Interchannel Interference Considerations in Angle-Modulated Systems

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This paper considers the deterioration in performance of angle-modulated systems resulting from interchannel interference. We show that with band-limited white gaussian noise modulation (simulating modulation by a frequency division multiplex signal), we can derive an explicit expression for the spectral density of the baseband interchannel interference when two or more PM waves interfere with each other.

We show that, if the interference is co-channel, maximum interference occurs at the lowest baseband frequency present in the system and we can derive upper and lower bounds to this minimum baseband signal-to-interference ratio. For high enough modulation index, we show that this minimum signal-to-interference ratio is proportional to the cube of the modulation index and that phase modulation can be used with advantage in interference limited systems. We do not consider the effects of linear filters on angle-modulated systems, but give some results about the effect of adjacent channel interference when the interference is in the passband of the receiver.

I. INTRODUCTION

The properties of frequency and phase modulation with respect to exchanging bandwidth for signal-to-noise ratio are well known,[1,2] but the type of noise considered is almost always limited to be random gaussian noise. In the design of any system, where the noise is likely to be interference limited, it is necessary to consider other kinds of disturbances such as co-channel and adjacent channel interference corrupting the desired received signal.

Consider the following situation. In the frequency bands above 10 GHz where the signal attenuation resulting from rain could be very severe, close spacings of the repeaters are almost always mandatory for reliable communication from point-to-point and for all periods of time.[3,4] If low noise receivers are used in the system, it is possible that the total interference power received by the system may be very much larger than the noise power in the system. For all practical purposes, the performance of such a system is determined by the interchannel interference.[3,4] It is therefore desirable to evaluate the effect of co-channel and adjacent channel interference on the performance of any modulation system like FM or PM (or PCM) so that its advantages in combating interference can be determined, and any system parameters (such as rms phase deviation, channel separation, and so on) can be properly chosen to keep the baseband interference below a certain desired level. (It is possible to reduce adjacent channel interference by using suitable receiving filters, but co-channel interference occupies the same band as the signal.)

The problem of interference in angle-modulated systems has been considered by many authors.[5–12] In the analysis, most of these authors have given an approximate expression (the first term in the power series expansion) for the baseband interchannel interference, and have shown that it can be expressed as the convolution of the spectral densities of the angle-modulated waves. The accuracy in this approximation has not been determined previously. Also, in the calculation of interchannel interference in high index FM and PM systems, most of these authors use the quasistatic approximation, the accuracy of which is unknown.

We first consider a general method of evaluating the baseband interchannel interference when two angle-modulated waves interfere with each other. We assume that an ideal angle (frequency or phase) demodulator is used in the system. (An ideal angle demodulator does not respond to any variations in the amplitude of the wave. This can be achieved in practice by using an ideal limiter at the front end of the receiver. If $A(t)e^{i\Phi(t)}$ is the input to an ideal limiter, its output is given by $A_o e^{i\Phi(t)}$ where A_o is a constant.)

We obtain a general expression for the baseband interference when the modulating wave is gaussian. This expression can be utilized even when the baseband signal is passed through a linear network (such as a pre-emphasis—de-emphasis network).

We are specifically interested in calculating the baseband interchannel interference between two or more waves phase modulated (without pre-emphasis) by band-limited white gaussian random processes. It has been found in practice that such a random gaussian noise of appropriate bandwidth and power spectral density adequately simulates (for some purposes) a variety of signals such as a frequency division multiplex (FDM) signal, a composite speech signal, and so on.[13] Since the determination of the power spectrum is fundamental to the evaluation of baseband interference, first we review briefly the methods of obtaining this spectrum for a wave phase modulated by band-limited white gaussian noise.

In the case of band-limited white gaussian noise modulation, if the bandwidths of the modulating waveforms for the desired and interfering carriers are the same, we show that the determination of baseband interference power is relatively simple, and requires only the computation of the spectral density of a phase-modulated carrier for a variety of values of rms phase deviation. For small values of interference and for band-limited white gaussian noise modulation, we also show that the first term in the series gives most of the contribution to the baseband interference, and that this first term can be used as a good approximation.

For a co-channel interferer, we show that maximum interference occurs at the lowest baseband frequency present in the system (we assume that this lowest frequency is $f = 0$)* and that we can derive upper and lower bounds to this minimum signal-to-interference ratio. For sufficiently high modulation index, we show that these bounds are proportional to the cube of the modulation index, and that phase modulation can be used to advantage in combating interference.[14]

We show that maximum interference with an adjacent channel interferer occurs at the highest baseband frequency present in the system if the carrier frequency separation f_d between the two channels is relatively large compared with the baseband bandwidth W. For a set of values of f_d/W and for different modulation indexes of the two channels, we compute this minimum signal-to-interference ratio and give the results in graphic form.

We then consider the case in which more than one interferer may corrupt the desired received carrier and show that we can derive an expression for the spectral density of the resulting baseband interference. This expression is in the form of an infinite series and for its evaluation, in the case of band-limited white gaussian noise modulation and equal modulation bandwidths, it is only necessary to be able to compute the spectral density of a sinusoidal carrier phase modulated by gaussian noise. In case all these interferers are co-channel and all of them have the same (high) modulation index Φ, we show that we can derive upper and lower bounds to the minimum baseband signal-to-interference ratio.

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* We do not imply that maximum baseband interchannel interference always occurs at $f = 0$ for any general system angle modulated by gaussian noise.

II. INTERFERENCE BETWEEN TWO ANGLE-MODULATED WAVES

We first assume that there is only one interfering wave corrupting the desired received signal, and that both of them are angle modulated by two independent gaussian random processes. Let the desired angle-modulated wave be given by

$$\begin{aligned} s(t) &= A \cos [\omega_o t + p(t) * \varphi(t)] \\ &= \operatorname{Re} A \exp \{j[\omega_o t + p(t) * \varphi(t)]\}, \end{aligned} \tag{1}$$

where A is the amplitude of the wave, $f_o = \omega_o/2\pi$ its carrier frequency, $p(t)$ the impulse response of the pre-emphasis network, and $\varphi(t)$ is a stationary gaussian random process with mean zero, and covariance function $R_\varphi(\tau)$. (We only assume that $p(t)$ is the impulse response of a linear network through which $\varphi(t)$ may be passed. Only for convenience, we refer to it as the impulse response of the pre-emphasis network.) The notation $A(x)*B(x)$ represents the convolution of function $A(x)$ with $B(x)$.

Let the interfering wave $i(t)$ be given by

$$\begin{aligned} i(t) &= R_i A \cos [\omega_i t + p_i(t) * \varphi_i(t) + \mu_i] \\ &= \operatorname{Re} AR_i \exp \{j[\omega_i t + p_i(t) * \varphi_i(t) + \mu_i]\}, \end{aligned} \tag{2}$$

where AR_i is its amplitude (R_i is the relative amplitude of the interfering wave with respect to the desired wave), ω_i is its angular frequency, $p_i(t)$ is the impulse response of its pre-emphasis network, and $\varphi_i(t)$ is a stationary gaussian random process with mean zero and covariance function $R_{\varphi_i}(\tau)$.

Since $s(t)$ and $i(t)$ usually originate from two different sources, it seems reasonable to assume that μ_i is a uniformly distributed random variable with probability density $\pi_{\mu_i}(\mu)$ where

$$\pi_{\mu_i}(\mu) = \begin{cases} \dfrac{1}{2\pi}, & 0 \leqq \mu < 2\pi \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Further, we assume that $\varphi(t)$ and $\varphi_i(t)$ are independent of each other and independent of μ_i. (Reference 13 treats of the case in which μ_i is a deterministic constant, and $\varphi(t)$ and $\varphi_i(t)$ are not independent of each other.)

If we assume that $s(t)$ and $i(t)$ are both in the passband of the receiver used in the system, the total signal $r(t)$ incident at the receiver is given by†

$$\begin{aligned} r(t) &= \text{Re}\, A(\exp\{j[\omega_o t + p(t) * \varphi(t)]\} \\ &\quad + R_i \exp\{j[\omega_i t + p_i(t) * \varphi_i(t) + \mu_i]\}) \\ &= \text{Re}\, A(1 + R_i \exp\{j[(\omega_i - \omega_o)t + p_i(t) * \varphi_i(t) - p(t) * \varphi(t) + \mu_i]\}) \\ &\quad \cdot \exp\{j[\omega_o t + p(t) * \varphi(t)]\} \\ &= \text{Re}\, A a(t) e^{j\lambda(t)} \exp\{j[\omega_o t + p(t) * \varphi(t)]\} \\ &= \text{Re}\, A a(t) \exp\{j[\omega_o t + p(t) * \varphi(t) + \lambda(t)]\}, \end{aligned} \tag{4}$$

where

$$a(t)e^{j\lambda(t)} = 1 + R_i \cdot \exp\{j[(\omega_i - \omega_o)t + p_i(t) * \varphi_i(t) - p(t) * \varphi(t) + \mu_i]\}. \tag{5}$$

Notice from equation (4) that the (excess) phase angle $\eta(t)$, as detected by an ideal angle demodulator, is given by

$$\eta(t) = \varphi(t) + \lambda(t). \tag{6}$$

(The gain—or proportionality factor—of the phase demodulator has been assumed to be unity.) Therefore, the spectral density of $\eta(t)$ can be written as

$$S_\eta(f) = \int_{-\infty}^{\infty} R_\eta(\tau) e^{-j2\pi f\tau}\, d\tau, \tag{7}$$

where $R\eta(\tau)$ is the covariance function of $\eta(t)$, and

$$R_\eta(\tau) = \langle \eta(t)\eta(t + \tau) \rangle. \tag{8}$$

(The notation $\langle x \rangle$ represents the ensemble average of random variable x.) If there is no interference, and if $q(t)$ is the impulse response of the de-emphasis network used in the system, the detected phase angle $\Omega(t)$ can be written as

$$[\Omega(t)]_{R_i = o} = q(t) * p(t) * \varphi(t). \tag{9}$$

If $R_i \neq 0$,

$$\Omega(t) = q(t) * p(t) * \varphi(t) + q(t) * \lambda(t). \tag{10}$$

Now if we assume that the de-emphasis network is the inverse of

† In this paper we do not consider the effects of linear filters usually used in receiving systems on the interchannel interference between two (or more) angle-modulated systems.

the pre-emphasis network, we have

$$q(t) * p(t) = \delta(t), \tag{11}$$

and

$$\Omega(t) = \varphi(t) + q(t) * \lambda(t), \tag{12}$$

where $\delta(t)$ is the Dirac delta function.

From equation (5), we have

$$\lambda(t) = \operatorname{Im} \ln (1 + R_i \exp \{j[\omega_d t + p_i(t) * \varphi_i(t) - p(t) * \varphi(t) + \mu_i]\}), \tag{13}$$

where

$$\omega_d = \omega_i - \omega_o . \tag{14}$$

Notice that

$$\ln (1 + z) = \sum_{m=1}^{\infty} (-1)^{m+1} \frac{z^m}{m}, \qquad |z| < 1, \tag{15}$$

where z is any complex number.

Therefore, for $R_i < 1$, we have†

$$\begin{aligned} \lambda(t) &= \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m} \\ &\quad \cdot R_i^m \left[\frac{\exp \{jm[\omega_d t + p_i(t) * \varphi_i(t) - p(t) * \varphi(t) + \mu_i]\}}{2j} \right. \\ &\quad \left. - \frac{\exp \{-jm[\omega_d t + p_i(t) * \varphi_i(t) - p(t) * \varphi(t) + \mu_i]\}}{2j} \right] \\ &= \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m} R_i^m \sin \{m[\omega_d t + p_i(t) * \varphi_i(t) - p(t) * \varphi(t) + \mu_i]\}. \end{aligned} \tag{16}$$

Since $\varphi(t)$, $\varphi_i(t)$, and μ_i are statistically independent random variables and since $\langle \exp (jk\mu_i) \rangle = 0$ with $k \neq 0$, we can show from equations (6), (8), (13), and (16) that

$$R_\eta(\tau) = R_p(\tau) * R_\varphi(\tau) + \sum_{m=1}^{\infty} \frac{R_i^{2m}}{2m^2} \cos m\omega_d \tau \cdot \exp (-m^2\{[R_{\varphi p}(0) - R_{\varphi p}(\tau)] + [R_{\varphi_i p_i}(0) - R_{\varphi_i p_i}(\tau)]\}), \tag{17}$$

† For $R_i < 1$, notice that $a(t) > 0$.

where‡

$$R_p(\tau) = \int_{-\infty}^{\infty} p(t)p(t+\tau)\,dt, \tag{18}$$

$$R_{p_i}(\tau) = \int_{-\infty}^{\infty} p_i(t)p_i(t+\tau)\,dt, \tag{19}$$

$$R_{\varphi p}(\tau) = R_p(\tau) * R_\varphi(\tau), \tag{20}$$

and

$$R_{\varphi i p_i}(\tau) = R_{p_i}(\tau) * R_{\varphi i}(\tau). \tag{21}$$

Therefore, the spectral density of the output is given by

$$S_\Omega(f) = S_\varphi(f) + \frac{1}{|H_p(f)|^2}\sum_{m=1}^{\infty}\frac{R_i^{2m}}{4m^2}[T_m(f-mf_d)+T_m(f+mf_d)], \tag{22}$$

where $H_p(f)$ is the Fourier transform of $p(t)$, and

$$T_m(f) = \int_{-\infty}^{\infty} \exp\,(-m^2\{[R_{\varphi p}(0) - R_{\varphi p}(\tau)] + [R_{\varphi i p_i}(0) - R_{\varphi i p_i}(\tau)]\})e^{-i2\pi f\tau}\,d\tau. \tag{23}$$

From equation (23), we can show that

$$T_m(f) = U_m(f) * V_m(f) \tag{24}$$

where¶

$$U_m(f) = \int_{-\infty}^{\infty} \exp\,\{-m^2[R_{\varphi p}(0) - R_{\varphi p}(\tau)]\}e^{-i2\pi f\tau}\,d\tau, \tag{25}$$

and

$$V_m(f) = \int_{-\infty}^{\infty} \exp\,\{-m^2[R_{\varphi i p_i}(0) - R_{\varphi i p_i}(\tau)]\}e^{-i2\pi f\tau}\,d\tau. \tag{26}$$

Equation (22) gives a general expression for the baseband interchannel interference when two angle-modulated waves interfere with each other. To calculate this interchannel interference, equations (22) through (26) show that it is essential to determine the RF spectral density of a wave angle modulated by gaussian noise. Methods of

‡ Since $\varphi(t)$ and $\varphi_i(t)$ are assumed to be gaussian, $p(t) * \varphi(t)$, and $p_i(t) * \varphi_i(t)$ are also gaussian.[2,15] Notice also that the Fourier transform of $R_p(\tau)$ is equal to $|H_p(f)|^2$, if $H_p(f)$ is the Fourier transform of $p(t)$.

¶ Notice that $U_m(f)$ and $V_m(f)$ are the RF spectral densities of waves angle modulated by gaussian noise.

calculating this spectrum for low and medium index modulation are generally available, and the quasistatic approximation has been used for high index modulation.[2,13–16] Since the accuracy in the quasistatic approximation cannot often be determined, some rigorous methods of evaluating this spectrum for high index modulation have recently been developed.[2,16]

III. SPECTRAL DENSITY OF A PM WAVE

In this paper, we are specifically interested in determining the interchannel interference between two or more waves phase modulated by band-limited white gaussian random processes. Hence, we now review briefly the methods of obtaining the RF spectrum of such a wave. A sinusoidal wave of constant amplitude A phase modulated by a signal $n(t)$ can be written as

$$w(t) = A \cos [\omega_0 t + n(t) + \theta], \tag{27}$$

$$= \text{Re } A \exp \{j[\omega_0 t + n(t) + \theta]\}, \tag{28}$$

where $f_o = \omega_o/2\pi$ is the carrier frequency of the wave, and θ is a random variable with probability density function

$$\pi_\theta(\theta) = \begin{cases} 1/2\pi, & 0 \leqq \theta < 2\pi \\ 0, & \text{otherwise.} \end{cases} \tag{29}$$

If the modulating waveform is band-limited and white, its spectrum $S_n(f)$ is given by (see Fig. 1)

$$S_n(f) = \begin{cases} \Phi^2/2W, & |f| < W, \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

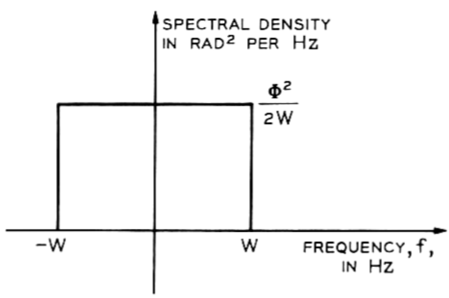

Fig. 1 — Spectral density of modulating wave.

Notice that Ref. 16 treats, in detail, the methods of obtaining the spectral characteristics of a sinusoidal carrier phase modulated by such a signal. From equation (30) we can show that (see Fig. 2)

$$R_n(\tau) = \Phi^2 \frac{\sin 2\pi W\tau}{2\pi W\tau}. \tag{31}$$

For $\Phi^2 \gg 1$ and for low frequencies, the quasistatic approximation yields[2,15]

$$S_V(f) \approx \exp(-\Phi^2)\,\delta(f) + \frac{1}{\Phi W}\left(\frac{3}{2\pi}\right)^{\frac{1}{2}} \exp\left[-\frac{3}{2}\frac{1}{\Phi^2}\left(\frac{f}{W}\right)^2\right]. \tag{32}$$

One can show that the approximation given by equation (32) is only good at low frequencies and that it is too small for large f.[16]

For large modulation indexes ($\Phi > 1.7432$ rad) and for all frequencies, we can show that[16]

$$\begin{aligned} S_V(f) = {} & \exp(-\Phi^2)\left\{\delta(f) + \frac{\Phi^2}{2W}\left[u_{-1}(f+W) - u_{-1}(f-W)\right]\right\} \\ & + \frac{1}{2\pi W}\exp\left[-2\Phi^2\left(\cosh^2\frac{y_s}{2} - \frac{\sinh y_s}{y_s}\right)\right]\mu, \end{aligned} \tag{33}$$

where

$$u_{-1}(x) = \begin{cases} 1, & x > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{34}$$

$$\left(\frac{2\pi}{\Phi^2 A_2}\right)^{\frac{1}{2}}(1 - C) < \mu < \left(\frac{2\pi}{\Phi^2 A_2}\right)^{\frac{1}{2}}(1 + D), \tag{35}$$

$$\frac{\cosh y_s}{y_s} - \frac{\sinh y_s}{y_s^2} = \frac{f}{\Phi^2 W}, \tag{36}$$

and

$$A_2 = \frac{\sinh y_s}{y_s} - \frac{2}{y_s}\frac{f}{\Phi^2 W}. \tag{37}$$

We can also show that C and D, appearing in equation (35), are less than 8 per cent for $\Phi > (10)^{\frac{1}{2}}$ rad. Further, for all f, one can show that[16]

$$C < 2\% \quad \text{for} \quad \Phi > 5 \text{ rad}, \tag{38}$$

and

$$D < 2\% \quad \text{for} \quad \Phi > 5 \text{ rad}. \tag{39}$$

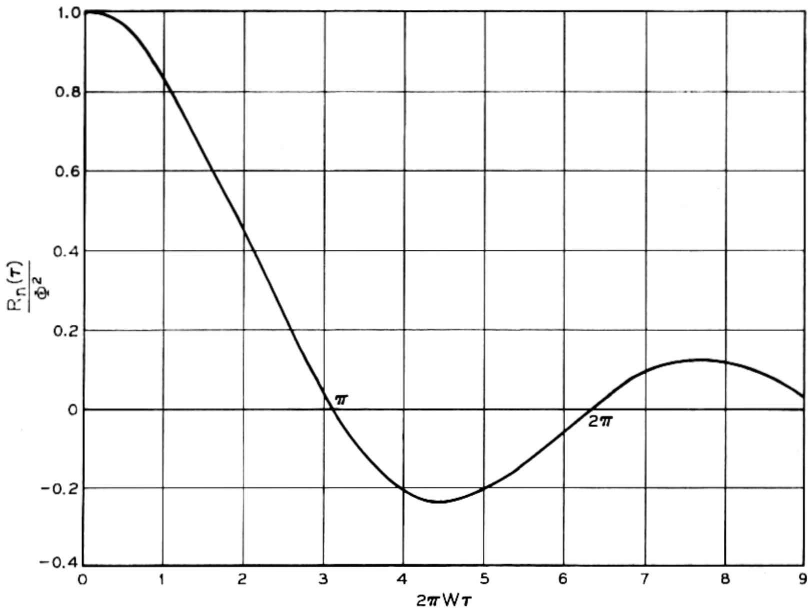

Fig. 2 — Covariance function $R_n(\tau)$. Since $R_n(\tau)$ is an even function of τ, we only show $R_n(\tau)$ for $\tau \geqq 0$.

Hence, we can say that

$$\mu \approx \left(\frac{2\pi}{\Phi^2 A_2}\right)^{\frac{1}{2}}, \tag{40}$$

and that the fractional error in this approximation is very much less than unity (less than 2 per cent, $\Phi > 5$ rad).

For $f = 0$, from equations (33) through (37) we can show that

$$0.92\left(\frac{3}{2\pi}\right)^{\frac{1}{2}}\frac{1}{\Phi W} < S_V(f) - \exp\left(-\Phi^2\right)\,\delta(f) < 1.08\left(\frac{3}{2\pi}\right)^{\frac{1}{2}}\frac{1}{\Phi W}, \qquad \Phi > (10)^{\frac{1}{2}}\text{ rad}. \tag{41}$$

For any f and Φ, the determination of the spectral density $S_V(f)$ from equations (33) through (40) is rather simple. For any given f, Φ^2, and W, we calculate y_s from equation (36), and A_2 from equation (37). The spectral density $S_V(f)$ is then calculated from equations (33) and (40).

IV. INTERFERENCE BETWEEN TWO PM WAVES

We now assume that $\varphi(t)$ and $\varphi_i(t)$ in Section II are band-limited white gaussian random processes with the same bandwidth W and rms phase deviations Φ and Φ_i. We also assume that $p(t) = p_i(t) = \delta(t)$, or that no pre-emphasis—de-emphasis networks are used in the system. Therefore, we have

$$R_\varphi(\tau) = \Phi^2 \frac{\sin 2\pi W\tau}{2\pi W\tau}, \tag{42}$$

and

$$R_{\varphi_i}(\tau) = \Phi_i^2 \frac{\sin 2\pi W\tau}{2\pi W\tau}. \tag{43}$$

From equations (22), (23), (42), and (43) we can write†

$$S_\Omega(f) = S_\varphi(f) + \sum_{m=1}^{\infty} \frac{R_i^{2m}}{m^2} G_m(f), \tag{44}$$

where

$$G_m(f) = \tfrac{1}{4}[H_m(f - mf_d) + H_m(f + mf_d)], \tag{45}$$

and

$$H_m(f) = \int_{-\infty}^{\infty} \exp\left[-m^2(\Phi^2 + \Phi_i^2)\left(1 - \frac{\sin 2\pi W\tau}{2\pi W\tau}\right)\right] e^{-j2\pi f\tau}\, d\tau. \tag{46}$$

Notice that $G_m(f)$ is the spectral density of a sinusoidal carrier (at carrier frequency mf_d, and having unit amplitude) phase modulated by a band-limited white gaussian random process having mean square phase deviation $m^2(\Phi^2 + \Phi^2)$. Section III gives methods of obtaining this spectrum for all values of f; hence, $S_\Omega(f)$ can easily be calculated. In order to evaluate $S_\Omega(f)$ from equation (44), we must be able to determine the spectral density of a carrier phase modulated by gaussian noise for any arbitrary modulation index. In the case of band-limited white gaussian noise modulation the technique presented in Ref. 16 is very convenient to calculate this spectrum. The series method of determining this spectral density can become rather tedious when Φ or Φ_i is large.

When there is no interference, the signal as detected by an ideal phase demodulator is given by $\varphi(t)$, and its spectral density by $S_\varphi(f)$. Therefore, from equation (44), the spectral density $S_I(f)$ of the base-

† Notice that in this case $\Omega(t) = \eta(t)$, since $p(t) = p_i(t) = \delta(t)$.

band interchannel interference can be written as

$$S_I(f) = S_\Omega(f) - S_\varphi(f), \tag{47}$$

or

$$S_I(f) = \sum_{m=1}^{\infty} \frac{R_i^{2m}}{m^2} G_m(f). \tag{48}$$

Figure 3 is a graph of $S_I(f)$ for $f_d/W = 0$, 1, and 5; $\Phi = 3$ rad, and $\Phi_i = 2$ rad. Notice that, for $f_d/W = 1$, $S_I(f)$ is maximum at $f = 0$ or that maximum interchannel interference occurs at the lowest baseband frequency present in the system.

In practice the quantity of interest is usually the ratio of the average signal power to average interchannel interference power. In this case this signal-to-interference ratio $\sigma(f)$ can be written as

$$\sigma(f) = \frac{S_\varphi(f)\,\Delta f}{S_I(f)\,\Delta f} = \frac{S_\varphi(f)}{S_I(f)}, \tag{49}$$

where Δf is the spot frequency band of interest. Clearly, $\sigma(f)$ is a function of f and in designing an angle-modulated system one is usually

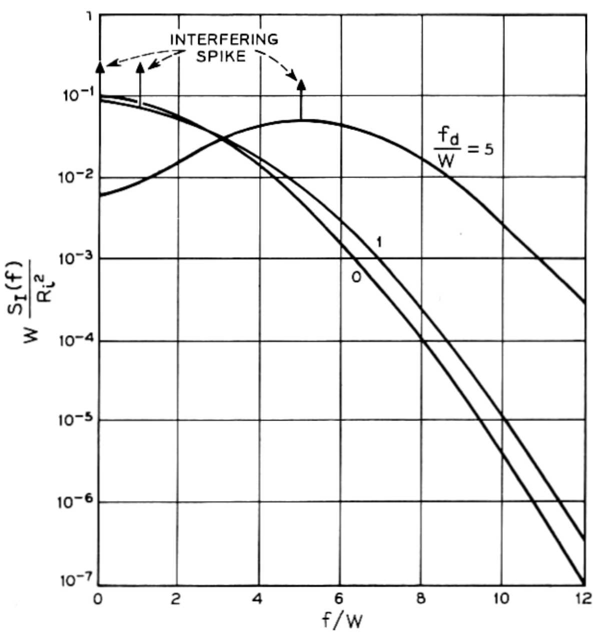

Fig. 3 — Spectral density $S_I(f)$ of baseband interference. $\Phi = 3$ rad; $\Phi_i = 2$ rad.

interested in the minimum value of $\sigma(f)$ for $0 < |f| \leqq W$. We denote the minimum value of this signal-to-interference ratio by S/I. In practice a phase demodulator is followed by a linear low pass filter. We assume that this filter is ideal and that it removes all the frequency components outside the desired signal frequency band $0 < |f| \leqq W$.

4.1 *Interference Between Two Co-Channel PM Waves*

In general, one can show (see Fig. 3) that $S_I(f)$ contains a (nonzero) Dirac delta function (corresponding to a line spectrum) at the frequency $\pm f_d$, and that the frequency division multiplex channel corresponding to this frequency may not be usable.‡ In case the interference is co-channel, $f_d = 0$, and the line spectrum lies at the frequency $f = 0$. In systems usually encountered in practice, there is no frequency division multiplex channel at dc even though the lowest frequency present in the baseband signal may approach a frequency arbitrarily close to zero.[14]

Notice from equation (48) and Fig. 3 that, in the case of co-channel interference between two PM waves, maximum baseband interference occurs at the lowest frequency present in the system; we assume that this lowest baseband frequency lies arbitrarily close to zero. In this case the minimum signal-to-interference ratio therefore occurs at $f = 0$ and

$$S/I = \frac{\Phi^2}{2W} \frac{1}{S_I'(0)}, \tag{50}$$

where

$$S_I'(0) = \sum_{m=1}^{\infty} \frac{R_i^{2m}}{2m^2} \{H_m(f) - \exp[-m^2(\Phi^2 + \Phi_i^2)]\,\delta(f)\}_{f=0}. \tag{51}$$

Since the interference is co-channel we further assume that $\Phi = \Phi_i$ so that the rms phase deviations in the two PM waves are the same. We can now write

$$S_I'(0) = \sum_{m=1}^{\infty} \frac{R_i^{2m}}{2m^2} [H_m(f) - \exp(-2m^2\Phi^2)\,\delta(f)]_{f=0}. \tag{52}$$

Consider the case $\Phi > (5)^{\frac{1}{2}}$ radians. In this case one can show that[16]

$$[H_m(f) - \exp(-2m^2\Phi^2)\,\delta(f)]_{f=0} \approx \frac{1}{2m\Phi W}\left(\frac{3}{\pi}\right)^{\frac{1}{2}}, \tag{53}$$

and that the error in this approximation is less than 8 per cent. Hence,

‡ We do not put any lower limit on the width of any frequency division multiplex channel present in the baseband signal.

we have

$$\frac{0.23}{\Phi W}\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\sum_{m=1}^{\infty}\frac{R_i^{2m}}{m^3} < S_I'(0) < \frac{0.27}{\Phi W}\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\sum_{m=1}^{\infty}\frac{R_i^{2m}}{m^3}, \quad \Phi > (5)^{\frac{1}{2}}\text{ rad}. \tag{54}$$

It can be shown that

$$\sum_{m=1}^{\infty}\frac{R_i^{2m}}{m^3} = Q(R_i^2) = \int_0^{\infty}\frac{t^2\,dt}{e^t - R_i^2}. \tag{55}$$

Therefore, the signal-to-interference ratio at $f = 0$ is bounded by

$$\frac{1}{0.46}\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\frac{\Phi^3}{Q(R_i^2)} > S/I > \frac{1}{0.54}\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\frac{\Phi^3}{Q(R_i^2)}, \qquad \Phi > (5)^{\frac{1}{2}}\text{ rad}. \tag{56}$$

For any value of $R_i < 1$, equation (56) gives upper and lower bounds to S/I. We shall now investigate whether we can derive simpler upper and lower bounds to $Q(R_i^2)$.

From equation (55)

$$\sum_{m=1}^{\infty}\frac{R_i^{2m}}{m^3} = R_i^2 + \sum_{m=2}^{\infty}\frac{\exp\left[-m\ln\left(1/R_i^2\right)\right]}{m^3}. \tag{57}$$

Now one can show (see Fig. 4) that

$$0 < \sum_{m=2}^{\infty}\frac{\exp\left[-m\ln\left(1/R_i^2\right)\right]}{m^3} < \int_1^{\infty}\frac{\exp\left[-x\ln\left(1/R_i^2\right)\right]}{x^3}\,dx$$
$$= E_3[\ln\left(1/R_i^2\right)], \tag{58}$$

where†

$$E_3(z) = \int_1^{\infty}\frac{e^{-zt}}{t^3}\,dt, \qquad z > 0. \tag{59}$$

We can show that for $R_i < 1$, ($\ln 1/R_i^2 > 0$), (see Ref. 17)

$$0 < E_3(\ln 1/R_i^2) \leqq \frac{R_i^2}{2 + \ln\left(1/R_i^2\right)}, \tag{60}$$

or

$$Q(R_i^2) < R_i^2\left[1 + \frac{1}{2 + \ln\left(1/R_i^2\right)}\right]. \tag{61}$$

Since

† The function $E_3(z)$ is tabulated in Ref. 17 (see pp. 228–248). Notice also the inequality $E_n(z) \leqq e^{-z}/(z + n - 1)$ on p. 229.

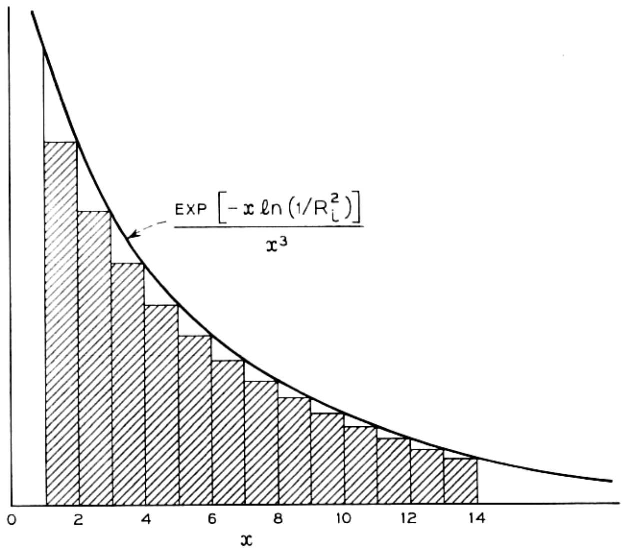

Fig. 4. — Function $\exp[-x \ln(1/R_i^2)]/x^3$ and $\Sigma_{m=2}^{\infty} R_i^{2m}/m^3$. The area in the shaded region is less than the area under the curve from $x = 1$.

$$\sum_{m=1}^{\infty} \frac{R_i^{2m}}{m^3} < \sum_{m=1}^{\infty} \frac{R_i^{2m}}{m} = -\ln(1 - R_i^2), \qquad Q(R_i^2) < -\ln(1 - R_i^2). \quad (62)$$

We are thankful to W. T. Barnett for having suggested another upper bound $R_i^2/(1 - R_i^2)$ to $Q(R_i^2)$.

One can show that the bound given in equation (62) is tighter than that given in equation (61) if

$$R_i < R_0 = 0.695573. \quad (63)$$

Let us write

$$U(R_i^2) = \begin{cases} 1 + \dfrac{1}{2 + \ln(1/R_i^2)}, & R_0 < R_i < 1, \\ -\dfrac{\ln(1 - R_i^2)}{R_i^2}, & 0 < R_i < R_0, \end{cases} \quad (64)$$

so that

$$R_i^2 < Q(R_i^2) < R_i^2 U(R_i^2), \qquad 0 < R_i < 1. \quad (65)$$

For carrier-to-interference ratio of 10 dB or for $R_i^2 = 0.1$

$$R_i^2 < \sum_{m=1}^{\infty} \frac{R_i^{2m}}{m^3} < 1.0536\, R_i^2 . \quad (66)$$

From equations (56) and (65), we next write

$$\frac{1}{0.46}\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\frac{1}{R_i^2}\Phi^3 > S/I > \frac{1}{0.54}\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\frac{1}{R_i^2}\frac{\Phi^3}{U(R_i^2)},$$

$$\Phi > (5)^{\frac{1}{2}}\text{ rad}, \quad R_i < 1. \tag{67}$$

Since the physical characteristics of elements used in a PM receiver are far from being ideal, and since thermal noise (which is always present) further deteriorates the performance of any PM receiver, we often find that $R_i^2 < 0.1$ in systems currently in use. Equations (66) and (67) show that the error introduced in truncating the series at $m = 1$ is less than 5.36 percent if $R_i^2 < 0.1$. For any $R_i \ll 1$, we therefore need take only the $m = 1$ term in equation (54) to estimate the baseband interference. Equation (67) gives upper and lower bounds to S/I for any $R_i < 1$. Also, note from equation (67) that co-channel interference can be suppressed in PM systems by using a large modulation index Φ.[14]

4.2 *Interference between Two Adjacent-Channel PM Waves*

As mentioned in Section II we do not consider the effects of linear filters on angle-modulated systems. We assume that the desired and interfering wave are both in the passband of the PM receiver used in the system, and that no filters are used to reduce the adjacent channel interference.

In any multichannel angle-modulated system generally encountered in practice there is usually both adjacent channel and co-channel interference. Protection against adjacent channel interference is often obtained by proper choice of the channel separation frequency and the required (linear) filters generally used in such systems. The assumptions made in this section are, therefore, a little unrealistic; hence, the results given may serve only as a guide in the actual calculation of adjacent channel interference.

For $0 < f_d/W < 1$, one can show that $S_I(f)$ contains a (nonzero) Dirac delta function (corresponding to a line spectrum) at the frequency $\pm f_d$ and that the frequency division multiplex channel corresponding to f_d/W may not be usable.

For $f_d \neq 0$ we can show, from equations (44) through (46), that

$$S_I(f) = \sum_{m=1}^{\infty} \frac{R_i^{2m}}{m^2} G_m(f), \tag{68}$$

where

$$G_m(f) = \tfrac{1}{4}[H_m(f - mf_d) + H_m(f + mf_d)], \tag{69}$$

and

$$H_m(f) = \int_{-\infty}^{\infty} \exp\left[-m^2(\Phi^2 + \Phi_i^2)\left(1 - \frac{\sin 2\pi W\tau}{2\pi W\tau}\right)\right] e^{-j2\pi f\tau}\, d\tau. \qquad (70)$$

For $0 \leqq |f| < W$, and $|f_d| \gg W$, one can show (by numerical methods) that $S_I(f)$ reaches its maximum at $f = W$, or that maximum baseband interchannel interference occurs at the highest frequency present in the baseband signal. For other values of channel separation frequency, this maximum is to be determined from equations (68) through (70).

For $(\Phi^2 + \Phi_i^2)^{\frac{1}{2}} > (30/\pi)^{\frac{1}{2}}$ rad, the saddle-point method of calculating $G_m(f)$ is very convenient;[16] and this method can be applied in a straightforward manner to estimate $S_I(f)$. (Since one can show that the saddle-point approximation reduces to the quasistatic approximation for $f_d/W \ll (\Phi^2 + \Phi_i^2)^{\frac{1}{2}}$, the quasistatic approximation may be used for convenience if this condition is satisfied. However, the error introduced as a result of the use of quasistatic approximation cannot often be estimated.) For $R_i \ll 1$, we can also show that we need take only the $m = 1$ term in equation (68) to estimate S/I with a very small fractional error (less than 5.36 percent for $R_i < 0.1$).

For $f_d/W = 2, 4, 6, 8$, and 10 and for a set of values of Φ and Φ_i, we have calculated this minimum signal-to-interference ratio; Figs. 5 through 9 give these results. For any value of f_d/W and for any S/I, the required values of Φ and Φ_i may be obtained from these figures. Since the effects of linear filters on adjacent channel interference has not been taken into account in this paper, these values of Φ and Φ_i may serve only as a guide in the design of any angle-modulated system.

V. INTERFERENCE BETWEEN L+1 PM WAVES

We now assume that there are L interfering waves, and that all of them are phase modulated by mutually independent gaussian random processes.† Let the desired PM wave be given by

$$s(t) = \text{Re}\, A \exp\{j[\omega_o t + \varphi(t)]\}. \qquad (71)$$

Let the kth interfering wave be represented as

$$i_k(t) = \text{Re}\, R_{ik} A \exp\{j[\omega_k t + \varphi_{ik}(t) + \mu_k]\}, \qquad 1 \leqq k \leqq L. \qquad (72)$$

† The analysis given in this section can suitably be modified for angle modulation by general gaussian random processes (see Section II).

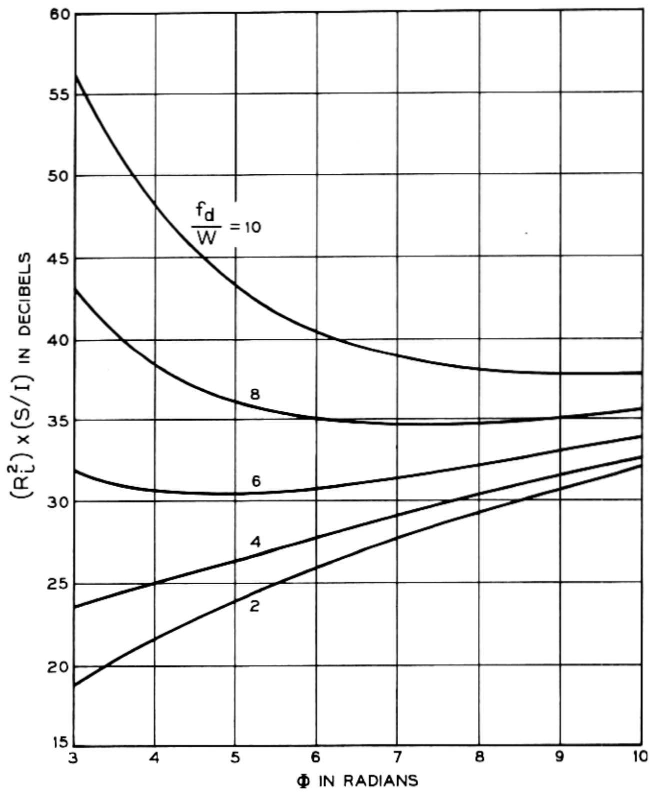

Fig. 5 — Signal-to-interference ratio as a function of rms phase deviations and channel separation for $\Phi_i = 2$ rad.

Since the L interfering waves are assumed to originate from L different sources, we assume that the μ_k's are independent of each other, and that μ_k, $1 \leqq k \leqq L$ has a uniform probability density function $\pi_{\mu_k}(\mu)$ where

$$\pi_{\mu_k}(\mu) = \begin{cases} 1/2\pi, & 0 \leqq \mu < 2\pi, \quad 1 \leqq k \leqq L, \\ 0, & \text{otherwise}. \end{cases} \tag{73}$$

We further assume that $\varphi(t)$, the $\varphi_k(t)$'s, and the μ_k's (with $1 \leqq k \leqq L$) are mutually independent random variables.

If $s(t)$ and the $i_k(t)$'s are all in the passband of the PM receiver used in the system, the total signal incident at the receiver can be written as

$$r(t) = s(t) + \sum_{k=1}^{L} i_k(t)$$
$$= \operatorname{Re} A\left(1 + \sum_{k=1}^{L} R_{ik} \exp\{j[\omega_{dk}t + \varphi_{ik}(t) - \varphi(t) + \mu_k]\}\right)$$
$$\cdot \exp\{j[\omega_0 t + \varphi(t)]\}, \tag{74}$$

where

$$\omega_{dk} = \omega_k - \omega_o = f_{dk}/2\pi. \tag{75}$$

From equation (74), we can show that the output $\theta(t)$ of an ideal phase demodulator can be represented as

$$\theta(t) = \varphi(t) + \operatorname{Im} \ln\left(1 + \sum_{k=1}^{L} R_{ik} \exp\{j[\omega_{dk}t + \varphi_{ik}(t) - \varphi(t) + \mu_k]\}\right). \tag{76}$$

Next we write

$$\ln\left(1 + \sum_{k=1}^{L} R_{ik} \exp\{j[\omega_{dk}t + \varphi_{ik}(t) - \varphi(t) + \mu_k]\}\right)$$
$$= \sum_{m=1}^{\infty} \frac{(-1)^{m+1}}{m}\left(\sum_{k=1}^{L} R_{ik} \exp\{j[\omega_{dk}t + \varphi_{ik}(t) - \varphi(t) + \mu_k]\}\right)^m$$

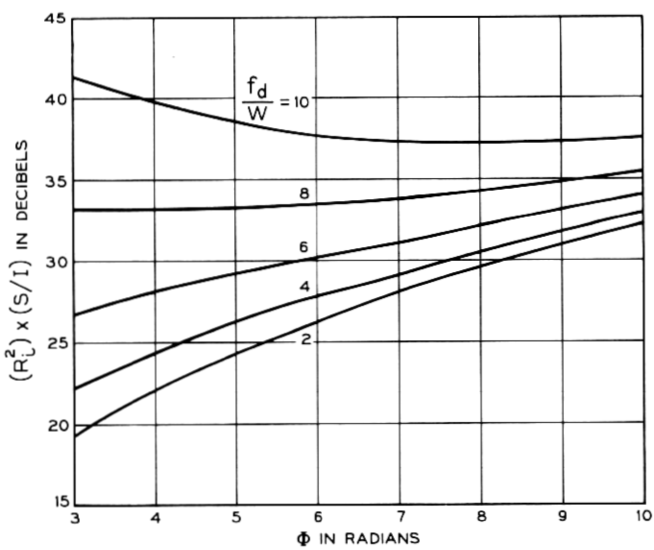

Fig. 6 — Signal-to-interference ratio as a function of rms phase deviations and channel separation for $\Phi_i = 4$ rad.

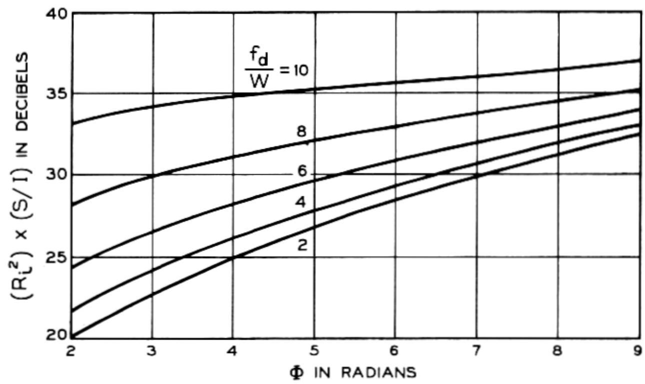

Fig. 7 — Signal-to-interference ratio as a function of rms phase deviations and channel separation for $\Phi_i = 6$ rad.

$$\sum_{k=1}^{L} R_{ik} < 1. \tag{77}$$

By the multinomial theorem, we have

$$\left(\sum_{k=1}^{L} R_{ik} \exp \{j[\omega_{dk}t + \varphi_{ik}(t) - \varphi(t)]\}\right)^m$$

$$= \sum \frac{m!}{\prod_{r=1}^{L} a_r!} \prod_{r=1}^{L} R_{ir}^{a_r} \exp \{ja_r[\omega_{dr}t + \varphi_{ir}(t) - \varphi(t) + \mu_r]\}, \tag{78}$$

where the a_r's are a set of nonnegative integers such that

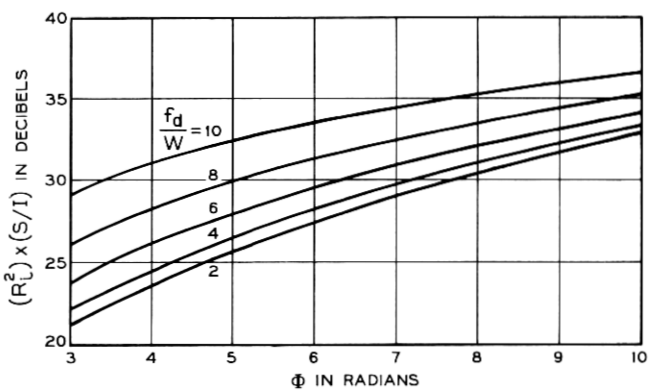

Fig. 8 — Signal-to-interference ratio as a function of rms phase deviations and channel separation for $\Phi_i = 8$ rad.

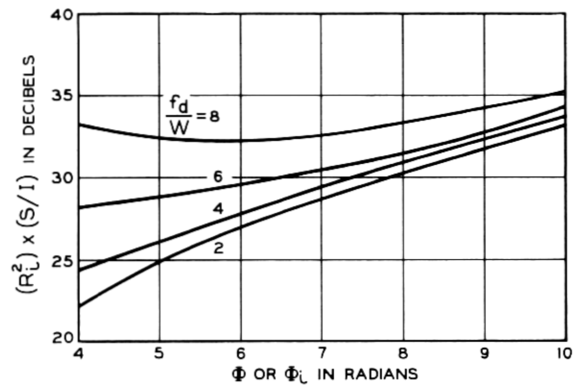

Fig. 9 — Signal-to-interference ratio as a function of rms phase deviations and channel separation for $\Phi = \Phi_i$.

$$\sum_{r=1}^{L} a_r = m. \tag{79}$$

From equations (73) and (76) through (79), one can show that the covariance function $R_\theta(\tau)$ of $\theta(t)$ can be written as

$$\begin{aligned} R_\theta(\tau) &= \langle \theta(t)\theta(t+\tau) \rangle \\ &= R_\varphi(\tau) + \sum_{m=1}^{\infty} \frac{1}{2m^2} \exp\{-m^2[R_\varphi(0) - R_\varphi(\tau)]\} \\ &\quad \cdot \left[\sum \left(\frac{m! \prod_{r=1}^{L} R_{ir}^{a_r}}{\prod_{r=1}^{L} a_r!} \right)^2 \exp\left(-\sum_{r=1}^{L} a_r^2 [R_{\varphi r}(0) - R_{\varphi r}(\tau)] \right) \right. \\ &\quad \left. \cdot \cos\left(\tau \sum_{r=1}^{L} a_r \omega_{dr} \right) \right]. \end{aligned} \tag{80}$$

If the random gaussian noise is band-limited and white, and if all the modulating waveforms have the same bandwidth W, we have

$$R_\varphi(\tau) = \Phi^2 \cdot \frac{\sin 2\pi W\tau}{2\pi W\tau}, \tag{81}$$

and

$$R_{\varphi k}(\tau) = \Phi_k^2 \frac{\sin 2\pi W\tau}{2\pi W\tau}, \qquad 1 \leqq k \leqq L. \tag{82}$$

In this case, equation (80) can be written as

$$R_\theta(\tau) = R_\varphi(\tau) + \sum_{m=1}^{\infty} \frac{1}{2m^2} \exp\left[-m^2\Phi^2\left(1 - \frac{\sin 2\pi W\tau}{2\pi W\tau}\right)\right]$$

$$\cdot \left\{ \sum \frac{(m!)^2 \prod_{r=1}^{L} R_{ir}^{2a_r}}{\prod_{r=1}^{L} (a_r!)^2} \exp\left[-\left(1 - \frac{\sin 2\pi W\tau}{2\pi W\tau}\right) \sum_{r=1}^{L} a_r^2 \Phi_r^2\right]\right.$$

$$\left. \cdot \cos\left(\tau \sum_{r=1}^{L} a_r \omega_{dr}\right)\right\}. \tag{83}$$

Therefore, the spectral density of baseband interchannel interference is given by

$$S_I(f) = \sum_{m=1}^{\infty} \frac{1}{4m^2} \left\{ \sum_s \frac{(m!)^2 \prod_{r=1}^{L} R_{ir}^{2a_r}}{\prod_{r=1}^{L} (a_r!)^2}\right.$$

$$\left. \cdot \left[T_{ms}\left(f - \sum_{r=1}^{L} a_r f_{dr}\right) + T_{ms}\left(f + \sum_{r=1}^{L} a_r f_{dr}\right)\right]\right\}, \tag{84}$$

where

$$T_{ms}(f) = \int_{-\infty}^{\infty} \exp\left[-\left(1 - \frac{\sin 2\pi W\tau}{2\pi W\tau}\right)\left(m^2\Phi^2 + \sum_{r=1}^{L} a_r^2\Phi_r^2\right]\right) e^{-j2\pi f\tau} d\tau. \tag{85}$$

Next notice that the methods given in Section III can be used to calculate $T(f)$ for all values of Φ, and Φ_{ik}'s (with $1 \leqq k \leqq L$); hence, we can calculate $S_I(f)$ for all values of R_{ik}'s such that $\sum_{k=1}^{L} R_{ik} < 1$. The minimum signal-to-interference ratio S/I can then be obtained from equation (49).

Now assume that we have L co-channel interferers and that all have the same rms phase deviation Φ, or

$$\Phi_r = \Phi, \qquad 1 \leqq r \leqq L. \tag{86}$$

In this case equation (84) yields

$$S_I(f) = \sum_{m=1}^{\infty} \frac{1}{2m^2} \left\{ \sum_s \frac{(m!)^2 \prod_{r=1}^{L} R_{ir}^{2a_r}}{\prod_{r=1}^{L} (a_r!)^2} G_{ms}(f) \right\}, \tag{87}$$

where

$$G_{ms}(f) = \int_{-\infty}^{\infty} \exp\left[-\left(1 - \frac{\sin 2\pi W\tau}{2\pi W\tau}\right)\Phi^2\left(m^2 + \sum_{r=1}^{L} a_r^2\right)\right]e^{-j2\pi f\tau}\, d\tau. \tag{88}$$

From equations (87) and (88) and Refs. 2 and 16, one can show that the continuous part of $S_I(f)$ reaches its maximum at $f = 0$, and that†

$$0.92\left(\frac{3}{2\pi}\right)^{\frac{1}{2}} \frac{1}{\left(m^2 + \sum_{r=1}^{L} a_r^2\right)^{\frac{1}{2}}} \frac{1}{W\Phi} < G_{ms}(0)$$

$$< 1.08\left(\frac{3}{2\pi}\right)^{\frac{1}{2}} \frac{1}{\left(m^2 + \sum_{r=1}^{L} a_r^2\right)^{\frac{1}{2}}} \frac{1}{W\Phi}, \qquad \Phi > (5)^{\frac{1}{2}} \text{ rad}. \tag{89}$$

The expression $G_{ms}(0)$ in equation (89) does not include the delta function contained in $G_{ms}(f)$ at $f = 0$.

Since $\sum_{r=1}^{L} a_r = m$, one can prove that

$$\frac{m^2}{L} \leqq \sum_{r=1}^{L} a_r^2 \leqq m^2. \tag{90}$$

From equations (89) and (90) we have

$$0.46\left(\frac{3}{\pi}\right)^{\frac{1}{2}} \frac{1}{m\Phi W} < G_{ms}(0) < 0.54\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\left(\frac{2L}{L+1}\right)^{\frac{1}{2}} \frac{1}{m\Phi W}. \tag{91}$$

Next

$$S_I(0) < \sum_{m=1}^{\infty} \frac{1}{2m^2} \left[\sum \frac{(m!)^2 \prod_{r=1}^{L} R_{ir}^{2a_r}}{\prod_{r=1}^{L} (a_r!)^2} 0.54\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\left(\frac{2L}{L+1}\right)^{\frac{1}{2}} \frac{1}{m\Phi W}\right]. \tag{92}$$

If all x_i's are nonnegative, one can show that

$$\sum x_i^2 \leqq \left(\sum x_i\right)^2. \tag{93}$$

Using equation (93), equation (92) yields

$$S_I(0) < \sum_{m=1}^{\infty} 0.27\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\left(\frac{2L}{L+1}\right)^{\frac{1}{2}} \frac{1}{\Phi W} \frac{1}{m^3} \left[\left(\sum \frac{m! \prod_{r=1}^{L} R_{ir}^{a_r}}{\prod_{r=1}^{L} a_r!}\right)^2\right]$$

† We consider only the continuous part of $S_I(f)$.

$$= 0.27\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\left(\frac{2L}{L+1}\right)^{\frac{1}{2}} \frac{1}{\Phi W} \sum_{m=1}^{\infty} \frac{\left(\sum_{r=1}^{L} R_{ir}\right)^{2m}}{m^3} \tag{94}$$

or

$$S_I(0) < 0.27\left(\frac{3}{\pi}\right)^{\frac{1}{2}}\left(\frac{2L}{L+1}\right)^{\frac{1}{2}} \frac{1}{\Phi W} Q(b^2), \tag{95}$$

where

$$b^2 = \left(\sum_{r=1}^{L} R_{ir}\right)^2 < 1. \tag{96}$$

We have shown in Section IV that

$$\sum_{m=1}^{\infty} \frac{b^{2m}}{m^3} < b^2 U(b^2), \qquad b^2 < 1. \tag{97}$$

Therefore, the minimum baseband signal-to-interference ratio is bounded by

$$S/I > \frac{1}{0.54}\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\left(\frac{L+1}{2L}\right)^{\frac{1}{2}} \frac{\Phi^3}{b^2 U(b^2)}, \qquad \Phi > (5)^{\frac{1}{2}} \text{ rad}, \quad b < 1. \tag{98}$$

From equation (87) we can also show that

$$S_I(0) > \frac{1}{2}\left(\sum_{r=1}^{L} R_{ir}^2\right) G_{1s}(0). \tag{99}$$

Equations (89) and (99) yield

$$S_I(0) > 0.23\left(\frac{3}{\pi}\right)^{\frac{1}{2}} \frac{1}{W\Phi}\left(\sum_{r=1}^{L} R_{ir}^2\right), \tag{100}$$

or

$$S/I < \frac{1}{0.46}\left(\frac{\pi}{3}\right)^{\frac{1}{2}} \frac{\Phi^3}{\sum_{r=1}^{L} R_{ir}^2}. \tag{101}$$

Hence we have

$$\frac{1}{0.46}\left(\frac{\pi}{3}\right)^{\frac{1}{2}} \frac{\Phi^3}{\sum_{r=1}^{L} R_{ir}^2} > S/I > \frac{1}{0.54}\left(\frac{\pi}{3}\right)^{\frac{1}{2}}\left(\frac{L+1}{2L}\right)^{\frac{1}{2}} \frac{\Phi^3}{b^2 U(b^2)},$$

$$\Phi > (5)^{\frac{1}{2}} \text{ rad}, \quad b = \sum_{k=1}^{L} R_{ik} < 1. \tag{102}$$

For any set of values of R_{ik}'s, $1 \leqq k \leqq L$, and for any Φ, bounds to the signal-to-interference ratio S/I can be calculated from equation (102), and a proper Φ can then be chosen to keep the baseband interference below any desired level.

Notice that the upper bound is a function of the total interference power, and the lower bound a function of the sum of the amplitudes of all the interfering carriers. In such cases, the distribution of R_{ik}'s generally determines the closeness of the two bounds. However, it may be observed that both these bounds are proportional to the cube of the modulation index Φ (for a high index system).

VI. RESULTS AND CONCLUSIONS

In this paper we consider the effect of interchannel interference on angle-modulated systems. We also derive an expression for the baseband interchannel interference when two (or more) waves angle modulated by gaussian noise interfere with each other. This formula can be used even when the baseband signal is passed through a linear network such as a pre-emphasis—de-emphasis network. We show that the calculation of the RF spectral density is essential to the evaluation of the baseband interchannel interference.

We then consider band-limited white gaussian noise modulation and show that, in the case of co-channel interference, maximum baseband interference occurs at the lowest baseband frequency present in the system. For moderately high modulation index, we show that we can derive upper and lower bounds to this minimum signal-to-interference ratio and that these bounds are proportional to the cube of the modulation index. It therefore follows that co-channel interference in PM systems can be reduced by expanding bandwidth, and that phase modulation can be used with advantage in combating interference. We also show that the first term in the power series expansion for the baseband interchannel interference gives most of the contribution if the carrier-to-interference ratio is greater than about 10 dB (the error is less than 5.36 per cent for a carrier-to-interference ratio greater than 10 dB).

In this paper we also give some results about the effects of adjacent channel interference on angle-modulated systems. We assume that all the incident signals at the receiver are in the passband of the PM receiver used in the system. This assumption is justified in the case of co-channel interference, but is not realistic in the case of adjacent channel interference. However, we feel that the results given in this paper for the adjacent channel interference may serve as a guide in determining the deterioration in performance produced by adjacent channel interference.

VII. ACKNOWLEDGMENT

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