# Interference in a Dense Radio Network

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*Radio systems operating at frequencies above 10 GHz are limited to short hops by rain attenuation. A severe interference problem arises from the fact that in a given area many repeaters may mutually interfere through through the back and side lobe responses of their antennas. Introducing the concept of frequency spectrum conservation by maximizing the communication flow through an area, a model of a dense network of radio systems has been studied to determine the effect of antenna discrimination and bandwidth expanding modulation methods upon the total communication capacity in a given frequency band. We conclude that for efficient performance, bandwidth expansion is required; we also conclude that communication capacity can be considerably increased by improving the near side lobe performance of antennas.*

## I. INTRODUCTION

The major factor influencing the design of radio systems above 10 GHz is the attenuation caused by rainfall; the principal result, from which many other prameters are determined, is that the repeaters must be closely spaced compared with lower frequency systems.[1,2] Consequently, the number of exposures to co-channel interference may be large—requiring interference resistant modulation. Such solutions generally use considerable bandwidth, thus raising questions about the efficiency of frequency occupancy.

The uses for short hop systems are such that many systems may exist in the same area on the same frequency assignment. Among the possible applications are broadband exchange area service, *Picturephone*® visual telephone distribution, and communication between mobile telephone concentrators. The important property with respect to efficient spectrum usage is the total "communication capacity through the area" in a given bandwidth. It is conceivable that, in terms of the concept of capacity through an area, interference resistant wideband modulation is more efficient than the narrowband methods now used. The purpose of this work is to explore and answer this question. The approach is to set up a two-dimensional system model, compute the total interference at each repeater site, and explore the consequences of the RF signal-to-interference ratio (S/I) on the RF bandwidth required to meet the baseband interference criteria.

## II. INTERFERENCE PARAMETERS

The three most important system parameters affecting interference are antenna discrimination, the modulation method used to transmit information, and the frequency assignment plan.

### 2.1 *Antenna Discrimination*

Increased antenna discrimination reduces interference, but a finite limit on antenna size and performance is set by practical considerations such as antenna cost, stability of the supporting structure, and the repeater antenna environment. The short wavelengths resulting from the very high radio frequency greatly reduce the overall size and bulk of an antenna having a given gain and beamwidth, and thus make feasible new, more precise construction techniques. This is one of the few areas where the problems become easier as frequency increases. Hence in the calculation of interference it is admissible to assume well-shielded antennas with very small back and far sidelobe radiations and an aperture illumination which is tapered to control off-axis radiation near the main beam; this requires increased sophistication in antenna design. Some loss of antenna gain and a slight increase in repeater cost results; the increase in efficiency with which the frequency spectrum and the area of the countryside are used are well worthwhile.

Since there are practical but no natural limits on antenna beamwidth, the performance criteria selected for use in an interference study is, in part, a matter of judgment. One possibility is to demand only what has been obtained with antennas currently in production. This appears much too restrictive since it makes no allowance for new designs, changed requirements, and new construction techniques.

In the past antenna gain has been a dominant design parameter. For short hop systems operating in an environment of severe interference this is no longer justified; the angular response is much more important. For maximum communication capacity through an area, systems should be designed to be interference limited under conditions of normal propagation; thermal noise is controlling only during heavy rain. Hence, it is reasonable to expect new antennas having better off-axis discrimination; the question is how much better. In this paper three antenna patterns are used, ranging from the theoretical limit, which can be approached but not achieved, to the patterns of existing antennas.

In addition to the antenna pattern, the environment in which the antenna is placed is also of importance. Reflections from buildings, trees, rocks, and so forth, can distort an otherwise ideal antenna pattern and considerably reduce the discrimination obtained in an ideal environment. This problem is well recognized in that most antenna testing ranges are chosen to be free of such obstacles. However, it is unlikely that many repeater locations for short hop systems of the type considered here will be so ideal. For these reasons there is a practical upper limit to the amount of discrimination which can be obtained even with an "ideal" antenna. In our calculations we assume the antenna response falls off in accordance with its diffraction pattern until it reaches the level limited by the environment and levels off at this value.

### 2.2 *Modulation Methods*

A second important aspect of system design which affects our ability to achieve maximum use of a frequency assignment in the area in which the systems are located is the modulation method used. Signals passing through a radio system are subjected to degradations caused by a variety of sources of noise and interference from the same and other systems. If maximum traffic is to be carried, the system must be resistant to interference so that systems can be densely packed. Part of this resistance can be obtained by using well-designed narrow beam antennas.

Additional resistance to interference can be obtained by using modulation methods such as PCM and large index phase or frequency modulation which exchange bandwidth for improved signal-to-interference ratio. Increased resistance to interference will allow more systems to be built in a given area but will require more bandwidth per channel so that an optimum is eventually reached beyond which total capacity begins to decrease.

Communication capacity is an involved function of many parameters including some related to the cost of providing a given level of sophistication in apparatus performance. These parameters change with time as technology evolves and our understanding improves, so we do not attempt to calculate a grand maximum. Instead we study a somewhat arbitrary but sophisticated network which we believe demonstrates that, by properly exchanging bandwidth for interference resistance, the maximum communication flow through an area can be achieved.

In this paper only large index phase modulation is studied, although both digital and analog methods of expanding bandwidth are of interest.[1] The relations between bandwidth expansion and interference must be known for this work. For large index phase (or frequency) moduation, these relations are well known.[3,4] However, multilevel phase-shift keyed modulation, the digital method of interest, has not been so completely analyzed. The deterioration in performance produced by one or more co-channel interferers can be evaluated, and while some excellent work has been done on the spectral properties of phase-shift keying, we do not yet have sufficient information to determine the bandwidth required for multilevel phase-shift keying systems with realizable baseband pulses.[5–7]

In some circumstances it is possible to use each frequency assignment twice by use of orthogonal polarizations. The efficacy of this approach to doubling the capacity of the network depends upon the cross-polarization discrimination of the antenna, the tower stability, and the bandwidth expansion factor used in the method of modulation. In this paper the frequency assignment is used only once.

### 2.3 *Frequency Plan*

A two-frequency plan is assumed, at each site one frequency is transmitted and the other received; two-way transmission is accomplished. This is believed to be the most efficient plan and is the one used for most microwave systems. A single frequency plan is not feasible because the maximum repeater gain exceeds the coupling between the transmitting and receiving antennas. Use of the same frequency in the receiver and transmitter could result in an unstable repeater.

## III. ANALYSIS

### 3.1 *The Network Model*

There is no completely satisfactory way of choosing a network of systems on which to base an interference study. Regular geometric networks are artificial and unrealistic. A closer approach to field conditions could be obtained by choosing a "typical" area of the country and by using topographical maps, laying out several interesting systems, and then studying the resulting interference. The trouble comes in choosing a typical area; for every situation, favorable or not, that one chooses someone else can find a counter example. There is no "typical" area. A compromise approach suggested by R. Kompfner is to assume that possible repeater sites occur at random locations and that on the average, investigation of two sites is required to locate one repeater. One result of this approach is shown in Fig. 1.

The points shown on Fig. 1 were located by using two successive random 5-digit entries from the "Table of a Million Random Digits," as X, Y coordinates of repeater locations.[8] This approach has the advantage that sites are located randomly but their locations are precisely known so that interference can be computed. Figure 1 has a total of 11 north-south systems and 11 east-west systems, using 160 sites from 250 possible locations. Each of these systems is two-way; a two-frequency plan is assumed.

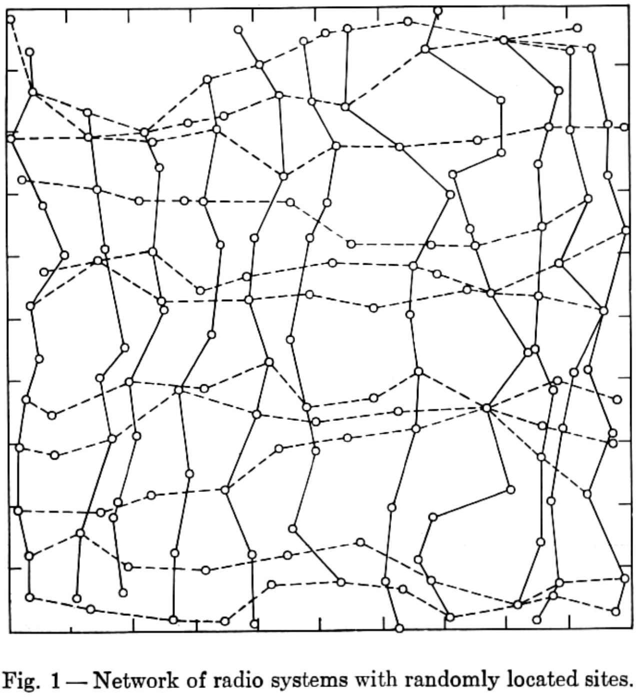

Fig. 1 — Network of radio systems with randomly located sites.

### 3.2. *Signal-to-Interference Ratios*

Let the antenna have gain $G(\theta)$ where $\theta$ is the angle in azimuth measured from the beam axis. The transmission between any two antennas can be written[9]

$$P_R = P_T\left(\frac{\lambda}{4\pi r_{TR}}\right)^2 G(\theta_{TR})G(\theta_{RT}) \tag{1}$$

where

$P_T$ is the power transmitted from one antenna,
$P_R$ is the power received in the other,
$\lambda$ is the free space wavelength,
$r_{TR}$ is the distance between antennas $T$ and $R$, and
$\theta_{TR}$ is the angle between the axis of the beam of antenna $T$ and the direction of antenna $R$.

In an interference computation, the quantity of interest is the ratio of the received signal power to the total interference power in each receiver. From (1) the received signal power in the $i$th receiver is

$$S_i = P_T\left(\frac{\lambda}{4\pi r_{TR}}\right)^2 G(0)^2, \tag{2}$$

where $r_{\mathrm{TR}}$ is the distance between transmitter and receiver. The total interference power in the $i$th receiver is given by

$$I_i = P_T\left(\frac{\lambda}{4\pi}\right)^2 \sum_j \frac{G(\theta_{ij})G(\theta_{ji})}{r_{ij}^2}, \tag{3}$$

and the summation is over all sources of interference. The ratio of interference-to-signal power in the $i$th receiver is therefore

$$\frac{I_i}{S_i} = \sum_j \frac{G(\theta_{ij})G(\theta_{ji})}{G(0)^2}\left(\frac{r_{TR}}{r_{ij}}\right)^2. \tag{4}$$

As expected, the signal-to-interference ratio is a function of relative antenna response and the ratios of antenna separations. The signal-to-interference ratios discussed in this paper were computed for the network of Fig. 1 from expression (4), and in accordance with the following comments.

(*i*) Each receiver receives interference from the adjacent sites in the same system because two signals are transmitted from each adjacent site on the same frequency; one is the desired signal, the other is interference.

(*ii*) There is no interference in the $i$th receiver from the $i \pm 2$ sites because these sites do not transmit on the interfering frequency.

(*iii*) When systems cross at the same site the distance is $r_{ij} = 0$ and expression (4) cannot be used. In this instance we assume separate antennas with coupling limited by the environment.

(*iv*) Only half of the repeaters transmit on the same frequency. If it is known which repeaters transmit on a given frequency, then only those sites need be considered in the computation. By the proper choice of frequency assignment within each system, the interference can be minimized. Such a procedure requires extensive and, perhaps, unmanageable frequency coordination. We assume that no frequency coordination is required and, for computational purposes, all repeaters transmit on both frequencies except as just noted. The resulting interference is therefore somewhat greater than would otherwise be the case.

The three antenna patterns used are shown in Fig. 2. Curves A, B,

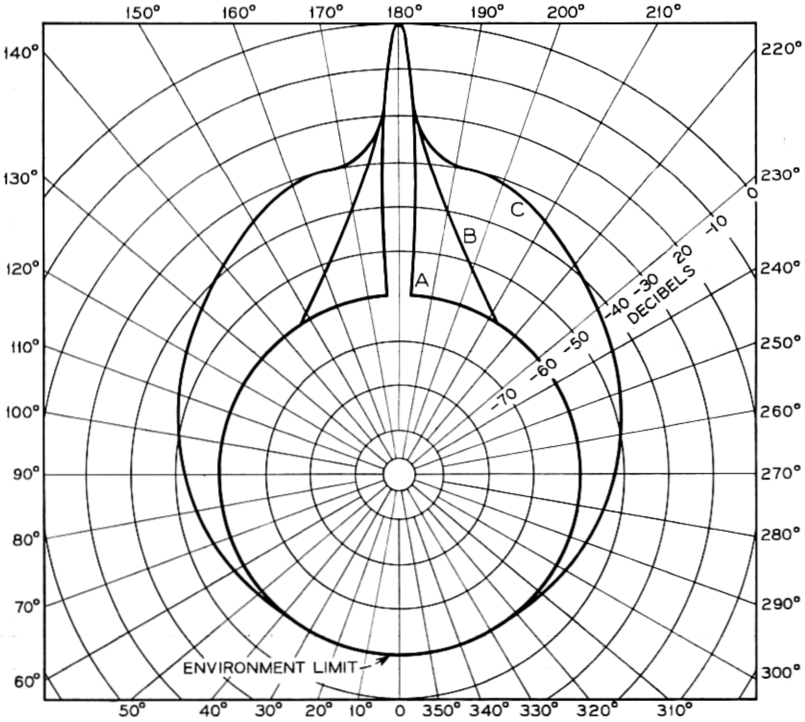

Fig. 2 — Antenna patterns used in the computations.

and C represent the envelopes of the responses. The patterns are for a beamwidth of two degrees and an environment limitation of −60 dB as discussed in Section 2.1. These responses were used for all computations; the beamwidths and environmental limitations are parameters.

Antenna A is the limiting case described by Dolph and approximated by Taylor,[10,11] Antenna C is an approximation to the type of antenna described by Crawford and Turrin;[12] the envelope of the measured response is approximated by the function

$$\frac{G(\theta)}{G(0)} = \frac{1}{1 + \left(\frac{\theta}{\theta_0}\right)^4} + \frac{10^{-3}}{1 + \left(\frac{\theta}{25\theta_0}\right)^4}. \tag{5}$$

As shown in Fig. 2 this function levels off at the assumed environment limit. Antenna B was chosen as a compromise between the theoretical limiting antenna A and the existing antenna C. Its response is given by

$$\frac{G(\theta)}{G(0)} = \frac{1}{1 + \left(\frac{\theta}{\theta_0}\right)^4}. \tag{6}$$

### 3.3 *Bandwidth Expansion and Interference Resistance*

As mentioned in Section 2.1, resistance to interference can be improved by suitably increasing RF bandwidth. At the present time there is enough knowledge about the bandwidth requirements of analog angle-modulated systems to determine the necessary relations between interference and bandwidth expansion. For this reason, large index analog phase modulation is discussed in this paper.

Prabhu and Enloe have derived the relationships between the signal-to-interference ratio at a receiver input and the corresponding signal-to-interference ratio in the baseband output.[3] In their work the modulating baseband signal was a flat band of gaussian noise extending from 0 to $W$ Hz. For large modulation index and for one co-channel interferer, they have shown that the worst baseband interference occurs at $f = 0$, and that this minimum signal-to-interference ratio $S_o/I_o$ is given by

$$\frac{S_o}{I_o} \approx 2\varphi^3(S/I), \qquad \varphi \geqq 2.5, \tag{7}$$

where

$\varphi$ is the rms phase index in radians, and
$S/I$ is the signal-to-interference ratio at the receiver input.

Ref. 13 shows that when there is more than one co-channel interferer, the problem of estimating the baseband interchannel interference can be reduced to that resulting from a single equivalent co-channel interferer. The power in this one interferer can be expressed in terms of the powers of the individual interferers and the total number of interferers.

The RF bandwidth required for a large-index, phase-modulated baseband signal with uniform spectral density in the range 0 to $W$ Hz, and with a gaussian amplitude distribution, is given by Carson's rule. (The efficacy of Carson's rule is demonstrated in Ref. 4.)

$$B = 2W\left[1 + \frac{4\varphi}{\sqrt{3}}\right]. \tag{8}$$

The relationship between the bandwidth expansion factor and the RF signal-to-interference ratio is found by eliminating $\varphi$ from (7) and (8).

$$B/W \approx 2 + \frac{8}{\sqrt{3}}\left[\frac{1}{2}\frac{(S_o/I_o)}{(S/I)}\right]^{\frac{1}{3}}, \tag{9}$$

where

$B/W$ is the bandwidth expansion factor,
$S_o/I_o$ is the baseband signal-to-interference ratio at the receiver output, and
$S/I$ is the RF signal-to-interference ratio at the receiver input.

Figure 3, which has been drawn from (9), illustrates the relationship between the bandwidth expansion factor and $S/I$ with the baseband signal-to-interference ratio as a criterion.

### 3.4 *Communication Capacity*

We assume that if any repeater in a system does not meet the requirement on $S_o/I_o$, the system is inoperative. For any $S/I$, and therefore for any $B/W$, the percentage of systems in operation is designated by $\epsilon$. The relative communication capacity of the network, $C_R$, is defined as

$$C_R = \frac{\epsilon W}{B}. \tag{10}$$

This definition satisfies our intuitive ideas concerning communication capacity; it is proportional to the baseband bandwidth $W$ and the percentage of operating systems and is inversely proportional to the

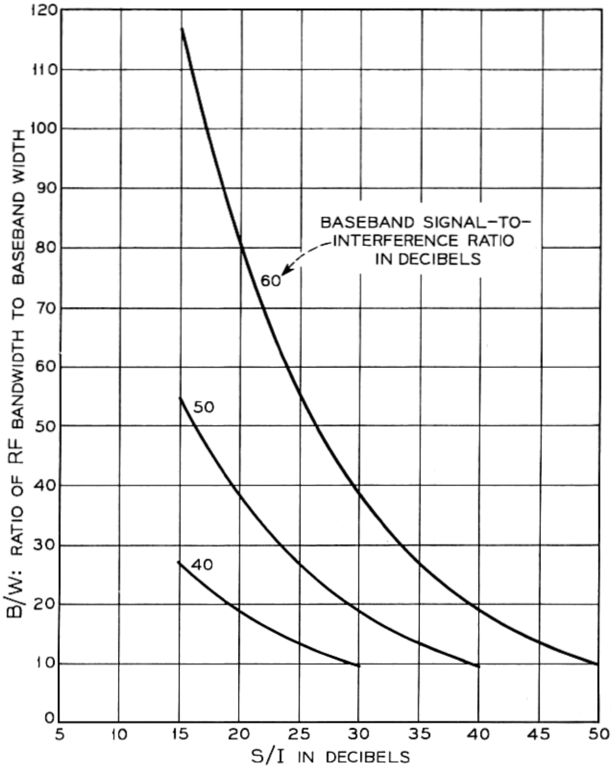

Fig. 3 — Bandwidth expansion as a function of signal-to-interference ratio for large index phase modulation.

RF bandwidth $B$ required for successful transmission. The computations performed on the network model were to determine $C_R$ as a function of antenna response and the bandwidth expansion factor.

## IV. COMPUTATIONS AND RESULTS

### 4.1 *Computations*

In accordance with the foregoing analysis the total $S/I$ at each of the 222 receivers was computed for the network of Fig. 1, for the antenna responses of Fig. 2, for six 3 dB beamwidths, and for three values of environmental limit. The results, presented in several ways in the following sections, illustrate the interference behavior of the network and demonstrate the effects of antenna response, beamwidth, and bandwidth expansion on the total capacity of the network.

### 4.2 *Maximum Interference*

Figures 4, 5, and 6 show the lowest total *S/I* computed for the network for various bandwidths and environment limits and for the three antennas of Fig. 2. If all systems in the network are to operate successfully, the modulation method must meet the baseband interference criterion at the *S/I* given in Figs. 4, 5, and 6.

For large beamwidths the results are insensitive to the environment limit and antenna response. Conversely, for small beamwidths, the results depend strongly on antenna response and the environment limit. For antenna A there is no reason to have a beamwidth of less than 0.02 radians, whereas, a smaller beamwidth is beneficial for antenna C. These relationships can be seen more clearly in Fig. 7 where the data for the three antenna patterns and an environment limit of −60 dB are plotted from Figs. 4, 5, and 6.

Figure 7 shows that narrowing the antenna beamwidth improves the network performance as expected. As the beamwidth becomes smaller, the antenna response pattern dominates the behavior; as the beamwidth decreases further the network performance becomes independent of antenna response and is determined by the environment limit. The best result occurs for the narrowest beamwidth for all

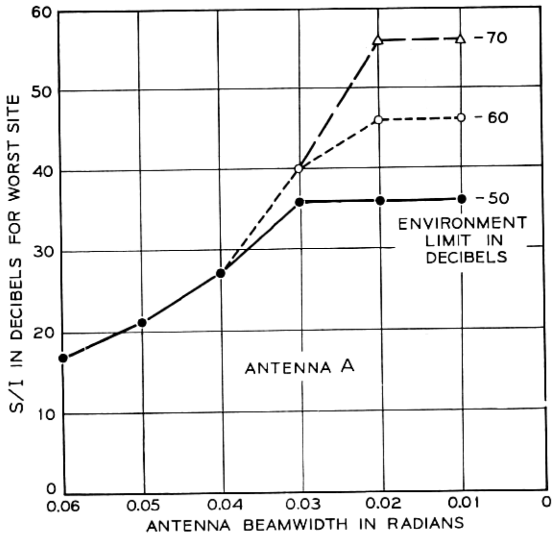

Fig. 4 — Minimum signal-to-interference ratio as a function of antenna beamwidth and environment.

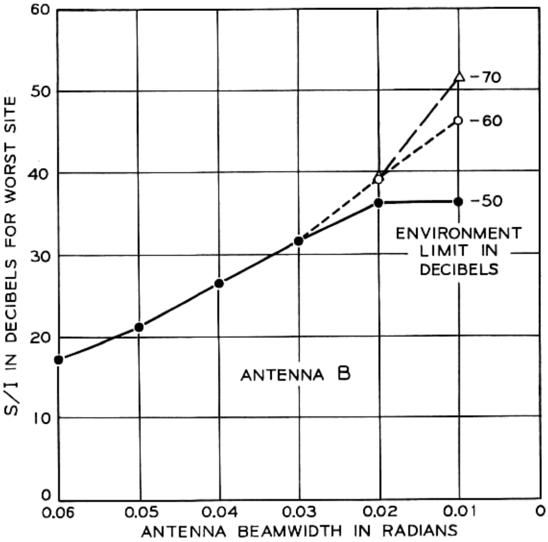

Fig. 5 — Minimum signal-to-interference ratio as a function of antenna beamwidth and environment.

antenna patterns. However, there are other constraints such as antenna size and tower stability which limit the minimum beamwidth of antennas for practical systems. Figure 7 also illustrates the important result that for a given $S/I$ criterion, the antenna with the best response pattern would allow a wider beamwidth to be used, thus

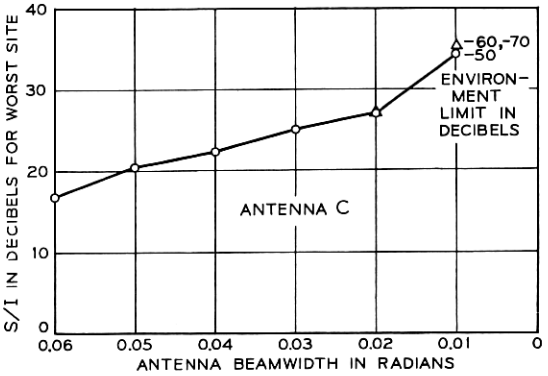

Fig. 6 — Minimum signal-to-interference ratio as a function of antenna beamwidth and environment.

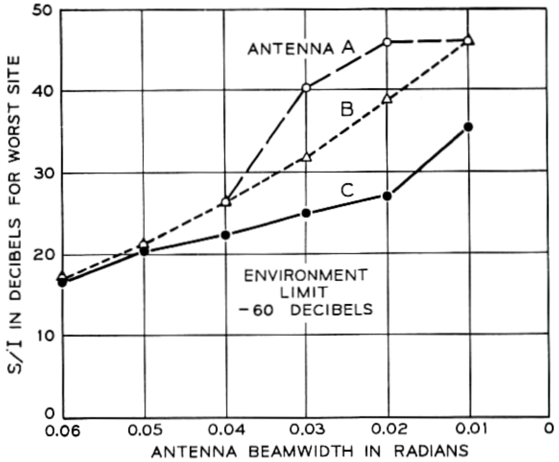

Fig. 7 — Worst site signal-to-interference ratio for three antennas and an environment limit of —60 dB.

reducing the requirements on tower stability and permitting the use of a smaller antenna.

### 4.3 *Distributions of the Number of Sites With Respect to Interference*

The previous remarks concerned only the $S/I$ at the worst site in the network. This section illustrates the way in which the interference is distributed throughout the network. A beamwidth of 0.03 radians and an environment limit of $-60$ dB were chosen.

Figure 8 shows the number of receiver sites having a given $S/I$ in intervals of one dB. For example, 6 percent of the sites have $S/I$ in the interval $35 \text{ dB} \leqq S/I < 36 \text{ dB}$ for antenna $C$. The same data are plotted in the form of distribution curves in Fig. 9.

It is probably safe to speculate on the basis of Figs. 8 and 9 that simple modifications in the network would result in substantial improvement in communication capacity for all antenna patterns.

### 4.4 *Communication Capacity of the Network*

For computation of the relative communication capacity, the distribution of the number of systems as a function of $S/I$ is required. As discussed in Section III and illustrated in Fig 1, there are 11 east-west and 11 north-south systems. Transmission is in both directions in each system. The distributions for the three antenna patterns are shown in Fig. 10 for an antenna beamwidth of 0.03 radians. In con-

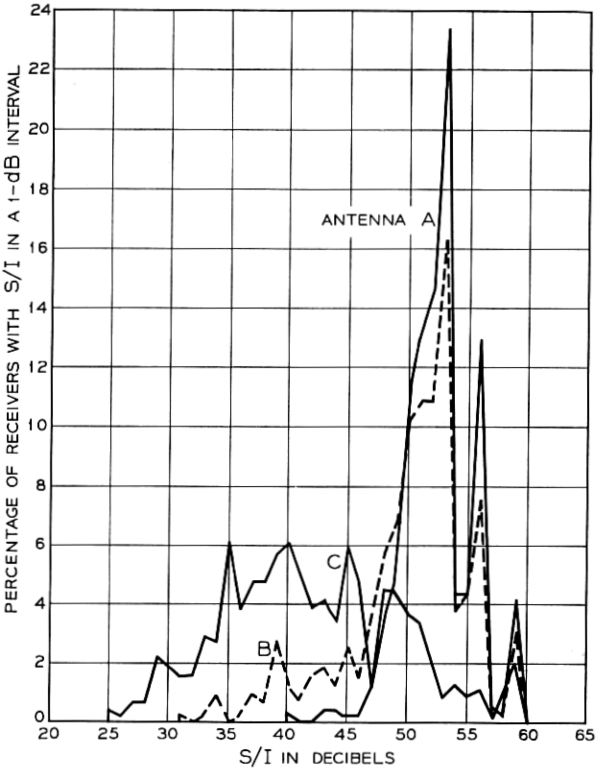

Fig. 8 — Density of the number of sites as a function of signal-to-interference ratio for three antennas with 0.03 radian beamwidth and an environment limit of −60 dB.

trast to the distributions of sites, the distribution of systems has a clear and specific interpretation. If, for example, a modulation index is satisfactory for a $S/I = 39$ dB or greater, then Fig. 10 indicates all systems meet requirements with antenna A, 50 percent with antenna B, and none with antenna C. The quality, $\epsilon$, in expression (10) is found from Fig. 10.

The relative communication capacity is shown in Fig 11 for the three antenna patterns of Fig. 2 and a beamwidth of 0.03 radians. The points on these curves are computed from (10) with the aid of Figs. 3 and 10. The baseband requirement is $S_0/I_0 = 60$ dB. (A baseband signal-to-interference ratio of 60 dB per hop would allow systems with about 100 hops to meet Bell System transmission objectives.)

It was mentioned in Section 3.3 that all computations were based on a single interferer. It has been shown that for a given interference power the single interferer results in the least baseband interference;[3] the worst case occurs when the interference power is divided equally among the maximum number of interferers. If the exact number and power distribution of the interferers were used in the computations the effect would be to shift the results with respect to the *S/I* scales. We believe that none of the general conclusions are affected by the computing for a single interferer.

## V. CONCLUSIONS

For future applications of radio systems above 10 GHz it may be desirable to serve a large number of customers in a limited area. To make the best use of the available frequency spectrum, many systems in the same frequency band must co-exist in this area. Optimum de-

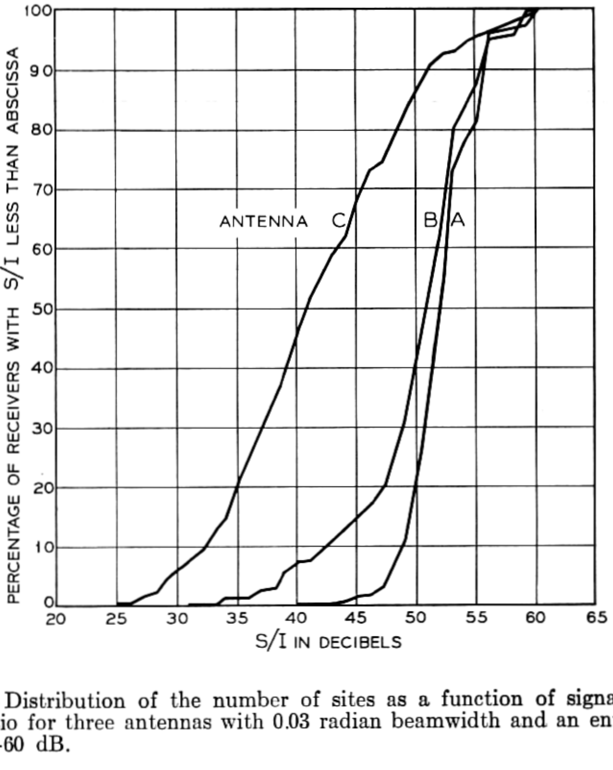

Fig. 9—Distribution of the number of sites as a function of signal-to-interference ratio for three antennas with 0.03 radian beamwidth and an environment limit of −60 dB.

sign of these systems must include the concept of communication capacity through an area. This paper analyzes a dense radio network of this type and describes the effects of antenna response patterns and bandwidth expansion on communication capacity.

The limitations of the network model are clear: the earth isn't plane, sites are not chosen at random, and such a square network is highly artificial. We believe, however, that important conclusions are demonstrated by the results and that the directions for further research are plainly indicated.

(*i*) The importance of antennas with good off-axis discrimination is amply demonstrated. The potential improvements in communication capacity is large and offers incentive for better antenna design.

(*ii*) Modulation methods which expand bandwidth are not, in themselves, wasteful of frequency spectrum in the sense of total communication through an area. To the contrary, the results of computations in a simulated field environment indicate that bandwidth expansion is required to achieve efficient results.

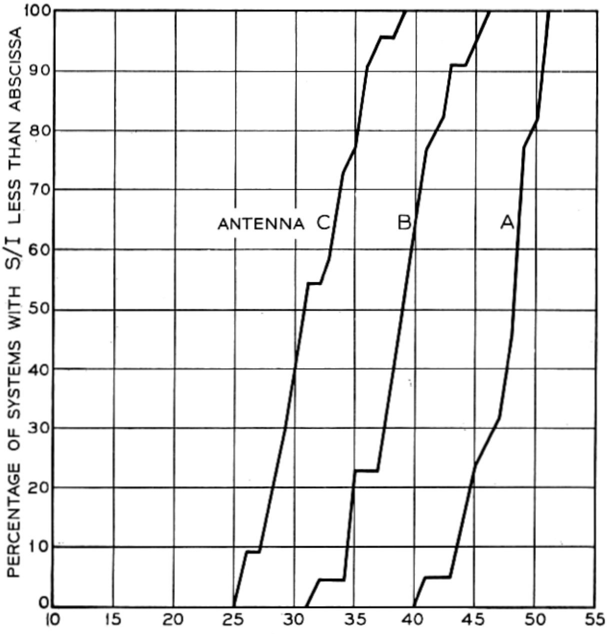

Fig. 10 — Distribution of the number of systems as a function of signal-to-interference ratio for three antennas with 0.03 radian beamwidth and an environment limit of −60 dB.

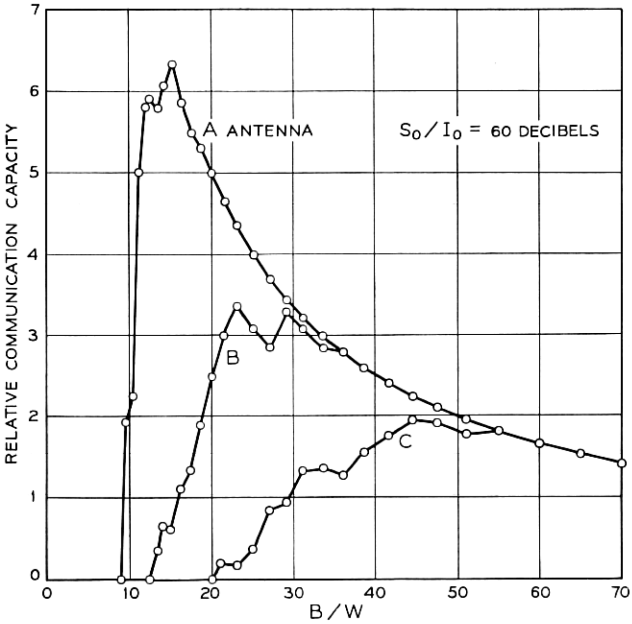

Fig. 11 — Network efficiency as a function of bandwidth expansion ratio for three antennas with 0.03 radian beamwidths, an environment limit of —60 dB, and a baseband signal-to-interference criterion of 60 dB.

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