A 50 Hz–250 MHz Computer-Operated Transmission Measuring Set

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A computer-operated transmission measuring set has been developed for the 50 Hz to 250 MHz frequency range. Use of the computer in this system has significantly effected the test set design and the measurements obtainable.

Compared with previously available transmission measuring sets, the computer-operated set increases speed more than 300 : 1. This speed, along with state-of-the-art accuracy and increases in operating range, flexibility, and convenience, enables the set to be used for types and quantities of measurements previously not practical. It has already been applied to laboratory and production testing with resulting improvements in the quality and reliability of manufactured product designs.

In addition to the directly measured quantities of insertion loss and phase, the set provides insertion delay, impedance, and two-port parameters as derived quantities. The two-port data conversion program provides H, Y, Z, G, T, S, ABCD *and* ABCD^{-1} *parameters with a number of useful options. Results of transmission measurements, impedance measurements, and two-port measurements are presented. Some of the error mechanisms and means of measuring them are discussed. Further development of centralized measuring facilities, with the computer operated set as a basic element, is discussed.*

I. INTRODUCTION

The development of communication systems for the Bell System has, in the past, required large numbers of transmission measurements. These measurements have been costly and time-consuming. With the trend toward more complex systems, the volume and accuracy of measurements must be increased. At the same time, the increased use of computers in modeling and design requires flexibility in the types of data obtained. For example, two-port characterization of devices such as transistors is becoming particularly important.

This paper describes a computer-operated transmission measuring set developed for laboratory use in the 50 Hz to 250 MHz frequency range (see Fig. 1). Compared with previously available sets, this set provides increased speed, operating range, accuracy, flexibility, and convenience, and the capacity for types and quantities of measurements previously not practical.

The computer-operated measuring set is a part of the centralized measuring facilities being developed for Bell Laboratories. With the addition of various appliques now being developed, the measurement centers will provide additional measurement facilities in the 50 Hz to 250 MHz range, with environmental control, and in the three microwave radio bands at 4, 6, and 11 GHz. The set is also being used for production testing by the Western Electric Company.

II. MEASUREMENTS SET CHARACTERISTICS

2.1 *General*

The small, general-purpose digital computer with fast and precise digital-analog components and with broadband analog components has had a significant effect on the measurements obtainable.

Increased operating speed results from the use of computer control, memory, and computation as well as fast measuring set components. The operating speed is 10 to 300 times faster than manual test sets (per measurement point), depending on the output media and the test frequency.

Computer control also makes wide operating ranges practical. By automatic control of test set level patterns, insertion loss can be measured over a range from −40 to +100 dB. By automatic control of signal sources and frequency dependent elements, operation over nearly seven decades is obtained. This frequency range previously required three separate test sets.

Accuracy is, of course, limited by accuracy of the test set standards. Computer operation, however, does play a significant part in the test set accuracy. First, test set errors can be comprehensively evaluated to a degree only practical with a high speed set. Second, the speed, memory, and computation capability permits the correction of data for known errors and the averaging of random errors. Correction of "zero line" errors is particularly important with the complex transmission paths used in the test set. Finally, the operating rules

Fig. 1 — Computer-operated transmission measuring set.

are set up so as to maintain near optimum levels during the measurements. In this way, tolerable compression errors can be deliberately introduced to improve signal-to-noise ratio.

The computer-operated set provides flexibility in a number of ways.

(*i*) By using software rather than hardware control, operation of the set can be readily changed.

(*ii*) Special measurements can be made which include system maintenance tests and periodic measurement of test set error sources.

(*iii*) Measured data can be converted into more useful forms. For example, meaningful acceptance criteria, which may be too complex for manual application, can be used for GO, NO GO tests of measured networks.

The computer-operated set is convenient in the ease of setting up measurements, particularly repeated measurements, and in the ability to yield measured or derived parameters on various output media.

2.2 *Transmission Measurements*

The basic quantities measured by the transmission-measuring set are insertion loss (or gain) and insertion phase shift as a function of frequency. Internal switching provides for direct measurement of insertion loss and phase at a number of impedance levels. Insertion delay is automatically obtained by calculation from two phase measurements separated by an appropriate frequency interval.

Table I summarizes test set performance.

2.3 *Impedance Measurements*

Fixtures are provided to permit connection of one port (or terminated two-port) networks into the transmission measuring circuits. By measuring suitable impedance standards (three needed) along with the network and processing data on a computer, the impedance, reflection coefficient, or return loss can be obtained.[1] With a change of program, the test set computer can be used for processing the data. In this case, the derived parameters can be output on the test set output equipment.

Measurement accuracy is a function of the impedance measured but will be better than 1 percent over an impedance range of 10^4. Section 5.2 has a more detailed discussion of impedance measurement accuracy.

2.4 *Two-Port Characterization Measurements*

Coaxial terminals, coaxial fixtures, and dc bias supplies are provided which permit the linear characterization of two-port networks and of devices such as transistors. For unbiased networks, the fre-

TABLE I — TEST SET PERFORMANCE

Characteristic	Range	Best accuracy
Test frequency	50 Hz to 250 MHz, adjustable to 0.01–0.08 Hz	3 parts in 10^8
Gain measurements	0 to 40 dB	0.001 dB
Loss measurements	0 to 100 dB	0.001 dB*
Phase measurements	0 to 360°	0.01°
Delay measurements†	39.999 ms to 39.999 nsec, full scale	$55/\Delta f \mu$sec

Impedance levels: 50, 75, 135 (balanced or unbalanced), 600 (balanced or unbalanced), arbitrary Z, and probe mode.

* Loss and phase accuracies decrease as loss or gain increases. For example, the random error in loss measurement is $0.001\ (1 + 0.2 \times 10^{L/20})$ dB for losses <40 dB and $0.01\ (1 + 0.2 \times 10^{(L-40)/20})$ dB for losses >40 dB.

† Computed from two phase measurements separated by Δf Hz.

quency range is 50 Hz to 250 MHz and for biased networks the range is 50 kHz to 250 MHz. Two-port characterization data, including calibration data, is processed by computer (such as the IBM 7094) to provide any of the standard two-port matrix representations such as H or Y parameters. Accuracy of the output parameters is parameter dependent but errors in S parameters will be less than 0.02 for most unknowns. Section 5.3 has a more detailed discussion of parameter accuracy.

2.5 *Test Set Input-Output*

Figure 2 is a simplified block diagram of the computer operated test set from the human operator-test set interface. Information required by the computer to control a particular set of measurements is contained in the computer memory, in switch positions on the operator control panel, or possibly on punched paper tape. The information in computer memory is inserted either with the tape reader or the typewriter. The operator control panel can be used both to set up and start the desired set of measurements or to modify the sequence after it has started.

Outputs are selected on the operator control panel. Visual readout is always present, and the typewriter, tape punch, and X–Y plotters can be independently selected. Plotting parameters are part of the input data, and output readings are scaled by the computer. Points are plotted to an accuracy of about 0.1 percent. If no output is selected

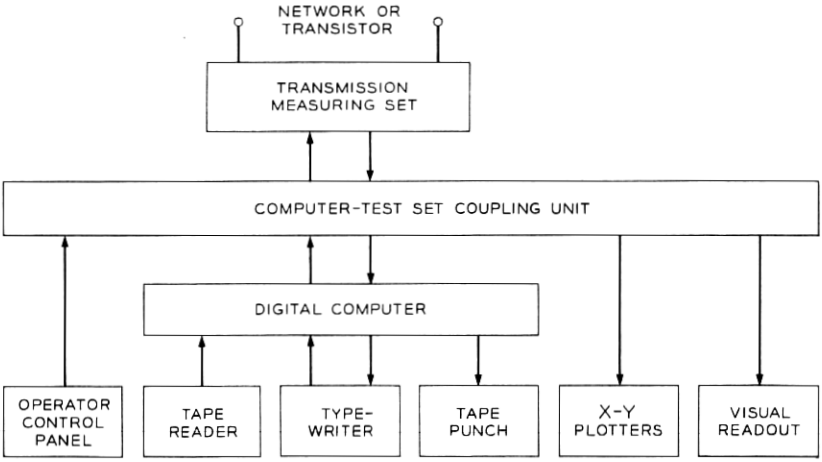

Fig. 2 — Operator-test set interface.

or if the output switches are turned off during a measurement sequence, the test set will provide continuous loss and phase measurements with the test frequency kept constant.

2.6 *Test Set Speed*

Measurement speed varies with a number of factors, particularly the number of measurements averaged to reduce random errors. However, Table II gives a useful summary of typical measurement times.

III. BLOCK DIAGRAM DESCRIPTIONS

3.1 *Overall Description*

Figure 3 is a simplified block diagram of the measuring set and connections to the computer. At this interface, control and readout of the measuring set is entirely digital.

Under computer control, the signal oscillator supplies the test signal to the comparison unit for excitation of the circuit to be measured and to the reference path frequency converter. The local oscillator provides the proper frequency to the measurement path and reference path converters for translating the test frequency to a fixed intermediate frequency. The loss standard is adjusted to have a loss equal to that of the measured circuit using error signals provided by the loss detector. The phase meter provides a measurement of the phase difference between the measurement path and reference path inputs. With suitable switching, the difference between two readings provides the desired phase measurement.

The measuring set block configuration is similar to others previously reported.[2, 3] However, when the elements in the configuration are realized with the components to be described and then controlled by a computer, the advantages cited in Section I result.

3.2 *Gain-Loss Measurements*

Figure 4 shows the loss measuring circuit with details added which are esential to the discussion.

3.2.1 *Comparison Circuit*

The comparison circuit rapidly interchanges the unknown path and standard path between the signal source and the heterodyne detector. This produces amplitude and phase variations in the signal at the switching rate which correspond to differences in transmission between the unknown and standard paths. The unknown path con-

TABLE II — TYPICAL MEASUREMENT TIMES

Type of output	Time per point (seconds)	
	(Freq. <2 kHz)	(Freq. >2 kHz)
Magnetic core	2	0.2
$X - Y$ plotters	2	0.2
Paper tape	3	1
Typewriter	9	7

tains either the switching unit for two port characterization measurements or a path connecting the network for insertion loss and phase measurements. Additional switches, not shown, provide connections for the various impedance levels.

The standard path contains either a low loss transmission line or a 40 dB pad. The 40 pad is always inserted in the standard path when the unknown path is being measured. When the standard path is being measured, the transmission line is inserted for losses less than 40 dB. For losses equal or greater than 40 dB, the 40 dB pad is inserted in the standard path and a 40 dB preamplifier is inserted in the level adjust unit.

Use of the 40 dB pad and preamplifier in this way has several

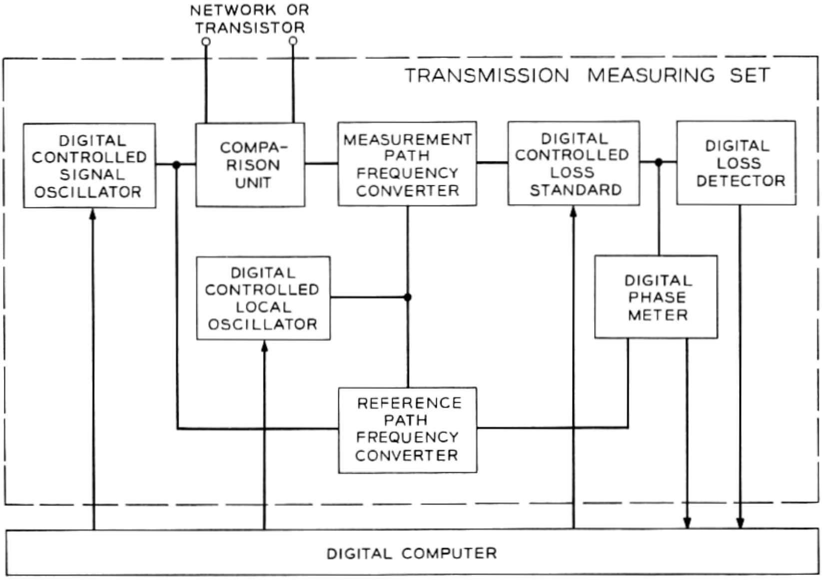

Fig. 3 — Simplified block diagram of measuring set.

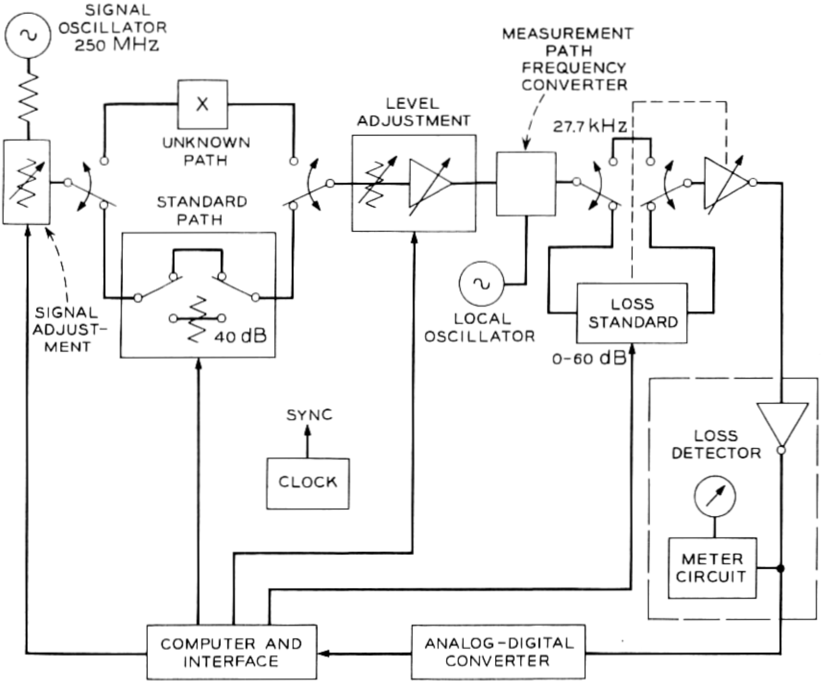

Fig. 4 — Loss measurement.

advantages: the crosstalk requirement in the comparison switches is reduced by 40 dB and the required dynamic range of the measurement path frequency converter is reduced by 40 dB. Use of the preamplifier also improves the system noise figure by about 20 dB.

Since the 40 dB pad has a significant frequency characteristic, some method of correction is necessary. This is conveniently accomplished by initiating an additional comparison cycle whenever the 40 dB pad is used. During this cycle, the two elements in the standard path are compared and the difference is measured on the IF loss standard. The 40 dB pad is, therefore, a transfer standard.

3.2.2 *Signal and System Level Adjustments*

The attenuators following the signal oscillator adjust the signal level into the measurement circuit. The attenuators are switched by the computer under manual or program control. In either case, if the unknown has enough gain to overload the test set detector, the signal level is automatically reduced and the signal level actually used is typed out.

The level-adjust circuit adjusts the level into the measurement path converter to minimize certain errors. When measured loss or gain is low, the level into the converter is maintained for best signal-to-noise ratio consistent with 0.001 dB linearity. At high loss, the level into the converter (not yet programmed) is increased to improve the signal-to-noise ratio without producing significant linearity errors.

3.2.3 *IF Loss Standard and Detector*

Amplitude and phase variations produced in the test signal by the comparison unit are linearly translated to the final IF of 27.777 kHz. Here the loss standard is switched in synchronism. For loss, the IF loss standard and the unknown are switched out of phase and for gain they are switched in phase.

The loss standard contains relay switched, precision attenuators ranging from 0 to 59.99 dB in 0.01 dB steps. Complementary gain is provided in the common output path so that the output is constant (±1 dB) when the standard is correctly balanced. Hence the loss detector and phase detector are operated at nearly constant levels.

The amplifier-detector in the loss detector has a logarithmic characteristic which provides a dc output proportional to the input amplitude in dB. During the balance sequence, a measure of the difference in loss between the loss standard and the measured network (loss unbalance) is obtained from the difference of two readings with the analog-digital converter. After the loss balance is completed, the analog-digital converter readings provide the 0.001 dB decade indication to the computer for automatic readout.

In order for the analog-digital converter to have ±20 dB balancing range and yet provide the ±0.001 dB indication, the equivalent of 16 bits is required. This was achieved with a 13 bit converter and a switched (5X) preamplifier. For loss standard balancing, the amplifier is out and the least significant bit is 0.005 dB. When a balance is achieved, the amplifier is switched in and the least significant bit is 0.001 dB.

3.3 *Phase Measurements*

Figure 5 is a block diagram of the phase measuring circuit. The phase changes produced by comparison switching are the changes between the unknown and standard paths at test frequency and between the loss standard and "strap" at intermediate frequency. The net change in phase is the change between the unknown and standard paths since

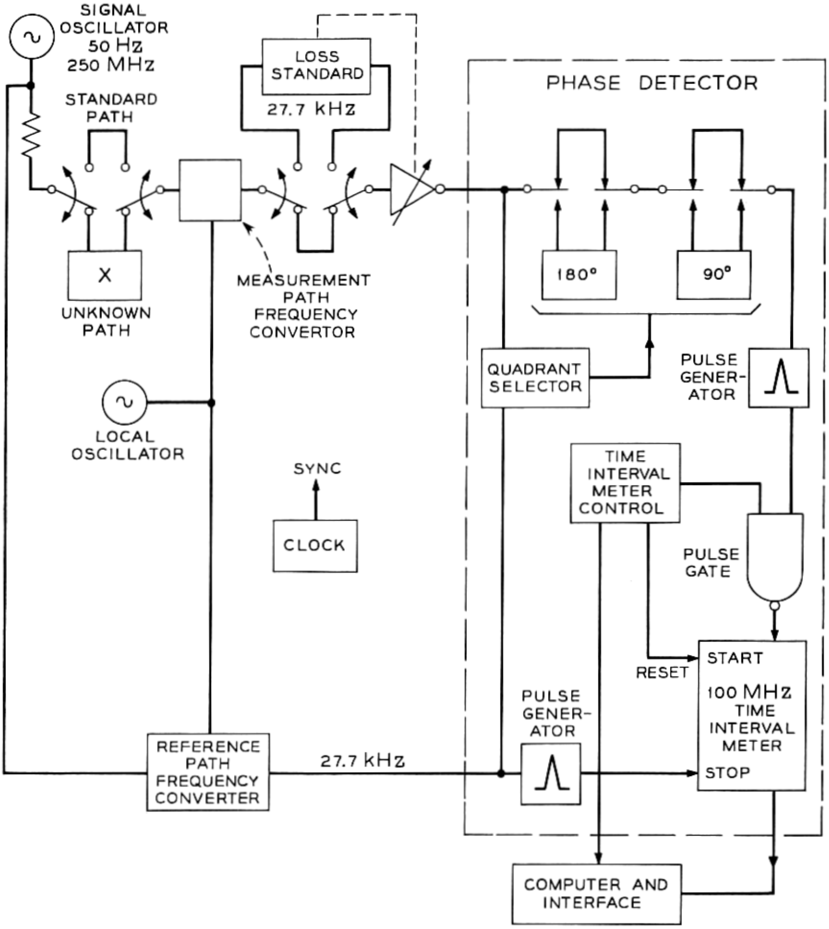

Fig. 5 — Phase measurement.

the two paths at intermediate frequency are adjusted to have phase differences less than ±0.002°. The reference path signal provides a constant phase reference during the switching cycle.

The phase measuring technique used cannot measure phase at exactly 0°. Phase equalization of the measurement path and the reference path over the 250 MHz frequency range is not possible with fixed networks. The quadrant select circuit together with the 180° and 90° networks select a quadrant such that the relative phase into the phase measuring circuit is within 180° ± 135° for both positions of the comparison switch. Operation of the quadrant select circuit does not add to the measurement time since it operates during an early portion of the switching cycle before transients have settled to the point where precision phase measurements can begin.

The pulse generators produce pulses on the positive zero crossing of the IF signals. The pulse from the measurement path generator starts the time interval meter measurement and the reference path pulse generator stops the time interval measurement. The time interval is measured by counting the number of pulses from a 100 MHz source that occur between the start and stop pulses. In a single period the time interval meter can resolve. 0.1°

$$\left(\text{that is, } \frac{27.777 \text{ kHz}}{100 \text{ MHz}} = \frac{1}{3600} \text{ period}\right).$$

The opportunity for increasing the resolution exists in the system. (See Ref. 3 and the Appendix.) By locking the 100 MHz source to the 1 MHz crystal source in the frequency synthesizer, by a proper choice of the intermediate frequency, and by taking 100 readings, resolution can be increased by a factor of up to 100. Because of compromises in this system, resolution was increased by a factor of about 20, to 0.005°.

The time interval meter control provides gating and reset signals so that 100 readings can be taken and provides timing signals to the computer. Level dependence of the phase meter is not a problem since the loss balance made before phase measurements is within 0.01 dB and the level-to-phase conversion of the pulse generators is less than 0.03° per dB.

3.4 *Signal Frequency Generation and Conversion*

Automatic control of signal sources and frequency converters provides operation over nearly seven decades. Fig. 6 is a simplified block diagram of the elements, including switches, for signal generation and conversion.

3.4.1 *Signal Oscillator and Local Oscillator*

The signal oscillator provides a sinusoidal (harmonics less than 40 dB) test signal to the measurement and reference paths, and the local oscillator provides the signal to tune the heterodyne detector. Each oscillator is composed of a frequency synthesizer and a frequency multiplier (including filters) which together produce output frequencies from 50 Hz to 250 MHz in response to digital signals. Be-

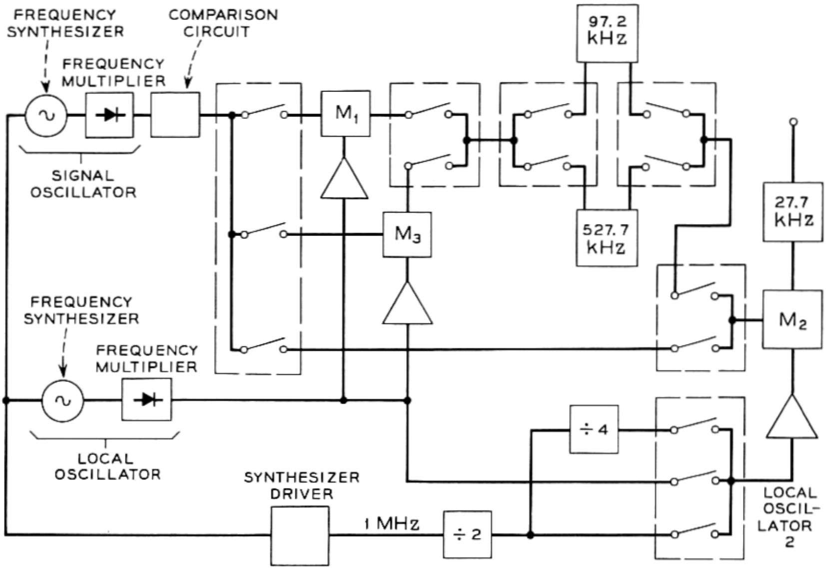

Fig. 6 — Frequency generation and conversion.

Signal frequency range	Frequency multiplier	Modulators used	Local oscillator frequency synthesizer offset (kHz)	Local oscillator 2 input (kHz)
0.05–2 kHz	X1	M1, M2	97.223	125
2–100 kHz	X1	M1, M2	527.777	500
0.1–5 MHz	X1	M2	27.777	Local oscillator
5–50 MHz	X1	M3, M2	527.777	500
50–100 MHz	X2	M3, M2	527.777/2	500
100–200 MHz	X4	M3, M2	527.777/4	500
200–250 MHz	X8	M3, M2	527.777/8	500

low 50 MHz, frequencies are set to a precision of 0.01 Hz within 1 ms after signalling. Above 50 MHz, frequencies are set to a precision ranging from 0.02 to 0.08 Hz within 10 ms. Absolute accuracies of the output frequency, frequency changes of either oscillator, and frequency differences between the two oscillators are all within 3 parts in 10^8. The accuracy of these oscillators eliminate test frequency uncertainty as a source of measurement error and permit the use of very narrow detection bandwidths to reduce errors caused by noise. In the test set operation the oscillators are tuned while the previous data point is being read out so the 10 ms tuning period is negligible.

3.4.2 *Frequency Converters*

The frequency converters must provide linear translation of the 50 Hz to 250 MHz test signal to a fixed intermediate frequency of 27.777 kHz and maintain a satisfactory signal-to-interference ratio at the output.

Since level differences produced by the comparison unit (up to 60 dB) are transmitted through the converters, all elements in the converters including amplifiers and filters must be linear within 0.001 dB. To measure 20 dB loss to ±0.001 dB, the converter noise must be 100 dB and spurious products 80 dB below the maximum linear output level.

In order to achieve this performance, four bands were used. When two stages of conversion are used, the second local oscillator frequency is derived from the 1 MHz standard in the frequency synthesizer. This provides a final IF accurate to 3 parts in 10^8 and a precision within ±0.04 Hz, the precision required for the phase measuring circuit.

3.5 *Impedance Measurements*

Impedance and return loss measurements on one port (or terminated two port) networks can be obtained by making the appropriate connections, making the required sequence of measurements, and processing the measurement data. The frequency range for these measurements is from 50 Hz to 250 MHz.

The three practical connections used for impedance measurements are shown in Fig. 7. The connections are implemented with such networks as a coaxial tee, connector boxes furnished with the test set, or other means such as hybrid networks.

The measured transmission obtained when the network is connected has been shown to be a bilinear function of the network impedance.[4] If four measurements are made, one with the measured network and the other three with "known" impedance standards, the impedance of the network can be determined in terms of the impedance standards.

The equations relating the measurements and the standards are:

$$T_x = \frac{\text{Detector voltage, switches in upper path}}{\text{Detector voltage, switches in lower path}}; \quad Z_x \text{ connected}$$

$$T_x = \frac{T_\infty Z_x + T_0 Z_i}{Z_x + Z_i} = \frac{T_\infty + T_0}{2} + \frac{T_\infty - T_0}{2} \rho_x$$

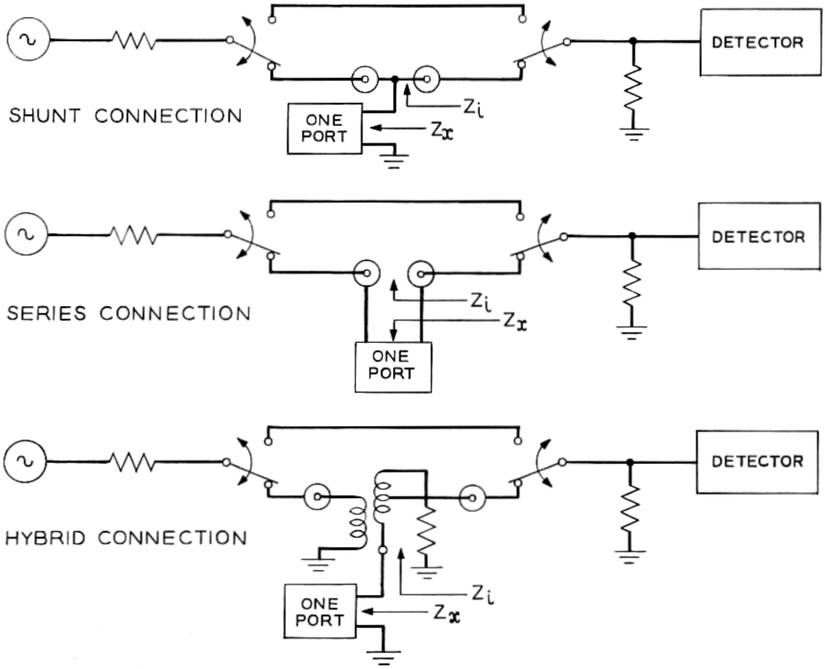

Fig. 7 — Impedance measurements.

Where:

$$T_\infty = T_x \Big|_{Z_x \to \infty}, \qquad T_0 = T_x \Big|_{Z_x = 0}, \qquad \rho_x = \frac{Z_x - Z_i}{Z_x + Z_i}.$$

If an impedance standard Z_s is used and $T_s = T_x\,|_{Z_x = Z_s}$

$$Z_x = Z_s \frac{(T_0 - T_x)(T_s - T_\infty)}{(T_x - T_\infty)(T_0 - T_s)}$$

and

$$\rho = \frac{Z_x - Z_s}{Z_x + Z_s} = \frac{(T_x - T_s)(T_\infty - T_0)}{(T_\infty - T_s)(T_x - T_0) + (T_s - T_0)(T_\infty - T_x)}$$

Accuracy of the method depends on the accuracy of the standards and on the accuracy of the individual transmission measurements. As indicated in these equations, the expressions for impedance and reflection coefficient involves differences in measurements, and the error in the computed result is a function of impedance as well.

Processing of the impedance measurement data can be done on the system computer (PDP-7) by replacing the operating program with one of the available conversion programs. These programs permit computation of real and imaginary components of impedance and the reactance for a series or parallel equivalent (two element) circuits. Angle and magnitude of reflection coefficient with respect to an arbitrary reference impedance can also be computed. Output from the program can either be on tele-typewriter or X-Y plotters.

Some impedance measurement results are discussed in Section 6.2 and illustrated in Figs. 21 through 23.

3.6 *Two-Port Measurements*

3.6.1 *General*

Linear characterization of two-port networks can be obtained by connecting the device (for example, the transistor) to the appropriate jig or coaxial cable, making the required sequence of measurements, and processing the measurement data. Devices not requiring bias can be measured from 50 Hz to 250 MHz and devices requiring bias can be measured from 50 kHz to 250 MHz. The bias can be voltage-regulated from 0 to 150 volts or current regulated from 0 to 1 ampere. Transistor case temperatures can be controlled from 0° to 95° C for dissipation up to 10 watts.

To the extent that the imperfections in the calibration standards and the capacitance added by the temperature control unit are accurately known, it is possible to reduce the measurement data to device parameters which are independent of the measurement environment.[5, 6] The data reduction program provides H, Y, Z, G, T, S, ABCD and $(\mathrm{ABCD})^{-1}$ parameters with any one of the three device terminals grounded. The terminal grounded in the output parameters is not necessarily the same terminal which was grounded during the measurements.

3.6.2 *Measurement and Calibration Techniques*

The technique which is used to characterize two-port networks is obtained from four transmission measurements which are a combination of two ordinary transmission measurements and two impedance measurements (shunt connection) as described in Section 3.5. Figure 8 shows the four connections needed for the measurements of forward and reverse gain and of forward and reverse bridging loss.

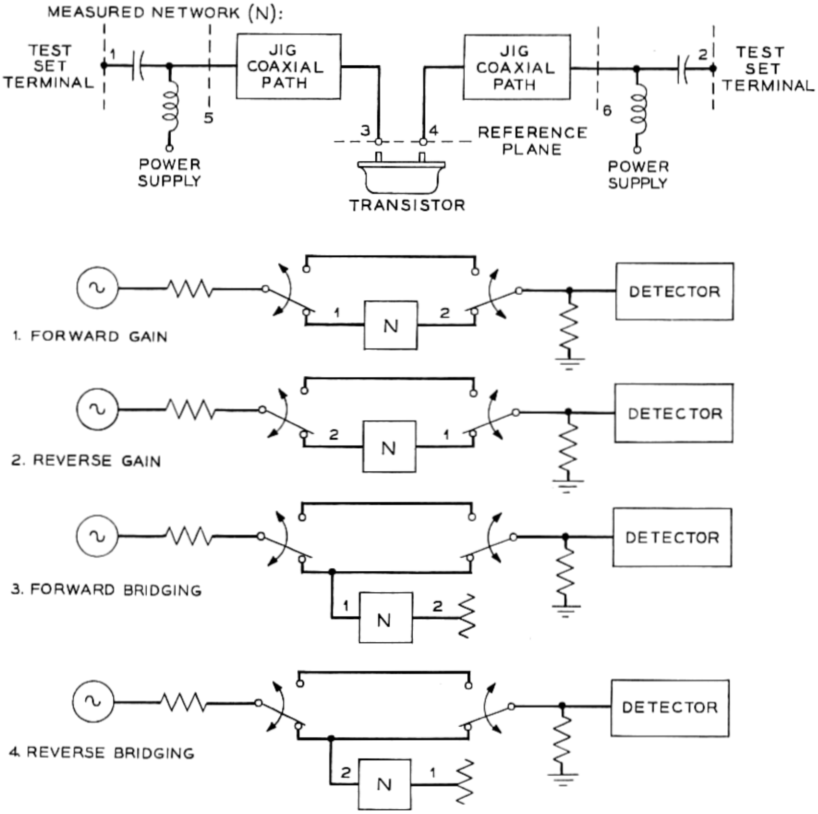

Fig. 8 — Transistor measurement technique.

In automating this sequence of four measurements, the constraint was applied that dc bias paths through the measured device must not be interrupted during the sequence. This means that the minimum network which can be measured when characterizing a transistor is that shown between ports 1 and 2. Even with an accurate test set, accurate device measurements can be made only if the complete network between the test set terminals is characterized or if the reference plane (ports 3 and 4) can be characterized.

Each approach has been used in a previous implementation. In the first case, available 7 mm coaxial standards were used to calibrate ports 5 and 6 (Fig. 8).[1] Then four jigs containing an open circuit, short circuit, reference impedance, and the measured device were successively plugged in to obtain the necessary measurements. The disadvantage of this technique is that symmetry in the jigs must be assumed and that the device bias must be set four times.

In the second case, 21 mm coaxial standards are used to calibrate ports 1 and 2 (Fig. 8).[7] Other coaxial standards (0.054-inch bore) are used to characterize the network between ports 1 and 3, and 4 and 2 at specified frequencies. When the device is inserted, the total network between ports 1 and 2 is measured and the device characteristics are extracted by calculation. The disadvantages of this technique are that calibration data is only available at the specified frequencies and that the network must be manually connected for each of the four measurements.

3.6.3 *Implementation of Automatic Two Port Measurements*

In order to automatically switch the network into the four measurement configurations, considerable coaxial relay switching was required. The return losses that these relays present are low enough so that, when "seen" from the measurement ports 3 and 4, appreciable errors will occur if corrections are not made. It is also desirable to be able to measure at any frequency in the range thus ruling out calibration at a fixed number of points.

It was decided to fabricate "small bore" standards to calibrate ports 3 and 4. Short circuit and 50 ohm standards were developed and the open circuit "standard" is obtained by open circuiting the ports and correcting for fringing capacitance. A 50 ohm "strap" was developed to give "zero line" measurements for the forward and reverse gain measurements. The "strap" also is used to measure the impedance into port 3 with port 4 and vice-versa. Table III lists the measurement and calibration sequence for characterization of a device.

The same technique is used for coaxial unknowns (for example, 14 mm connectors). Open, short, and 50 ohm standards are commercially available and the strap measurement is simply made by connecting the cables from the two terminals together.

3.6.4 *Data Reduction*

Reduction of the data obtained in the measurement sequence just described is a sizable data reduction problem. At present, measurement data is processed on the IBM 7094 computer.

The data reduction program first computes scattering parameters referred to the physical test set impedances. The next step is to trans-

TABLE III — TRANSISTOR MEASUREMENT AND CALIBRATION SEQUENCE

REQUIRED ELEMENTS
Transistor sample and jig
"Small bore" coaxial standards:
50 ohm termination
Strap
Short
Open

MEASUREMENT PROCEDURE

Insert	Measure	Number of measurements
Test set calibration		
Strap	FWD & REV gain, FWD & REV bridging	4
50 ohm in port 1	Bridging*	2
50 ohm in port 2		2
Short in port 1		2
Short in port 2		2
Open in port 1		2
Open in port 2		2
Transistor measurement		
Insert transistor, set bias	FWD & REV gain FWD & REV bridging*	6

* Bridging measurements made with and without quarter wave networks inserted.

form to scattering parameters referred to 50 ohms. This sequence is considerably simpler than initially calculating 50 ohm scattering parameters. The 50 ohm S-parameters can then be transformed into H, Y, Z, G, T, S, ABCD or $(\mathrm{ABCD})^{-1}$ parameters.

A number of useful options are also included. The measurement plane for a measured device can be translated through an arbitrary length of 50 ohm transmission line. Capacitance in shunt with any pair of terminals (for example, the transistor temperature control unit adds about 3 pF from case to ground when used) can be removed by computation and either of the noncommon terminals in the measurement can be made the common terminal in the output data.

The derived parameters can be output in cartesian or polar coordinates or in magnitude (dB) and angle. The output data forms include both tabulated and plotted data. Some two-port measurement results are discussed in Section 6.3 and illustrated in Figs. 24 and 25.

IV. OVERALL CONTROL DESCRIPTION

4.1 *Operating Description*

As Fig. 2 indicates, the primary means of input are the operator control panel (mode selector), the tape reader, and the typewriter. Inputs are also made from the computer console when loading tapes and in special instances such as maintenance tests. Output is obtained from the visual readout, X-Y plotters, tape punch, and typewriter.

In some of the operating modes, information previously entered via the typewriter or tape reader is stored in computer memory and can be used to make or repeat a measurement sequence. In other modes, the information is not stored and must be reinserted for each measurement sequence.

4.1.1 *Operator Control Panel*

The operator control panel, or mode selector, is shown on Fig. 9. Switch positions provide for selecting the desired measurement and readouts. The panel also provides for operator interaction during a measurement.

The frequency selection switch selects the method by which test

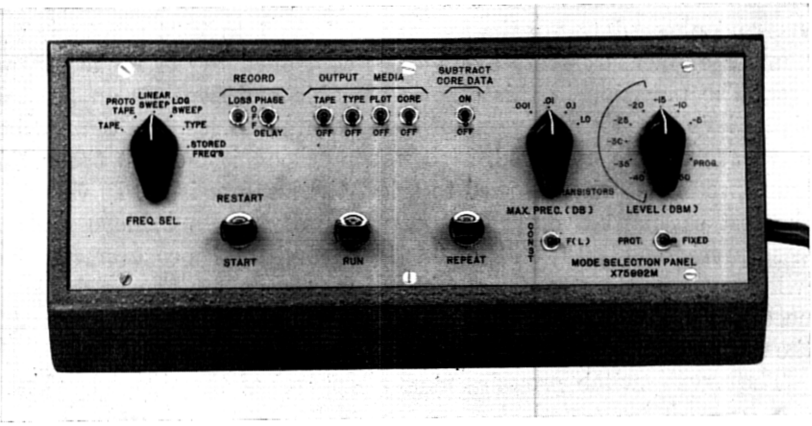

Fig. 9 — Operator control panel.

frequencies are chosen. The frequencies may be a sequence read from punched tape, a linear or logarithmic sequence generated by the computer, a list of frequencies internally stored, or single values entered on the typewriter.

The RECORD switches are used to select the parameters to be measured. This provides independent selection of loss-gain measurements and phase or delay measurements. In the usual case the measured values are displayed on the output media. In the PROTOTAPE mode, the displayed values are the measured values minus the values stored in the paper tape. This, of course, provides the means to compare a measured network with either a previously measured physical prototype or a "mathematical" prototype.

Visual readout is always provided. The OUTPUT MEDIA switches are used to select other desired readouts. The TAPE, TYPE, and PLOT switches provide output on the punched tape, typewriter, and X-Y plotters, respectively. The CORE switch provides output to the computer memory. On any measurement run, the contents of the core can be subtracted from the run being made by operating the SUBTRACT CORE DATA switch. This is a very useful option. Some of its uses are illustrated in Sections 5 and 6 on measurement results and on measurement of test set errors. If none of the output media switches are operated, a state is produced where continuous measurements are made at the prevailing test frequency.

Precision of the output data is controlled by two switches. The switch labeled CONST-F(L) determines whether the measurement precision is to be variable or constant. In the F(L) position, the output loss and phase data are the average of from 1 to 1024 readings. The number of measurements averaged is controlled by an input parameter. The precision in this case is variable and depends on the test signal level, the loss (or gain) of the measured device, and the number of measurements averaged. If CONST precision is desired, the maximum precision switch is used to select the loss precision. In this case the test set must average enough measurements to achieve the desired precision (1, 0.1, 0.01, or 0.001 dB) and total measurement time will vary according to the number of measurements averaged. If the necessary averaging exceeds the allowed limit of 1024 measurements, the estimated precision will be typed out.

The LEVEL switch provides manual control of the signal level into the device being tested. A programmed position is also provided to permit program control of signal levels.

The START-RESTART, RUN, and REPEAT switches provide control over the program. The switches permit the measurement run to be stopped, to be resumed, to return to the beginning with or without new parameter entry, and to repeat the measurement run one or more times.

4.1.2 *Parameter Insertion*

The parameters required for a measurement depend on the settings of the mode selector switches. For example, if a linear frequency sweep is selected, the minimum frequency, frequency increment, and maximum frequency must be stored in the computer. If plotting is to be done, scaling parameters must be stored.

The initial input of a set of parameters is usually made with the typewriter. If a parameter change is being considered, the parameter code is typed in and the current value is automatically typed out. The parameter is either changed by typing in a new value or retained by typing the appropriate symbol. Typing the list request code will cause the entire parameter list and current values to be typed out.

A parameter list can be retained for later use by typing in a "dump" code. The list is then stored on punched paper tape. When the measurement is repeated, the list can be read in on the paper tape reader by typing in a "read tape" code. This appreciably reduces the measurement set-up time.

4.2 *Program Description*

The stored program in the computer provides the necessary control for data input, processing, and output; for operation of the transmission measuring set; and for dialogue with the operator. The dialogue occurs when parameters are being entered, when networks are being connected for two-port measurements, and when trouble indications occur.

4.2.1 *Data Input, Processing, and Output*

Figure 10 is a block diagram showing the input-output options available for data used in the operation of the set. Measurement parameters are entered with the typewriter or tape reader. In a later stage of development, measurement parameters will also be entered from punched cards or magnetic tape.*

The test frequencies are generated by the program from stored measurement parameters, stored test frequencies, or from specific frequencies entered by typewriter, card reader, or tape reader.

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* Dashed lines indicate not yet operational.

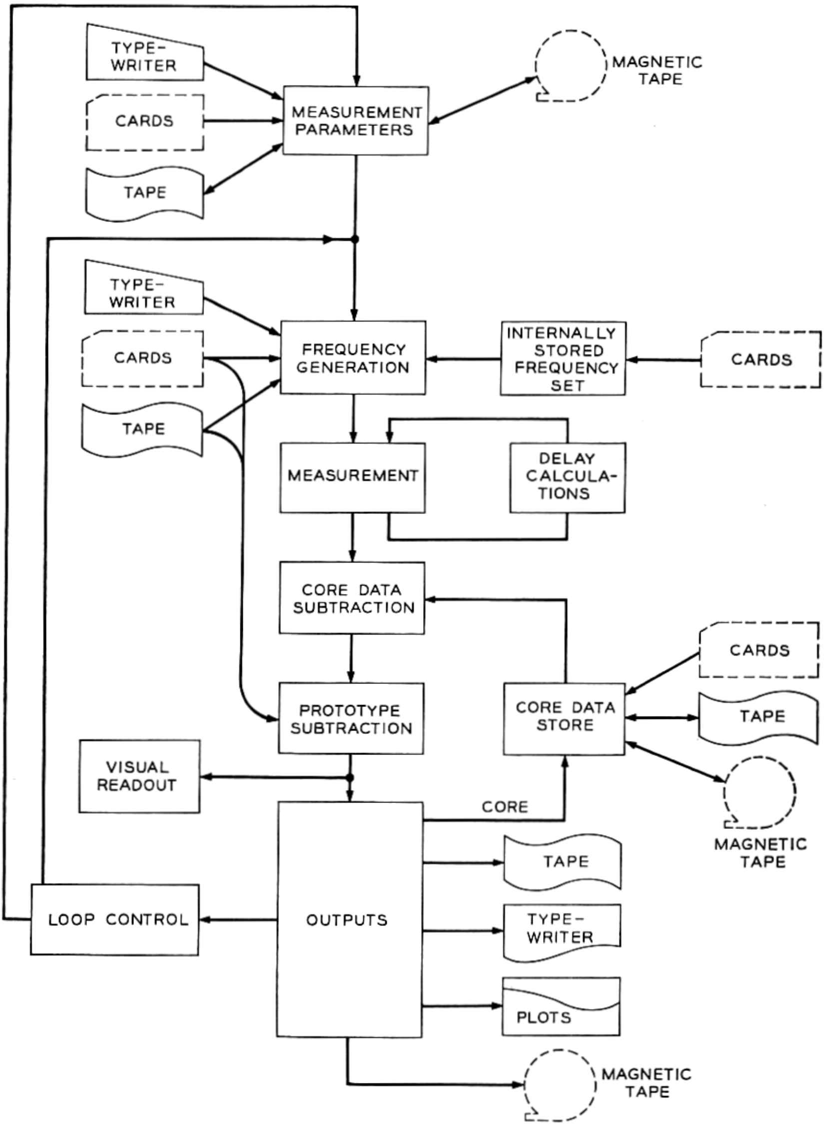

Fig. 10 — Input-output options.

After each measurement, which may include delay calculations, core data is subtracted if indicated by the mode selector. Prototype values on tape or cards can also be subtracted before data is output.

Output is always obtained visually and can also be obtained in core and on paper tape, typewriter, X-Y plotters, and magnetic tape. After output, the loop control returns the program to obtain new parameters or set a new test frequency.

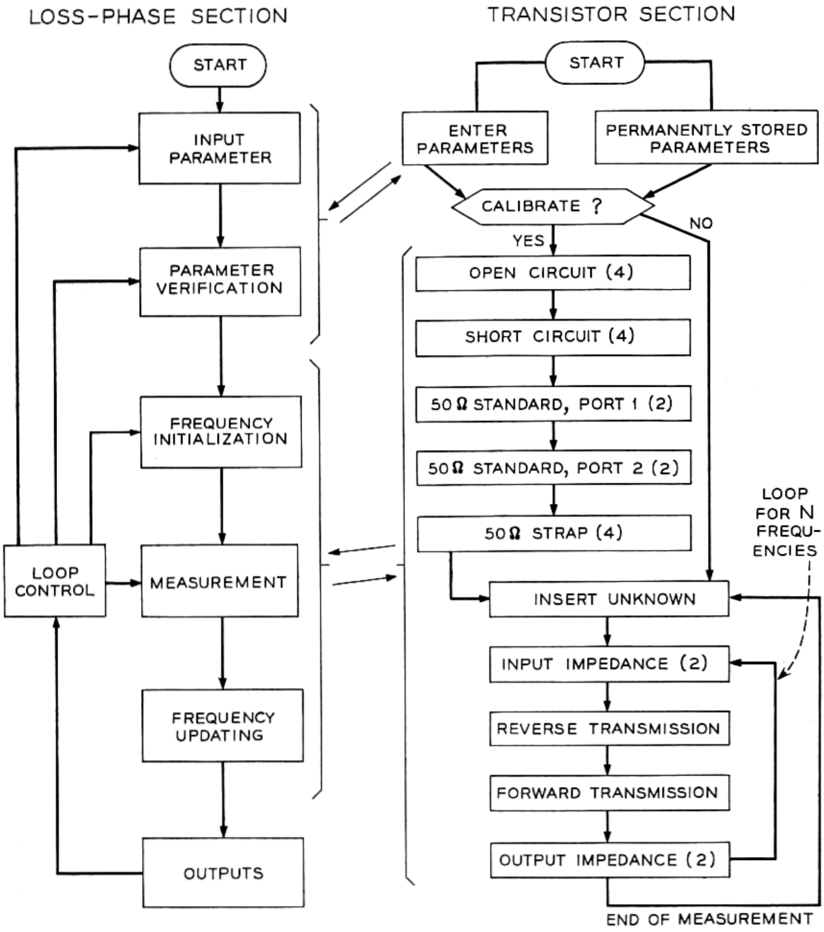

Fig. 11 — Control program.

4.2.2 *Control Program for Test Set Operation*

Figure 11 is a flow chart of the test set control program. Selection between the loss-phase section and the transistor section is made by choice of program starting address. The loss-phase section provides the subroutines for parameter entry, measurement, and data output. Notice that the frequency updating (change to a new frequency) occurs before data output. This gives a maximum time for the measured circuit to reach steady state before the next measurement.

The transistor section of the program provides the necessary control of switching, generates instructions for the operator, and provides the necessary connections to the subroutines in the loss-phase section. After entering the transistor section, there is the option of using permanently stored parameters or manually entering parameters. The next option is that of calibrating or not. Calibration is required if maximum accuracy is needed but adds 16 measurements at each frequency. With either option, the typewriter types instructions to indicate the standard or unknown network to be inserted and the particular connection to be made. The computer also makes the necessary connections in the measuring circuit. When the unknown is connected, six measurements are made (only four independent) and the four which provide the most accurate data are saved. Data is normally output on punched tape for subsequent processing to produce corrected, two-port parameters.

V. MEASUREMENT ERRORS

Measurement errors are evaluated in two complementary ways. The first is to directly evaluate each error-producing element in the system. The second is to measure networks with predictable properties. If the second group of measurements gives satisfactory results, confidence is gained that all of the important elements were evaluated in the first group.

5.1 *Test Set Errors*

The error sources in the test set that must be considered are: spurious signals, amplitude compression, mistermination errors, errors in standards, errors in detectors, circuit drifts, switching transients, and quantization errors. In order to measure these error sources efficiently and to approach the ideal of complete self-testing as closely as possible, measurements are made automatically on the test set wherever practical.

One example of automatic measurement of error sources is in the measurement of spurious signals. Any spurious signals present at the inputs of the loss detector or phase detector and some types of spurious signals at the inputs to nonlinear system elements can cause measurement error. Spurious signals considered are crosstalk, noise, modulation products, power frequency pickup, and test signal harmonics.

By the introduction of several auxiliary circuit elements in the test set transmission paths (Fig. 12), spurious products such as crosstalk, noise, 60 Hz harmonic pickup, and IF carrier leak as small as 100 dB down can be automatically measured and plotted as a function of frequency and level configuration. With the connections, A–A′, the converter and IF circuits are evaluated; with the connections B–B′, the RF circuits are evaluated.

Figure 13 plots the detector signal to interference ratio for a particular system configuration. For the range where the ratio is above 80 dB, random and systematic errors for low losses will be less than 0.001 dB and 0.01° (neglecting other error causes). Techniques have also been devised to automatically measure the other listed error sources with the test set.

5.2 *Impedance Measuring Errors*

Accuracy of the impedance measurements derived from transmission methods (Fig. 7) depends upon test set accuracy and accuracy of the impedance standards.

An error is made by the test set when each of the four transmission

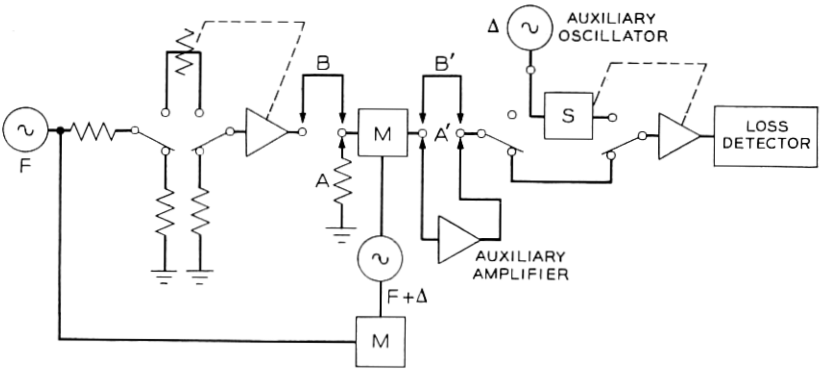

Fig. 12 — Measurement of spurious products.

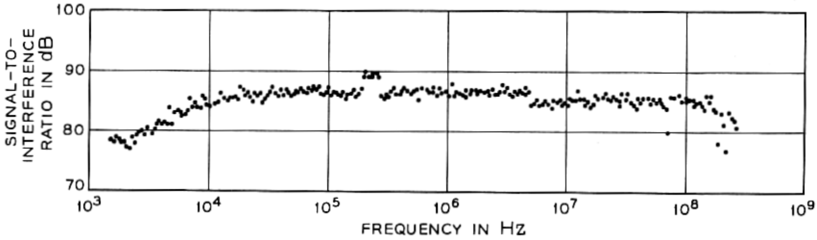

Fig. 13 — Test set signal to interference ratio.

measurements (that is, unknown and three standards) is made. These errors depend on the impedance of the unknown and are caused primarily by test set noise and finite measurement precision. The expression for the error in the calculated reflection coefficient resulting from test set errors is given in ref. 6. For the case of measuring a small reflection with a coaxial tee, the error, $\Delta\rho$, is typically less than 0.004.

The unknown is defined in terms of the standards used. Imperfections in the standards therefore cause errors unless the standards are adequately characterized. At high frequencies, characterization of the standards includes making a precise definition of the measurement.

When appropriate coaxial connectors are used to connect coaxial unknowns and standards, the reference plane and the measurement are well defined. In the case of unknowns with pig tail leads, the measurement is not well defined unless the unknown and the standards have precisely the same geometry so that field patterns are identical in the four measurements. If the geometries are not uniform, the unknown and standards can be placed in a shield but then the resulting parasitics must be estimated.

5.3 *Two-Port Measurement Errors*

Errors in determining two-port parameters include errors in measuring impedance as well as errors in measuring transmission. In the data reduction program used, two-port measurement data is always reduced first to s parameters (50 ohm reference). The first order error expressions for the s parameters are given in Ref. 6.

A worst case error for the s parameters of a precision air line would be:

$$|\Delta s_{11}(50, 50)| = |\Delta s_{22}(50, 50)| \leqq 0.004$$

$$\left|\frac{\Delta s_{21}(50, 50)}{s_{21}(50, 50)}\right| = \left|\frac{\Delta s_{12}(50, 50)}{s_{12}(50, 50)}\right| \leqq 0.002.$$

Estimation of errors in the s parameters is tedious, even when using a worst case estimate. The most promising approach advanced so far is to calculate errors by modifying the data reduction program. In the modified program each measured value obtained in the measurement sequence would be perturbed a small amount to obtain the parameter sensitivities to the measurement. Then the known systematic errors and the calculated random error (a standard test set output) would be applied to the sensitivity factors already obtained. Proper summing of the error terms would then give a good estimate of the parameter error expectation.

Two-port measurement data obtained so far indicates errors that are perhaps one fourth of the worst case errors indicated by the formulas just referred to.

VI. MEASUREMENT RESULTS

As indicated previously, the basic quantities measured by the transmission measuring set are insertion loss (or gain) and insertion phase shift as a function of frequency. Insertion delay, impedance, and two-port parameters are quantities derived from transmission measurements.

6.1 *Transmission Measurements*

Ideally, transmission measurement accuracy would be confirmed with measurement standards whose properties were precisely known. No complete set of standards exist, but some networks are available where knowledge of their behavior gives reliable indications of the test set precision and accuracy.

6.1.1 *Precision Coaxial Attenuators*

The uniform transmission properties of precision coaxial attenuators over the low frequency range makes possible meaningful comparisons between dc and ac measurements. A 6 dB and 14 dB attenuator were measured individually and in tandem on the test set and compared with measurements made on a dc ratio bridge capable of 0.0001 dB accuracy.

Results of the measurements are given on Figs. 14 through 17. Figure 14 shows the discrepancy between two measurements of the same 6 dB attenuator taken 15 minutes apart. Figures 15 and 16 give the data for the individual measurements on the 6 dB and 14 dB attenuators. In midband, discrepancies between ac and dc loss values are less than those which occur with a 2°F change in ambient temperature. Midband

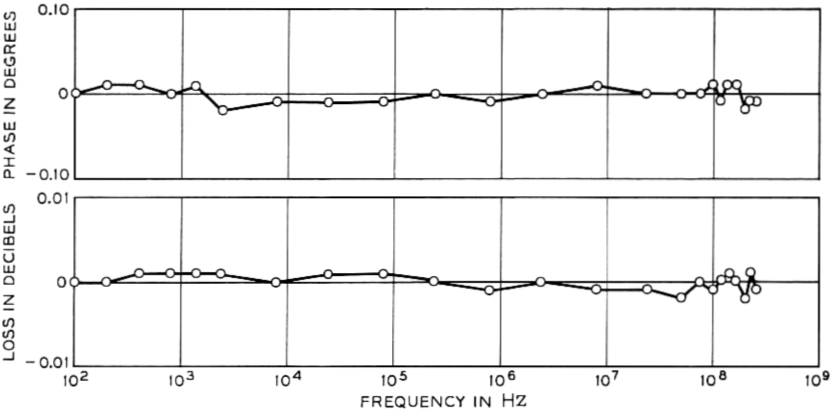

Fig. 14 — Repeatability of 6 dB pad measurements.

phase shift of the attenuators is within ±0.01° of zero. Figure 17 shows the discrepancy between the sum of the 14 dB and 6 dB measurements and the measurement of the two attenuators in tandem.

Errors are seen to increase at the low frequency end of the test set range. The error magnitudes correspond to signal-to-interference ratios measured over the same range by the method described in Section 5.1.2. The deviations at the high frequency end in Figs. 15 and 16 result primarily from changes in the insertion loss and phase of the attenuators.

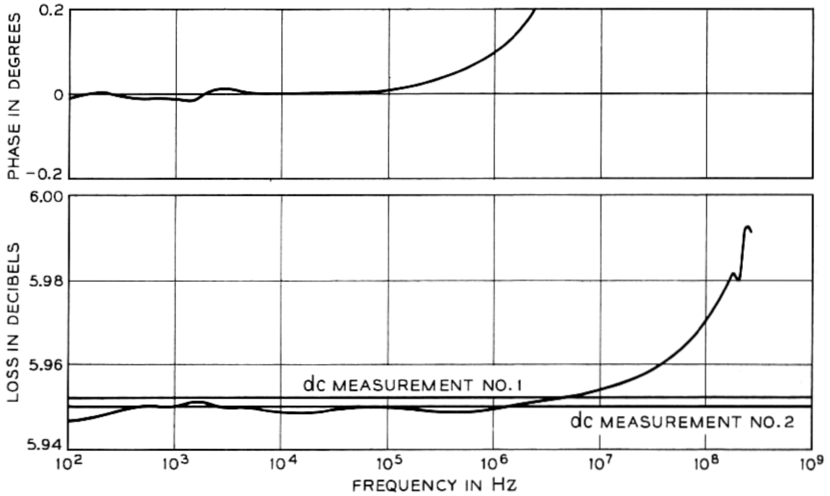

Fig. 15 — 6 dB coaxial pad measurement.

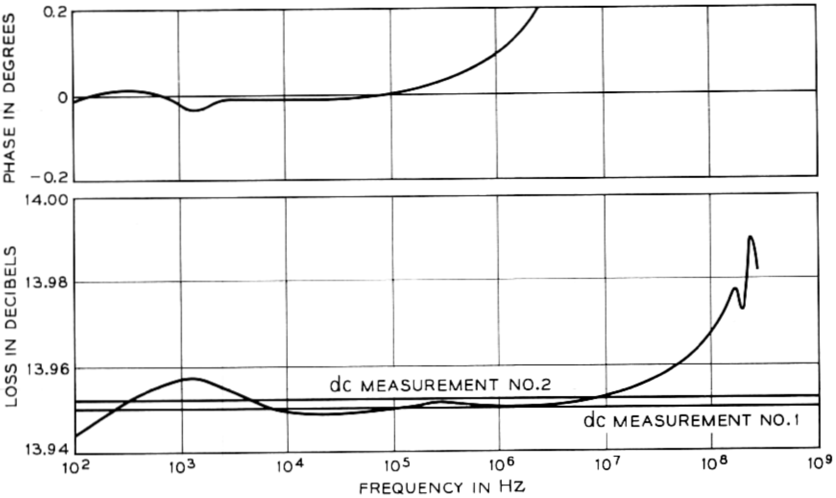

Fig. 16 — 14 dB coaxial pad measurement.

6.1.2 *Precision 30 cm Air Line*

A well-made precision air line is essentially ideal up to 250 MHz except for skin effect loss. The 30 cm line was first measured at a list of frequencies for comparison of measured loss and phase shift with theoretical values. Table IV gives this data. The discrepancies above 100 MHz are within the variation which could be caused by the ±0.02 cm tolerance in line length.

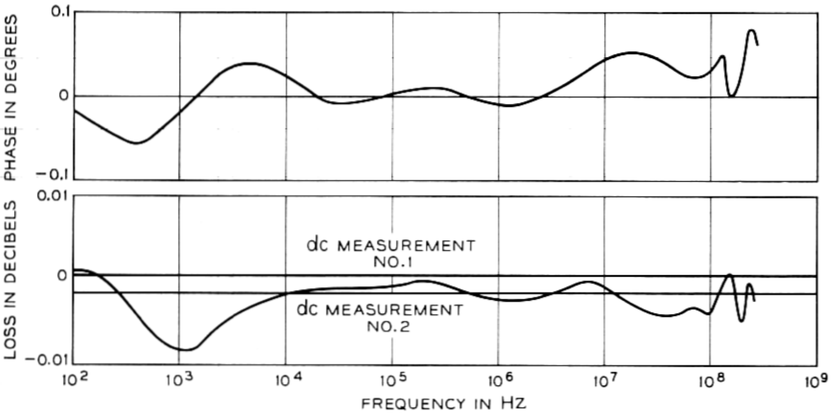

Fig. 17 — 14 dB and 6 dB coaxial pad addition.

TABLE IV — PHASE MEASUREMENTS ON PRECISION 30 CM AIR LINE

Frequency (Hz)	Phase		$\theta_c - \theta_m$
	Measured θ_m (degrees)	Calculated* θ_c (degrees)	
27,777.77	−0.01	0.0115792	0.02
55,555.55	0.02	0.0222333	0
83,333.33	0.04	0.0327353	−0.01
111,111.11	0.05	0.0431584	−0.01
138,888.89	0.05	0.0535312	0
166,666.67	0.06	0.0638682	0
194,444.45	0.09	0.0741732	−0.02
222,222.23	0.07	0.0844666	0.01
250,000.01	0.08	0.0947376	0.01
277,777.79	0.12	0.1049939	−0.02
277,777.78	0.09	0.1049939	0.01
555,555.56	0.21	0.2070624	0
833,333.34	0.29	0.3086497	0.02
1,111,111.12	0.41	0.4099878	0
1,388,888.90	0.51	0.5111667	0
1,666,666.68	0.62	0.6122325	−0.01
1,944,444.46	0.70	0.7132126	0.01
2,222,222.24	0.80	0.8141248	0.01
2,500,000.02	0.91	0.9149817	0
2,777,777.80	1.02	1.0157921	0
2,777,777.78	1.02	1.0157921	0
5,555,555.56	2.02	2.0223333	0
8,333,333.34	3.02	3.0273524	0.01
11,111,111.12	4.04	4.0315841	−0.01
13,888,888.90	5.04	5.0353121	0
16,666,666.68	6.02	6.0386825	0.02
19,444,444.46	7.04	7.0417819	0
22,222,222.24	8.04	8.0446667	0
25,000,000.02	9.06	9.0473762	−0.01
27,777,777.80	10.05	10.0499389	0
27,777,777.78	10.06	10.0499389	−0.01
55,555,555.56	20.08	20.0706242	−0.01
83,333,333.34	30.09	30.0864966	0
111,111,111.12	40.09	40.0998777	0.01
138,888,888.90	50.13	50.1116667	−0.02
166,666,666.68	60.15	60.1223247	−0.03
194,444,444.46	70.17	70.1321258	−0.04
222,222,222.24	80.20	80.1412484	−0.06
250,000,000.02	90.22	90.1498166	−0.07

* $\theta_c = 90/250\ F_{\mathrm{MHz}} + 0.134\ (F_{\mathrm{MHz}}/200)^{\frac{1}{2}}$.

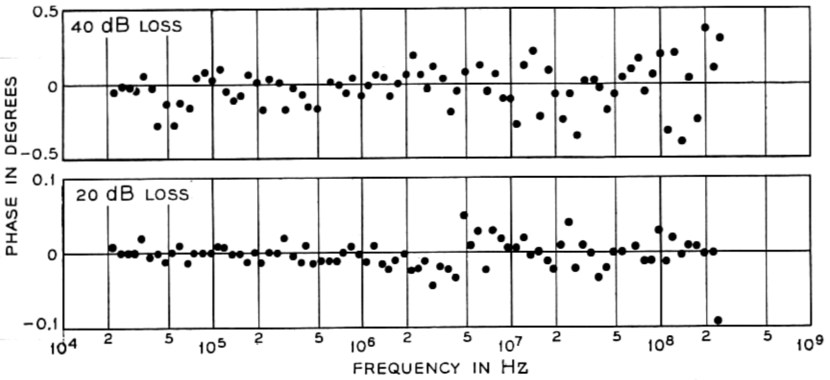

Fig. 18 — Effect of loss on fixed and random phase errors; —20 dBm loss level, 16 measurments averaged.

The same line was remeasured in tandem with 20 dB and 40 dB of loss. Figure 18 shows the effect of loss on the fixed and random phase errors. Figure 19 shows a curve of the measured delay of the line. The ±0.01 nsec variation in the measured value corresponds to a ±0.01° phase error with the 5 MHz frequency interval used.

6.1.3 *9 MHz Band Pass Crystal Filter*

Networks with especially "difficult" properties also provide useful information on the test set capabilities. Phase measurements in the passband of a 9 MHz filter with a 3 kHz bandwidth are especially sensitive to FM in the signal source.

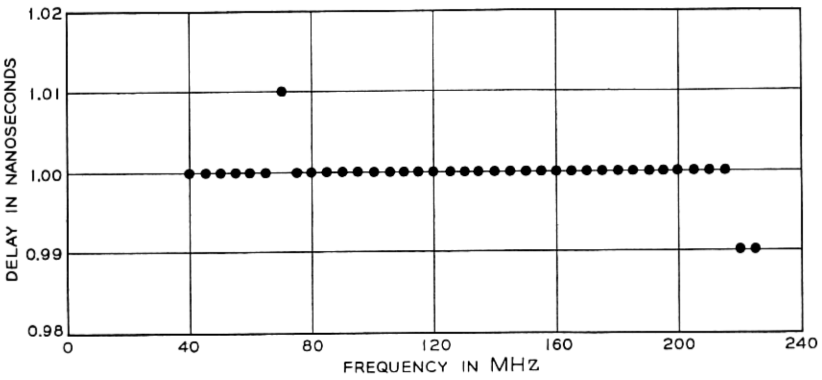

Fig. 19 — Measured delay of precision 30 cm air line.

The upper curve on Figure 20 shows the measured delay of the filter with an in-band delay of about 0.5 ms. The lower curve shows the difference of two successive delay measurements. Mid-band values repeat to ±0.001 ms, or ±0.02° with the 100 Hz frequency interval used.

6.2 *Impedance Measurements*

Resistors, capacitors, and inductors were measured to confirm the accuracy of impedance measurements. In the results given, the 50 ohm terminals and a 50 ohm impedance standard were used. The results were compared with measurements made on precision bridges.

Resistor measurements were made from 2 kHz to 10 MHz. For resistors (pigtail) of 100, 400, and 2500 ohms, the measurements are as shown in Fig. 21. The maximum deviation from the dc value of the resistors is less than 0.1 percent.

Capacitors with nominal values of 10, 100, 1000, and 10,000 pF were measured in the 2 kHz to 3 MHz range which provided a reactance range from 50 ohms to 500 k ohms. These results were compared with bridge measurements made at 100 kHz (less than 0.1 percent error) and the results are shown in Fig. 22.

Inductors with nominal values of 0.3, 1, 10, 100, and 1000 μh were

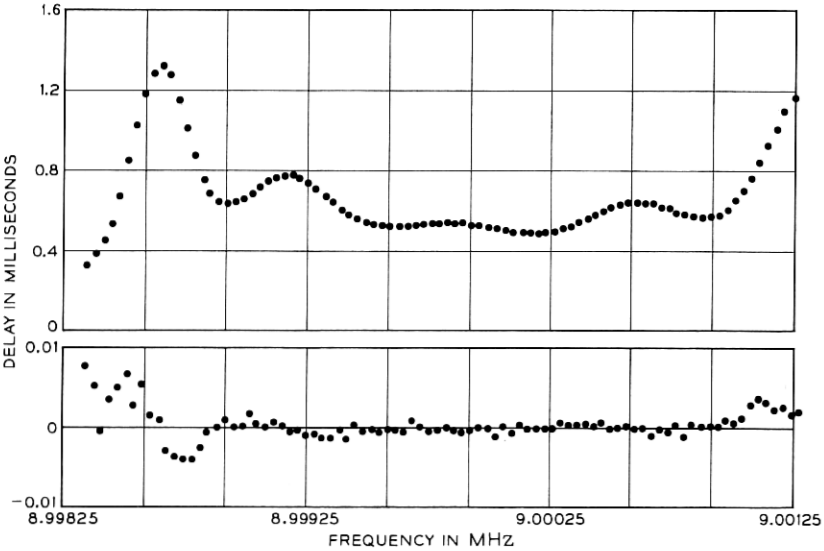

Fig. 20 — Measured delay of narrow band crystal filter.

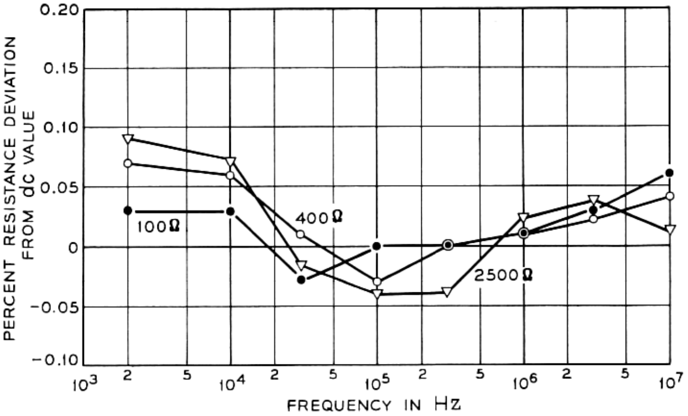

Fig. 21 — Deviations of measured resistance from dc values.

measured in the frequency range from 3 kHz to 10 MHz over an impedance range from 0.006 to 600 ohms. The results are shown in Fig. 23 with the indicated deviations being from bridge measurements (less than 0.05 percent error).

Typical agreement between impedance measurements on the set and the bridge is within 0.2 percent in favorable impedance ranges and the worst errors are less than 1 percent.

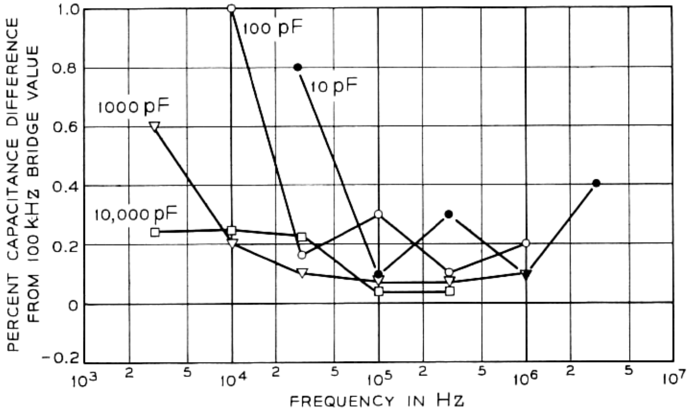

Fig. 22 — Deviations of measured capacitance from bridge measurements.

6.3 *Two-Port Measurements*

As an example of two-port characterization of a precise network, a precision 30 cm air line was measured.[6] Deviations of the measured characteristics from an ideal line result from measurement errors and skin effect. To emphasize these deviations, the characteristics of an ideal 30 cm line were mathematically removed from the measured data during processing. Figures 8 through 11 of Ref. 6 show the measured characteristics (normalized) compared with theoretical values. The S12 curves show the 0.001 dB and 0.01° resolution of the test set. Figure 24 shows curves of |S11| and |S12| for the same air line. The solid lines are the same as in the previous curves and the broken lines show the results that would be obtained if corrections were not made for test set misterminations.

Transistor characterization data is shown in Fig. 25. In this case, the transistor was measured in the common emitter, common base, and common collector modes and data from all three sets of measurements were transformed to common emitter h parameters. The curves shown are for magnitude and angle of $h21$ (beta).

VII. FURTHER DEVELOPMENT

The speed and flexibility of the computer operated transmission measuring set offers the opportunity for further development in a number of areas. Those being considered include increasing the frequency range, provision for measurements under environmental control and at remote locations, development of fault detection and location tests, and provision for interaction with a larger computer.

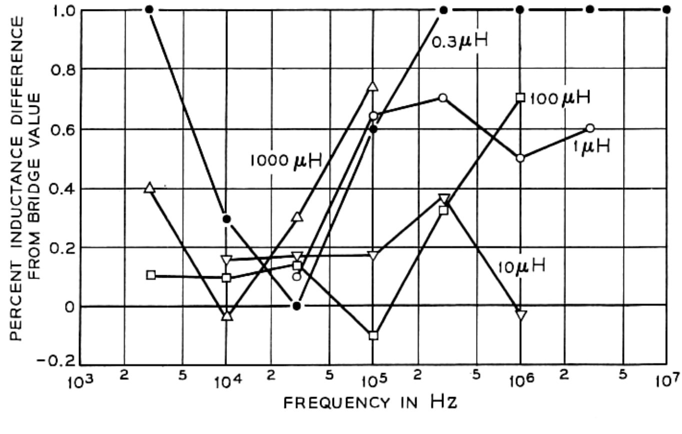

Fig. 23 — Deviations of measured inductance from bridge measurements.

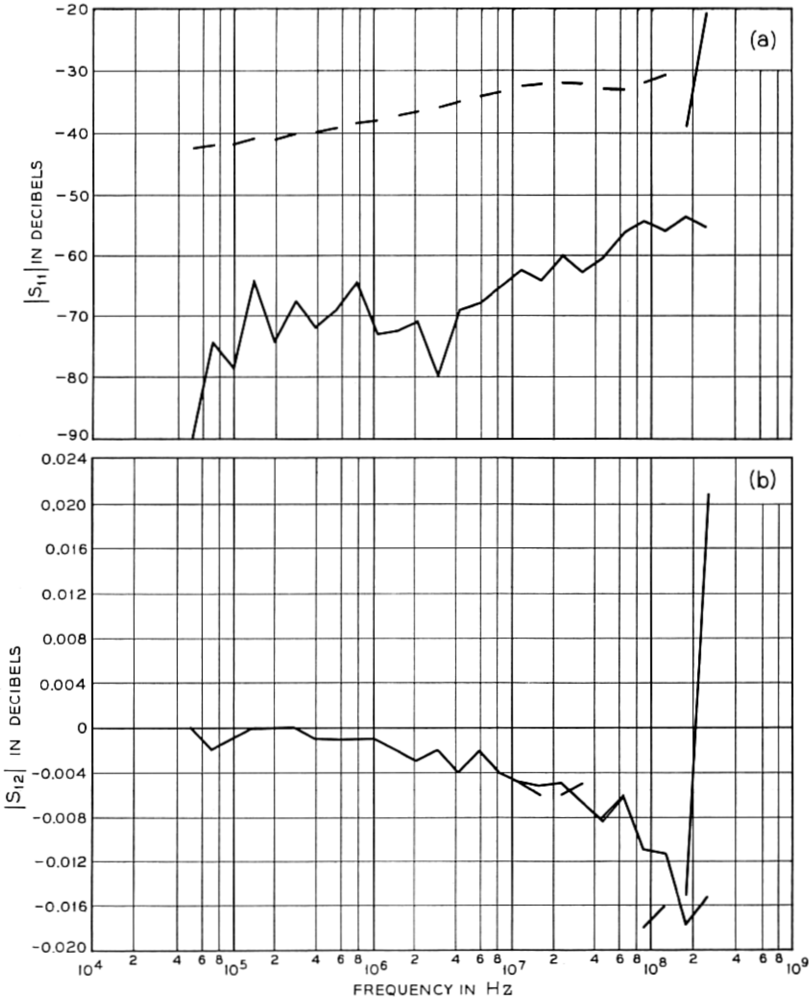

Fig. 24 — Normalized S parameters of precision 30 cm air line normalized to an ideal line of zero length (a) with and (b) without corrections for misterminations.

7.1 *Additional Measurement Capabilities*

One applique now under development will measure networks placed in an environmental chamber having temperature and humidity controls. After the environmental conditions have stabilized, computer control will transfer the necessary portions of the basic test set to the

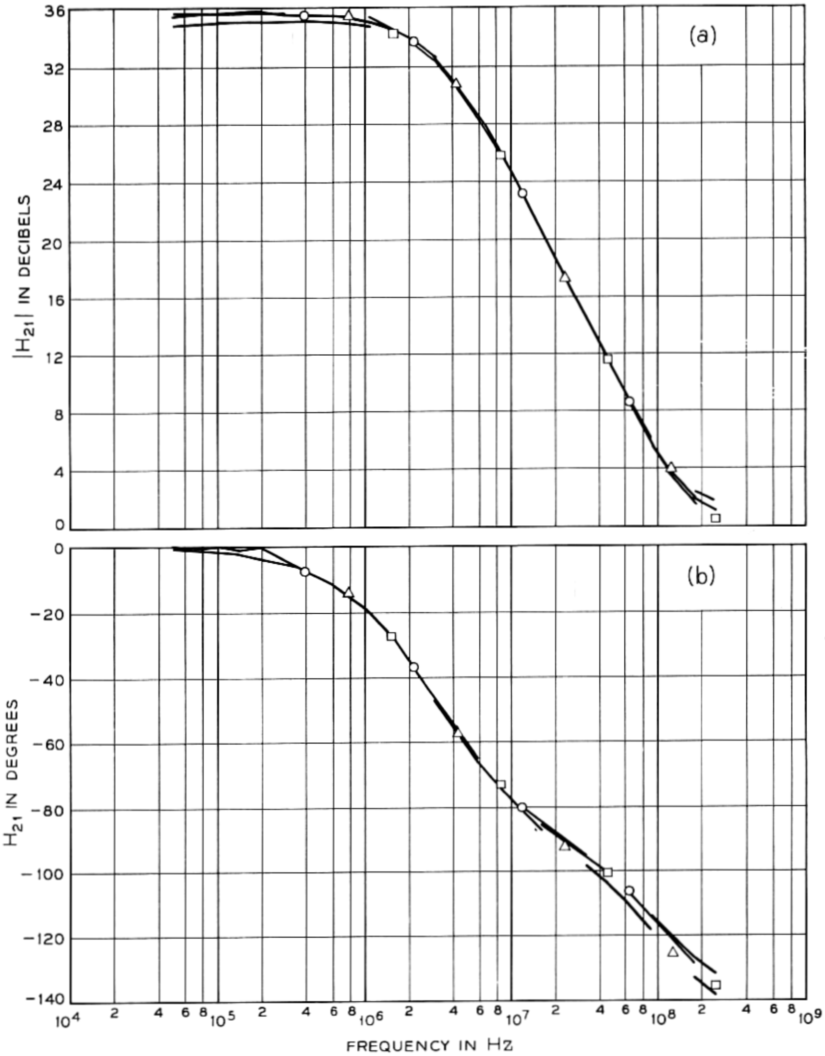

Fig. 25 — Transistor *h* parameters measured in the common emitter, common base, and common collector modes. ○ = 20C measured common emitter. △ = measured common base and computed common emitter. □ = measured common collector and computed common emitter.

applique for making measurements during idle periods on the basic set. The applique will have a 50 Hz to 250 MHz frequency range.

Other appliques are being developed to provide transmission measurements in the three microwave radio bands at 4, 6, and 11 GHz. These appliques will provide the microwave sources, control and switching, and down-converters. In the applique mode, about 80 percent of the basic set hardware is used.

Figure 26 is a block diagram of the 4 GHz applique which was recently completed.

A remote unit has been developed to permit transmission measurements hundreds of feet away from the basic set. Capabilities are not fully evaluated but 0.01 dB accuracy has been obtained over a 100 kHz to 100 MHz band at a distance of 275 feet from the basic set. Capabilities of the environmental applique and microwave appliques are estimated to be about 0.005 dB and 0.03°.

In the completed measurement center consisting of the basic set and appliques, it is planned to store data and control programs on a magnetic tape system. Then operation can be transferred from one applique to another in a few seconds.

7.2 *Maintenance Tests*

The question of test set accuracy is raised not only when a test set design is proven, but is a continuing question. The increased work load on the measurement center and the increased complexity of the total system inherent in the applique approach makes rapid and reliable fault detection and location vital. It is planned that the computer will enter an automatic test sequence whenever the measuring sets are idle. The test sequence would include tests for the computer as well as for the measuring sets.

7.3 *Frequency Extension*

Modular components are now available which will permit extension of the basic set operation up to 1000 MHz. Transmission circuits and frequency multipliers have been modified to operate to 1000 MHz, and a frequency converter to operate from 5 to 1000 MHz is being developed.

7.4 *Computer Interaction*

Coupling the measurement center to a larger computer via a data link is being considered. Two benefits of this connection are apparent.

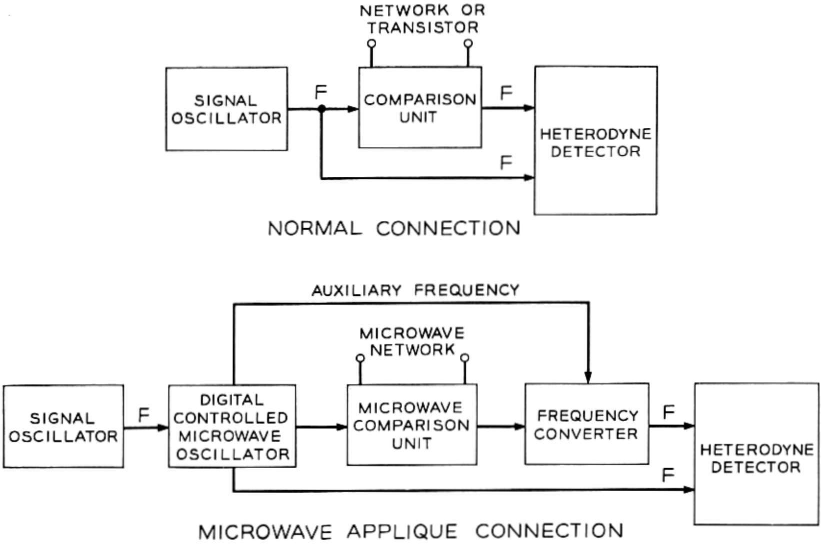

Fig. 26 — Extension of automated measurements to microwave frequencies.

The processing of two-port parameters now involves a considerable turn-around time. Direct connection to a larger computer could provide processed data in a few minutes. The problem of adjusting networks (particularly with interacting adjustments) might be substantially simplified if measurement data could be processed by a larger computer and adjustment information be returned to the test set.

VIII. SUMMARY

The computer-operated transmission measuring set provides high accuracy over a wide range of levels and frequency. The memory, control, and data processing capabilities of the computer provide the means to improve accuracy, operate at high speed, and provide versatility in the forms of output data.

The measurement capability of the set will make measurements an increasingly important part of the transmission system design process and, along with design aids such as computer analysis and modeling, improve the quality of systems being developed.

The flexibility inherent in the computer-operated set provides the opportunity for further development to increase its capability.

IX. ACKNOWLEDGMENTS

The computer-operated transmission measuring set hardware was developed by the Measuring Systems Design Department at Bell Laboratories. P. E. Rosenfeld suggested using the computer and developed the digital control and detection circuitry. J. G. Evans developed the transistor and two-port network characterization circuitry and technique (see Ref. 6). The computer program was developed by the Transmission Analysis Department at Bell Laboratories. We express particular appreciation to P. S. Meigs.

APPENDIX

Increased Resolution in Time Interval Method of Phase Measurements

A.1 *Basic Measurement*

A simplified block diagram of the time interval method of phase measurement is shown in Fig. 27. The measurement desired is $\theta_2 - \theta_1$. The pulse generators detect zero crossings; the first zero crossing occurs at $t_1 = \theta_1/\omega_1$, the second at $t_2 = \theta_2/\omega_1$. The exact time interval, Δt, is

$$\Delta t = t_2 - t_1 = \frac{\theta_2 - \theta_1}{\omega_1}$$

or in degrees

$$\Delta t = \frac{\varphi_2 - \varphi_1}{360 f_1}.$$

To measure the interval Δt with a counter using the pulse source F_s, the number of pulses, n, gated into the counter are: $2\pi(n \pm 1) = \omega_s \Delta t$.

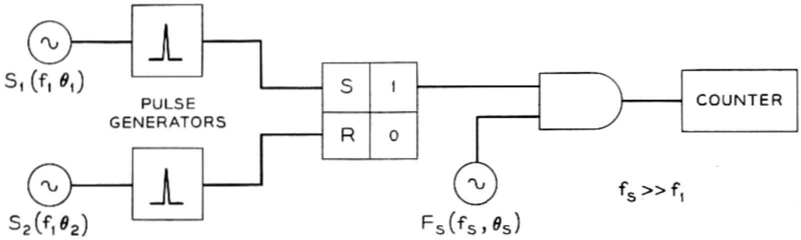

Fig. 27 — Time interval method of phase measurement.

Then:

$$\varphi_2 - \varphi_1 = 360 \frac{f_1}{f_s} (n \pm 1)^\circ .$$

For the case where $f_s = 100$ MHz and $f_1 = 27.777$ kHz,

$$\varphi_2 - \varphi_1 = 0.1n^\circ \pm 0.1^\circ .$$

This gives a resolution of 0.1°.

A.2 *Vernier Technique*

It will be shown that if f_s and f_1 are suitably related in frequency, multiple readings will provide increased resolution.

The circuit above can be modeled by integrating the product of two time functions. The gate opening is represented by $f(t)$, where

$$f(t) = \begin{cases} 1 & Kt_1 \leqq t < Kt_1 + \Delta t \\ 0 & Kt_1 + \Delta t \leqq t < (K+1)t_1 \end{cases}$$

and where

$$T = \text{the total measurement period}$$

$$K = 0, 1, 2, \cdots, \left(\frac{T}{t_1} - 1\right).$$

The pulse train from the source F_s is represented by $g(t)$, where:

$$g(t) = \sum_{m=o}^{\infty} \delta(t - \tau_o - m\tau)$$

and where: $\tau = 1/f_s$ and τ_o accounts for the relative phase between $f(t)$ and $g(t)$.

The total number of counts into the counter is represented by N, where

$$N = \int_0^T f(t)g(t)\, dt.$$

If we let the ratio of t_1/τ be I and I is an integer, the resolution (as in Section A.1) will be $360^\circ/I$. Integrating the product of $f(t)$ and $g(t)$ we obtain,

$$N = \sum_{K=0}^{(T/t_1-1)} \int_{Kt_1}^{Kt_1+\Delta t} \sum_{m=o}^{\infty} \delta(t - \tau_o - m\tau)\, dt.$$

It is convenient to let $m = m' + Kt_1/\tau = m' + KI$, where m' indexes the delta function relative to the beginning of each gate opening.

$$N = \sum_{K=0}^{(T/I\tau-1)} \int_{KI\tau}^{KI\tau+\Delta t} \sum_{m=o}^{\infty} \delta(t - \tau_o - (m' + KI)\tau)\, dt.$$

In each integration, the delta functions will contribute when $KI\tau \leqq \tau_o + m'\tau + KI\tau < KI\tau + \Delta t$, or $-(\tau_o)/\tau \leqq m' < (\Delta t)/\tau$, which is independent of K. Thus the average of multiple readings will be the same as one reading. An example will illustrate this. If $t_1/\tau = I = 3600$ and $\Delta t = 1.5\tau$(that is, 0.15°), two cases are evident, as shown by Fig. 28.

In the first case there will be one count during each opening and in the second there will be two counts during each opening. Since the gate opening is periodic in t_1, the count will not vary and the reading will be $\Delta t \pm 0.5\tau$.

If the ratio of t_1/τ is varied so that the measurement is periodic in T, the precision is increased by T/t_1. In this example, during each counting period the delta function will move $(t_1/T)\tau$ seconds relative to $f(t)$ or a total of τ seconds during the measurement. This will provide a count of 1 during half the periods and a count of two in the other half. The average will be the correct number of 1.5.

To show this in the analysis, we define a slightly different frequency f'_s in the pulse source frequency so that $t_1/\tau' = I/(1 + s)$, where s is a term to give t_1/τ' a noninteger value. As before, T/t_1 has an integer value. Then

$$g'(t) = \sum_{m=o}^{\infty} \delta(t - \tau_o - m\tau') = \sum_{m=o}^{\infty} \delta(t - \tau_o - m\tau(1 + s))$$

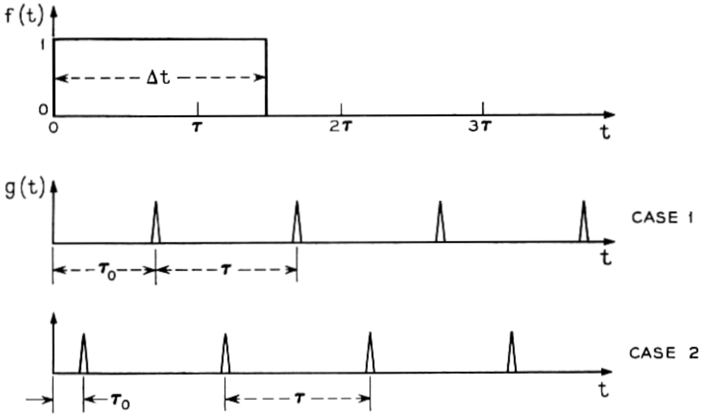

Fig. 28 — Example of average multiple reading being the same as one reading.

and

$$\begin{aligned} N' &= \int_0^T f(t)g'(t)\,dt \\ &= \sum_{K=o}^{(T/I\tau-1)} \int_{KI\tau}^{KI\tau+\Delta t} \sum_{m=o}^{\infty} \delta(t - \tau_o - m\tau(1+s))\,dt. \end{aligned}$$

By substituting $m = m' + KI$ as before and letting $s = \tau/T = t_1/(IT)$

$$N' = \sum_{K=o}^{(T/I\tau-1)} \int_{KI\tau}^{KI\tau+\Delta t} \sum_{m=o}^{\infty} \delta\left(t - \tau_o - (m' + KI)\left(1 + \frac{t_1}{IT}\right)\tau\right) dt. \tag{1}$$

The delta functions make contributions when

$$KI\tau \leqq \tau_o + (m' + KI)\left(1 + \frac{t_1}{IT}\right)\tau < KI\tau + \Delta t.$$

Solving for the integers m', two inequalities are obtained.

$$m' \geqq \frac{\dfrac{\tau_o}{-\tau} - K\dfrac{t_1}{T}}{1 + \dfrac{\tau}{T}} \tag{2}$$

and

$$m' < \frac{\dfrac{\Delta t}{\tau} - \dfrac{\tau_o}{\tau} - K\dfrac{t_1}{T}}{1 + \dfrac{\tau}{T}}. \tag{3}$$

For the ranges $0 \leqq (\tau_o)/\tau < 1$ and $0 \leqq K \leqq [(T/t_1) - 1]$, the values of m' provided by equation 2 are:

$$m' \geqq 0 \qquad \text{for all } K$$

$$m' = -1 \qquad \text{for } K \leqq \frac{T}{t_1}\left(1 + \frac{\tau}{T} - \frac{\tau_o}{T}\right).$$

Equation 3 depends on the measured quantity Δt and on the counting period, K. Using the previous example where $t_1/\tau = 3600$ and $\Delta t/\tau = 1.5$ assume that T/t_1 (the increase in resolution) $= 100$ and that $\tau_o/\tau = 0.5$.

Then from equation 2,

$$m' \geqq 0 \quad \text{for} \quad 0 \leqq K \leqq 99$$

$$m' \geqq -1 \quad \text{for} \quad 51 \leqq K \leqq 99.$$

Then the sum of the integrals yield

$$N' = \underbrace{100(1)}_{m'=0} + \underbrace{49(1)}_{m'=-1} = 149 \text{ for an average of } 1.49 \text{ counts.}$$

As another example let $t_1/\tau = 3600$, $T/_{t1} = 100$, $(\tau_o)/\tau = 0.9$, and $(\Delta t)/\tau = 3599.9$ (that is, 359.99°).

Then from equation 2

$$m' \geqq 0 \qquad \text{for} \quad \text{all} \quad K$$

$$m' \geqq -1 \qquad \text{for} \quad 10 < K < 99.$$

And from equation 3

$$m' = 0, 1, \cdots, 3598 \quad \text{for all} \quad K.$$

Then the sum of the integrals yield

$$N = \underbrace{100(3599)}_{m'=0 \text{ to } 3598} + \underbrace{89(1)}_{\substack{m'=-1 \\ K>10}} = 100(3599.89)$$

for an average of 3599.89 counts.

In each case the resolution is increased from 0.1° to 0.001°.

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