Resonances in Waveguide Antennas with Dielectric Plugs

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This paper discusses an analysis of the radiation from a parallel-plate waveguide to determine the effects of loading the waveguide with dielectric plugs near the aperture. We devote special attention to the situation in which the higher order modes, generated by the aperture discontinuity, propagate inside the dielectric plug but are evanescent in the unloaded waveguide region. We show that the dielectric plug may function as a resonant cavity for this type of wave mode. When one of these modes is at resonance, it is strongly excited by the incident wave; the presence of this resonance is manifested by the appearance of sharp spikes in the reflection coefficient either as a function of the frequency or the plug thickness. We also discuss the relation between the resonances in a single waveguide and in array configuration.

I. INTRODUCTION

The radiation from a parallel-plate waveguide with infinitesimally thin walls is one of the relatively few electromagnetic boundary value problems for which the Wiener-Hopf integral equation technique may be applied to obtain a closed form solution.[1] Unfortunately, this elegant mathematical technique quickly loses its usefulness even when rather minor modifications of the physical system are introduced, such as, for example, by allowing the waveguide to have finite wall thickness or loading the waveguide with a dielectric material.

The somewhat simpler problem of determining the radiation admittance of a waveguide terminated in an infinite conducting plane has been treated by several workers using the variational technique.[2,3] The field of the incident wave is used to approximate the true aperture field in these calculations. The results thus obtained appear adequate for engineering purposes. The implication is that the radiation admittance of an empty waveguide is rather insensitive to the approximation used for the aperture field distribution. There is no way, however, to ascertain without more elaborate calculations how well the aperture field is approximated by that of the incident wave.

The variational technique has also been widely used in a broad class of scattering problems. Although it seems that useful approximate answers are often obtainable even when rather crude approximations are used for the trial funnctions, there are numerous instances, notably in the area of phased arrays[4] and in problems involving dielectric material,[5] wherein it has been found that good approximations of the trial functions are necessary to obtain meaningful results. An important factor contributing to this knowledge undoubtedly is the widespread availability of high speed electronic computers, which have made it possible to perform elaborate computations hitherto regarded as too time-consuming and costly to be practical.

In this paper, we discuss the radiation properties of a waveguide which is loaded with dielectric plugs near the aperture and is terminated in an infinite conducting plane. A waveguide antenna has the advantage that it can be flush mounted. This feature makes it attractive for applications such as missile and aircraft antennas. Dielectric plugs, moreover, provide convenient covers to protect the antenna feed system against environmental influences. The introduction of dielectric material, however, makes it possible to excite the wave modes which have a surface wavelike field distribution within the waveguide because they propagate inside the dielectric plug but are evanescent in the empty waveguide region. (This excitation is caused by the aperture discontinuity.)

We show that because of the excitation of this type of wave mode, the antenna impedance (or the reflection coefficient) exhibits resonance characteristics versus both the frequency and the thickness of the dielectric plug. These resonances occur when the parameters are such that the impedances of a surface wavelike mode (or "ghost mode") satisfy a transverse resonance condition. The implication of this observation is that the dielectric plug acts like a resonance cavity for the surface wavelike modes. When the combination of the parameters is such as to permit one of these modes to resonate, the effect is to cause rapid variation in the radiation impedances (or reflection coefficient) which are manifested as sharp spikes.

The radiation patterns generally show smooth variations versus the angle of observation. Only when the parameters are in the close vicinity of a resonance such that the higher order mode is exceedingly strongly excited do pattern dips appear. Moreover, the dips are rather broad and shallow. It is therefore necessary to exercise extreme care in order to detect resonances by examining the patterns alone.

An earlier analysis of phased arrays using the waveguide at hand as the radiation element has revealed that resonance characteristics also exist in both the reflection coefficients and the mutual coupling of the array.[6,7] These resonances are related in certain ways to those of the present problem. We briefly discuss the relationship with the view toward using a single waveguide for the detection of the resonances in an array configuration.

The boundary value problem is formulated in two ways, one in a pure integral equation with the tangential magnetic field as the unknown and the other in an integro-differential equation with the aperture electric field as the unknown. It appears that no known analytical method is available for solving either equation. It is possible, however, to use numerical technique to determine approximate but accurate solution from the latter equation. We discuss the method of obtaining solutions by the method of moments; we also point out certain salient features with regard to the formulation.

II. FORMULATION OF THE PROBLEM

Consider a parallel-plate waveguide, terminated in an infinite conducting plane as illustrated in Fig. 1. The waveguide is loaded with a dielectric plug (or window) near the aperture. We consider the system to be excited by the lowest TE mode incident upon the aperture from the waveguide side, and assume the fields to be invariant with respect to y. Under these conditions, it is easily shown that the scat-

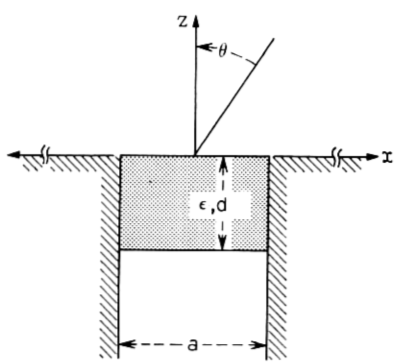

Fig. 1 — A flush mount parallel-plate waveguide with dielectric plug.

tered fields consist of TE modes alone. We determine the radiation characteristics of the antenna by using the integral equation approach.

2.1 *Integral Equations*

The problem may be formulated in terms of integral equations having as the unknown function either the tangential electric field or the tangential magnetic field in the plane $z = 0$. In order to do so, we must first introduce suitable representations for the tangential fields in the regions both inside and outside the waveguide. The application of boundary conditions using these field representations across the common $z = 0$ plane then leads to the desired integral equations. We derive first the equation with the tangential electric field as the unknown.

2.2 *Integro-Differential Equation for Aperture Electric Field*

By virtue of the equivalence principle,[8] the fields in $z \geqq 0$ may be derived from an equivalent magnetic dipole $\mathbf{M} = \mathbf{E}_t \times \hat{\mathbf{z}}$ situated above a perfectly conducting plane, where $\mathbf{E}_t$ denotes the tangential electric field which exists at the aperture and $\hat{\mathbf{z}}$ is a unit vector in the z direction. According to the image theorem, these fields are equal to twice the fields produced by the same equivalent source in free space. Since $\mathbf{E}_t = \hat{\mathbf{y}} E_y(x', 0)$, $\mathbf{M} = \hat{\mathbf{x}} E_y(x', 0)$. The vector potential due to this source distribution may be determined easily to be

$$\mathbf{F} = \hat{\mathbf{x}} \frac{-j}{2} \int_A H_0^{(2)}(kR) E_y(x', 0)\, dx', \tag{1}$$

where A denotes the waveguide aperture, $H_0^{(2)}(\mu)$ is the zeroth order Hankel function of the second kind, and $R = [(x - x')^2 + z^2]^{\frac{1}{2}}$. We use the time convention $\exp j\omega t$, which is suppressed for brevity.

The electromagnetic fields in $z \geqq 0$ may be derived from $\mathbf{F}$ by

$$\begin{aligned} \mathbf{E} &= -\nabla \times \mathbf{F}, \\ \mathbf{H} &= \frac{1}{j\omega\mu_0} [k^2 \mathbf{F} + \nabla(\nabla \cdot \mathbf{F})]. \end{aligned} \tag{2}$$

In particular, we find that the tangential field components are given by

$$\begin{aligned} E_y(x, z) &= \frac{j}{2} \int_A E_y(x', 0) \frac{\partial}{\partial z} H_0^{(2)}(kR)\, dx', \\ H_x(x, z) &= -\frac{1}{2\omega\mu_0} \left(k^2 + \frac{\partial^2}{\partial x^2} \right) \int_A E_y(x', 0) H_0^{(2)}(kR)\, dx'. \end{aligned} \tag{3}$$

Notice that the integrals in equation (3) have to be evaluated carefully when z approaches 0. In particular, the differentiation and integration in the second equation may not be interchanged when $z \to 0$, because in doing so the integral becomes divergent.

The fields inside the waveguide are most conveniently expressed in terms of the waveguide modal functions. The presence of dielectric plugs near the aperture may be accounted for by using appropriate modal admittances which are derivable by applying the transmission line theory. Assuming that the incident wave originating in the region $z < -d$ has unit modal voltage, we may write the tangential electromagnetic fields at the aperture as

$$\begin{aligned} E_y(x, 0) &= \sum_{n=1}^{\infty} \bar{V}_n \varphi_n(x), \\ H_x(x, 0) &= -2\bar{Y}_1 \varphi_1(x) + \sum_{n=1}^{\infty} \tilde{Y}_n \bar{V}_n \varphi_n(x), \end{aligned} \tag{4}$$

where φ_n are the orthonormal modal functions, and

$$\begin{aligned} \tilde{Y}_n &= Y_n^D \frac{Y_n + jY_n^D \tan \alpha_n^D d}{Y_n^D + jY_n \tan \alpha_n^D d}, \\ \bar{Y}_1 &= \frac{Y_1 Y_1^D}{Y_1^D \cos \alpha_1^D d + jY_1 \sin \alpha_1^D d}, \end{aligned} \tag{5}$$

with

$$Y_n = \frac{\alpha_n}{\omega \mu_0} \quad \text{and} \quad Y_n^D = \frac{\alpha_n^D}{\omega \mu_0}$$

(α_n^D and α_n being the nth propagation constants in the waveguide region with and without a dielectric, respectively). The $\bar{V}_n$ are the modal voltages at the aperture. When the modal voltages V_n in the empty waveguide region are desired, they may be obtained by using the following formula

$$V_n = \frac{Y_n^D}{Y_n^D \cos \alpha_n^D d + jY_n \sin \alpha_n^D d} \bar{V}_n + j \frac{2Y_1 \sin \alpha_1^D d}{Y_1^D \cos \alpha_1^D d + jY_1 \sin \alpha_1^D d} \delta_{1n} \tag{6}$$

where δ_{1n} is the Kronecker delta. The reflection coefficient R is obtainable from

$$1 + R = V_1 . \tag{6a}$$

The orthonormality of the waveguide modal functions may be applied to the first equation of (4) to obtain

$$\bar{V}_n = \int_A E_y(x', 0)\varphi_n(x')\, dx'.$$

When the result is substituted into the second equation of (4), we find

$$H_x(x, 0) = -2\bar{Y}_1\varphi_1(x) + \sum_{n=1}^{\infty} \bar{Y}_n\varphi_n(x) \int_A \varphi_n(x')E_y(x', 0)\, dx'. \tag{7}$$

Notice that the summation and integration in equation (7) are not interchangable. The reason is that when the summation is brought under the integral sign, the resulting kernel has a singularity of the form $1/(x - x')^2$, which is nonintegrable in the usual sense. In order to circumvent this difficulty and to put equation (7) into a form suitable for combination with equation (3) when the boundary condition is applied, we use the following relation

$$\tilde{Y}_n\varphi_n(x)\varphi_n(x') = \left(\frac{\partial^2}{\partial x^2} + k^2\right) \frac{\tilde{Y}_n}{\alpha_n^2}\, \varphi_n(x)\varphi_n(x'). \tag{8}$$

Equation (7) may then be written as

$$H_x(x, 0) = -2\bar{Y}_1\varphi_1(x) + \left(\frac{\partial^2}{\partial x^2} + k^2\right) \cdot \int_A \left[\sum_{n=1}^{\infty} \frac{\tilde{Y}_n}{\alpha_n^2}\, \varphi_n(x)\varphi_n(x')\right] E_y(x', 0)\, dx'. \tag{8a}$$

An application of the continuity condition on H_x across the aperture leads to

$$2\bar{Y}_1\varphi_1(x) = \left(\frac{\partial^2}{\partial x^2} + k^2\right) \int_A \left[\sum_{n=1}^{\infty} \frac{\tilde{Y}_n}{\alpha_n^2}\, \varphi_n(x)\varphi_n(x') + \frac{1}{2\omega\mu_0} H_0^{(2)}(k \mid x - x' \mid)\right] E_y(x', 0)\, dx' \quad \text{for} \quad x \,\varepsilon\, A. \tag{9}$$

This is the integral equation having as the unknown function the tangential electric field which is nonvanishing only over the aperture region.

Notice that the step introduced in equation (8) to facilitate the interchange of integration and summation is not essential in our later application of moment method for solution. The procedure, however, enables us to obtain a compact integro-differential equation from which a pure integral equation may be derived, thus permitting a solution by different techniques.

2.3 *Integral Equation for Tangential Magnetic Field*

We next consider the integral equation using the tangential magnetic field at $z = 0$ as the unknown function. The derivation in this case follows the same procedure as discussed in Section 2.2. We first recognize that the fields in $z \geqq 0$ may be expressed in terms of the tangential magnetic field as follows

$$\begin{aligned} E_y(x, z) &= -\frac{\omega\mu_0}{2} \int_{-\infty}^{\infty} H_0^{(2)}(kR) H_x(x', 0)\, dx', \\ H_x(x, z) &= -\frac{j}{2} \int_{-\infty}^{\infty} \frac{\partial}{\partial z} H_0^{(2)}(kR) H_x(x', 0)\, dx', \\ H_z(x, z) &= \frac{j}{2} \int_{-\infty}^{\infty} \frac{\partial}{\partial x} H_0^{(2)}(kR) H_x(x', 0)\, dx'. \end{aligned} \tag{10}$$

The limits of integration extend from $-\infty$ to ∞ because $H_x(x', 0)$ has values over the entire $z = 0$ plane. The fields inside the waveguide are given by

$$\begin{aligned} H_x(x, 0) &= \sum_{n=1}^{\infty} \bar{I}_n \varphi_n(x), \\ E_y(x, 0) &= -2\bar{Z}_1 \varphi_1(x) + \sum_{n=1}^{\infty} \tilde{Z}_n \bar{I}_n \varphi_n(x), \end{aligned} \tag{11}$$

where

$$\tilde{Z}_n = 1/\tilde{Y}_n\,, \qquad \bar{Z}_1 = \frac{Z_1 Z_1^D}{Z_1^D \cos \alpha_1^D d + jZ_1 \sin \alpha_1^D d}.$$

Again, the $\bar{I}$'s are the modal currents defined at the aperture, and the modal currents I_n for the empty waveguide region are related to $\bar{I}_n$ by

$$I_n = \frac{Z_n^D}{Z_n^D \cos \alpha_n^D d + jZ_n \sin \alpha_n^D d} \bar{I}_n + j \frac{2Z_1 \sin \alpha_1^D d}{Z_1^D \cos \alpha_1^D d + jZ_1 \sin \alpha_1^D d} \delta_{1n}\,. \tag{12}$$

The reflection coefficient may be calculated using

$$1 - R = I_1\,. \tag{12a}$$

Equation (11) may be rewritten by making use of the orthonormality relation between the φ's. Thus,

$$E_y(x, 0) = -2\bar{Z}_1\varphi_1(x) + \int_A \left[\sum_{n=1}^{\infty} \tilde{Z}_n\varphi_n(x)\varphi_n(x')\right] H_x(x', 0)\, dx'. \tag{13}$$

In obtaining equation (13), the integration and summation have been interchanged. This is permissible because the kernel

$$\sum_{n=1}^{\infty} \tilde{Z}_n\varphi_n(x)\varphi_n(x')$$

behaves like $\ln | x - x' |$ so that the integral is absolutely convergent for physically acceptable solution H_x.

We are now ready to derive the integral equation by applying the boundary condition using equations (10) and (13). The limits of integration in equation (13) may be extended from A to $(-\infty, \infty)$ with the understanding that the φ's are defined to be identically zero outside the aperture. We thus obtain

$$2\bar{Z}_1\varphi_1(x) = \int_{-\infty}^{\infty} \left[\sum_{n=1}^{\infty} \tilde{Z}_n\varphi_n(x)\varphi_n(x') + \frac{\omega\mu_0}{2} H_0^{(2)}(k \mid x - x' \mid)\right] H_x(x', 0)\, dx' \qquad -\infty < x < \infty. \tag{14}$$

Notice that equation (9) and (14) may be cast into variational form for the input impedance and admittance, respectively.

III. SOLUTIONS OF THE INTEGRAL EQUATIONS

Equations (9) and (14) constitute a pair of alternative integral equations for the radiation from a parallel-plate waveguide into a half space. One of the equations has as the unknown function the tangential electric field, while the other has as the unknown function the tangential magnetic field. Since there is no known method for solving these equations analytically, we have to resort to approximate techniques. Because of the infinite limits associated with the equation for the magnetic field, which is usually rather difficult to handle numerically, the one for the electric field is much preferred.

Strictly speaking, equation (9) is an integro-differential equation. We may derive from it a pure integral equation in a similar vein as Hallen did for the dipole antenna. The usefulness of this approach is currently being investigated. We discuss solutions of equation (9) directly by the method of moments.[9,10] To do so, we first approximate the aperture electric field by the following representation

$$E_y(x', 0) \approx \sum_{n=1}^{N} b_n U_n(x'), \tag{15}$$

where $U_n(x')$ is a set of linearly independent functions which are chosen to satisfy the boundary conditions on E_y at both ends of the aperture, that is

$$U_n(0) = U_n(a) = 0. \tag{16}$$

Substituting equation (15) into equation (9) gives

$$2\bar{Y}_1\varphi_1(x) \approx \sum_{n=1}^{N} b_n\left(\frac{\partial^2}{\partial x^2} + k^2\right) \int_A \left[\sum_{n=1}^{\infty} \frac{\tilde{Y}_n}{\alpha_n^2} \varphi_n(x)\varphi_n(x') + \frac{1}{2\omega\mu_0} H_0^{(2)}(k \mid x - x' \mid)\right] U_n(x')\, dx'. \tag{17}$$

We next require the difference between the left and right sides of equation (17) to be orthogonal to another set of functions

$$W_n(x), \qquad n = 1, 2, \cdots, N$$

with $W_n(0) = W_n(a) = 0$ (for reasons to become apparent presently). This last step then converts the integral equation into a set of algebraic equations

$$\sum_{p=1}^{N} A_{qp} b_p = f_q, \qquad q = 1, 2, \cdots, N, \tag{18}$$

where

$$A_{qp} = \sum_{n=1}^{\infty} \tilde{Y}_n(W_q, \varphi_n)(\varphi_n, U_p) + \frac{1}{2\omega\mu_0} \int_A dx\, W_q(x)\left(\frac{\partial^2}{\partial x^2} + k^2\right) \int_A dx'\, H_0^{(2)}(k \mid x - x' \mid) U_p(x'), \tag{19}$$

$$f_q = 2\bar{Y}_1 \int_A dx\, \varphi_1(x) W_q(x),$$

with

$$(W_q, \varphi_n) = \int_A dx\, W_q(x)\varphi_n(x).$$

For the evaluation of A_{qp}, it is desirable that $U_p(x)$ be chosen such that the integration of $U_p(x)$ and $H_0(k \mid x - x' \mid)$ can be carried out in closed form. Unfortunately, such functions which will also satisfy the boundary conditions (16) are not easy to find. This being the case, we shall manipulate the expression in equation (19) into forms which are more convenient to implement for numerical integration. Thus, by interchanging one differentiation with the integral with respect to x', and then integrating by parts twice (once with respect to x' and once with respect to x), we find

$$\int_A dx\, W_q(x) \frac{\partial^2}{\partial x^2} \int_A dx'\, H_0^{(2)}(k \mid x - x' \mid) U_p(x')$$
$$= -\int_A dx \frac{dW_q(x)}{dx} \int_A dx'\, H_0^{(2)}(k \mid x - x' \mid) \frac{dU_p(x')}{dx'},$$

where we have used the relation

$$\frac{\partial}{\partial x} H_0^{(2)}(k \mid x - x' \mid) = -\frac{\partial}{\partial x'} H_0^{(2)}(k \mid x - x' \mid)$$

and the fact that the integrated terms vanish on account of the boundary conditions.

Using this result, we may rewrite equation (19) as

$$\begin{aligned} A_{qp} = {} & \sum_{n=1}^{\infty} \tilde{Y}_n(W_q, \varphi_n)(\varphi_n, U_p) \\ & + \frac{1}{2\omega\mu_0}\left[k^2 \int_A dx\, W_q(x) \int_A dx'\, H_0^{(2)}(k \mid x - x' \mid) U_p(x') \right. \\ & \left. - \int_A dx \frac{dW_q(x)}{dx} \int_A dx'\, H_0^{(2)}(k \mid x - x' \mid) \frac{dU_p(x')}{dx'}\right]. \end{aligned} \tag{20}$$

The double integrals in equation (20) may be converted into single integrals by a transformation of variables. If the waveguide modal functions are chosen as both the basis and testing functions and if the fact that only modes of even symmetry with respect to yz plane are excited is accounted, we obtain

$$A_{qp} = \tilde{Y}_q\, \delta_{qp} + \frac{1}{2\omega\mu_0} \int_A ds\, H_0^{(2)}(ks) F_{qp}(s), \tag{21}$$

where

$$F_{qp}(s) = \begin{cases} \dfrac{2}{(q^2 - p^2)\pi}\left[k_p'^2 q \sin\dfrac{p\pi}{a}s - k_q'^2 p \sin\dfrac{q\pi}{a}s\right] & q \neq p \\[2ex] \dfrac{1}{a}\left\{k_p'^2(a - s)\cos\dfrac{p\pi}{a}s + \left[k^2 + \left(\dfrac{p\pi}{a}\right)^2\right]\dfrac{\sin\dfrac{p\pi}{a}s}{\dfrac{p\pi}{a}}\right\} & q = p \end{cases}$$

with

$$k_p'^2 = k^2 - \left(\frac{p\pi}{a}\right)^2.$$

The last integrals in equation (21) may be evaluated numerically. We have found that a fast, accurate, and yet economical way is to apply the Simpson's rule with the values of the Hankel function obtained from the Tschebycheff representation.[11]

After the matrix elements are calculated, the set of equations (14) is ready for a solution. An advantage of choosing the waveguide modal functions as both the basis and testing functions in the application of the moments method is that the solutions are expressed directly in terms of the modal coefficients of the aperture field. The reflection coefficients are then easily calculated by using equations (6) and (6a).

The radiation patterns of the antenna may be obtained from equation (3). Introducing the asymptotic expression for large arguments for the Hankel function, we find that the electric field in the far zone is approximated by

$$E_y(r,\theta) \approx \left(\frac{k}{2\pi r}\right)^{1/2} e^{-i(kr-3\pi/4)} \cos\theta \int_A E_y(x',0)e^{ik\sin\theta x'}\,dx'. \tag{22}$$

It is easy to show that the magnetic field in the far zone is related to the electric field through the free space admittance. Thus,

$$H_\theta(r,\theta) = \eta_0 E_y(r,\theta), \quad \text{for} \quad kr \gg 1,$$

where η_0 is the characteristic admittance of free space. For comparison, it is often desirable to normalize the radiation patterns. A commonly used normalization is to make the amplitude unity in the direction of maximum radiation. We use a different normalization here, however. Our patterns are normalized such that the integral of the square of the amplitudes gives the radiated power when a unit power is supplied to the incident wave. This way of displaying the patterns is more advantageous because it shows the normalized radiation intensity in addition to the information contained in the usual pattern presentation; this provides a basis of comparison when the frequency is varied. Thus, using expression (15) with $\{U_p(x)\} = \{W_p(x)\} = \{\phi_p(x)\}$ and equation (22), we obtain for the normalized radiation pattern

$$T(\theta) = \frac{2k}{(2\pi\alpha_1)^{1/2}} \cos\theta \sum_{n=1,3,\cdots}^{N} b_n \frac{\cos\left(k\frac{a}{2}\sin\theta\right)}{\left(\frac{n\pi}{a}\right)\left[1 - \left(\frac{2a}{n\lambda}\sin\theta\right)^2\right]}, \tag{23}$$

where α_1 is the propagation constant of the incident wave.

IV. RESULTS

We now present numerical results obtained by the method described in Section III. The computations are actually performed with exp $-j\omega t$ time convention. Table I shows the type of convergence one may expect for the reflection coefficient R versus N, the number of modes used to approximate the aperture electric field. The parameters used in this calculation are $\epsilon = 6$, $\lambda/a = 1.5$, and $d/a = 0.544$. This represents one of the worst situations encountered. Nevertheless, we find the convergence is quite rapid.

The variation of the reflection coefficients versus the thickness of the dielectric plug is considered first. Figure 2 shows such a calculation for $\lambda/a = 1.5$ and $\epsilon = 6$. With this value of a/λ, only one mode can propagate in an unloaded waveguide. The dielectric constant is chosen so that the third order mode is propagating inside the dielectric. (The second order mode will also be propagating; but this mode cannot be excited because of the symmetry in the geometry.)

The reflection coefficient shows a smooth standing wave like variation versus d/a over the entire range of d considered except in the vicinities of $d/a \approx 0.54$ and $d/a \approx 1.31$ (where sharp spikes appear). Figure 3 shows the details of the reflection coefficient near these spikes.

The maxima (or minima) of the standing wavelike pattern are equally displaced at a distance given by π/α_1^D, where α_n^D is the propagation constant of the nth mode of a dielectric loaded waveguide. The separa-

TABLE I—CONVERGENCE OF R VERSUS N

N	$\lvert R\rvert$	Phase of R (degrees)
1	0.8031	−162.8
3	0.9213	−169.8
5	0.9306	−169.2
7	0.9348	−168.9
9	0.9372	−168.6

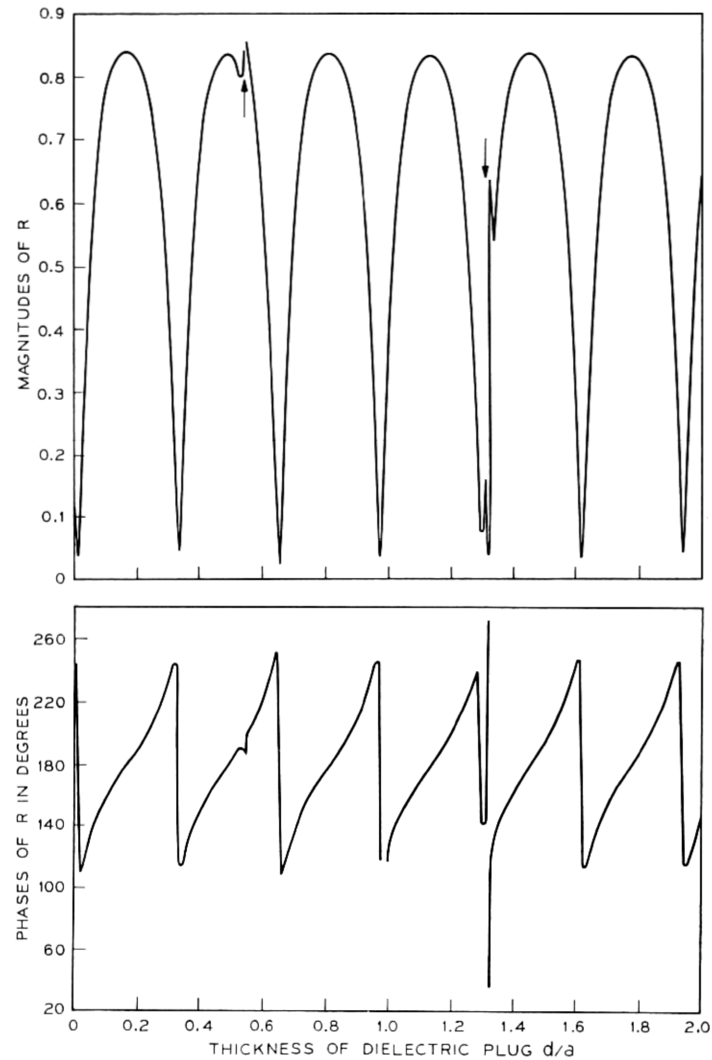

Fig. 2 — Reflection coefficient of a waveguide antenna with dielectric plug ($\epsilon = 6$ and $\lambda/a = 1.5$).

tion between the two spikes Δd is obtainable from the relation $\alpha_3^D(\Delta d) = \pi$. (Notice that the sharp spikes are frequently preceded by deep dips such that they may appear like close-by double spikes as displayed by the one at $d/a \approx 1.31$. See Fig. 3.) Figure 4 presents another calculation using a higher dielectric constant $\epsilon = 13$. Since the propagation

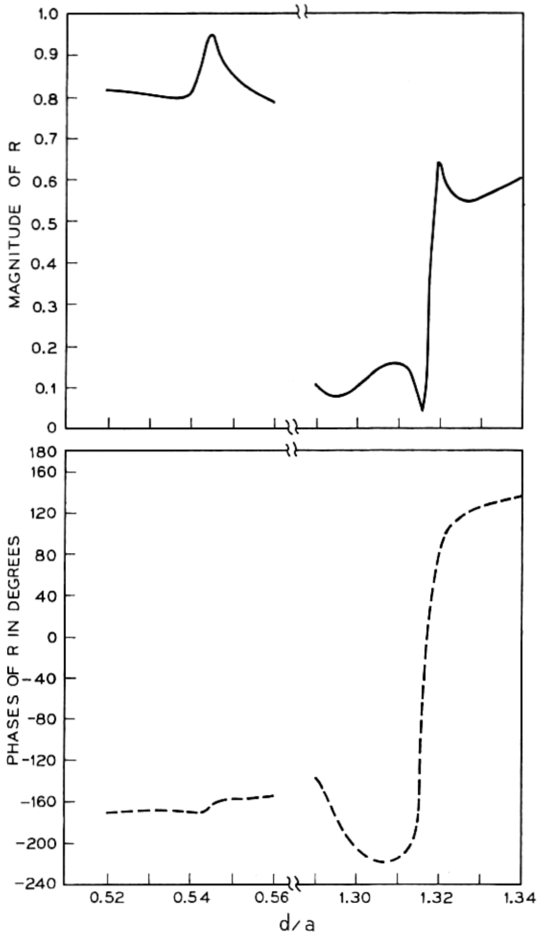

Fig. 3 — Details of R versus d/a for $\epsilon = 6$ and $\lambda/a = 1.5$.

constants α_1^D and α_3^D are larger when a higher dielectric constant is used, the maxima (or minima) and the spikes become more closely spaced. Otherwise, the relation stated above remains valid. This observation suggests that ordinarily the third order mode is only weakly excited so that the radiation impedance of the waveguide is determined primarily by the fundamental mode. Only when the dielectric plug has a certain thickness is the third order mode excited strongly enough to influence the reflection coefficient of the fundamental mode. Figure

5 shows the solutions for the third order modal coefficients versus d to demonstrate that indeed this is the case.

From the regularity of the intervals between the spikes at which the third order mode is excited sufficiently strongly to influence the radiation of the waveguide, it seems reasonable to assume that the dielectric plug forms a cavity for the third order mode. This cavity goes into resonance only at proper combinations of the wavelength and the thickness of the dielectric plug. To verify this conjecture we applied the transverse resonance technique at the waveguide aperture using the admittances pertinent to the third order mode. Let $\vec{Y}$ be the radiation admittance when a completely loaded waveguide is excited in the third order mode. The admittance looking toward the negative z direction, that is, into the waveguide is given by the appropriate modal admittance:

$$\overleftarrow{Y} = Y_3^D \frac{Y_3 + jY_3^D \tan \alpha_3^D d}{Y_3^D + jY_3 \tan \alpha_3^D d}.$$

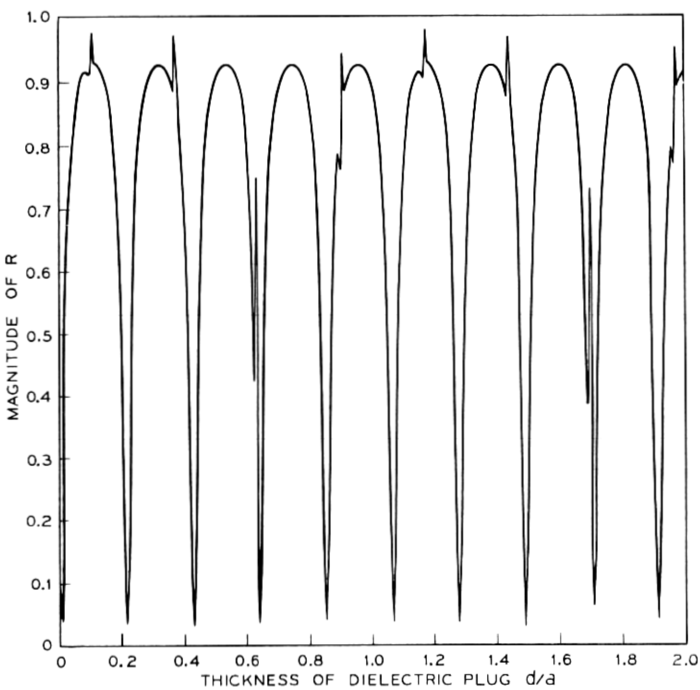

Fig. 4 — R versus d/a for $\epsilon = 13$, $\lambda/a = 1.5$.

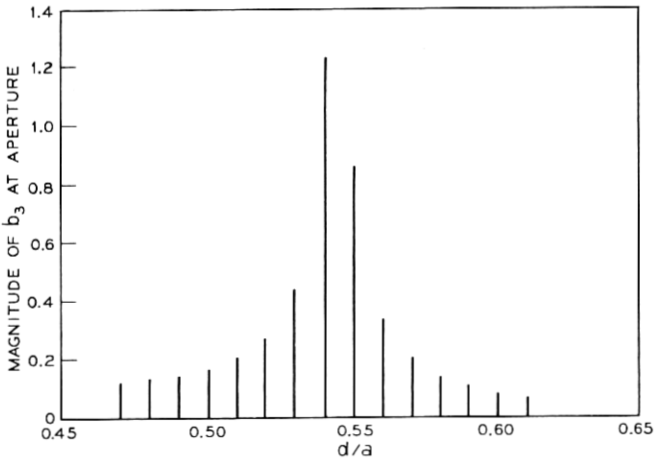

Fig. 5 — Magnitude of third model coefficients versus d/a for $\epsilon = 6$ and $\lambda/a = 1.5$.

The condition of resonance is given by*

$$\text{Im}\,(\overleftarrow{Y} + \overrightarrow{Y}) = 0.$$

Figure 6 shows a calculation of the imaginary parts of $\overleftarrow{Y}$ and $\overrightarrow{Y}$ as functions of d. The graph clearly demonstrates that there are intersections occurring at the values for which resonance behavior is exhibited in the reflection coefficients.

We next consider the variation of the reflection coefficient when the frequency is varied. Figure 7 gives a calculation using $\epsilon = 6$ and $d/a = 0.55$. That there are two frequencies at which the reflection coefficient displays abrupt variations is quite evident. The details of one of the variations are illustrated in expanded scale in the inset. Examination of the admittances pertinent to the third order mode again shows that the transverse resonance condition is satisfied at both of these frequencies. Another salient feature shown in this calculation is that there are several frequencies at which the reflection coefficients are practically zero. Therefore, when the parameters are judiciously chosen, the use of a dielectric plug does not necessarily degrade the match characteristic of the antenna.

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* Strictly speaking, because of the radiation from the waveguide aperture, the resonance condition should be $(\overleftarrow{\text{Y}} + \overrightarrow{\text{Y}}) = 0$. Since our interest is to obtain the condition for maximum excitation of the third order mode as d is varied, this should be a good approximation.

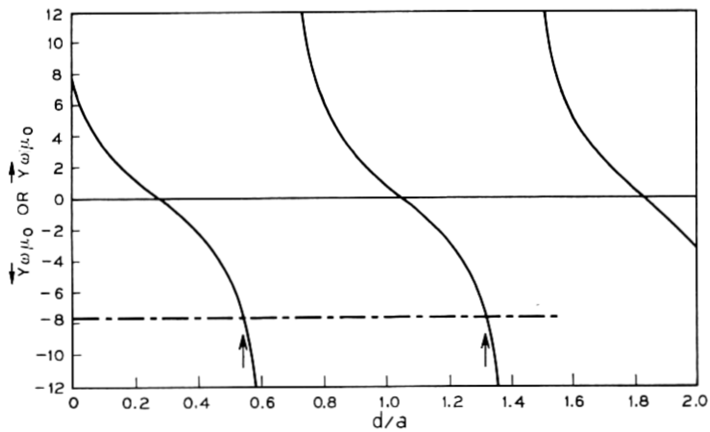

Fig. 6 — $\overrightarrow{Y}$ and $\overleftarrow{Y}$ of the third order mode for $\epsilon = 6$ and $\lambda/a = 1.5$ (——— $\overleftarrow{Y}$; —·—·— $-\text{Im}\,\overrightarrow{Y}$).

The radiation patterns of the antenna have also been computed for the various values of parameters considered. The results in general display smooth variation versus the angle of observation θ. Only when the parameters are such that the resonating higher order mode is exceedingly strongly excited do dips appear in the radiation patterns. Figure 8 gives some typical results for smoothly varying patterns and Figure 9 illustrates the patterns with dips. Notice that the pattern dips are exhibited only over a very narrow range of the parameter

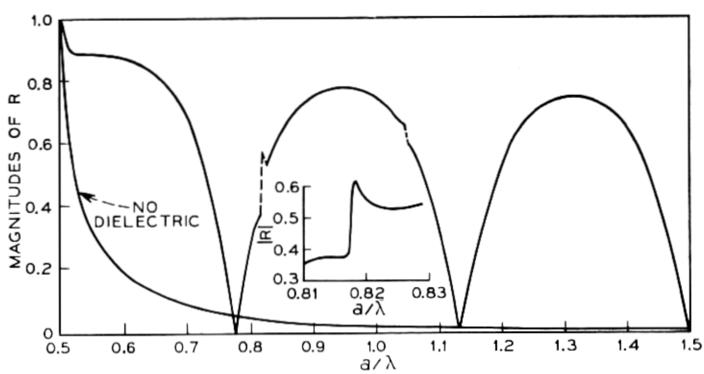

Fig. 7 — Variation of R with frequency for $\epsilon = 6$ and $d/a = 0.55$.

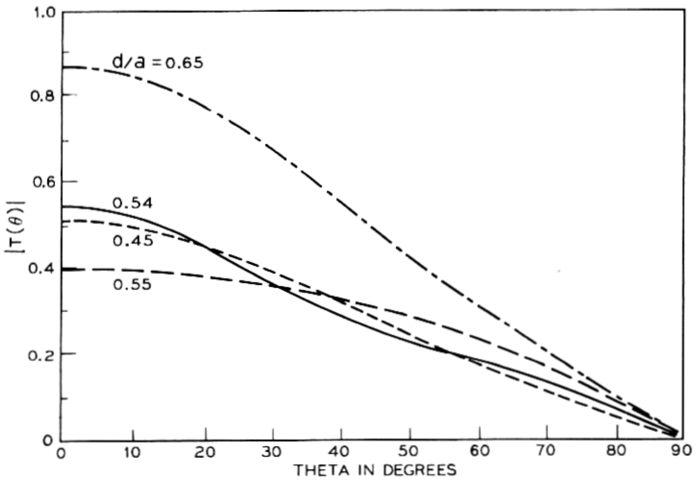

Fig. 8 — Normalized radiation patterns $T(\theta)$ of a waveguide with dielectric plug for $\epsilon = 6$ and $\lambda/a = 1.5$.

d/a. Moreover, the dips are rather broad and shallow because the aperture is small in wavelength, $\frac{1}{2} < a/\lambda < 1$.

Figure 9 also shows the patterns for the situation when the waveguide is *completely* loaded with a dielectric and is excited in the first or the third order mode. The aperture field in such situations consists primarily of the incident wave. We observe that a relatively small aperture with an aperture field distribution of the third order mode is capable of producing a dip in the radiation pattern. Now, when the third order mode is at resonance inside the dielectric plug so that it is strongly excited, the aperture field contains high content of both the incident dominant mode and the third order mode. The relative amplitudes and phases of these two modes determine the shape of the radiation pattern. The combination sometimes may be such as to generate a pattern which exhibits a considerably suppressed radiation in the broadside direction as shown in the curve for $d/a = 0.545$.

V. CONCLUSIONS AND DISCUSSIONS

The investigation of the effects of dielectric plugs on the radiation from a flush mounted waveguide has shown that dielectric plugs can function as a resonant cavity for the wave modes which are propagating inside the dielectric but evanescent in the unloaded waveguide region. Such wave modes have interesting effects on the radiation impedances of the antenna. When one of these modes is at resonance, it is strongly excited by the incident wave; the presence of the resonance is manifested in the form of sharp spikes in the reflection coefficient.

Resonances have also been observed in the analysis of phased arrays using the present waveguide with dielectric plugs as the radiating elements. They appear in both infinite and finite arrays. The occurrence of these resonances may be identified by the conditions of total reflection of the incident power in infinite arrays[6] and rapid variation of the coupling coefficients in finite arrays.[7] Although there has been considerable discussion on array resonances in general, it appears that no consensus has been reached yet about the basic mechanism of this phenomenon. We hope that observation of resonances and our analysis of their causes may shed some light on this problem.

Another aspect which deserves some comment is the use of a single array element for the detection of potential difficulty due to resonances. This question is particularly important in array designs using antenna elements which are less susceptible to analysis. We realize that this is an ambitious question which cannot be answered completely without a more elaborate analysis. The calculation so far, however, has indicated that resonances observed in array configurations are often not exhibited by the radiation characteristics of a

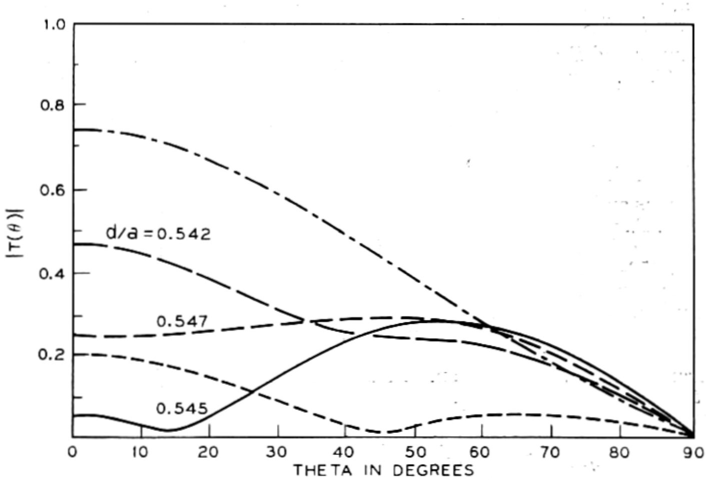

Fig. 9 — Pattern dips due to strong higher order mode excitation for $\epsilon = 6$ and $\lambda/a = 1.5$ (—·— 1st mode excitation; · · · · 3rd mode excitation).

single element. For example, in arrays of waveguides with dielectric plugs such as the one considered here,[6] resonances which are found to occur as a result of the interaction with the resonating second order mode are not displayed by a single element because this mode is usually not excited in the latter situation on account of geometric symmetry. When the dielectric constant is large enough to permit the third order mode to resonate, it is possible that the resonance conditions resulting from this mode may be uncovered. Even so, resonances which are caused by the second order mode are still undetectable. Moreover, there are other situations in which resonances do occur without the use of dielectrics such as planar arrays of rectangular and circular waveguides.[12,13] It therefore appears that it is not suitable to use a single element in the prediction of potential array resonances.

VI. ACKNOWLEDGMENT

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