Television Transmission of Holograms With Reduced Resolution Requirements on the Camera Tube

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This paper proposes a technique for the television transmission of a hologram of a two-dimensional transparency. The spatial resolution required on the camera tube is reduced by a factor of four compared with the transmission of a conventional off-axis reference beam hologram. The resolution required is therefore no higher than that required for the direct transmission of the transparency itself. Implementation of the proposed arrangement should be easy. Three holograms formed with an on-axis reference beam are transmitted. The phase of the reference beam assumes the values 0°, 120°, *and* 240° *for the first, second, and third hologram, respectively. The carrier-frequency hologram is "synthesized" from these three on-axis holograms at the receiver. The technique has the further advantage that the undesirable zero-order terms are eliminated.*

Holograms of two-dimensional transparencies have been transmitted via television.[1] The hologram is first formed on the face of the camera tube with an off-axis reference beam and is then transmitted. The main difficulty with this scheme is the high spatial resolution requirement for the camera tube. If the object wavefront has spatial frequencies between $-W$ and $+W$, then the spatial frequencies of the unwanted zero-order terms extend from $-2W$ to $2W$. The spatial frequency of the reference beam therefore has to be at least $3W$; the highest spatial frequency to be resolved by the camera tube is $4W$. (The conditions mentioned, and further discussed in Ref. 2, are well known.) This is higher by a factor of 4 than the highest spatial frequency of the original two-dimensional transparency which is unfortunate because television camera tubes are of rather limited resolution.

Two scanning schemes have recently been proposed which reduce the resolution requirement for the camera tube by a factor of 2 and 4.[3] It is the purpose of this paper to point out that a reduction by a factor of 4 can also be achieved by the adaption of a scheme described by Burckhardt and Doherty.[4] The idea to be described should be easier to implement than the heterodyne scanners proposed in Ref. 3 and has the advantage that it allows the use of charge storage camera tubes. Since a factor of 4 is saved in resolution requirement, the resolution of the camera tube has to be no higher than that required for the direct transmission of the original transparency.

Figure 1 shows the adaption of the idea of Reference 4 to hologram transmission via television. The hologram is formed with an on-axis reference beam on the camera tube. This hologram is scanned and transmitted; at the receiver, the received electrical signal is multiplied by a cosinusoidal signal and displayed on a kinescope. The phase plate at the transmitter is then switched electro-optically to give a phase shift of 120° in the reference beam; correspondingly the cosinusoidal signal at the receiver is shifted by 120° in temporal phase. The hologram is again scanned, transmitted, multiplied by the cosinusoidal signal, and displayed. This procedure is repeated once more. It will now be shown that the intensities of the three scans add up to give a carrier frequency hologram on the kinescope.

Let the complex-valued amplitude of the subject wavefront be called A and the real-valued amplitude of the reference beam be called B. For the intensity I_1 on the camera tube during the first scan we then have

$$I_1 = (A + B)(A^* + B) = AA^* + B^2 + AB + A^*B. \qquad (1)$$

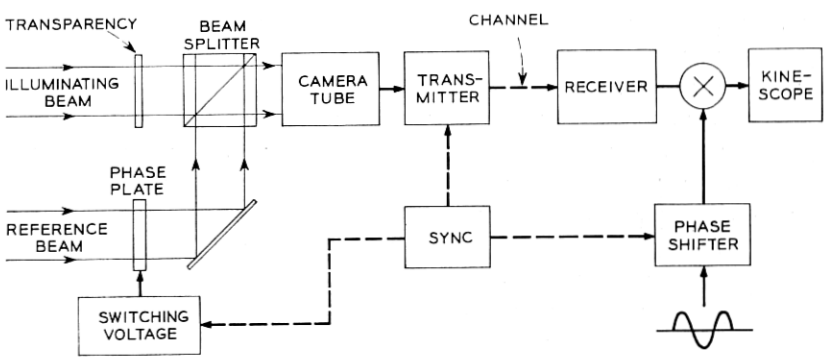

Fig. 1 — Hologram transmission via television, with reduced resolution requirement on the camera tube.

Suppose now that the transmitter is linear and that the voltage U_1 arriving at the receiver is proportional to I_1,

$$U_1 = K_1 I_1 , \tag{2}$$

where K_1 is the constant of proportionality. The voltage U_1 is now multiplied by a cosinusoidal signal to give the voltage U_1',

$$\begin{aligned} U_1' &= U_1 \cos \omega t = K_1 I_1 \cos \omega t \\ &= K_1 \cos \omega t (AA^* + B^2 + AB + A^*B). \end{aligned} \tag{3}$$

We now assume that the display is also linear and that the intensity I_{K1} on the kinescope is given by

$$I_{K1} = K_o + K_2 U_1' . \tag{4}$$

The constant bias term K_o is necessary because U_1' assumes both positive and negative values. The term K_2 is a constant of proportionality. Combining equations (3) and (4) we obtain for the intensity I_{K1} on the kinescope

$$I_{K1} = K_0 + K_1 K_2 \cos \omega_s x (AA^* + B^2 + AB + A^*B). \tag{5}$$

The term ω_s is the spatial frequency which corresponds to the temporal frequency ω in equation (3).

During the second scan the total amplitude on the camera tube is $A + B \exp (j2\pi/3)$ because the phase of the reference beam is now shifted by 120°. The intensity I_2 therefore is

$$\begin{aligned} I_2 &= [A + B \exp (j2\pi/3)] \cdot [A^* + B \exp (-j2\pi/3)] \\ &= AA^* + B^2 + AB \exp (-j2\pi/3) + A^*B \exp (j2\pi/3). \end{aligned} \tag{6}$$

We now multiply the voltage arriving at the receiver by $\cos (\omega t + 2\pi/3)$ and obtain for the intensity I_{K2} on the kinescope

$$\begin{aligned} I_{K2} &= K_0 + K_1 K_2 I_2 \cos (\omega_s x + 2\pi/3) \\ &= K_0 + \tfrac{1}{2} K_1 K_2 I_2 [\exp (j\omega_s x + j2\pi/3) + \exp (-j\omega_s x - j2\pi/3)] \\ &= K_0 + \tfrac{1}{2} K_1 K_2 [\exp (j\omega_s x + j2\pi/3) + \exp (-j\omega_s x - j2\pi/3)] \\ &\quad \cdot [AA^* + B^2 + AB \exp (-j2\pi/3) + A^*B \exp (j2\pi/3)]. \end{aligned} \tag{7}$$

The intensity I_{K3} on the kinescope during the third scan is obtained in an analogous way. For the total intensity $I_{K\text{tot}}$ we then obtain

$$
\begin{aligned}
I_{K\text{tot}} &= I_{K1} + I_{K2} + I_{K3} \\
&= 3K_0 + \tfrac{1}{2}K_1K_2 \sum_{n=0}^{2} \{[\exp(j\omega_s x + jn2\pi/3) \\
&\qquad + \exp(-j\omega_s x - jn2\pi/3)] \\
&\qquad \cdot [AA^* + B^2 + AB \exp(-jn2\pi/3) + A^*B \exp(jn2\pi/3)]\} \\
&= 3K_0 + (\tfrac{3}{2})ABK_1K_2 \exp(j\omega_s x) + (\tfrac{3}{2})A^*BK_1K_2 \exp(-j\omega_s x).
\end{aligned}
\tag{8}
$$

The last two terms of this expression are the real and virtual image terms modulated onto different spatial carriers. Notice that the undesirable zero-order terms do not occur in equation (8). This is because we multiplied the voltage arriving at the receiver with a bipolar electrical signal. If the subject wavefront at the camera tube has spatial frequencies extending from $-W$ to W, the spatial carrier frequency at the kinescope can be chosen as W. The positive spatial frequencies of the kinescope display then extend from 0 to $2W$. (Since the intensity on the kinescope is a real function, a knowledge of the positive frequencies is sufficient.)

Some bandwidth considerations are appropriate. If the positive spatial frequencies of the amplitude transmitted through the original transparency extend from 0 to W, the hologram displayed at the receiver has a bandwidth of $2W$. This increase by a factor of 2 occurs because the hologram contains information about amplitude and phase. The system just described transmits three holograms, each with a bandwidth W. This is equivalent to transmitting one hologram with a bandwidth $3W$. The minimum bandwidth of an off-axis hologram is $4W$; therefore, our scheme requires less bandwidth than transmitting an off-axis hologram. Since the bandwidth of the hologram on the kinescope is $2W$, the amount of information to be transmitted in our scheme is still higher by a factor 3/2 than what it necessarily has to be. The scheme described in Section IV of Ref. 3 only transmits a hologram of bandwidth $2W$ therefore avoiding this increase.

In our discussion we have used three subholograms and phase shifts of 120°. In the Appendix we derive the general equations and show that three subholograms is the minimum required number.

It might be mentioned that our scheme can be modified such that it only transmits one hologram of bandwidth $2W$. In this case all the processing is done at the transmitter and the final hologram of bandwidth $2W$ is transmitted. A scheme for doing this is shown in Fig. 2.

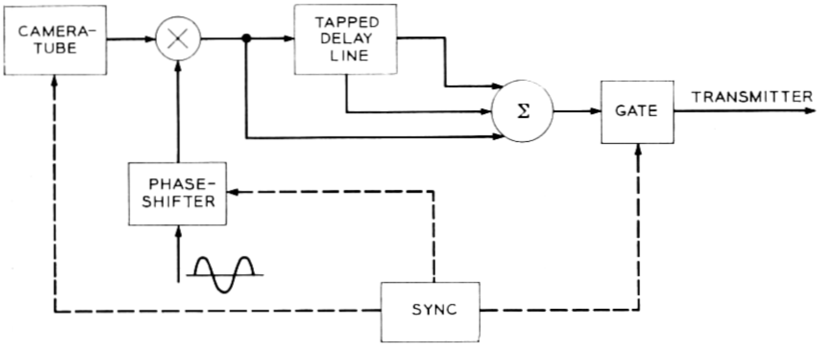

Fig. 2 — Modification of the arrangement of Fig. 1 to reduce the amount of information to be transmitted by one third.

The voltage from the scan is multiplied by the sinusoidal signal and then stored in a tapped delay line. The delay line delays the first scan by the time needed for two scans and the second scan by one scanning time. The gate opens during the third scan. The channel is only used during one third of the time. In order that anything be gained the channel has, of course, to be used for something else during the remaining two thirds of the time. Alternatively, the output of the gate can be stored in a buffer memory (for example, magnetic tape) and transmitted at a slower rate. It is seen that the scheme of Fig. 2 is quite a bit more complex than the scheme of Fig. 1.

APPENDIX

General equations

Here we present the general equations which must be satisfied for the N subhologram case and show that the least number required is $N = 3$.

The general expression for the spatially varying part of the intensity on the kinescope corresponding to equation (8) is

$$\begin{aligned} I_{tot} &= \mathrm{Re} \sum_{n=0}^{N-1} [\{1 + AA^* + A \exp(-j\beta_n) \\ &\qquad + A^* \exp(j\beta_n)\} \exp(j\omega_s x + j\gamma_n)] \\ &= \mathrm{Re} \sum_{n=0}^{N-1} [\{[1 + AA^*] \exp(j\gamma_n) + A \exp(j\gamma_n - j\beta_n) \\ &\qquad + A^* \exp(j\gamma_n + j\beta_n)\} \exp(j\omega_s x)] \end{aligned} \tag{9}$$

where β_n is the relative phase-shift of the plane-wave reference beam; ω_s and γ_n are the spatial frequency and phase of the grating produced by the electrical carrier introduced at the receiver. In order to simplify expressions, we have equated the multiplying factors K_1, K_2 and the magnitude of the reference beam to unity. Notice that the quantity within the { } braces in the second equation of (9) represents the beam which would be diffracted at the angle ω_s when a hologram is reconstructed. This term is the coefficient of exp $(j\omega_s x)$. We desire to superimpose N exposures, each having the form of Eq. (9), to accomplish the following:

(*i*) Force the complex coefficient of $1 + AA^*$ to zero. This prevents components from the direct beam, during reconstruction, from being diffracted at angle ω_s.

(*ii*) Force the complex coefficient of A^* to zero. This prevents components of the conjugate wave from being diffracted at ω_s.

(*iii*) Force the complex coefficient of A to some nonzero value. This reconstructs the desired object wavefront at angle ω_s.

In order to control these 3 complex coefficients, we need a minimum of 6 independent variables to adjust.† Each exposure of the form equation (9) has 2 variables to adjust, β_n and γ_n. Thus, we need a minimum of 3 subholograms.

The equations which must be satisfied are

$$\sum_{n=0}^{N-1} \exp(j\gamma_n) = 0 \tag{10a}$$

$$\sum_{n=0}^{N-1} \exp(j\gamma_n - j\beta_n) \neq 0 \tag{10b}$$

$$\sum_{n=0}^{N-1} \exp(j\gamma_n + j\beta_n) = 0, \tag{10c}$$

where $N = 3$ is the minimum value. Equation (10a) can be satisfied if for the γ_n's we simply pick the N roots of $\exp(j\theta)$ according to the well-known theorem of De Moivre, that is,

$$\gamma_n = \frac{\theta + 2\pi n}{N}$$

† There is always a possibility that the equations for these three complex coefficients are not themselves independent, and that as a consequence only four independent variables are required to control them. In order to rule out this case, we let $N = 2$ in equations (10) and define $s_n \equiv \exp(j\gamma_n)$ and $z_n \equiv \exp(j\beta_n)$. Equations (10) then reduce to $s_0 = -s_1$, $s_0^*(z_0 - z_1) \neq 0$ and $s_0(z_0 - z_1) = 0$. We see that these equations cannot be satisfied simultaneously.

where $n = 0, 1, 2, \ldots N - 1$. If we then set $\beta_n = \gamma_n$, equations (10b) and (10c) are then automatically satisfied.

As an example, for the minimum number of subholograms $N = 3$, we may pick $\theta = 0$ without loss of generality since the absolute phase of the reference beam is unimportant. Then we have from De Moivre's theorem $\gamma_0 = \beta_0 = 0$, $\gamma_1 = \beta_1 = 2\pi/3$, $\gamma_2 = \beta_2 = 4\pi/3$. Thus, for three subholograms we shift the reference beam and grating producing electrical carrier phase by 120°

REFERENCES

1. Enloe, L. H., Murphy, J. A. and Rubinstein, C. B., "Hologram Transmission via Television," B.S.T.J., *45*, No. 2 (February 1966), pp. 333–335.
2. Leith, E. N. and Upatnieks, J., "Reconstructed Wavefronts and Communication Theory," J. Opt. Soc. Amer., *52*, No. 10 (October 1962), pp. 1123–1130.
3. Enloe, L. H., Jakes, W. C., Jr., and Rubinstein, C. B., "Hologram Heterodyne Scanners," B.S.T.J., *47*, No. 9 (November 1968), pp. 1875–1882.
4. Burckhardt, C. B. and Doherty, E. T., "Formation of Carrier-Frequency Holograms with an On-Axis Reference Beam," Appl. Opt., *7*, No. 6 (June 1968), pp. 1191–1192.