# Measured Quantizing Noise Spectrum for Single-Integration Delta-Modulation Coders

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(Manuscript received October 14, 1969)

*We give experimental verification, for idle-channel and sinusoidal inputs, of a recently developed quantizing noise theory for asymmetrical, single-integration delta-modulators.*

A recent paper by Iwersen described a procedure for calculating quantizing noise for single-integration delta-modualtion coders employing unequal positive and negative integrator step sizes.[1] The purpose of this note is to provide experimental verification of the theory.

Measured quantizing noise for both idle-channel and sinusoidal inputs is given and the idle-channel noise spectrum is calculated.

Defining the positive, $\sigma_+$, and negative $\sigma_-$, integrator step sizes as

$$\sigma_+ \equiv \sigma + \epsilon$$

$$\sigma_- \equiv -\sigma + \epsilon \tag{1}$$

where $\sigma$ is the average step size, an error wave is generated by the

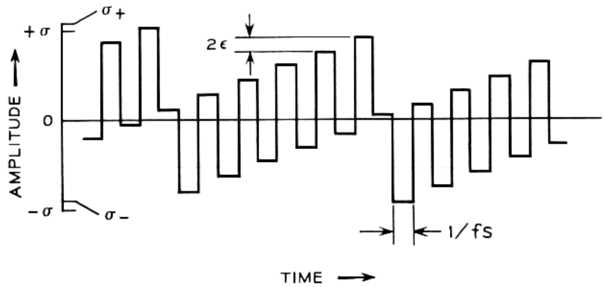


Fig. 1 — Asymmetrical integrator output for an idle-channel input, shown for $|\sigma_+| > |\sigma_-|$.

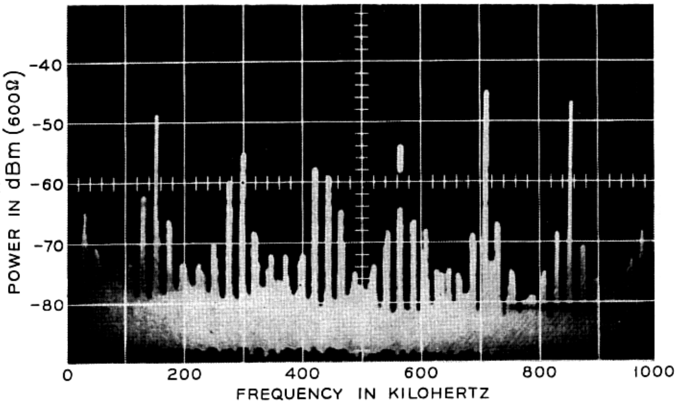


Fig. 2 — Observed idle-channel noise spectrum, $f_s = 1.56$ MHz, $\vartheta = 0.0937$.

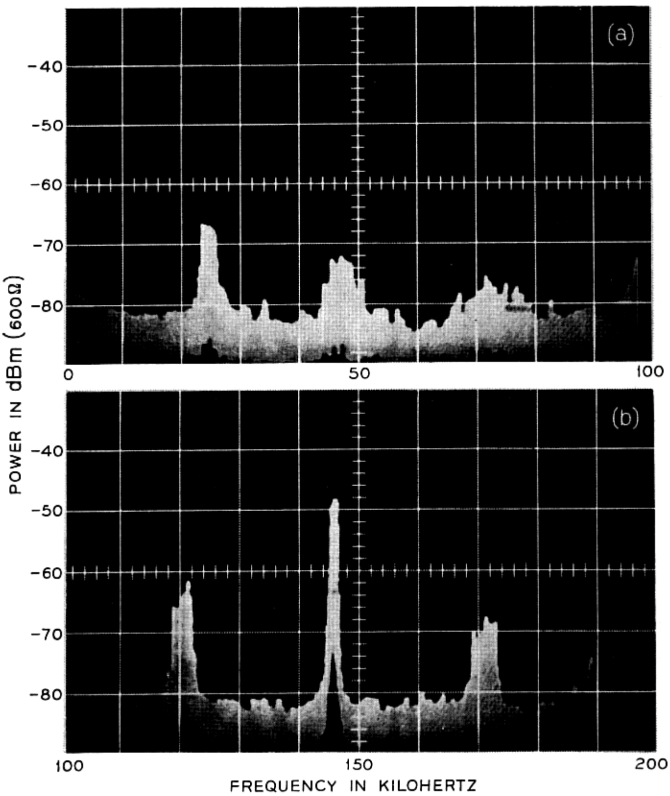


Fig. 3 — Expanded idle-channel noise spectrum: (a) 0 — 100 kHz, (b) 100 — 200 kHz.

integrator for idle-channel inputs as shown in Fig. 1. The quantizing noise spectrum resulting from the error wave is a line spectrum, and the line frequencies, $f_l$, for a one-sided spectrum from zero to one-half the sampling frequency are given as a function of the integer index $l$ by[1]

$$f_l = | Q[l(1 - \vartheta)/2]f_s | \qquad (2)$$

where

$$Q(\alpha) = \alpha - N(\alpha),$$

$$N(\alpha) = \text{integer nearest } \alpha$$

and $\vartheta$ is the integrator step imbalance $\epsilon/\sigma$ and $f_s$ the sampling frequency.

The power at the frequency of index $l$ is calculated from

$$P_l = 2\sigma^2/\pi^2 l^2. \qquad (3)$$

The resulting noise-spectral lines will subsequently be referred to as $l$-lines (1-line, 2-line, 14-line, and so on).

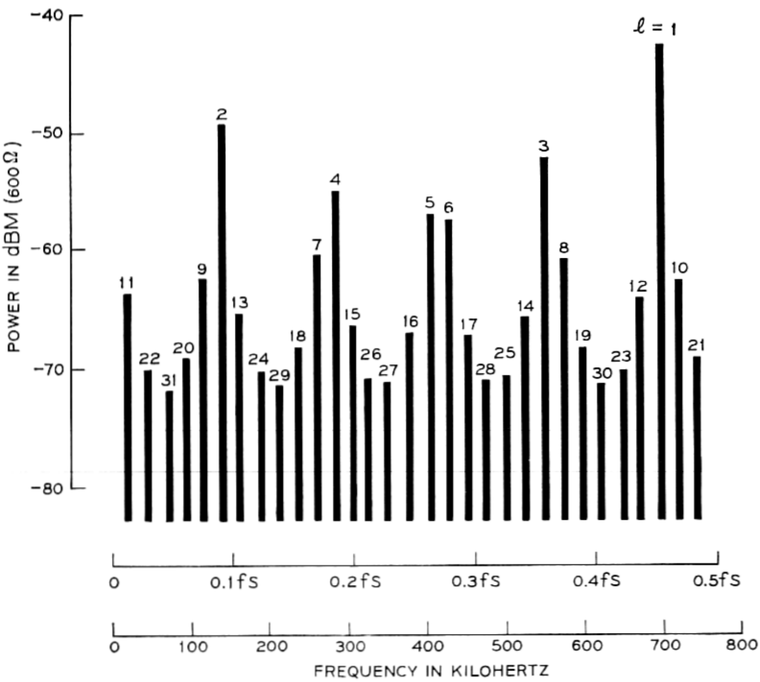


Fig. 4 — Calculated idle-channel noise spectrum, $f_s = 1.56$ MHz, $\vartheta = 0.0937$.

Measurements of the quantizing noise spectrum were made using a delta-modulation coder designed for telephone switching applications.[2] A 1.56 MHz sampling frequency and an average integrator step size of 13 millivolts were used for the measurements. Figure 2 shows the observed idle-channel spectrum of the coder for a frequency range from 0 to 1 MHz. The region near the 2-line is expanded in Figs. 3a and 3b where the noise spectrum is shown for frequencies from 0 to 100 kHz and 100 to 200 kHz respectively.

The calculated spectrum from 0 to $f_s/2$ (0 to 0.78 MHz) is shown in Fig. 4 for $\vartheta = 0.0937$. Excellent correlation can be observed with the measured spectrum in Fig. 2. For a more detailed comparison, Table I gives the calculated and measured frequencies and powers of the $l$-lines for the band from $0 - 200$ kHz. With respect to frequency, the agreement is within experimental error. However, measured peak powers of higher order $l$-lines fall below the calculated values. This discrepancy is believed to be due to modulation broadening of the lines by a low-level noise input of unknown origin.

Figures 5a and 5b show the effect of sinusoidal inputs on the coder noise spectrum. As suggested by Iwersen, inputs to the coder phase-modulate the idle-channel lines and force the frequency band occupied by each $l$-line group, $\Delta f$, to become proportional to the slope of the input signal, $2\pi A f_0$, and to the index of the $l$-line,[1]

$$\Delta f \approx 2\pi l A f_0 \tag{4}$$

where $A$ is the amplitude and $f_0$ the frequency of the input signal. This is illustrated in the figures where broadening of the 1-line, 2-line, 3-line and 4-line as a function of signal amplitude is clearly visible.

Table I—Comparison of Measured and Calculated Noise Spectrum for $0 - 200$ kHz

| | Measured | | Calculated | |
|---|---|---|---|---|
| $l$-line | $f_l$ | $P_l$ | $f_l$ | $P_l$ |
| 2 | 146 kHz | −48 dBm | 146 kHz | −48 dBm |
| 9 | 121 | −61 | 121 | −61 |
| 11 | 24 | −66 | 24 | −63 |
| 13 | 171 | −67 | 170 | −64 |
| 20 | 98 | −69 | 98 | −66 |
| 22 | 48 | −72 | 48 | −69 |
| 24 | 194 | −74 | 194 | −70 |
| 31 | 73 | −75 | 74 | −72 |

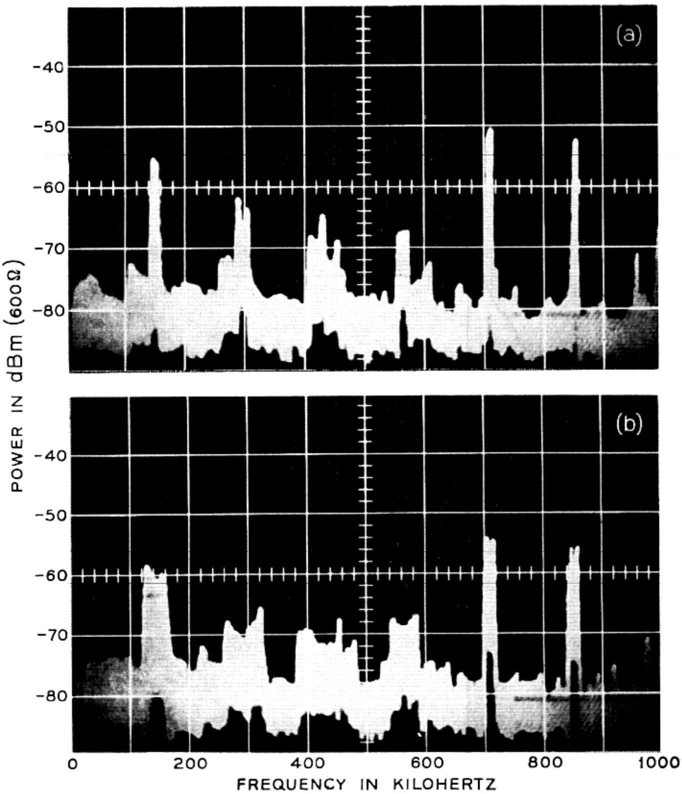


Fig. 5 — Effect of 1 kHz sinusoidal inputs on noise spectrum; (a) —40 dBm input, (b) —30 dBm input.

Additional discussion of the noise characteristics as well as a description of the design of the delta-modulation coder will be presented in a future paper.[2]

REFERENCES

1. Iwersen, J. E., "Calculated Quantizing Noise of Single-Integration Delta-Modulation Coders," B.S.T.J., *48,* No. 7 (September 1969), pp. 2359–2389.
2. Laane, R. R., and Murphy, B. T., "Delta-Modulation Coder for Telephone Transmission and Switching Applications," unpublished work.