

Reed-Contact Switch Series for the I.F. Band

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A series of switches using a miniature dry-reed sealed contact in a cable switch configuration has been developed to provide switching capability from dc to 100 MHz. We present a description of the development, performance characteristics, and mechanical design features.

I. INTRODUCTION

The nationwide network of transmission facilities is not only growing in number of routes and capacity but also in terms of service capability and administrative flexibility. Within the network there are usually alternate routes for providing service between two points. Interconnection between points may ultimately be controlled by a remote, centralized, real time machine that contains an accurate map of the state of the network.

The broadband restoration system, for example, can detect failures, make routine maintenance checks and report to a regional control center where an alternate route between the two points is selected. The control center then remotely operates the wideband switch at each junction of the route to effect a restoration of service.

One component group needed to implement these systems is a family of wideband switches capable of meeting the transmission requirements of low insertion loss, high isolation loss, high crosstalk loss, and having an impedance well matched to the 75-ohm system impedance.

The 266B (8×8 matrix), 274A (1×8), and 273B (1×2) switches have been developed to meet these requirements with low operate power, small size, and moderate cost. All of these codes use 237-type miniature dry-reed sealed contacts in a cable-switch¹ arrangement to provide an extremely high isolation loss in the open state and a low insertion loss and good impedance match in the closed state. Appropriate matrix configurations are achieved by interconnecting the cable switches with

stripline networks designed to provide good system performance from dc to 100 MHz. The requirements, performance characteristics, mechanical design features, and a description of the development of these new wideband switching matrices are presented in this paper.

II. REQUIREMENTS

The restoration transmission requirements for an 8×8 matrix of 64 crosspoints used to interconnect 75-ohm coaxial transmission paths over the frequency range of dc to 100 MHz are:

Insertion loss (closed-contact loss)	0.6-dB maximum
Isolation loss (open-contact loss)	95-dB minimum
Crosstalk loss	95-dB minimum
Return loss	28-dB minimum

Transmission requirements for the 1×2 matrix and the 1×8 matrix are identical to those of the 8×8 matrix except for the crosstalk loss requirement, which is not pertinent in single-level matrices. Where more than one switch is enclosed in the same housing, the crosstalk requirement will apply between switches.

Speed of switching is not a stringent requirement, and operation in the millisecond range is satisfactory. Compact matrix size and moderate manufacturing cost are additional features required for practical application in the restoration switching systems.

The switch arrays, 8×8 , 1×8 , and 1×2 are illustrated schematically in Fig. 1. Photographs of the three switch types are shown in Fig. 2. The major elements of the design in a transmission sense are the coaxial crosspoint developed from the cable switch, the input/output circuit boards, and the coaxial jacks. From the schematic diagram, one can see that in the 8×8 and 1×8 designs the closure of any crosspoint leaves seven open crosspoints connected by stubs on each associated circuit board or "tree." Because of the length of these stubs, the structural considerations become important design parameters that seriously affect performance. The design considerations in each of these elements are presented in the sections that follow.

III. CABLE SWITCH THEORY

An extremely high isolation-loss requirement in the megahertz range normally precludes the use of conventional electromechanical switches,

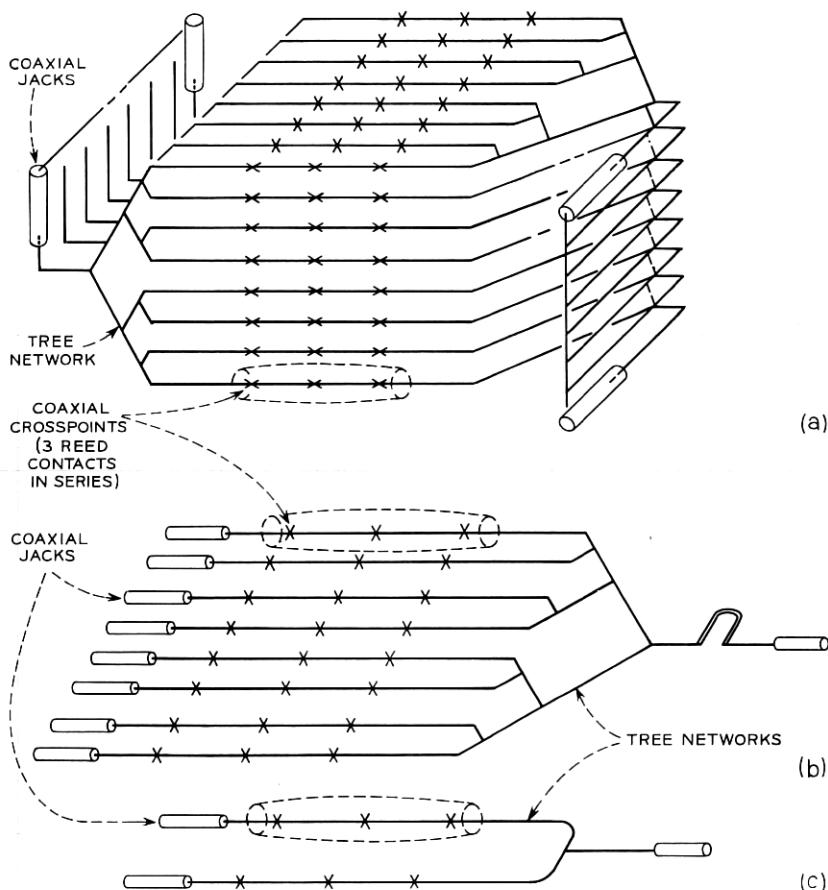


Fig. 1 — Array schematic: (a) 8×8 , (b) 1×8 , (c) 1×2 .

such as the wire spring and flat spring relay, the crossbar switch, and the ferreed, because of their generally large open-contact capacitance of between 0.1 and 1.0 pF. However, an arrangement of two or more conventional switching elements connected in series by a length(s) of low-loss transmission cable is particularly well suited for operation in broadband switching applications where extremely high isolation is required. This broadband switching crosspoint is called the cable switch.

Applying conventional lumped constant analysis to a string of open switches (that is, serially connected switches with substantially zero transmission paths between them) produces the following conventional and well known voltage divider approximation expression:

$$|V_{\text{out}}/V_{\text{in}}| = \omega CR/K + 1 \quad (1)$$

when $\omega CR \ll 1$ and where

- ω = angular frequency,
- C = open switch capacitance,
- $K + 1$ = number of switches, and
- R = the load impedance.

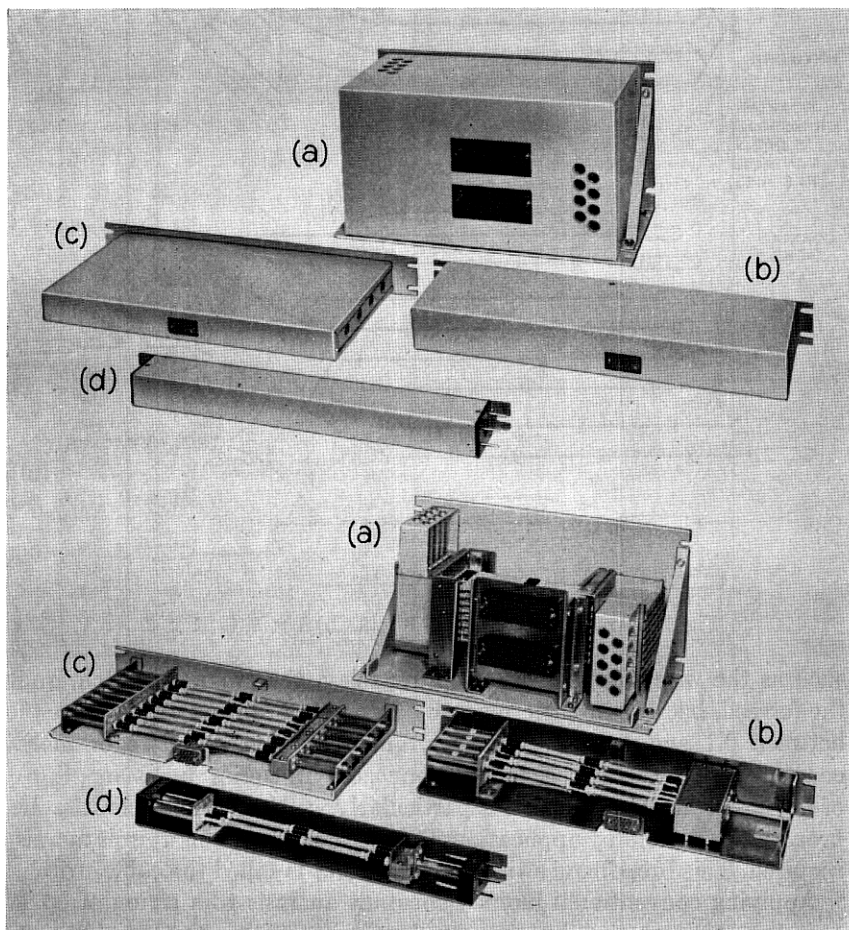


Fig. 2—Photograph of switch arrays: (a) 8×8 , (b) 1×8 , (c) 1×2 (4 switches per package), (d) 1×2 (1 switch).

In other words, each switch added to the string reduces the ratio by adding one to the denominator.

The above expression does not apply, however, when coaxial cables are connected between the switches. In particular, for $(K + 1)$ switches with K pieces of identical lossless coaxial cables interconnecting them, the following expression applies (see Appendix):

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{\omega CR}{A_K + (A - \omega CZ_0 \sin \beta d) \sum_{n=0}^{K-1} A^{K-1-n} A_n}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{\omega CR}{\sum_{n=0}^K A^{K-n} A_n} \tag{2}$$

for practical components where the values of A_n are given in the following table:

n	A_n
0	1
1	A
2	$2A^2 - 1$
3	$4A^3 - 3A$
⋮	⋮
K	$2AA_{K-1} - A_{K-2}$

and $A = \cos \beta d + \sin \beta d / 2\omega CZ_0$ for lossless lines where: $\beta = \omega(\epsilon_R \mu_R / c)^{\frac{1}{2}}$ (phase constant).

ϵ_R = relative dielectric constant of coaxial cable,

μ_R = relative permeability constant of coaxial cable,

c = velocity of light,

d = distance between contacts from switch to switch,

Z_0 = characteristic impedance of coaxial cable in ohms, and the remaining symbols have the same meanings as in equation (1).

When the length of transmission line, d , between two switching elements equals one-quarter wavelength, equation (2) indicates that the isolation loss in dB of the overall switch is twice the isolation loss of the individual switching element. However, a plot of equation (2) for the

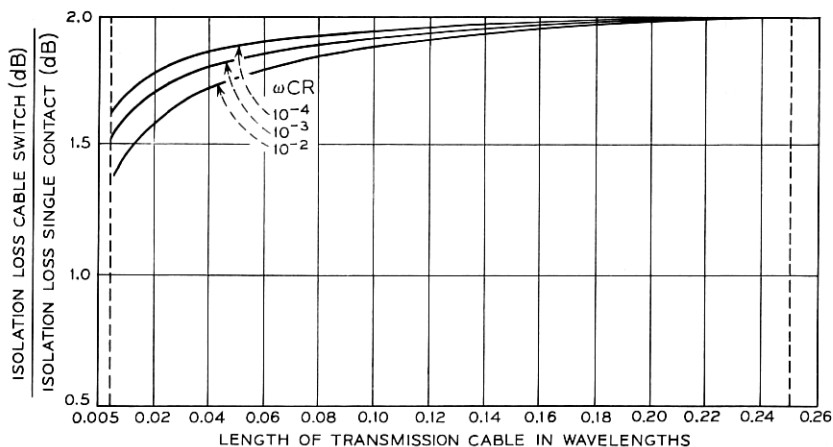


Fig. 3—Isolation loss improvement for the cable switch. Multiplying factor for the isolation loss in dB of a two-element cable switch as a function of cable length between switching elements. ($Z_0 = R$).

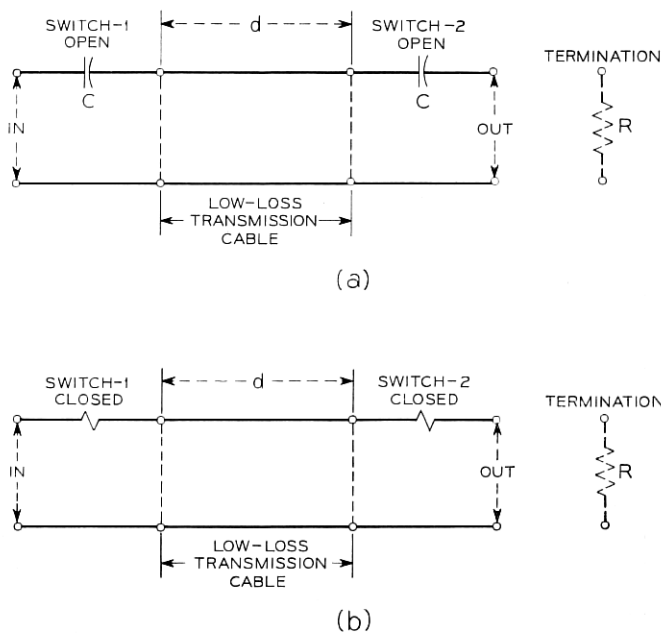


Fig. 4—Equivalent circuit for the cable switch: (a) open, (b) closed.

specific case of two switching elements (Fig. 3) shows that greater than one and one-half times the isolation loss in dB of a single switching element can generally be realized with a length of cable of only 0.005 wavelength. In addition, for short lengths of transmission cable the increase in isolation loss of the cable switch over that of a single switching element is relatively independent of frequency. This results in an extremely broad frequency bandwidth of operation.

As equation (2) indicates, a further increase in isolation loss can be obtained by adding more cable sections and switching elements to the structure. This, of course, increases the insertion loss as well as the physical length of the cable switch. Alternately, a choice of Z_0 less than the system impedance will result in a further increase in isolation loss. However, in practice Z_0 is chosen equal to the system impedance in order to avoid an impedance mismatch between the switch input and the termination when the switching elements are closed.

Figure 4 shows a schematic representation of the open and closed

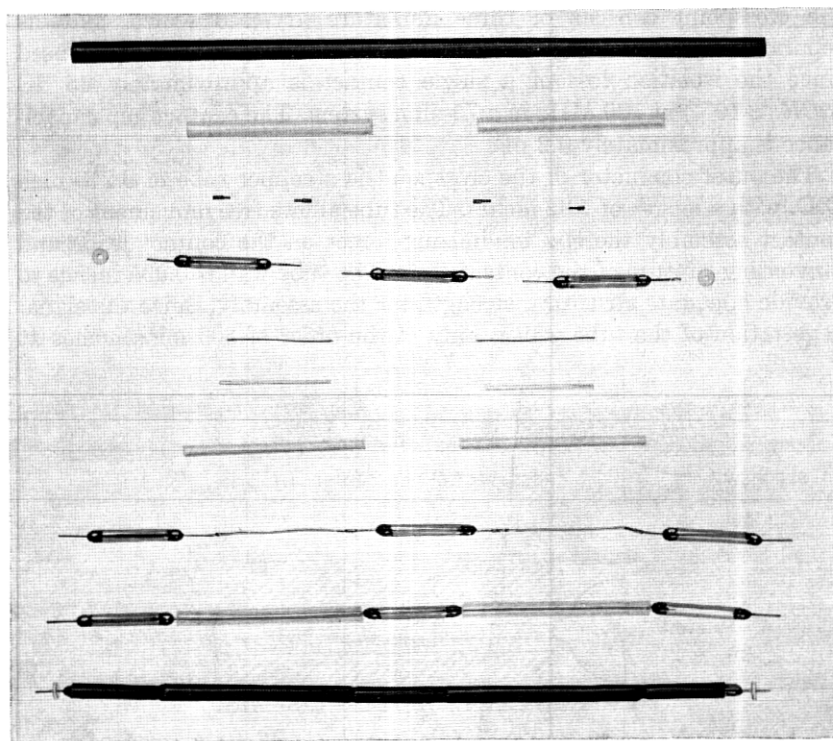


Fig. 5. — Crosspoint elements and assembly.

cable switch in which each switching element is approximated by a small closed-contact resistance. For the closed state, the insertion loss of the cable switch is about equal to the combined loss of the individual switching elements since the loss of the short transmission cable is negligible, and the impedance of the cable is generally equal to the system impedance. This factor of two in the insertion loss for a two-element cable switch is the only penalty in achieving the marked improvement in isolation loss desired.

The cable switch concept reduces crosstalk between various signal paths in the switching matrix because whenever the switch structure is effectively shielded, crosstalk will be defined in terms of the isolation loss of the individual crosspoints.

IV. COAXIAL CROSSPOINT DESIGN

The crosspoint assembly and its piece parts are shown in Fig. 5. A cross-sectional view of the assembly is shown in Fig. 6. As can be seen, the crosspoint consists of three miniature dry-reed sealed contacts separated by short lengths of coaxial line. Three contacts are used since the isolation loss of a single contact is approximately 45 dB ($\omega CR \approx 10^{-2}$) at 100 MHz in a 75-ohm system. The contact gap capacitance is approximately 0.2 pF.

The outer conductor of the crosspoint is a copper tube of 0.210 inch O.D. with a length of 7.82 inches. The tube allows free movement of the contact assembly thereby minimizing forces on the contact leads and preventing rupture of the contact seals. The tube wall is 0.005 inches to provide adequate structural strength for the assembly. Since the signal penetration of the tube wall is only of the order of 300 microinches at

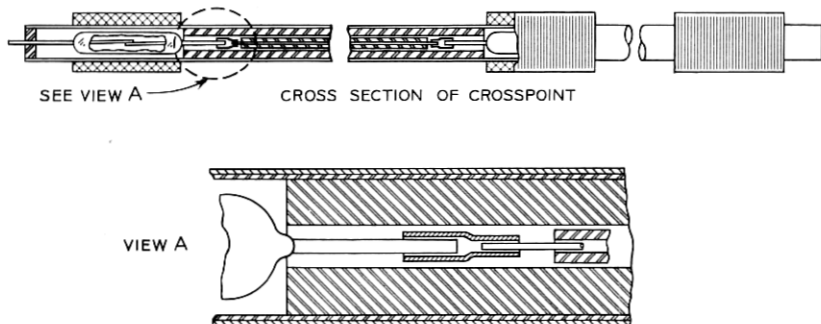


Fig. 6 — Cross-section of crosspoint assembly.

100 MHz, the integrity of the coaxial transmission line is preserved, and there is no crosstalk coupling to the control windings.

The tube length provides approximately three inches between the contact make points with two inches of coaxial line between the glass bottles of the contacts. Since this line segment is loaded with a molded polypropylene sleeve whose dielectric constant is 2.3, the equivalent electrical length is about three inches or 0.03 wavelength at 100 MHz. Equation (2) indicates that the open circuit isolation loss in dB of this three-switch crosspoint approaches 2.3 times that of a single contact.

The diameter of the center conductor and the dielectric material between the contacts proved to be one of the most easily changed variables, and a wide range of diameters and materials were evaluated. The diameter of the copper center conductor that gave the best return loss in the 8×8 matrix was found to be 0.010 inches ($Z_0 = 120$ ohms). The higher impedance (with respect to 75 ohms) of the center conductor section is required to offset the capacitance of the tree networks as discussed in Section 5.1.

The yield strength of the annealed copper conductor between contacts is reached with 1.25-lbs force so that any stresses applied to the crosspoint will be absorbed by the center conductor thereby protecting the contact seal.

In initial switch models, the center conductor was wrapped around

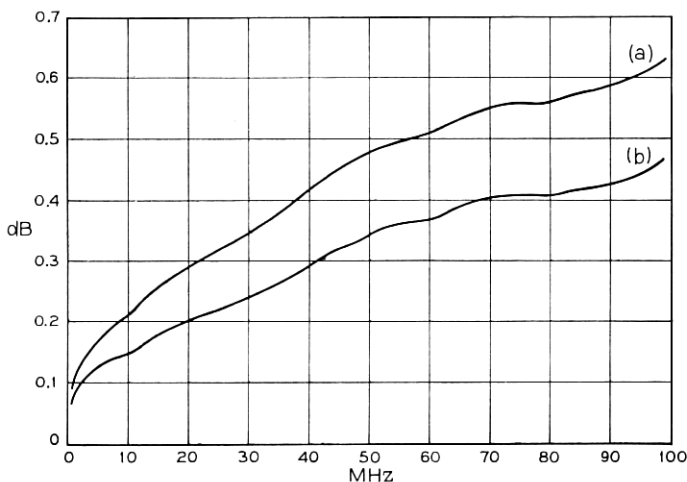


Fig. 7—Crosspoint insertion loss: (a) with 237B contacts, (b) with 237G contacts.

the contact leads and soldered. Although performance was adequate, experience has indicated the use of the connecting sleeve, Figs. 5 and 6, between the contact and the center conductor facilitates manufacture.

A dielectric end plug fits over the contact lead and into the copper tube. This plug supports the lead during the assembly process. A heat shrinkable polyethelene tube is shrunk over the entire assembly to provide mechanical stability in manufacture.

The insertion loss of the 237B contacts was found to be of the order of 0.15 dB at 100 MHz. This high loss characteristic occurs because only a relatively short length of the contact blade is plated with gold and silver. To provide a continuous surface conductor along the blade with a better conductivity than the nickel-iron blade alloy, a barrel plating process was developed by the Western Electric Company, Allentown Works. These barrel-plated blades are assembled in a recently coded contact, the 237G. The leads in this contact are solder dipped for 0.1 inch to provide easy assembly and a good bond with the

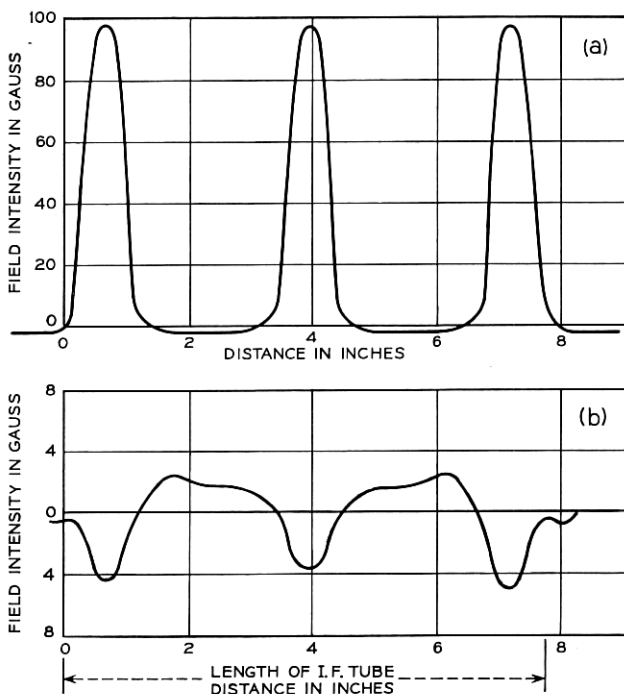


Fig. 8—Magnetic flux distribution along the axis of a crosspoint: (a) operated, (b) interference from diagonally operated crosspoints in an 8×8 array.

sleeve. The insertion loss characteristics of the cable switch when made from the standard 237B contact and the new 237G contact are contrasted in Fig. 7.

Each contact is driven by a 300 turn, 5 layer, 32 gage coil. The coils of a given crosspoint are series connected giving an overall resistance of

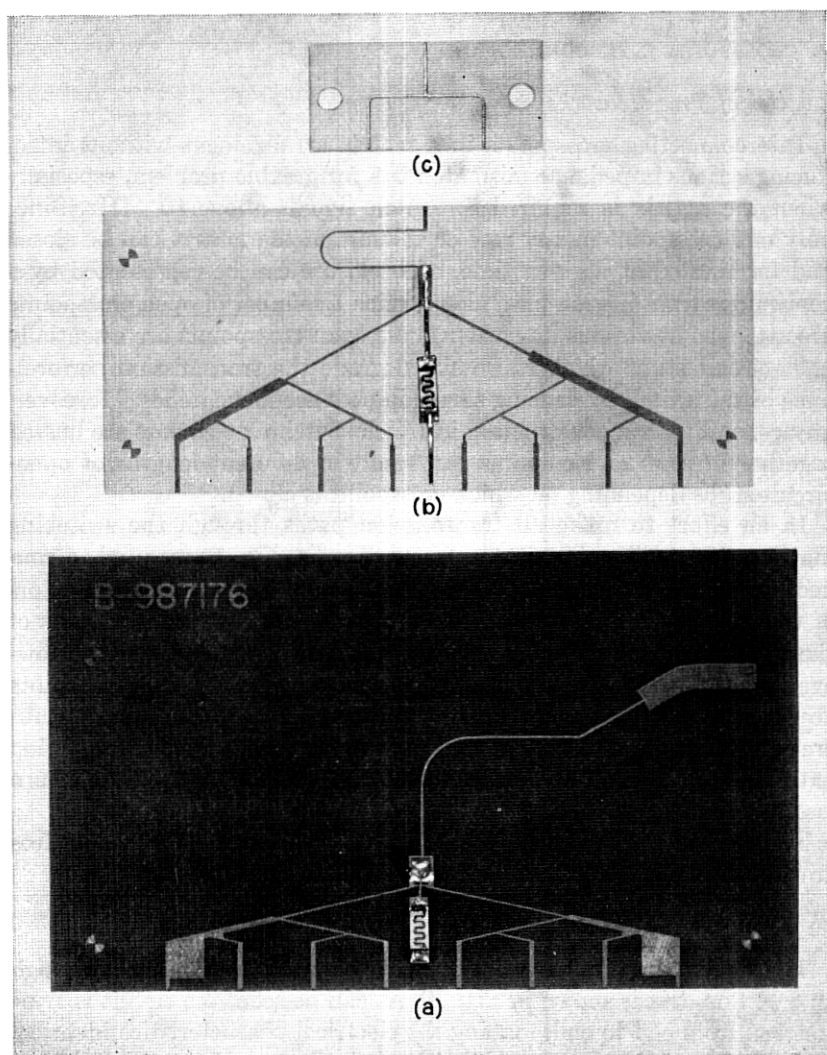


Fig. 9 — Tree networks: (a) 8×8 , (b) 1×8 , (c) 1×2 .

about 11 ohms. Figure 8a shows the flux distribution along the axis of an operated crosspoint. Figure 8b shows the interfering flux from diagonal crosspoints in the 8×8 array. Current reversal in alternate columns of the array provides an interfering flux cancellation so the resulting flux level is well below the contact operate levels when driven at the design level of 4.5 ± 1 volt.

V. CROSSPOINT INTERCONNECTION

5.1 Use of Tree Networks

Interconnecting crosspoints in a matrix arrangement without introducing serious impedance mismatch is a formidable problem, especially when the matrix is required to switch signals above 10 MHz. Since only one crosspoint in any row or column on the matrix can be closed and terminated at a given time, each closure can be represented by a continuous transmission path along which a number of open crosspoints are attached at various intervals. These open crosspoints are essentially open-circuit stubs which can easily degrade the transmission performance of the switch by causing severe impedance mismatches. Moreover, any general matrix construction in which switching elements are bussed together in rows and columns can result in different lengths of open-circuit stubs depending on which crosspoint is closed.

In an effort to make all transmission paths through the switching matrix appear alike electrically, tree networks, Fig. 1, are used to connect groups of crosspoints to the various input and output connectors of the matrix. Through use of tree networks the rows and columns of the matrix are formed so that no long open-circuited stubs exist. However, short stubs still exist at the positions where the open crosspoints are connected to the closed transmission path. Open-circuit stubs are equivalent to small capacitors short circuiting the closed transmission path and, unless carefully designed, usually preclude meeting the return loss requirement of 28 dB at frequencies above 50 MHz.

The tree networks for the 8×8 , the 1×8 and the 1×2 matrices are shown in Fig. 9.

5.2 Tree Network for 8×8 Matrix

The equivalent circuit of a single closed transmission path through an 8×8 matrix is shown in Fig. 10. As can be seen, the circuit is symmetrical from end to end, making the electrical characteristics identical for both directions of transmission through the matrix.

The open-circuit stubs represented by short capacitors C_1 , C_2 , and

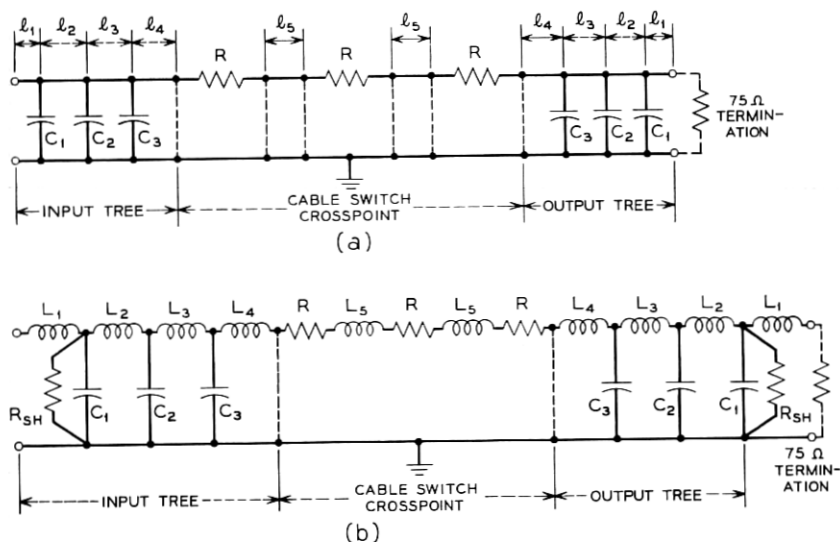


Fig. 10—Equivalent circuit of a closed connection through an 8×8 matrix: (a) untuned circuit (R = contact resistance of the 237G contact; C_1 , C_2 , C_3 = equivalent capacitance of open-circuited stubs; l_1 , l_2 , l_3 , l_4 , l_5 = 75Ω transmission-line lengths), (b) low-frequency approximation of the tuned circuit. (R_{SH} = shunt resistance to adjust real part of input impedance, L_1 , L_2 , L_3 , L_4 , L_5 = equivalent inductance of high-impedance transmission lines.)

C_3 in Fig. 10a can be effectively tuned out by

- (i) adjusting the lengths of l_1 , l_2 , l_3 and l_4 and increasing their characteristic impedance to a value higher than 75 ohms, and
- (ii) increasing the impedance of l_5 , the two inch coaxial line between the 237G contacts, above 75 ohms.

The low-frequency equivalent circuit of this tuned network is shown in Fig. 10b, where the high-impedance transmission lines are approximated by series inductors.

While this analysis provides physical insight into the factors affecting

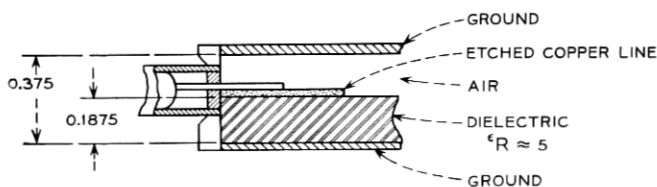


Fig. 11—Cross-section of tree network for an 8×8 matrix.

the design, it cannot provide a solution in closed form to the design problem. The organization of the 8×8 matrix was dictated by such factors as minimization of the stub lengths by minimizing the distance between crosspoints, symmetry in the array, interfering flux from adjacent crosspoint coils, and the availability of suitable materials for design.

The 0.375-inch dimension between crosspoints is a reasonable lower limit for mechanical assembly of the crosspoint array since the O.D. of the driving coils is approximately 0.31 inches. Circuit board material standard thickness is $\frac{3}{16}$ inch so that the buildup of the strip line tree circuits is readily accomplished. Flux interference was not a problem as seen in Fig. 8. The question became, then, whether utilizing the insights given by the above circuit analysis, the system design requirements could be met for the adopted physical configurations.

Initial efforts on the tree structure followed conventional stripline technology in which a planar center conductor is positioned between parallel ground planes by dielectric layers. Dielectric materials with a low loss and a uniform dielectric constant over the frequency band

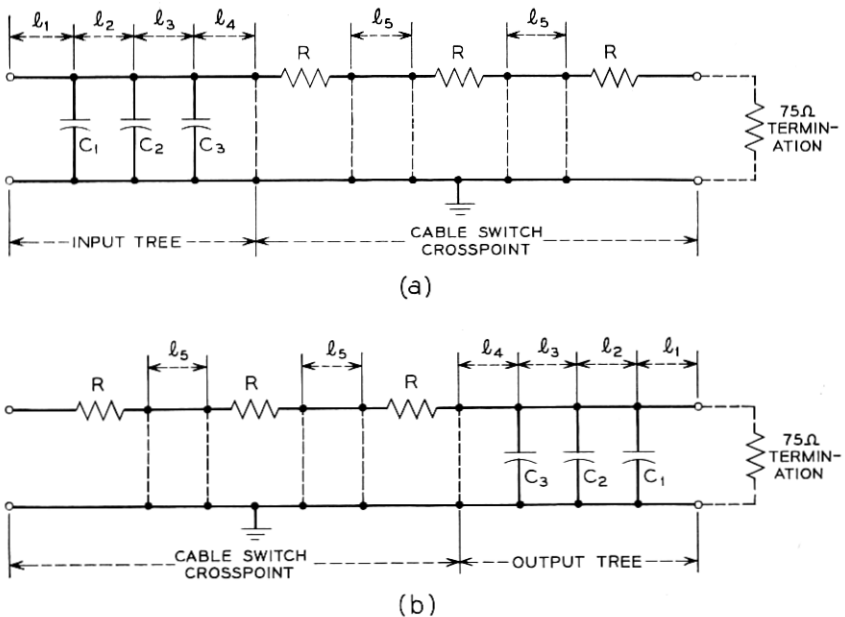


Fig. 12—Equivalent circuit of a closed connection through a 1×8 matrix: (a) 1-to-8 direction, (b) 8-to-1 direction

include the polyolefins and polyphenylene oxides. These are widely used for microwave (300 MHz to 300 GHz) printed circuits. However, the frequency band of concern is well below that for which these more expensive materials are designed, and their generally poorer mechanical stability and peel strength suggested an alternate approach be taken.

Epoxy glass has relatively better mechanical properties and peel strength and is more economical than the above materials. It also has considerably higher loss and dielectric constant.

The compromise solution, Fig. 11, is the use of $\frac{3}{16}$ -inch-thick epoxy glass board with one-ounce copper and an air space above the circuit to lower the effective overall dielectric constant. The impedance levels and capacitance of the various branches of the tree were adjusted experimentally with the pattern shown in Fig. 9 resulting.

A shunt resistance, as seen in Fig. 10b, was added at the input and output of the circuit to compensate for the series resistance of the tree and crosspoint. The resistor used, Fig. 9, is the 257A type ceramic with evaporated tantalum nitride film element. It is mounted directly on the board by its leads.

The ground plane spring is formed from a single stamped part of five thousandth-inch beryllium copper over-plated with fifty millionths of hard gold for corrosion resistance and sealing to the end plates and side rails of the switch assembly. The thickness of the material guarantees a minimum of 100 dB crosstalk loss through the circuit boards in the assembled switch. The spring's rolled edges compress when slid into the side rail slots while the front slotted edge seats firmly to the face of the switch. The circuit board assembly is forcibly held in position by the bracket which seats against the rear slotted edge. The result is a well defined geometry insensitive to the temperature ranges to which the switch will be exposed and efficiently sealed against crosstalk.

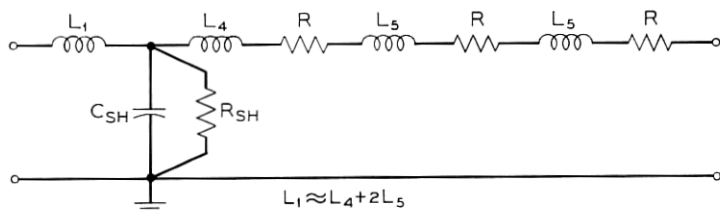


Fig. 13—Low-frequency approximation of a tuned path through a 1×8 matrix. (L_1 , L_4 , L_5 = equivalent inductance of high impedance transmission lines; C_{SH} = equivalent lumped capacitance of open-circuited stubs; R = contact resistance of the 237G contact; R_{SH} = shunt resistance to adjust real-part of input impedance.)

5.3 Tree Network for the 1×8 Matrix

The tree network for the 1×8 matrix is shown in Fig. 9. The tree differs from that used in the 8×8 matrix because the 1×8 matrix is physically asymmetric from end to end. In such an array the tree is used to either connect one input to one of eight possible outputs (1-to-8) or to connect one of eight possible inputs to one output (8-to-1). Fig. 12a shows the equivalent circuit of a closed transmission path through the matrix in the 1-to-8 direction, and Fig. 12b shows the equivalent circuit for the 8-to-1 direction. The only possible way to approach a good input impedance match for this matrix for both directions of operation is to:

- (i) minimize the lengths l_2 and l_3 so that C_1 , C_2 , and C_3 can be approximated by a single shunt capacitance, C_{SH} ,
- (ii) adjust the length of l_1 and the characteristic impedance of l_1 , l_4 , and l_5 such that the low-frequency equivalent series impedance of l_1 equals $l_4 + 2l_5$, and
- (iii) add a shunt resistance across C_{SH} to compensate for the series resistance of the tree and the crosspoint.

As can be seen from the low-frequency equivalent circuit of the resulting network, shown in Fig. 13, the input impedance is essentially the same when viewed from either end with the other end terminated in the 75-ohm system impedance.

The circuit board for the 1×8 matrix is shown in Fig. 9. In order to achieve the higher stripline impedance, a thinner epoxy glass board is used, and the ground plane spacing is increased as shown in Fig. 14.

5.4 Tree Networks for the 1×2 Matrix

The tree network for the 1×2 matrix is also shown in Fig. 9. The analysis parallels that presented for the 1×8 matrix. The simpler

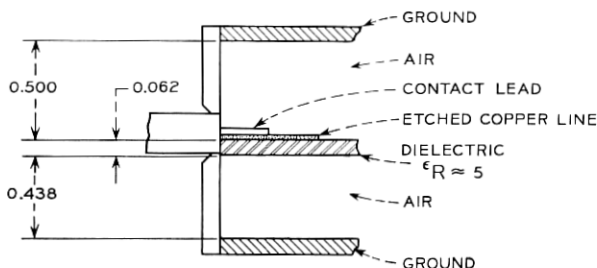


Fig. 14 — Cross-section of tree network for 1×8 and 1×2 matrices.

nature of the matrix was confirmed by the relative ease of obtaining an experimental solution to the design problems.

VI. CONNECTORS

Standard 478A and 477B jacks accommodating type 728A coaxial cable are used for signal interface. The 478A jack flange was modified to allow closer placement of the input jacks on the 8×8 array.

The crosspoint control windings for each crosspoint are individually terminated in standard commercially available connectors allowing wide latitude for control circuit design.

VII. STRUCTURE

Since the structure carries the signal ground, the integrity of the structure at each crosspoint is vital to the maintenance of high return loss. A failure of a tube joint will bring the return loss for that crosspoint to 10 dB even though all other joints are structurally sound. The structure must also be tight to prevent crosstalk. Gaps at the flanges of the connector, between the tree circuits and the tube face, or along the rail at the spring will quickly raise the crosstalk above the minimum required limit of 95 dB below the signal level.

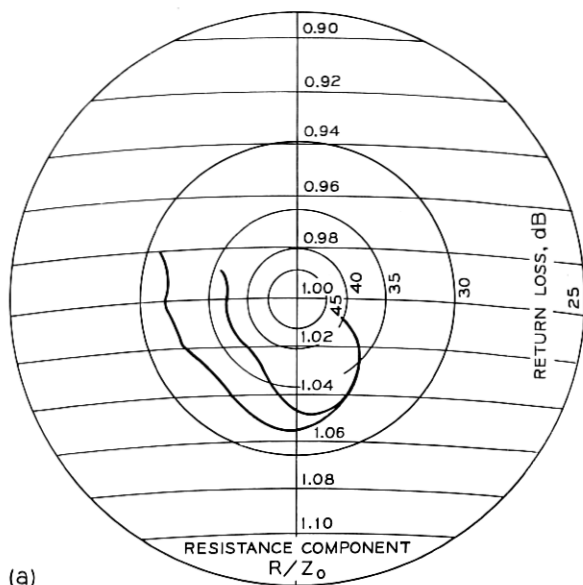
The tube array in each switch code is fixed to the base on one end and pinned on the other to provide axial freedom for thermal expansion. This degree of freedom is sufficient to protect the solder joints between the tubes and the end plates and those between the contact assembly and the circuit boards.

The switch assembly has been vibration tested over the range of 5 to 500 Hz. Resonant points were found for the structure in early models and modifications were made to provide a stiffer structure at those frequencies. Tests of the switch models shown in Fig. 2 led to the inclusion of shipping blocks in the 8×8 switch to provide damping of the pinned end of the tube assembly during transport. The switches otherwise withstand anticipated shock and vibration in normal shipping and installation.

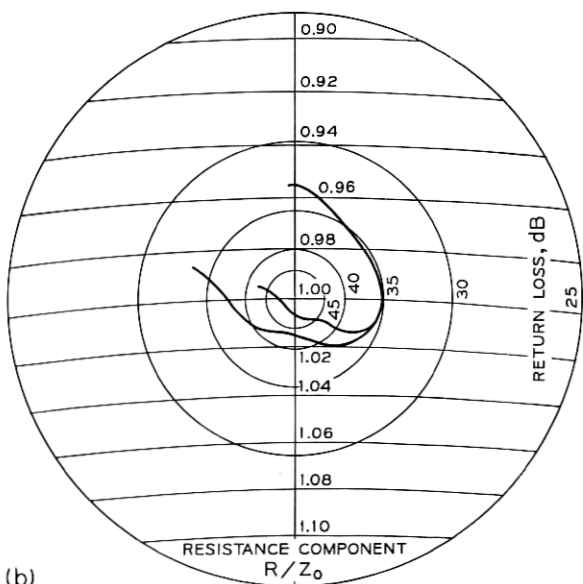
The switch has been cycled over the temperature range of 40°F to 140°F at relative humidities to saturation without incident.

VIII. PERFORMANCE

Switch performance has been measured at three Bell Laboratories locations: Merrimack Valley at North Andover, Massachusetts; Holmdel, New Jersey; and Columbus, Ohio. The switch has also been

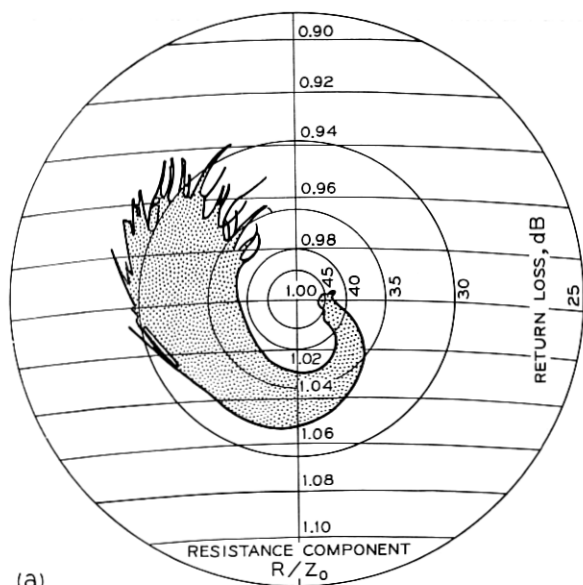


(a)

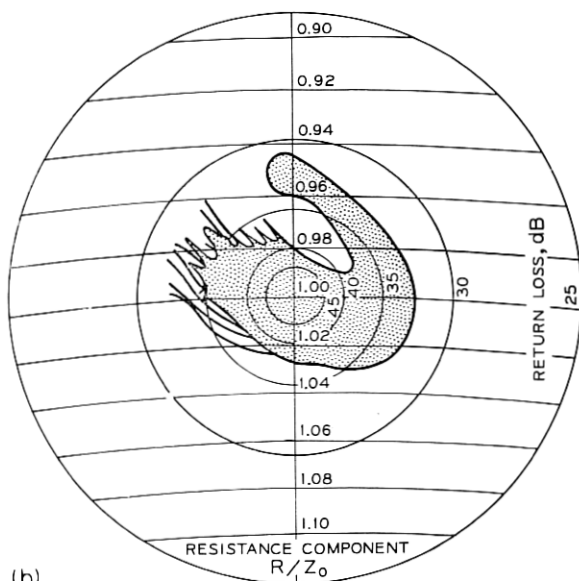


(b)

Fig. 15—Return loss for a single crosspoint in an 8×8 array: (a) 8250 Ω resistor, (b) 2740 Ω resistor.



(a)



(b)

Fig. 16—Return loss for the 8×8 array from dc to 100 MHz: (a) with 8200-ohm shunt resistance, (b) with 2740-ohm shunt resistance.

measured at the Western Electric Works in Kansas City where production has been scheduled.

Return loss results on a given piece of apparatus are reproducible to within 1 dB over the range from 28 to 48 dB. Insertion loss is reproducible to 0.02 dB. 75-ohm attenuators are used in bridge circuits as calibration standards for the measurements program.

8.1 Return Loss

The return loss for a single crosspoint of an 8×8 array is shown in Figure 15. The presentation is in the form of a Smith chart. The trace is swept from 100 kHz to 100 MHz with the output of the switch terminated in 75 ohms. Two curves are obtained for each crosspoint by testing each end as the input. Since the physical structure is mechanically symmetric, performance should be the same. The electrical asymmetry observed is the result of mechanical tolerances of switch elements with respect to the tuned circuit performance when seen from opposite ends.

The return loss characteristic for two 8×8 arrays is shown in Figure 16. The envelope of performance of all sixty-four crosspoints swept

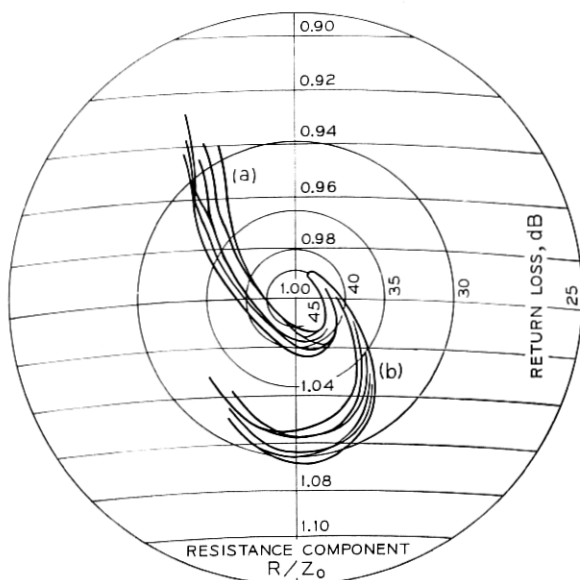


Fig. 17—Return loss for the 1×8 array from dc to 100 MHz: (a) 1-to-8 direction, (b) 8-to-1 direction.

from 100 kHz to 100 MHz from both ends, that is, 128 curves on each graph, is shown. The return loss is obviously better at the lower frequencies where the geometric factors are a smaller fraction of a wave length. No one crosspoint defines the envelope of the plot for more than a fraction of the frequency swept. The scatter at the higher frequencies is a function of both switch geometry and mechanical tolerances. The return loss of the switch with 8200-ohm shunt resistance is seen to be better than 30 dB over most of the band. The improved performance at the higher frequencies obtained with the 2740-ohm resistance is at the expense of the performance at the lower frequencies. In either case the performance limit of 28 dB is met.

The return loss for the 1×8 array is shown in Figure 17. The eight traces for either end as input indicates the design compromises required to meet the return loss objective. The switch could be optimized for either input at the expense of the return loss for the opposite end input.

The return loss for the 1×2 switch is shown in Figure 18. No shunt resistor was used in this design since it readily meets the requirements for either end as input.

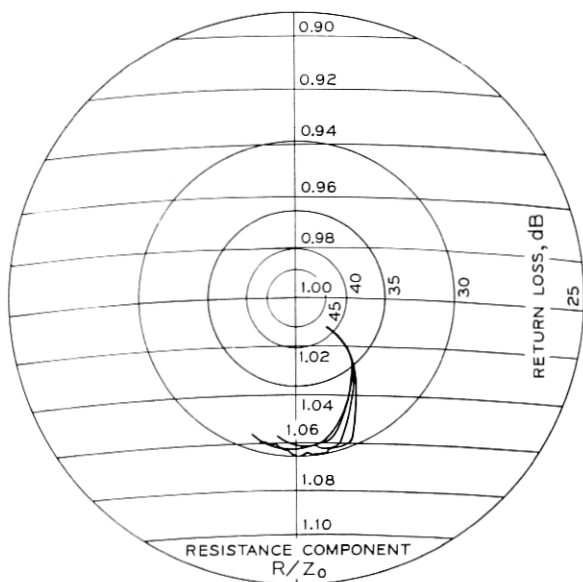


Fig. 18—Return loss for the 1×2 array from dc to 100 MHz in 1-to-2 direction and in 2-to-1 direction.

8.2 Insertion Loss

The insertion loss for the 8×8 array is shown in Fig. 19. Only the two crosspoints are shown which represent the upper and lower limits of the insertion loss for the switch. Fig. 19b indicates the loss for the switch with shunt resistors of 8200 ohms. Fig. 19a indicates the loss for the switch with 2740-ohm shunt resistors. The higher insertion loss of the second switch is seen to be the penalty for the improved return loss as shown in Figs. 16a and 16b.

Figures 20a and 20b indicate the insertion loss for the 1×8 and 1×2 arrays respectively.

8.3 Isolation and Crosstalk Loss

These losses were found to exceed 105 dB across the band on all codes of switches. The crosstalk loss is at least as good as the isolation loss since the results of crosstalk loss are essentially the same as for isolation loss. There are no significant differences between near end, far end, terminated and unterminated crosstalk observations.

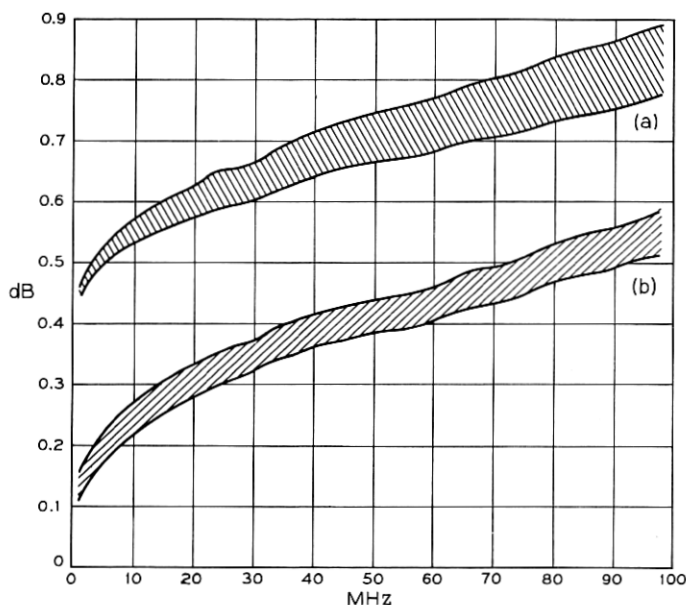


Fig. 19—Insertion loss for the 8×8 array from dc to 100 MHz: (a) with 2740-ohm shunt resistance, (b) with 8200-ohm shunt resistance.

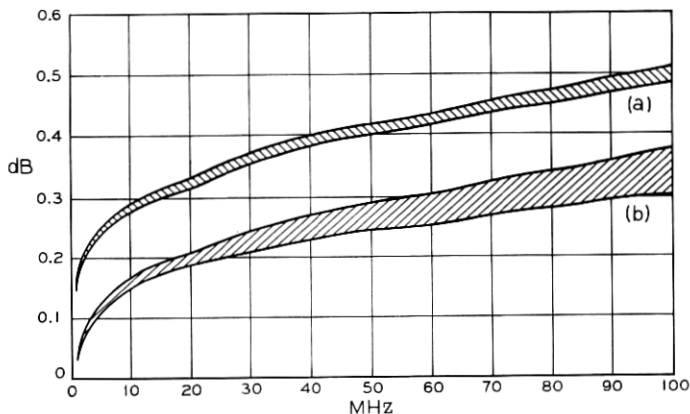


Fig. 20—Insertion loss from dc to 100 MHz: (a) 1 × 8 array, (b) 1 × 2 array.

IX. SUMMARY

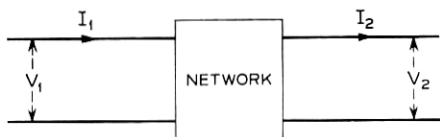
A series of switches designed to meet Bell System requirements across the dc to 100 MHz band have been designed and models built. Tests show the performance characteristics over the band meet system requirements and that the physical structures are mechanically satisfactory for the environmental conditions anticipated in transport and installation.

APPENDIX

Derivation of the V_{out}/V_{in} Relationship for the Cable Switch

A.1 Network Transfer Matrix

Any network can be described in terms of an *ABCD* transfer matrix:



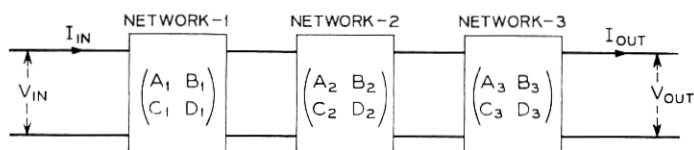
- V_1 = input voltage
- I_1 = input current
- V_2 = output voltage
- I_2 = output current

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \tag{3}$$

where $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is the *ABCD* transfer matrix.

A.2 Overall Matrix

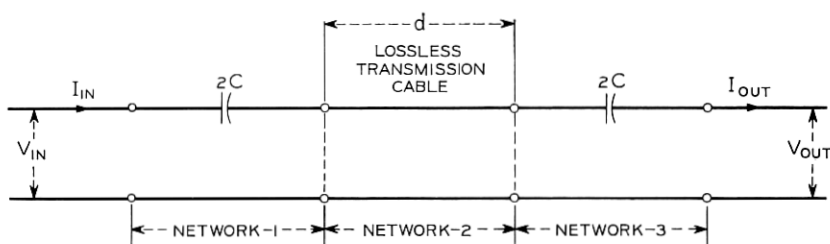
The overall $ABCD$ matrix of a series of networks in tandem is equal to the matrix multiplication of the individual network $ABCD$ matrices:



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}_{\text{overall}} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \quad (4)$$

A.3 Equivalent Matrix

The equivalent circuit for *one* of K identical sections of the cable switch is given by:



Now, the $ABCD$ matrices for the 3 networks in this circuit are:

Networks-1 and 3

$$\begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} = \begin{bmatrix} 1 & 1/j\omega 2C \\ 0 & 1 \end{bmatrix} \quad (5)$$

Network-2

$$\begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} = \begin{bmatrix} \cos \beta d & jZ_0 \sin \beta d \\ j \sin \beta d / Z_0 & \cos \beta d \end{bmatrix} \quad (6)$$

where $\beta = 2\pi/\lambda$

λ = wavelength

d = length of transmission line

Z_0 = characteristic impedance of the transmission line.

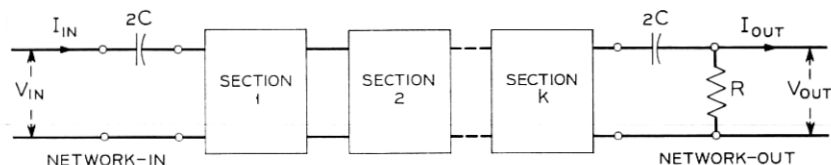
The $ABCD$ matrix of one of the K identical sections can be obtained by matrix multiplication:

$$\begin{pmatrix} A_{1s} & B_{1s} \\ C_{1s} & D_{1s} \end{pmatrix} = \begin{pmatrix} \cos \beta d + \frac{\sin \beta d}{\omega 2CZ_0} \frac{\cos \beta d}{j\omega C} + \frac{\sin \beta d}{j(\omega 2C)^2 Z_0} + jZ_0 \sin \beta d & \\ j \frac{\sin \beta d}{Z_0} & \cos \beta d + \frac{\sin \beta d}{\omega 2CZ_0} \end{pmatrix} \quad (7)$$

Note: $A_{1s} = D_{1s}$.

A.4 Switch Circuit

The equivalent circuit for the entire cable switch is given by:



The *ABCD* matrices for the networks in this circuit are:

Network-OUT

$$\begin{pmatrix} A_{out} & B_{out} \\ C_{out} & D_{out} \end{pmatrix} = \begin{pmatrix} 1 + 1/j\omega 2CR & 1/j\omega 2C \\ 1/R & 1 \end{pmatrix} \quad (8)$$

Network of *K* identical sections

$$\begin{pmatrix} A_K & B_K \\ C_K & D_K \end{pmatrix} = \begin{pmatrix} A_{1s} & B_{1s} \\ C_{1s} & D_{1s} \end{pmatrix}^K \quad (9)$$

Note: $A_K = D_K$.

By matrix multiplication:

<i>n</i>	A_n	B_n	C_n
0	1	$B_{1s}(0)$	$C_{1s}(0)$
1	A_{1s}	$B_{1s}(1)$	$C_{1s}(1)$
2	$2A_{1s}^2 - 1$	$B_{1s}(2A_{1s})$	$C_{1s}(2A_{1s})$
3	$4A_{1s}^3 - 3A_{1s}$	$B_{1s}(4A_{1s}^2 - 1)$	$C_{1s}(4A_{1s}^2 - 1)$
⋮	⋮	⋮	⋮
K	$2A_{1s}A_{K-1} - A_{K-2}$	$B_{1s} \sum_{n=0}^{K-1} A_{1s}^{K-1-n} A_n$	$C_{1s} \sum_{n=0}^{K-1} A_{1s}^{K-1-n} A_n$

(10)

Network-IN

$$\begin{pmatrix} A_{in} & B_{in} \\ C_{in} & D_{in} \end{pmatrix} = \begin{pmatrix} 1 & 1/j\omega 2C \\ 0 & 1 \end{pmatrix} \quad (11)$$

The overall $ABCD$ matrix for the cable switch can be obtained by matrix multiplication:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}_{\text{overall}} = \begin{pmatrix} A_K \left(1 + \frac{1}{j\omega CR}\right) + \frac{B_K}{R} + \frac{C_K}{j\omega 2C} \left(1 + \frac{1}{j\omega 2CR}\right) & \frac{A_K}{j\omega C} + B_K - \frac{C_K}{(\omega 2C)^2} \\ \frac{A_K}{R} + C_K \left(1 + \frac{1}{j\omega 2CR}\right) & A_K + \frac{C_K}{j\omega 2C} \end{pmatrix}. \quad (12)$$

A.5 Special Case: $I_{\text{out}} = 0$

For the case where $I_{\text{out}} = 0$:

$$\frac{V_{\text{out}}}{V_{\text{in}}}\bigg|_{I_{\text{out}}=0} = 1/A_{\text{overall}}.$$

Therefore, for the cable switch:

$$\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{A_K \left(1 + \frac{1}{j\omega CR}\right) + \frac{B_K}{R} + \frac{C_K}{j\omega 2C} \left(1 + \frac{1}{j\omega 2CR}\right)}. \quad (13)$$

Since $\omega CR \ll 1$ for practical values of C and R ,

$$\frac{V_{\text{out}}}{V_{\text{in}}} \approx \frac{j\omega CR}{A_K + jB_K\omega C + \frac{C_K}{j\omega 4C}}. \quad (14)$$

Substituting the expressions for B_K and C_K in (10) and the expressions for B_{1s} and C_{1s} in (7) into (14) gives:

$$\begin{aligned} \frac{V_{\text{out}}}{V_{\text{in}}} &= \frac{j\omega CR}{A_K + \left(\cos \beta d + \frac{\sin \beta d}{\omega 2CZ_0} - \omega CZ_0 \sin \beta d\right) \sum_{n=0}^{K-1} A_{1s}^{K-1-n} A_n} \\ \left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| &= \frac{\omega CR}{A_K + (A_{1s} - \omega CZ_0 \sin \beta d) \sum_{n=0}^{K-1} A_{1s}^{K-1-n} A_n}. \end{aligned} \quad (15)$$

Since $A_{1s} \gg \omega CZ_0 \sin \beta d$ for practical values of C and Z_0 ,

$$\left| \frac{V_{\text{out}}}{V_{\text{in}}} \right| = \frac{\omega CR}{\sum_{n=0}^K A^{K-n} A_n}. \quad (16)$$

REFERENCE

1. Church, D. S., and Kordos, R. W., "Coaxial Cable Switch," United States Patent No. 3,355,684, November 28, 1967.