Statistics on Attenuation of Microwaves by Intense Rain

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(Manuscript received June 12, 1969)

Heavy rainfall and associated attenuation at centimeter and millimeter wavelengths are discussed. Measured attenuations are combined with path-rainfall statistics obtained from a rain-gauge network to produce plots of attenuation versus path length for a given probability of fading. Under the assumption that the spatial behavior of heavy rain is similar at various locations, the path-average rainfall statistics are combined with highly resolved point rain rates for geographically separated places to produce attenuation data appropriate to those places. Dual parallel-path-diversity is also evaluated; it is shown to be a very advantageous arrangement.

I. INTRODUCTION

An important problem in designing wide-band radio-relay systems at frequencies exceeding 10 GHz is reliability. Propagation through heavy rain is the significant factor in determining realiability of the medium. Thus it is important to examine the spatial and temporal behavior of heavy rain and the resultant attenuation.

Recent measurements of progagation at 18.5 GHz and 30.9 GHz, and analysis of rainfall data from the Crawford Hill rain-gauge network of Bell Telephone Laboratories at Holmdel, New Jersey, have led to an improved understanding of the rain environment.[1-4] Those data are used here to provide information on attenuation by rain for use in system design. In particular, the improvement in performance obtained by use of path diversity is evaluated.[5]

II. SINGLE-PATH STATISTICS (NEW JERSEY)

2.1 *The Magnitude of the Attenuation*

First, one must ask: What is the magnitude of the attenuation caused by heavy rain at frequencies exceeding 10 GHz? Figure 1 is a plot of

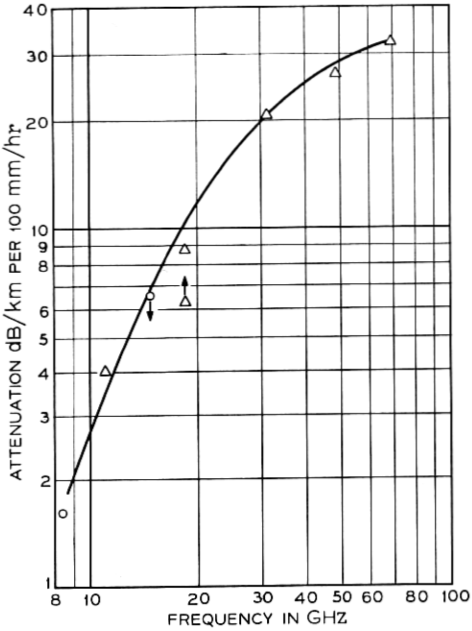

Fig. 1 — Attenuation measured during rain of rate 100 mm/hr (averaged over a 1 km path). The measurements at 8 and 15 GHz are from Ref. 6; 11 GHz from Ref. 7; 18 and 30 GHz from Refs. 1 and 2; and 50 and 70 GHz from Ref. 9. △ indicates Bell Telephone Laboratories data and ○ indicates DRB 1966 data—Canada (some extrapolation for both).

attenuation measured at a rain rate of 100 mm/hr (4 inches per hour) for a path length of 1 km. Data measured at 100 mm/hr, rather than at low rain rates, are used because path-average rain rates of this magnitude do indeed occur a significant percentage of the time in many places, including New Jersey.

Moreover, in the discussion that follows, we are concerned with attenuations caused by path-average rain rates of the order of 100 mm/hr, and the attenuations will be taken to be directly proportional to the path average rain rates; that is, proportional to the average density of rain along the path.* The curve in Fig. 1 serves as a benchmark by means of which attenuation is related to heavy path-average rain rates. Thus

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* From theoretical considerations, the attenuation γ, at frequencies of the order 10 GHz is believed related to the rain rate R by $\gamma = \alpha R^{\beta}$. α is a function of frequency as indicated and β is also a mild function of frequency with values near unity. Here we use values of α measured at high rain rates (since β is taken to be unity) to minimize errors in the event β departs from unity.

for heavy rains one has:

$$\gamma = 0.04\mathbf{R}d; \qquad \gamma = 0.1\mathbf{R}d; \qquad \gamma = 0.2\mathbf{R}d$$

for frequencies of 11, 18, and 30 GHz, γ being the attenuation in decibels and **R** the average rain rate on a path of length d.

2.2 *Dependence of Path-Average Rain Rate on Path Length*

The path analysis of rain rate discussed in Ref. 3 encompasses the heavy rains of 1967 taken on 100 rain gauges forming a $130(\text{km})^2$ grid in New Jersey. Obviously, there are many paths of various lengths in such a network and a relatively large amount of data is obtained for such paths from the several storms that occur during one year. Path average rain rates have been converted to a yearly base and are plotted in Fig. 2;* the curves show the probability of path-average rain rate with path length as parameter. At rain rates of the order 50 mm/hr, the probabilities are about the same for all path lengths, namely, about 0.01 percent; thus the probability of exceeding an average rate of 50 mm/hr on a 10.4 km path is about the same as at a point (path length—zero in Fig. 2). As the rain rate increases, the curves diverge. For example, the probability of a 100 mm/hr rain rate on a 10.4 km path is less by a factor of ten than that for a point; at 150 mm/hr, the factor is one hundred.

The data in Fig. 2 can be examined in another way. Consider a given probability, say, 0.001 percent (five minutes per year); the corresponding rain rate at a point is about 160 mm/hr, whereas for a 10.4 km path it is 80 mm/hr. This behavior tells us that heavy rains occur as localized showers. Of course, this behavior will show up in evaluating the attenuation on paths of various lengths.

2.3 *Dependence of Attenuation on Path Length*

The relationship between attenuation and path-average rain rate at various frequencies as given in Fig. 1, and the probability of occurrence of rain rates, Fig. 2, have been used to produce Figs. 3a and b. Two probability levels (0.01 percent, 50 min/yr, and 0.001 percent, 5 min/yr) and three frequencies (11, 18, and 30 GHz) have been chosen as representative of radio relay. These plots give computed attenuation that is exceeded for the percent of time indicated on the figure as a function of path length. Note that there is curvature in the plots. As one would expect, having looked at Fig. 2, the attenuation one obtains

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* From curves A in Fig. 28 of Ref. 3.

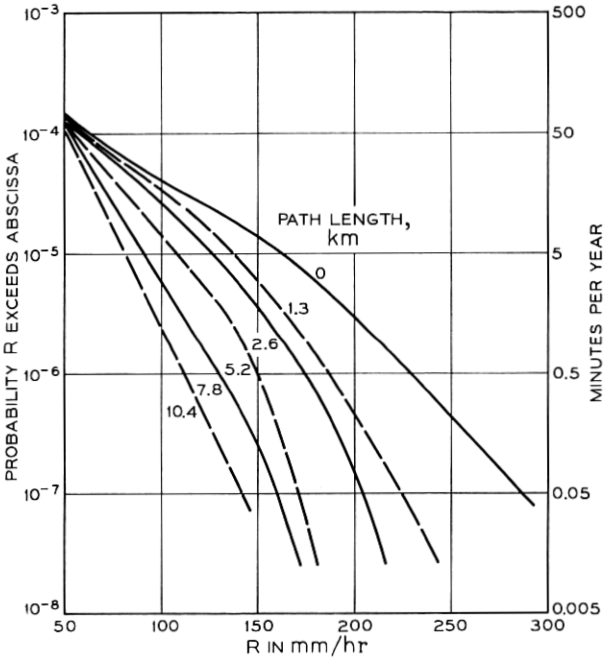

Fig. 2 — Probability of path-average rain rates for paths of various lengths; 1967 rain-gauge network data.

(for a low probability) on a 10 km path is less than one would expect by linearly extrapolating from the attenuation on a 1 km path.

The two propagation paths in operation within the Holmdel rain-gauge network are 1.9 and 6.4 km long at frequencies 30.9 and 18.5 GHz, respectively;[1,2] these lengths are indicated by arrows on the abscissas in Fig. 3.* Percent of time distributions of attenuation on these paths were measured throughout 1967 and 1968 and points taken at the indicated probability level are shown on the figures. For the 1.9 km path, the measured 30.9 GHz attenuations agree well with the computed curves for 30 GHz: somewhat higher in Fig. 3a and slightly lower in Fig. 3b.

Likewise, in Fig. 3a the points measured at 18.5 GHz (6.4 km) are in good agreement with, but are somewhat lower than, the computed curve for 18 GHz. In Fig. 3b the 18.5 GHz measurement for 1968 is somewhat below the computed curve; however, the 1967 measurement

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* The 18.5 GHz signal is vertically polarized and the 30.9 GHz signal is polarized 45° from vertical.

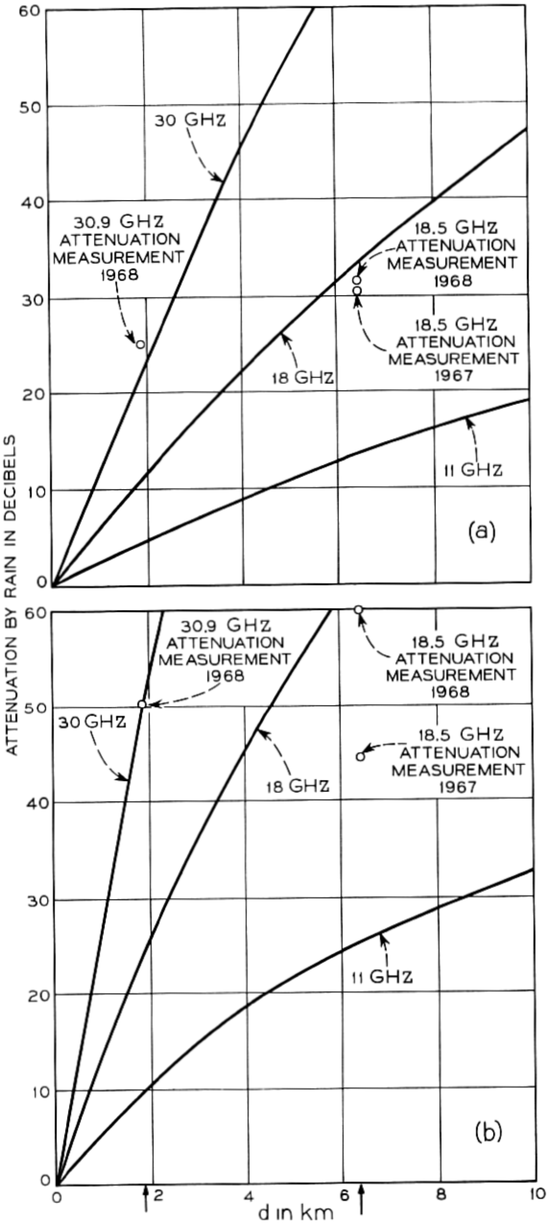

Fig. 3 — Attenuation as a function of path length at 11, 18 and 30 GHz (1967 network data) for (a) 0.01 percent probability (50 min/yr) and (b) 0.001 percent probability (5 min/yr) along with measurements on paths of length 1.9 km (30.9 GHz) and 6.4 km (18.5 GHz).

is considerably lower. Comparison of the 18.5 GHz attenuation distributions for 1967 and 1968 shows that heavy showers were more frequent on this path in 1968 than in 1967. The 18 GHz curves in Figs. 3a and b are apparently somewhat conservative.

Thus use of the attenuations in Fig. 1 to convert the pool of path-average rain rates from the rain-gauge network has led to a set of curves of attenuation versus path length that are consistent with independent measurements of attenuation. Accordingly, for design of a conventional tandem relay system at, say, 18 GHz, with a 30 dB margin, the repeater spacing is, from Fig. 3b, 2.5 km for 0.001 per cent probability on individual paths in coastal New Jersey.

III. SINGLE-PATH STATISTICS (OTHER LOCATIONS)

It is tempting to ask if the knowledge gained from the above studies can be used to say something about the attenuation environment in places other than coastal New Jersey. If certain assumptions are made concerning the spatial distribution in rain showers, that can be done.

3.1 *Point Rainfall Rates of High Resolution*

Distribution of point rain rates with high resolution have been measured in a few places, shown in Fig. 4. Four of the full curves were measured in the United states by the Illinois State Water Survey using a photographic method measured over the best part of a year; they form a consistent set of data.[6] This method is capable of measuring drops in a small volume during a short interval every ten seconds. The solid line for Bedford, England is from a four-year sample;[5] gauges with two-minute resolution were used. The dashed curve is the distribution for the pool of data taken during 1967 on the rain-gauge network at Holmdel, New Jersey;[3] gauges with a time constant less than one second were sampled every ten seconds.

For a given probability of occurrence, how much heavier does it rain at other locations than in New Jersey? Table I shows the point rain-rate intensity in other places relative to New Jersey for the 0.01 and 0.001 per cent levels; the Illinois state survey set of curves and the data from England in Fig. 4 are used in this comparison.

Thus in the regime of low probability (high rain rate), the rain intensity in New Jersey is about one quarter that of Miami, Florida, and five times that of Corvallis, Oregon. These data must now be linked with the spatial distributions obtained in New Jersey in order to determine the attenuations.

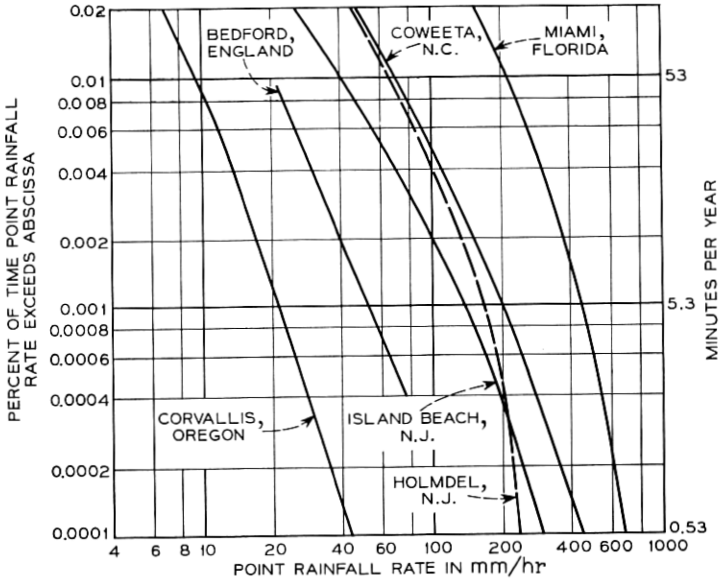

Fig. 4—Point rainfall rates measured in several places by instruments with rapid response.

3.2 *The Spatial Distribution of Rain Showers*

The data in Fig. 2 show that the probability of a given path-average rain rate decreases with increasing path length for heavy rains, a not too surprising result since one is dealing with rain cells of limited size. Likewise, for a given probability level, the path-average rain rate decreases with increasing path length as shown in Fig. 5. For relatively high probability (10^{-4}), this decrease does not amount to much; as shown by the lowest curve in Fig. 5, the average rain rate for a 10 km path is about the same as that for a point ($d = 0$). However, for example, on the upper-

TABLE I—RELATIVE INTENSITY OF POINT RAIN RATES

Probability Level	Miami Florida	Coweeta North Carolina	Island Beach New Jersey	Bedford England	Corvallis Oregon
(a) 10^{-4} (50 min/yr)	5	1.75	1	0.48	0.25
(b) 10^{-5} (5 min/yr)	3.5	1.55	1	0.42	0.15
AVERAGE of (a) & (b)	4.2	1.65	1	0.45	0.2

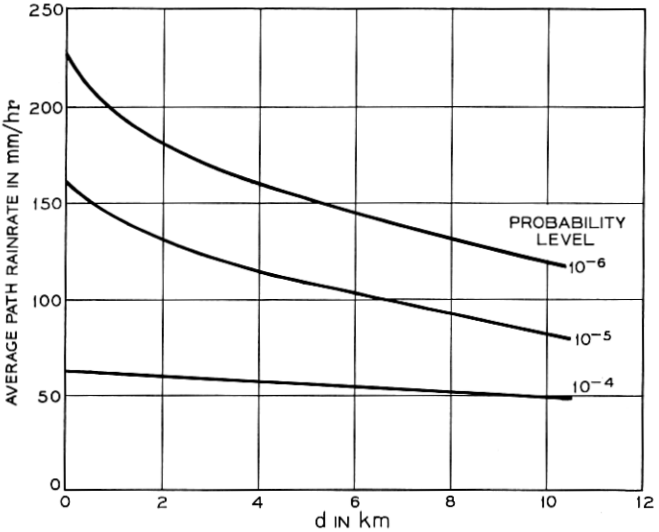

Fig. 5 — Average path rain rate in New Jersey versus path length for probability levels 10^{-4}, 10^{-5}, and 10^{-6}.

most curve in Fig. 2 (for 10^{-6} probability) the average rain rate on a 10 km path is only one half the rate at a point.

Assume that the spatial behavior of heavy rainfall is the same in other places as it is in New Jersey. This means that in a place with relatively low point rain rates (such as Oregon, Fig. 4), path-average rates are about the same as point rates (such as in the lowest curve in Fig. 5), that is, large-area rain. Whereas, where the point rain rates are very high (such as in Florida, Fig. 4), the path-average rates are much less than the point rates (such as in the uppermost curve in Fig. 5), that is, showers. To determine whether this assumption is warranted, one must await spatial measurements of rain rate in other places.

The data in Figs. 2 and 4 are used to construct Table II, a list of path-average rain rates for the various locations, as a function of path length, *d*. Some extrapolation of the curves in Fig. 2 was necessary to obtain the column for Miami, Florida.

Table II has been converted to attenuation at 18 GHz by way of the relationship discussed above as shown in Fig. 6. As one would expect, the attenuation for Oregon is linear with path length, whereas for places with heavy rain, there is considerable curvature. Figure 6 tells us that a single transmission path at 18 GHz with a 30 dB fading margin should not exceed 1, 2, 3, 6, and 15 km in Florida, North Carolina, New Jersey,

TABLE II—PATH-AVERAGE RAIN RATES IN MM/HR FOR THE 10^{-5} PROBABILITY LEVEL

d-km	Corvallis Oregon	Bedford England	Island Beach New Jersey	Coweeta North Carolina	Miami Florida
0	20	55	130	200	450
1.3	20	53	110	165	325
2.6	20	52	103	153	265
5.2	20	50	90	135	215
7.8	20	48	75	110	210
10.4	20	45	70	95	

Bedfordshire-England*, and Oregon, respectively, if a probability of 10^{-5} is stipulated.

One might argue that in Florida (for example) where the water vapor available for production of rain exceeds that of New Jersey, the dimension of a rain cell of given rain rate may exceed that of a cell of the same rain rate in New Jersey. If this were true, the attenuation for Florida and North Carolina in Fig. 6 would be somewhat higher than shown.

IV. PATH DIVERSITY (NEW JERSEY)

The analysis of the rain-gauge network data by Freeny and Gabbe[3] encompasses not only single paths of various lengths but also joint statistics for pairs of parallel paths separated by various distances.† These data are applicable to the design of path-diversity systems in that they are statistics of the percentage of time that the average rain rate on both paths exceeds given values. Of course, the idea in path diversity is to switch to the path with lowest attenuation.[5]

4.1 *Two Parallel Paths with a Given Separation*

An example of how path-average rain rates in the diversity arrangement convert to attenuation is given in Figs. 7a, b, and c for frequencies of 11, 18, and 30 GHz. The curves apply to the 0.001 percent probability level (5 min/yr) and a diversity separation of 5.2 km (3.25 miles). For comparison, the attenuation for a single path is shown by a dashed line

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* In a recent Committee Consultatif Internationale Radio document (United Kingdom Document IX/164-E, May 9, 1969), attenuation distributions for a 24 km path, and for the worst year (1968) observed to date, indicate that the path length appropriate to 0.001 percent probability and 30 dB attenuation is something less than 12 km in Bedfordshire at 18 GHz. This presumably means that, even in a relatively low rain-rate environment, the heavier rains do indeed occur as showers of limited size (see also Ref. 5). That being the case, the curve for England in Fig. 6 would have more curvature than indicated, that is, the curve in Fig. 6 would be quite conservative.

† See Fig. 28 of Ref. 3.

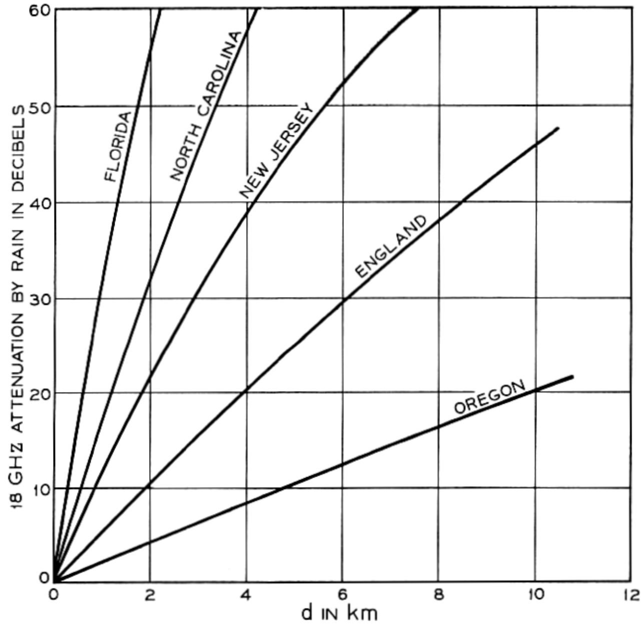

Fig. 6 — 18 GHz attenuation versus path length for various places; probability level, 10^{-5} (5 min/yr).

on each figure. If transmission paths at 18 GHz with a 30 dB fading margin are considered, Fig. 7b shows that the path length in the diversity arrangement with 5.2 km separation can be just over 5 km, compared with 2.5 km when no diversity is used.

4.2 *Relationship Between Interrepeater Path Length and Diversity Separation*

A somewhat more general question is: For a given attenuation margin and a given probability level, how does the inter-repeater path length change with diversity separation? As an example, 18 GHz, 30 dB, and 0.001 percent are chosen for the frequency, margin, and probability level; the data are plotted in Fig. 8. Note that the path length d for a diversity separation s of 7.5 km is 7 km, about thrice the path length (2.5 km), for the nondiversity arrangement ($d = 0$). Results such as these have considerable economic implications. The data can also be plotted as in Fig. 9 where 18 GHz attenuation is given as a function of path length with path separation a parameter. Apparently, for a given

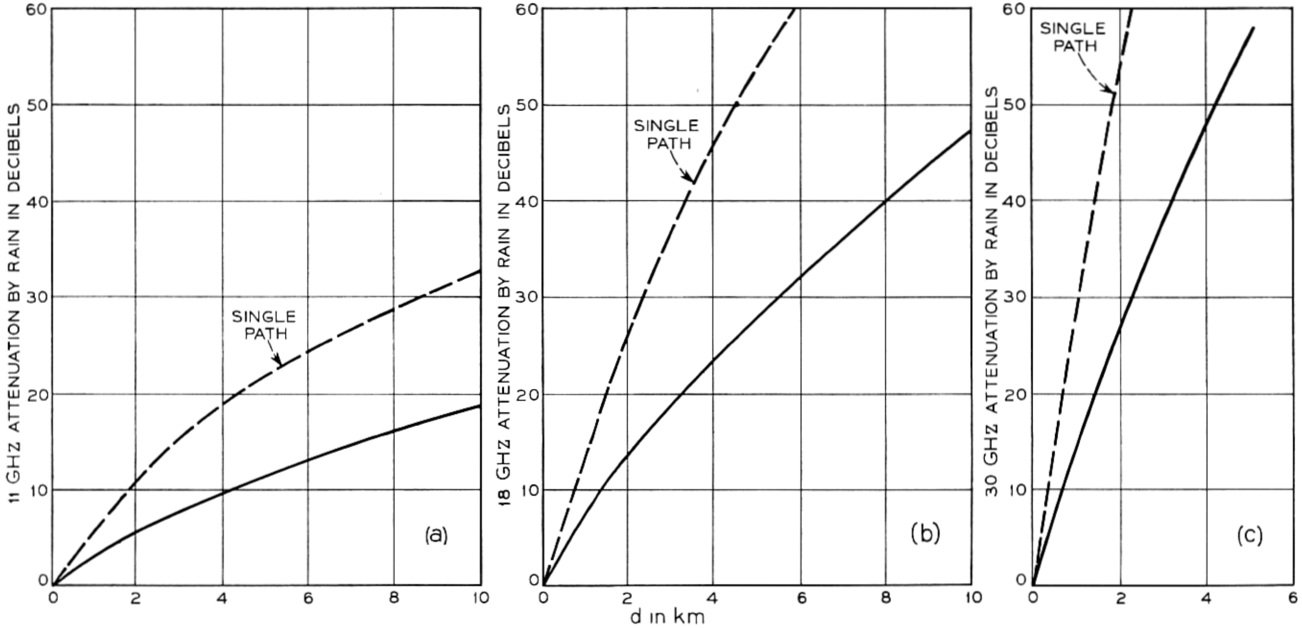

Fig. 7—Attenuation appropriate to a dual parallel-path-diversity separation of 5.2 km as a function of path length for frequencies (a) 11 GHz, (b) 18 GHz, and (c) 30 GHz. The dashed curves are for conventional (nondiversity) paths. (Attenuation exceeded 0.001 percent of the time jointly for two parallel paths spaced 5.2 km apart.)

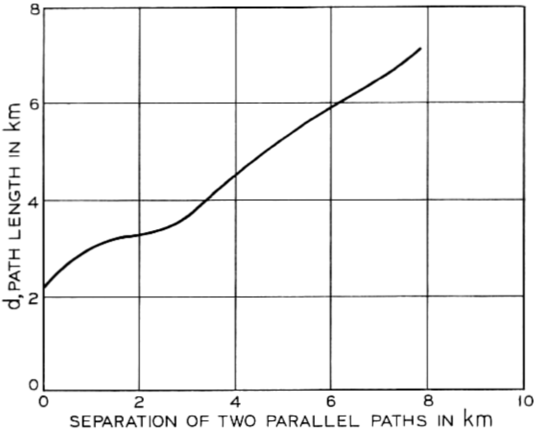

Fig. 8 — Path length as a function of dual parallel-path-diversity separation for a 0.001 percent probability level (5 min/yr) at 18 GHz with a 30 dB attenuation margin; 1967 network data.

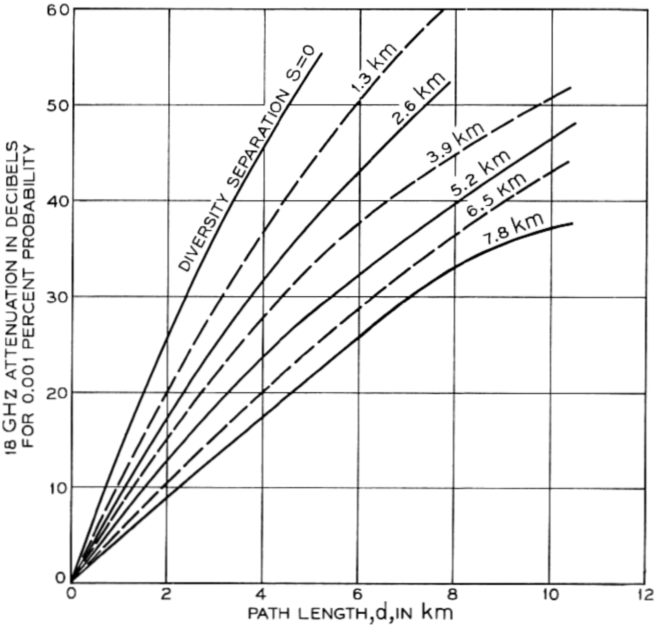

Fig. 9 — 18 GHz attenuation appropriate to 0.001 percent probability versus path length for various diversity separations; 1967 network data.

diversity separation, the advantage of diversity over nondiversity is not a strong function of the fading margin.

As yet we have no actual attenuation measurements on path diversity. However, the data of Figs. 8 and 9 are believed conservative in the same sense as those of Fig. 3; of course, they apply only to coastal New Jersey.

V. DISCUSSION

Although the data given here result in well-resolved design curves hopefully useful in design of radio systems, at least two important questions remain. A system is comprised of many paths in tandem forming a route of length l, whereas here only single paths have been discussed. If one has n such paths in tandem, is the probability P_l of attenuation by rain on the system simply nP_1 where P_1 is the probability for a single path? In other words, is there no correlation between heavy fades on tandem paths? Obviously, if a dense rain cell were centered on a repeater, there would be correlation of attenuation on the two paths associated with that repeater. From such considerations and examination of rainfall data, the relationship $P_l = nP_1$ is believed too conservative.

The other question is related to path diversity. We have only discussed the case of two (single) parallel paths separated by various distances. But in an actual system one deals with several paths in tandem on each leg of the route; these two legs must of course merge if one wishes to switch from one to the other. The path lengths for merge points lie between those given in Fig. 3 and those appropriate to a parallel path diversity arrangement.[5] Moreover, the diversity analysis here deals with two (single) parallel paths of given separation whereas in practice one would be dealing with a line of tandem paths parallel to, and displaced from, a second such set. In that case, the advantage gained by path diversity must be investigated beyond what we have done here.

Finally, it should be pointed out that the microwave systems to which the above discussion is pertinent would carry very wide bands of information. Clearly, the advantages of dual paths in providing equipment diversity (in addition to propagation reliability) would be considerable in such systems, especially from the viewpoint of maintenance.

VI. ACKNOWLEDGMENT

Discussion with R. A. Semplak, J. D. Gabbe, and Mrs. A. E. Freeny stimulated production of this paper.

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