Rain Attenuation and Radio Path Design

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This paper describes the application of the rain attenuation theory of Ryde and Ryde to the design of Radio Systems. It shows that an upper bound on the outage time due to rain attenuation can be computed from a measured point rain rate distribution. The paper also describes a suitable rain gauge.

I. INTRODUCTION

Heavy rainfall on a radio path absorbs and scatters power transmitted at frequencies above 10 GHz and causes large fading of received signals. At 20 GHz, for example, the attenuation due to a uniform rain rate of 100 mm/hr is about 10 dB/km. Rain attenuation is so severe at these frequencies that for some applications transmission paths must be restricted to a few kilometers or less rather than the tens of kilometers common at lower frequencies. Since the cost of a radio system increases with the number of repeaters it is important to use the longest path allowed by the transmission objectives. This path length can be determined accurately only if the fading outage due to rain attenuation can be predicted.

Bussey estimated fading statistics on a microwave path from point rain rate data.[1] He used the rain attenuation theory of Ryde and Ryde[2–4] to convert rain statistics to fading statistics and since 1950 his results have been used in the design of radio systems.[5] However, as operating frequencies increase and path lengths get shorter, increased precision of fading estimates is required for optimum radio system design.

Over the years a number of experiments have been performed in which attenuation was measured on a path at specific times and compared with values computed from rain rates measured by rain gauges spaced along the path near ground level. Here too, the theory of Ryde and Ryde was used. In general there is wide disagreement between computed and measured values. Also, Medhurst questions the validity of the application of the theory to a practical rainfall situation.[6] Since the theory was derived for uniform rain the question is a good one—rain on a microwave path is seldom uniform.

In this paper the theory of attenuation by uniform rain is applied to the practical rainfall situation. The radio path is defined as the volume of the first Fresnel zone and the rain attenuation is assumed proportional to the number of raindrops in this path volume. Of course, this expression reduces to that of Ryde and Ryde when the rain is uniform.

Rain rate is a vector which can be written as the product of a rain density and the velocity of raindrops. Rain density is proportional to the number of raindrops per unit volume.

Since rain rate is a vector the expression for attenuation in terms of rain density in the path volume can be transformed by the divergence theorem into an expression for attenuation as a function of the rain rate on the surface of the path volume. From this formulation the following results emerge:

(*i*) A natural definition of rain rate which is appropriate to the radio situation.

(*ii*) A time interval, T_o, exists during which no significant fade can occur. T_o is determined by the path length, the frequency of operation and the speed of raindrops.

(*iii*) A rain gauge is described which is suitable for measuring rain rate in accordance with the definition mentioned in (*i*).

By applying these results, an upper bound on the outage time due to rain attenuation is derived. The bound can be computed from a measured point rain rate distribution using the results of uniform rain theory. The bound can be made tight by the proper choice of rain rate integration time interval. A method of estimating this interval from measured path loss distributions is given.

II. GENERAL CONSIDERATIONS

2.1 *The Radio Path*

The radio link consists of two narrow-beam antennas pointing directly at each other over a distance of a few hundred to a few thousand meters. The space, or volume, of the path is taken to be the first Fresnel zone.[7] This means that only the energy confined to that volume contributes significantly to the total energy collected by the receiving antenna.

The first Fresnel zone is a long, thin, prolate ellipsoid of revolution. For a path of length L at wavelength λ, it has a major axis L and equal minor axes $(\lambda L)^{\frac{1}{2}}$ and is terminated at the ends by the antennas. The radio path is defined as the volume enclosed by the first Fresnel zone and the two antennas. Figure 1 is a sketch of the path. When we speak of rain falling on the path we mean rain falling through this volume.

2.2 *Rain Rate as a Vector*

A theory of rain attenuation has been formulated by Ryde and Ryde,[2–4] and others, and a good account of it is given by Medhurst.[6] The attenuation in a radio path depends upon the number and size of the raindrops and not explicitly upon their speed or direction. But the quantity usually measured is rain rate and it does depend on the speed and direction of the raindrops. Since rain rate is the product of a density and a velocity, it can be interpreted as a vector.

Let there be a uniform distribution of N_D drops of water per cubic centimeter in the space between two antennas. The drops are spherical with diameter D and velocity v_D . The fraction of volume occupied by water is defined as the rain density

$$\rho_D \equiv \frac{\pi}{6} N_D D^3. \tag{1}$$

Rain density is a dimensionless, real, nonnegative quantity. The rain rate for drops with diameter D and velocity v_D is

$$R_D \equiv \rho_D v_D .$$

The direction of the rain rate is the direction of travel of the drops.

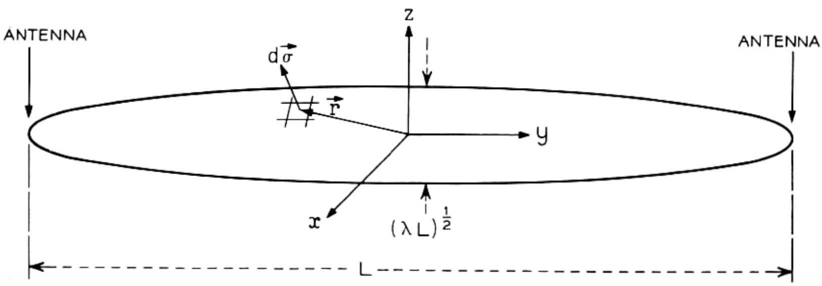

Fig. 1 — Radio path.

The vector expression for rain rate is, therefore,

$$\mathbf{R}_D = \rho_D \mathbf{v}_D , \tag{2}$$

where a boldface letter denotes a vector quantity.

In general a rain storm has drops of many diameters and the total rain rate is a summation over the drop diameters present.

$$\mathbf{R} = \sum_D \rho_D \mathbf{v}_D . \tag{3}$$

2.3 *Attenuation and Rain Density*

For a rain consisting of uniformly distributed drops with diameter D the attenuation of radio waves with wavelength λ is[6]

$$\text{Attenuation} = 4.343 \frac{N_D \lambda^2}{2\pi} A_D \times 10^5 \text{ dB/km} \tag{4}$$

where A_D is a function of the drop diameter, the wavelength, and the dielectric constant of water. Substituting from (1) the attenuation is

$$\alpha_D = k(\lambda, D) L \rho_D \text{ dB}, \tag{5}$$

where

$$k(\lambda, D) = 3 \times 4.343 \frac{\lambda^2 A_D}{\pi^2 D^3} \times 10^5 .$$

The result in (5) is extended to the case of nonuniform spatial distribution of raindrops by replacing the uniform rain density in (5) with the average rain density in the radio path. The path attenuation is therefore assumed to be

$$\alpha_D(t) = k(\lambda, D) \frac{L}{V} \iiint_V \rho_D(x, y, z; t) \, dV \text{ dB}. \tag{6}$$

This expression reduces to (5) for uniform rain density.

There are neither sinks nor sources of rain in the radio path so, for constant drop diameter D, the hydrodynamic equation of continuity which applies is[8]

$$\nabla \cdot (\rho_D \mathbf{v}_D) + \frac{\partial \rho_D}{\partial t} = o,$$

where the time and space variables have been omitted for convenience. Substitution into (6) gives

$$\frac{d\alpha_D(t)}{dt} = k(\lambda, D) \frac{L}{V} \iiint_V [-\nabla\cdot(\rho_D \mathbf{v}_D)]\, dV$$

and transforming to a surface integral by the divergence theorem[9] we get

$$\frac{d\alpha_D(t)}{dt} = k(\lambda, D) \frac{L}{V} \int_S [-\rho_D \mathbf{v}_D \cdot d\boldsymbol{\sigma}], \tag{7}$$

where the integral is over the surface enclosing the volume V.

Expression (7) relates point rain rate to path attenuation. The vector differential $d\boldsymbol{\sigma}$ has a magnitude equal to the differential area of the surface S, is normal to it and directed outward. Consider a small area on the surface S. The rain rate at this point is $\rho_D \mathbf{v}_D$; the component of this rain rate which is normal to the surface and directed into the path volume is $-\rho_D \mathbf{v}_D \cdot d\boldsymbol{\sigma}/|\, d\boldsymbol{\sigma}\,|$. The integral over the surface S is the increase in water density on the path in unit time which causes an increase in attenuation.

Let the surface S enclosing the radio path volume V be described by orthogonal parametric curves on the surface.[10] A point $P(x, y, z)$ on the surface can be written $P(u, v)$ where,

$$x = \frac{(\lambda L)^{\frac{1}{2}}}{2} \sin u \cos v$$

$$y = \frac{L}{2} \cos u$$

$$z = \frac{(\lambda L)^{\frac{1}{2}}}{2} \sin u \sin v.$$

With this transformation, and substituting R_D from (2), (7) becomes

$$\frac{d\alpha_D(t)}{dt} = k(\lambda, D) \frac{L}{V} \iint_S [-\mathbf{R}_D(u, v; t)] \cdot d\mathbf{s}, \tag{7a}$$

where $d\mathbf{s}$ is the transformed vector surface differential. Integrating both sides over time T and dividing by T results in the following equation.

$$\frac{\alpha_D(t+T) - \alpha_D(t)}{T} = k(\lambda, D) \frac{L}{V} \iint_S \left\{ -\left[\frac{1}{T} \int_t^{t+T} R_D(u, v; t)\mathbf{r}\, dt \right] \right\} \cdot d\mathbf{s}, \tag{8}$$

where the unit vector $\mathbf{r}$ is defined by $\mathbf{R}_D(u, v; t) \equiv R_D(u, v; t)\mathbf{r}$.

The left side of (8) is an approximation to the rate of change of attenuation since, by definition,

$$\frac{d\alpha_D(t)}{dt} \equiv \lim_{T \to o} \frac{[\alpha_D(t+T) - \alpha_D(t)]}{T}.$$

Whether attenuation or rain rate is measured, the measuring instrument has some time constant, or integration time, T. Since, in a microwave system, an outage of a millisecond may be significant, an important question arises—can we be sure that the measurement will give us all the important data? In other words does an integration time T_o exist such that for $T \leqq T_o$, (8) is a good approximation to (7a)? The existence of T_o is justified by the following physical argument.

For $t \ll t_o$ let there be no rain on the path. At $t = t_o$ let the rain rate at every point on the surface S be $R_o = \rho_0 v_0$ and be directed inward in the direction of the shortest line from the surface to the path axis. In order that a substantial fade occur, the rain will have to travel a substantial fraction of the shortest distance from the path surface to the path axis. The average of this distance is $(\lambda L)^{\frac{1}{2}}/3$ and the average time required for the rain to reach the axis is $(\lambda L)^{\frac{1}{2}}/3v_o$. An integration time $T_o \ll (\lambda L)^{\frac{1}{2}}/3v_o$ is therefore sufficient since substantial fades will not occur in times less than this. If T is chosen small enough, say $T = T_o$, (8) is a good approximation to (7a), and can be written

$$\frac{d\alpha_D(t)}{dt} \approx k(\lambda, D) \frac{L}{V} \iint_S \left\{ -\left[\frac{1}{T_o} \int_t^{t+T_o} R_D(u, v; t)\mathbf{r}\, dt \right] \right\} \cdot d\mathbf{s}. \quad (9)$$

The quantity in brackets is the rain rate at the point (u, v) averaged over time T_o and defines a suitable rain rate measurement. A rain gauge which measures rain rate in accordance with this definition is described in the Appendix. If the rain rate is known at every point on the surface of the radio path the time derivative of attenuation can be computed from (9).

Since rain rate cannot be measured at all points on the surface of the path we consider what can be done with a more reasonable experiment—one or more rain gauges near the ground in the vicinity of the path. No satisfactory theoretical expression has been derived for computing attenuation from rain rate measurements made near the ground in the vicinity of the path. And the wide disagreement between computed and measured attenuations in experiments of this type described in the literature support the conclusion that the empirical expressions used are also unsatisfactory.[6] Also, the requirements of the sampling theorem must be met if the attenuation is to be computed from sampled rain rates.[11] Visual observation of rainfall reveals a spatial structure so fine that an unreasonably close spacing of rain gauges would be required to meet these requirements.

III. POINT RAIN RATE AND PATH ATTENUATION DISTRIBUTIONS

3.1 *Important Assumptions*

Fortunately, the attenuation as a function of time is not required for the design of radio links; for radio link design the fraction of time that the path attenuation exceeds the fading margin is the important parameter. Thus, we need only a suitable statistic of path attenuation which can be related to a similar statistic of rain rate at ground level in the vicinity of the path. In this section a point rain rate distribution function is defined and related to the path attenuation distribution function.

Let the rain rate be $\mathbf{R}(u, v; t)$ on the surface S of the path. At some point near the path—on the ground beneath the path, for example—a rain gauge measures a sequence of rain rates given by

$$R_n(T, \mathbf{k}, x, y, z) = \frac{1}{T} \int_{t_o+nT}^{t_o+(n+1)T} [-\mathbf{R}(x, y, z; t) \cdot \mathbf{k}] \, dt \tag{10}$$

where $\mathbf{k}$ is a unit vector normal to the collecting surface of the rain gauge and x, y, z are the space coordinates of the rain gauge. Suppose that rain rate measurements have been made for a very long time. The data available is a large number of rain rates $R_n(T, \mathbf{k}, x, y, z)$, one for each interval T. The data is organized by choosing a rain rate R_o and computing the fraction of intervals T for which the rain gauge recorded rates less than R_o. This fraction is denoted by

$$P[R_n(T, \mathbf{k}, x, y, z) \leqq R_o], \tag{11}$$

and is a point rain rate distribution function; it is a function of R_o, the integration time T, the location of the rain gauge, and the pointing direction $\mathbf{k}$. The optimum direction for $\mathbf{k}$ is expected to be vertically upward in many regions; in any case, the rain gauge must be pointed in the direction $\mathbf{k}$ such that for the high rain rates of interest, that is, for $R_o > R_m$,

$$P[R_n(T, \mathbf{k}, x, y, z) \leqq R_0] \leqq P[R_n(T, \mathbf{1}, x, y, z) \leqq R_o],$$

where $\mathbf{1}$ is any unit vector.

For the radio systems of interest, the rain rate, R_m, may be chosen at least one order of magnitude greater than the mean rain rate. In this country the mean rain rate is on the order of 0.1 mm/hr whereas a reasonable value of R_m may be 10 mm/hr.

Let the radio path be divided into a large number, n, of volume elements such that the rain rate is uniform in each element. The average rain rate on the path is

$$R_{ave}(t) \equiv \frac{1}{n} \sum_{i=1}^{n} R_i(x_i, y_i, z_i; t). \tag{12}$$

In integral form this is written

$$R_{ave}(t) \equiv \frac{1}{V} \iiint_V R(x, y, z; t)\, dV. \tag{12a}$$

Two assumptions are made:

(*i*) In a region containing the radio path the point rain rate distribution function is independent of position and (11) can be written

$$P[R_n(T, \mathbf{k}, x, y, z) \leqq R_o] = P[R_n(T) \leqq R_o]. \tag{13}$$

(*ii*) For the rain rates of interest, that is, for $R_o > R_m$, and for integration time T, the distribution function of the average rain rate on the path is greater than, or equal to, the point rain rate distribution function.

$$P[R_{ave}(t, T) \leqq R_o] \geqq P[R_n(T) \leqq R_o], \tag{14}$$

where,

$$R_{ave}(t, T) \equiv \frac{1}{T} \int_t^{t+T} R_{ave}(t)\, dt.$$

The first assumption is that the point rain rate distribution function is the same whether it is measured below the path, on the path, or near the path. It does not mean that the rain rate at any time t is the same everywhere in the region—an essential distinction. The assumption does not mean that rain rate is a stationary random process in either the wide sense or the strict sense. In statistical language it means that the rain rate can be considered a first order stationary process over a small interval.[12] The second assumption reflects Bussey's observation that high rain rates extend over smaller areas than do lower rain rates.

3.2 *A Bound on the Path Attenuation Distribution*

If the speed of the raindrops does not change while in the radio path, the attenuation can be written in terms of rain rate. From (6),

$$\alpha_D(t) = \frac{k(\lambda, D)}{v_D} \frac{L}{V} \iiint_V R_D(x, y, z; t)\, dV. \tag{15}$$

Let the rain have the Laws and Parsons distribution of drop diameters. Then

$$R_D(x, y, z; t) = R(x, y, z; t)p_D , \tag{16}$$

where p_D is the fraction of water in the rain consisting of drops of diameter D. Substituting into (15) and using the definition (12a) the expression for attenuation is

$$\alpha_D(t) = \frac{k(\lambda, D)}{v_D} L p_D R_{ave}(t). \tag{17}$$

The total attenuation is

$$\alpha(t) = LR_{ave}(t) \sum_D \frac{k(\lambda, D)}{v_D} p_D . \tag{18}$$

The quantity represented by the summation has been computed by Ryde and Ryde and by Medhurst for the Laws and Parsons drop diameter distribution and for the terminal velocities of water drops in still air.[6]

In Section 2.3 we showed that negligible changes occur in $\alpha(t)$ in a time $T \leqq T_o$. Thus, (18) can be written

$$\alpha(t) \approx \alpha(t, T_o) = LR_{ave}(t, T_o) \sum_D \frac{k(\lambda, D)}{v_D} p_D , \tag{19}$$

where

$$\alpha(t, T) \equiv \frac{1}{T} \int_t^{t+T} \alpha(t)\, dt.$$

The path attenuation for a uniform rain rate R_o is, from (18),

$$\alpha_o = LR_o \sum_D \frac{k(\lambda, D)}{v_D} p_D . \tag{20}$$

The desired bound can be found by substituting from (19) and (20) into (14).

$$P[\alpha(t) \leqq \alpha_o)] \geqq P[R_n(T_o) \leqq R_o]. \tag{21}$$

This bound says that if the measured point rain rate distribution is converted to an attenuation distribution by (20), the outage time predicted is greater than, or equal to, the outage time that occurs on the path. The application of this bound is illustrated in Section V.

IV. EXPERIMENTAL DETERMINATION OF INTEGRATION TIME

It was shown in Section 2.3 that if the rain rate integration time T is short enough no significant fades will be missed; specifically if $T =$

$T_o \ll (\lambda L)^{\frac{1}{2}}/3v_o$, it is small enough. This determination of T_o is based on considerations as to what could happen on a path. Fades may occur slowly compared with T_o so it is worthwhile to determine whether a larger integration time may be practical.

Suppose that path attenuation and point rain rates have been measured with an integration time $T \leqq T_o$. Then the distributions computed from the measurements will satisfy the inequality (21) which can be written

$$P[\alpha(t, T) \leqq \alpha_o] \geqq P[R_n(T) \leqq R_o], \qquad T \leqq T_o. \tag{22}$$

Now, from (14), this inequality holds for any T and since $T \leqq T_o$ all fading of significance is included.

Let the attenuation distribution be computed from the path loss measurements for integration times of T, $2T$, $3T$, $\cdots$, as long as the distribution remains substantially unchanged. The point rain rate distributions computed for the same integration times are such that the inequality holds and

$$P[\alpha(t, mT) \leqq \alpha_o] \geqq P[R_n(mT) \leqq R_o], \tag{23}$$

for all $m = 1, 2, 3, \cdots$. Now suppose that $P[\alpha(t, mT) \leqq \alpha_o]$ remains substantially unchanged for all integers m up to M. Then all of the corresponding rain rate distributions result in valid upper bounds on outage time as shown by (23); one of these will be the least upper bound.

From experiments of this kind, practical values of integration time can be determined. It is expected (but not proven) that the integration time which results in the least upper bound will be the largest value for which the attenuation distribution remains unchanged, that is, MT.

V. DISCUSSION

Experimental rain rate distributions which meet the requirements of this theory are not available. For this reason a careful experimental verification cannot be made now. There is reason for optimism, however. The upper bound on outage time computed from Bussey's[1] one-minute point rain rate distribution is remarkably close to the one-minute attenuation distributions reported by Semplak and Turrin.[13,14] These distributions are shown in Fig. 2. Semplak and Turrin also report that the attenuation distribution remains unchanged for shorter integration times.[14]

The simplest application of this theory to the design of a radio path requires only a point rain rate distribution measured as described in

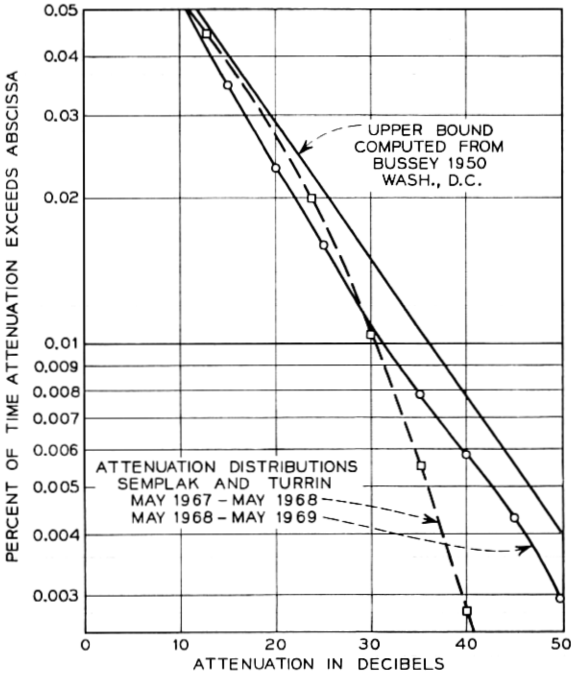

Fig. 2 — Comparison of computed and measured attenuation distributions for one minute integration time.

Section III. In this case the rain rate integration time is computed from the frequency, the path length, and the maximum raindrop velocity. This measured point rain rate distribution can be converted to an upper bound on attenuation outage time by use of expressions (20) and (21). The upper bound obtained this way is not necessarily the least upper bound whereas the least upper bound is required for optimum radio system design. The least upper bound can be computed from the point rain rate distribution if the corresponding optimum rain rate integration time is known; the optimum rain rate integration time can be determined from a path attenuation experiment as described in Section IV.

It is important, then, to determine the optimum integration times in those regions of the country where radio systems above 10 GHz may be used. The optimum integration time is a function of wavelength, path length, and the climate in which the path is located. A few path loss measurements, located in different climatic regions, would yield valuable results. In these experiments attenuation distributions would be measured as functions of path length and wavelength, and optimum integration times computed from this data and the measured point rain rate distributions. It may be, for instance, that the optimum integration times are about the same everywhere; if so, the path loss experiments would show it and, thereafter, only point rain rate measurements would be required.

The accuracy with which the least upper bound predicts the outage time on a radio path cannot be stated precisely until further experimental data is available. It may be anticipated, however, that the accuracy will decrease as the path length increases. For example, if the path is long compared with the dimensions of thunderstorms, the least upper bound prediction will probably be pessimistic, especially for large fades. On the other hand, for paths shorter than the dimensions of thunderstorms the least upper bound prediction may be accurate.

A rain gauge, from which the rain rate in each fixed integration interval can be determined, is suitable for the determination of the point rain rate distribution function defined in Section III. Morgan has built and described such a rain gauge recently, and another rain gauge proposed for this purpose is described in the Appendix.[15]

No attempt has been made to include the effects, on attenuation, of raindrop distortion, temperature, radio wave polarization, and so on, in this theory.

VI. ACKNOWLEDGMENT

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APPENDIX

An Instrument for Measuring Point Rain Rates

The instrument described measures rain rates in accordance with the definition of (9). Figure 3 illustrates the basic element in the rain rate instrument—a depth gauge consisting of a funnel, a cylindrical capacitor, and a shutter for draining the capacitor. The funnel is exposed to the rain for a specified time T and then covered. The rain which passed through the collecting area of the funnel drains into the cylindrical capacitor and forms a column of water as shown. The dielectric constant of water increases the capacitance—the greater the height of

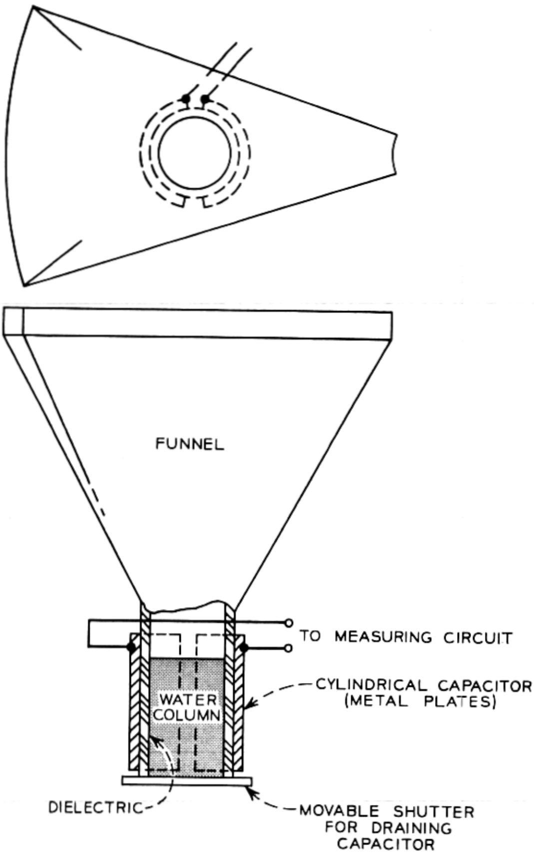

Fig. 3 — Capacitor depth gauge.

the water column, the greater the capacitance. When the funnel has completely drained, the capacitance (and hence the volume of water) is measured.

There are two features of importance about this depth gauge.

(*i*) The interval of exposure, or integration time, can be determined precisely by careful operation of the shield over the funnel, and is not dependent on the rain rate.

(*ii*) The time allowed to drain the funnel and measure the capacitance is independent of the exposure interval T. Sufficient time can be allowed to drain the funnel and stabilize the water column prior to measuring the capacitor. This eliminates fluctuations due to the random behavior of water flow on the surface of the funnel.

When the shield is over the depth gauge no rain is collected. The complete rain gauge consists of a number of depth gauges arranged as shown in Fig. 4. In this illustration, ten depth gauges are shown, with number 2 in position to collect rain. When the interval ends, the shield rotates, covering depth gauge number 2 and exposing number 3.

A rotating shutter for draining the depth gauge capacitors is shown in Fig. 4B. The shutter is fixed to a common shaft with the funnel shield of Fig. 4A. The operating sequence is as follows. There are ten time intervals of length T in a single rotation of the shaft. With the aid of Fig. 3 the following sequence can be seen to occur.

Time Interval Number	Status of Depth Gauge #2	Status of Depth Gauge #3
1	Collecting Water	Draining Capacitor
2	Draining Funnel	Collecting Water
3	|	Draining Funnel
4	|	|
5	|	|
6	|	|
7	|	|
8	Measure Capacitance	|
9	Draining Capacitor	Measure Capacitance
10	Draining Capacitor	Draining Capacitor

The rain shield steps rapidly from gauge to gauge. There is always one gauge collecting rain; one measurement is made in each time interval T. If the measurement starts at time t_o and the rain gauge points

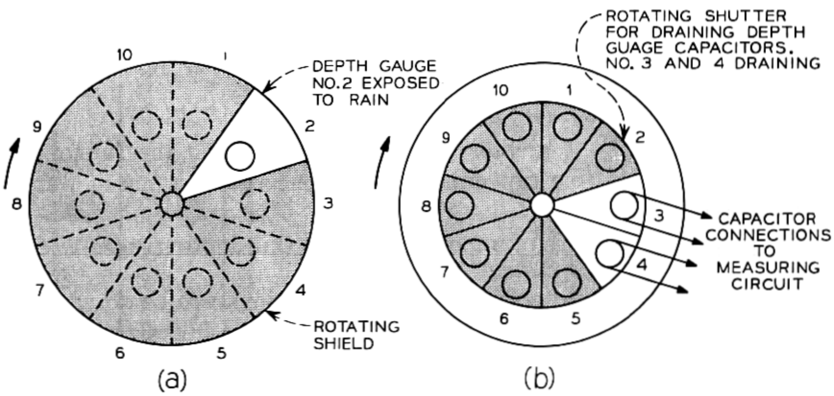

Fig. 4(a) Top view of rainrate gauge. (b) Section of rainrate gauge.

vertically upward, the output of the instrument is a sequence of measurements

$$R_n(t, x, y, z) = \frac{1}{T} \int_{t_0+nT}^{t_0+(n+1)T} R(t)\, dt.$$

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