly Varying Atmospheric Conditions

Date	Time	ΔN_F	t_Δ (sec)	$\Delta N_F / t_\Delta$
2/28/67	10:30 PM	7.23	900	.00804
7/14/67	3:00 PM	3.94	600	.00657
10/3/67	9:00 PM	−13.67	300	.0456
2/17/68	2:00 PM	13.63	300	.0455
3/29/68	3:30 PM	15.93	900	.0177
4/30/68	6:00 PM	7.55	300	.0252
6/3/68	4:00 PM	13.75	600	.0229
7/2/68	8:00 PM	22.18	900	.0246
7/24/68	2:30 PM	19.84	300	.0662
8/7/68	2:30 AM	18.67	600	.0311
8/15/68	4:00 AM	−31.75	1200	.0264
8/17/68	3:30 AM	6.18	300	.0206
8/22/68	9:40 PM	−2.42	300	.00807
11/29/68	2:00 AM	3.67	180	.0204
12/5/68	2:00 PM	14.18	180	.0788
6/13/69	2:30 PM	10.12	600	.0169
6/24/69	5:30 PM	7.49	300	.0250

$$\dot{\tau}_{\max} = 2.67 \times 10^{-13} S. \tag{7}$$

The maximum value S over which good correlation of N can occur will be taken as 500 km (this value will be discussed subsequently). From equation (7), $\dot{\tau}_{\max} \cong 1.36 \times 10^{-10}$ sec/sec. In terms of a 500 Mb system, $\dot{\tau}_{\max} \cong .0667$ b/s, or 1/16th of a bit per second.

The derived $\dot{\tau}_{\max}$ is thought to be larger than would be encountered in practice. Complete parallelism (or coincidence), as shown in Fig. 3, becomes highly improbable when the path length S approaches 500 km. Primary reasons are: (*i*) a front and a transmission path can be oriented at angles ranging over a large fraction of π radians; (*ii*) nature will not produce fronts composed of straight line segments, but rather of curves. Since a line-of-sight propagation route is made up of straight line segments, it will be improbable that a front would coincide with it. Consequently, it appears that the choice of $S = 500$ km is extreme, and that $\dot{\tau}_{\max} = 1/16$ b/s at 500 Mb is a good upper bound.

IV. TRANSIT-TIME VARIATIONS DUE TO RAIN

Letting the superscript r denote rain, then Δn^r is specified using computations of the medium refractive index for given rainfall rates under the assumption of a Laws & Parsons drop size distribution.[5] These computations include the index of refraction of the medium for 6, 16 and 30 GHz for the rain fall rates 0.25, 1.25. 2.5, 5.0, 12.5, 25.0, 50.0, 100.0, and 150.0 mm/hr. For rainfall rates exceeding 150 mm/hr, proportional scaling of the 150 mm/hr medium index will be used.

The nature of rainfall is such that higher fall rates are associated with smaller areas of coverage. For this reason, long and short path length transit-time variations will be treated differently. For path lengths of the order of hundreds of kilometers, we will assume that average-path rainrates of about 10 mm/hr can occur. Using a 10 mm/hr rainfall rate, Δn^r is equal to 0.83 N units at 6 and 16 GHz, and 0.67 N units at 30 GHz. The corresponding values of $\Delta \tau^r$ are plotted versus path length in Fig. 4. Comparison of Fig. 4 with Fig. 1 shows that the transit-time variations one expects from rain are more than an order of magnitude smaller than those expected from the atmosphere itself. This implies that as far as the dynamic range of compensation equipment in synchronized digital systems is concerned, atmospheric variations (other than rain) are the determining factor for long paths.

For short paths, rainfall rates much greater than 10 mm/hr often occur. Using results for New Jersey,[6] transit-time variations for short paths are computed using the refractive index values found in Table

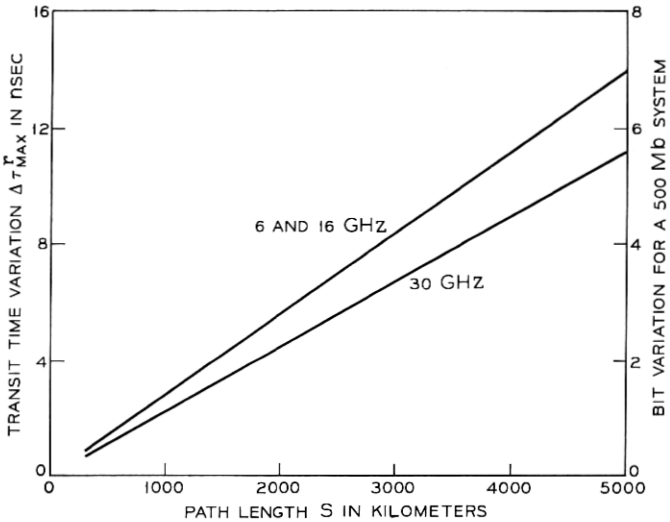

Fig. 4—Estimated maximum rain-caused transit-time variations for long path length line-of-sight communication systems.

IV; they are plotted in Fig. 5 with P as a parameter, where P equals the number of minutes per year the given variation will be exceeded. Variations to be exceeded 0.5 min/year and 5 min/year are shown, and as before, the rainfall variations are an order of magnitude smaller than the atmospheric variations (Fig. 2) for the corresponding path lengths.

When viewed in terms of the nature of convective showers (showers exhibiting high rainfall rates), Fig. 5 aids in estimating the upper bounds for the time derivative of rain-caused transit-time fluctuations. Convective showers can introduce large attenuation in a path rapidly with onsets of the order of tens of seconds. In Fig. 5 the intersection of the criterion line for a 500 Mb system with the 16 GHz curve for 0.5 min/yr occurs at a path length of 10 km. If the onset of the rain over this path were to occur in 10 seconds, then $\dot{\tau}_{max} = .01$ b/s, which is a factor of 6 smaller than the time derivative for the fronts considered previously.

IV. CONCLUSIONS

The maximum transit-time variations encountered in the troposphere are due primarily to changes in the gaseous atmosphere rather than rain, and may amount to 1.6 μs for a transcontinental path; this converts to 800 bits in a 500 Mb system. The maximum time derivative of the

TABLE IV—INDEX OF REFRACTION VARIATIONS EXCEEDED *P* MINUTES PER YEAR FOR VARIOUS PATH LENGTHS

(a) $P = 5$ min/yr.

Path Length (km)	Path Average Rainfall Rate (mm/hr)	Δn^r at 6 GHz (N units)	Δn^r at 16 GHz (N units)	Δn^r at 30 GHz (N units)
1.3	140	9.44	7.42	4.64
2.6	135	9.11	7.18	4.51
5.2	110	7.46	5.98	3.86
7.8	90	6.14	5.02	3.3
10.4	80	5.48	4.54	3.0

(b) $P = 0.5$ min/yr.

Path Length (km)	Path Average Rainfall Rate (mm/hr)	Δn^r at 6 GHz (N units)	Δn^r at 16 GHz (N units)	Δn^r at 30 GHz (N units)
1.3	190	12.8	10.1	6.21
2.6	175	11.78	9.3	5.72
5.2	150	10.1	7.9	4.9
7.8	130	8.78	6.94	4.38
10.4	110	7.46	5.98	3.86

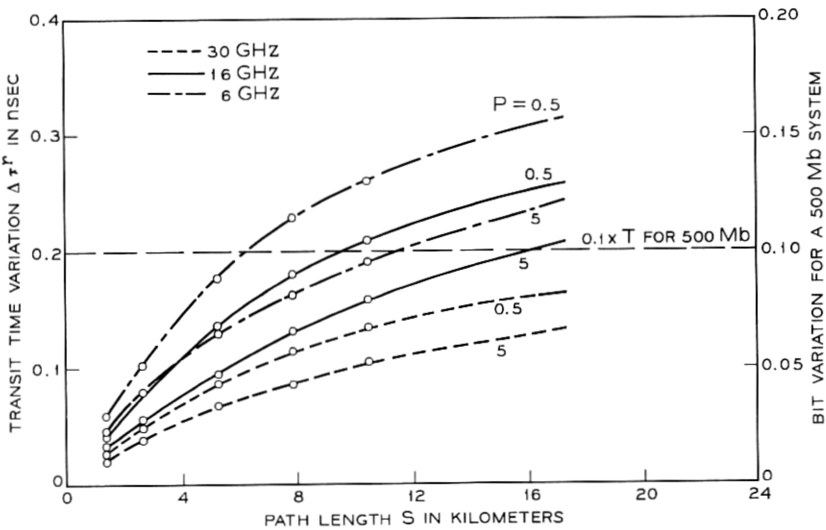

Fig. 5—Rain-caused transit-time variations for short path lengths. The parameter *P* equals the number of minutes/year the given variation will be exceeded.

transit-time variation is due to the motion of fronts, and has been derived herein: $\dot{\tau}_{max} = 1.36 \times 10^{-10}$ sec/sec which is equivalent to 1/16th of a bit at 500 Mb. Neither $\Delta\tau^a_{max}$ nor $\dot{\tau}_{max}$ appears so large as to prohibit the synchronization of digital systems transmitted over line-of-sight propagation paths.

V. ACKNOWLEDGMENTS

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