# An Automatic Equalizer for General-Purpose Communication Channels

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*The restriction imposed by linear distortion on the flow of information in a communication channel is well known. In the past, the effects of this distortion have been alleviated through the use of manually adjusted equalizing or compensating networks. The adjustment of these networks is too cumbersome a process for the user of a switched communication service to perform each time a new connection is established. Therefore, in present switched networks, control of linear distortion is imposed only on the individual links. Variation between links and variation of the number of links in tandem result in channels with distributed performance. Lower distortion can be achieved by equalizing the overall connection.*

*Recent developments have made automatic linear distortion removal (equalization) practical for synchronous data communication systems. Here an implementation is described wherein these techniques have been generalized so that automatic equalization can be provided for a communication channel independent of the signal format used in that channel. For a number of applications the speed of automatic equalization makes efficient end-to-end equalization practical in a switched network.*

*The implementation described affords automatic minimization of the discrepancy between a specified response and the actual response of a linear transmission medium. Thus, on the one hand, it permits the automatic reduction of transmission defects such as signal dispersion and echoes, and, on the other hand, it permits the mechanized synthesis of filters with specified transfer functions.*

*This paper reviews the general aspects of automatic equalization, describes an implementation of a general purpose automatic equalizer, discusses the theoretical performance of such an equalizer as determined from computer simulations, and lastly presents results for the equalization of real channels using the implementation described.*

## I. INTRODUCTION

Recent years have witnessed an increasingly intensive investigation of automatic equalization techniques.[1] *Equalization,* itself, is necessary because of the increased demand for efficient use of communication channels. Fixed compromise equalizers have been used in terminal equipment but they cannot remove all of the distortion because of variation between connections in a switched service. Two factors contribute to the distribution of distortion on different connections—differences in the characteristics of the individual links that may be switched together and differences in the number of links in a connection. Better equalization and, therefore, greater transmission efficiency can be achieved by individually equalizing each connection after it has been established. *Automatic* equalization provides a practical means for rapidly and efficiently equalizing each connection.

Several automatic equalization schemes have been published which provide equalization for specific, usually synchronous, communication systems. Some of the techniques for synchronous data transmission systems are those of Coll and George,[2,3] DiToro,[4] Funk et al,[5] and Lucky and Becker et al.[6,7,8] These techniques are very powerful for the synchronous data transmission systems for which they are intended. Furthermore, the implementations of these equalization strategies possess considerable economy of design because they rely upon the peculiarities of the particular synchronous transmission systems for which they are intended. But, their use is restricted to such systems.

The present paper is concerned with an equalization technique which is essentially independent of the transmission format to be used on the channel. The inclusion of such an equalizer in a communication channel is shown in Fig. 1 in the simplest form. A test signal is transmitted through the channel and the equalizer controller adjusts the equalizer until optimum equalization has been attained. The equalizer adjust-

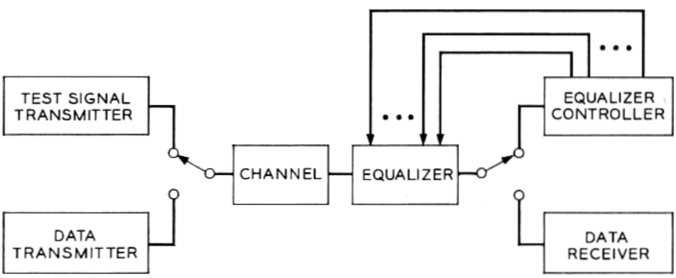



Fig. 1 — Preset mean-square channel equalizer.

ments are then locked and the equalized channel used for communication.

"Optimum" equalization is here defined as the minimization of the mean-squared difference between a specified channel response and the actual equalized channel response. Much work has been done on the problem of optimization under a mean-squared error criterion. The most famous of these is Wiener's classic paper.[9] A paper by Narendra and McBride[10] also relates to the present work.

In the equalization schemes for synchronous data transmission,[2–8] the data receiver is inside the equalizer control loop and the actual data transmitter is used to generate the test signal. Here, the equalizer can correct for distortion introduced by imperfections in the transmitter and receiver as well as in the channel, resulting in very effective equalization. A general purpose equalizer of the type described does not have this capability (by intent) but instead has the advantage that it is not tied to a single communication system. The equalized channel can be used by arbitrary information transmission systems. The equalization is generally carried out at passband frequencies and the control circuitry could be shared by a number of communication channels. Thus, the technique described may be an attractive one when it is necessary to provide equalization for a variety of customers whose communication channels terminate at a common location.

The equalizer described here uses a transversal filter to operate on the channel response so that the equalized channel response approximates the desired response in an optimum fashion. Again, the criterion used to determine this optimum fashion is the minimization of the mean-squared error.

In summary, this paper reviews some of the general aspects of automatic equalization, describes an implementation of a general purpose automatic equalizer, discusses the theoretical performance of such an equalizer, and presents results for the equalization of real channels using the implementation described. Some laboratory results are also presented for the application of these techniques to a network synthesis problem.

The present paper expands on two previous brief disclosures in the literature.[11, 12]

## II. THE TECHNIQUE

### 2.1 *The Basic Mathematics*

The notion of the mean-square equalizer starts in the frequency domain. Here, the channel transmission characteristic is equalized so that it best resembles the ideal transmission characteristic. This "best" fit is made using a mean-square error criterion. Thus, the distortion to be minimized is

$$E_1 = \int_{-\infty}^{\infty} | H(\omega) - G(\omega) |^2 \, d\omega, \tag{1}$$

where $H(\omega)$ is the equalized channel characteristic and $G(\omega)$ is the ideal channel characteristic. Notice that this error criterion includes both phase (and consequently delay) and amplitude information in the goodness of fit.

The error criterion given in (1) can be made more general by adding information concerning the relative importance of errors at various frequencies. For example, in most information transmission schemes the major portion of the signal energy is placed near the center of the band, so that the equalization should be most perfect there. Since relatively little signal energy is put near the band edges, the quality of equalization is not of as great concern in this region. Therefore, a real, nonnegative weighting function $|W(\omega)|^2$, which assigns a relative weight $|W(\omega)|^2$ to the equalization error at each frequency $\omega$, is included in the criterion. The resultant criterion is

$$E = \int_{-\infty}^{\infty} | H(\omega) - G(\omega) |^2 \, | W(\omega) |^2 \, d\omega. \tag{2}$$

Usually the ideal characteristic $G(\omega)$ would have flat amplitude and linear phase within the band of interest, while the spectral weighting function $|W(\omega)|^2$ would resemble the spectral density of the signal likely to be transmitted, if this spectral density is known beforehand. The system is shown in block diagram form in Fig. 2.

The equalized channel characteristic is the product of the unequalized channel characteristic $X(\omega)$ and the equalizer characteristic $C(\omega)$.

$$H(\omega) = X(\omega)C(\omega). \tag{3}$$

The frequency characteristic function of a $(2N+1)$ — tap transversal equalizer with tap gains $c_n$, $n = -N, \ldots\ldots, N$ spaced at $\tau$ second intervals is

$$C(\omega) = \sum_{n=-N}^{N} c_n e^{-jn\omega\tau}. \tag{4}$$

Notice that this response is periodic with period $2\pi/\tau$, the real part of the response being even about frequencies $2n\pi/\tau$ and the imaginary

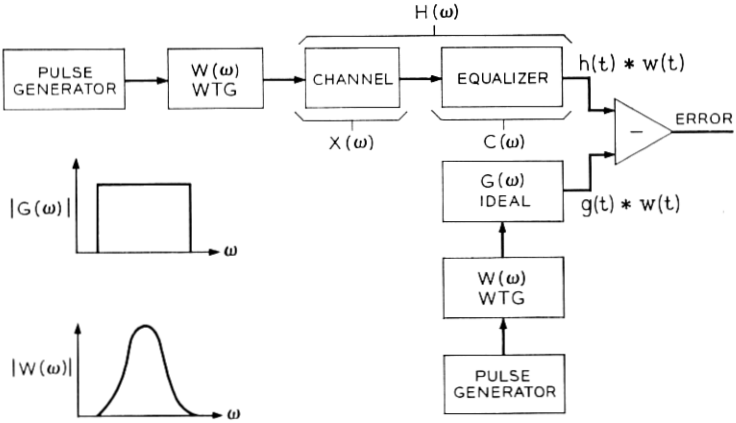



Fig. 2 — Mean-square equalizer.

part odd about these frequencies. Thus, the transversal equalizer offers independent control of the overall frequency response of the system over only one of the frequency intervals $n\pi/\tau \leqq \omega \leqq (n + 1)\pi/\tau$. The value of the tap spacing $\tau$ must be picked such that the desired equalization frequency range is included in one of these intervals. A frequent case is that where the channel is essentially low-pass in nature. Here the tap separation $\tau$ will be the Nyquist period $1/2W$, where $W$ is the highest frequency of interest. For bandpass channels, $\tau$ will generally have to be less than a Nyquist period.

The objective is the minimization of the distortion $E$ as a function of the $(2N + 1)$ variables $c_n$ in automatic fashion. Because this minimization is more easily carried out in the time domain, Parseval's theorem is used to obtain an equivalent form for (2):

$$E = \int_{-\infty}^{\infty} \{[h(t) - g(t)] * w(t)\}^2 \, dt. \tag{5}$$

In (5), $h(t)$, $g(t)$, and $w(t)$ are the impulse responses corresponding to the frequency responses $H(\omega)$, $G(\omega)$, and $W(\omega)$, respectively, and the * symbol is used to represent convolution.

If $x(t)$ is the impulse response of the unequalized channel, the equalizer output response is

$$h(t) = \sum_{n=-N}^{N} c_n x(t - n\tau) \tag{6}$$

and (5) can be written

$$E = \int_{-\infty}^{\infty} \left\{ \sum_{n=-N}^{N} c_n x(t - n\tau) * w(t) - g(t) * w(t) \right\}^2 dt. \tag{7}$$

It can easily be demonstrated that $E$ is a convex function of the tap gains $c_n$; $n = -N, N$. Thus, there is a single minimum of $E$ and this occurs when the $(2N+1)$ derivatives $\partial E/\partial c_n$ are zero. Setting these derivatives to zero gives $(2N+1)$ simultaneous linear equations which can be solved to effect a minimization of $E$. If the partial differentiation is carried out with respect to a particular tap setting (say $c_j$), the following relation is obtained:

$$\frac{\partial E}{\partial c_j} = 2 \int_{-\infty}^{\infty} \{h(t) * w(t) - g(t) * w(t)\}\{x(t - j\tau) * w(t)\}\, dt,$$

$$-N \leqq j \leqq N. \tag{8}$$

The set of (8) contains all the information required for automatic optimization. First, if these equations are set equal to zero and solved for the $c_n$'s, the desired tap coefficients are obtained. Second, if arbitrary values are chosen for the $c_n$'s, the set of (8) dictates the direction in which the coefficients must be changed to reduce the error $E$. Further, a comparison of the set of (8) with Fig. 3 yields a technique which facilitates the calculation of the partial derivatives which, in

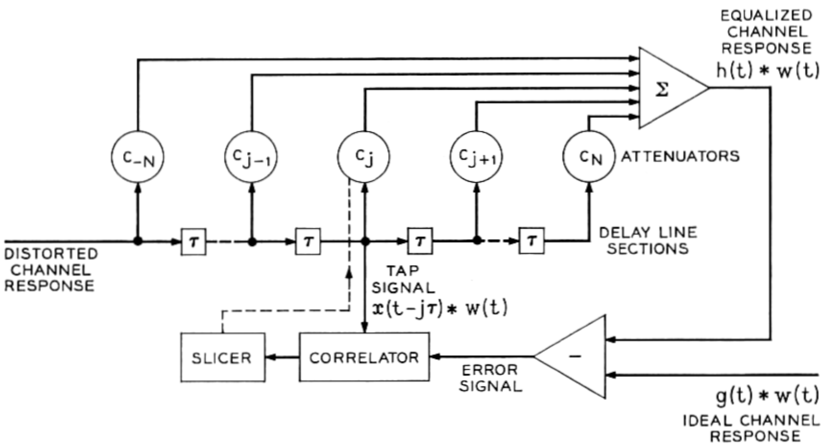



Fig. 3 — Mean-square transversal equalizer.

turn, provides the basis for an algorithm for automatic equalization under the mean-square error criterion.

### 2.2 *Basic Implementation*

The tapped-delay line structure which forms the basis of a transversal filter is shown in Fig. 3. The "attenuators" have the capability of supplying both positive and negative weights. The first term in the set of (8) is simply the error signal, or the difference between the equalized channel response and the ideal channel response. The second term, which multiplies the first, is the signal at the $j$th tap when (8) is written for $\partial E/\partial c_j$. Thus, the partial derivative of the distortion with respect to a particular attenuator setting is given by the time-integral of the product of the error signal and the signal at the particular tap being considered. In other words, the partial derivative is given by the cross-correlation of the error signal with the tap signal.

Coincident with the start of the equalization process, the various cross-correlation coefficients for all of the delay-line taps are calculated by the correlators. The polarity of a particular cross-correlation coefficient indicates the polarity of the partial derivative of the distortion with respect to the corresponding tap weighting coefficient. Because of the convexity of the criterion this polarity information indicates the direction in which the tap weight must be changed to reduce the distortion. When all cross-correlation coefficients become zero, no further adjustment of the weights can lower the distortion and the desired equalization is achieved.

Some feeling for the algorithm can be obtained from the following argument. The signals at the various taps contribute to the error signal in linear fashion. The best that the equalizer can expect to achieve is the elimination of any systematic contribution between the tap signals and the error signal. Under a mean-squared error criterion the measurement of such a systematic contribution is cross-correlation. When all the cross-correlation coefficients are zero, nothing further can be done to reduce the error.

### 2.3 *Related Applications*

In the course of equalization, an automatic equalizer is called upon to perform a network synthesis. Specifically, it synthesizes that network within its repertoire which results in the minimum mean-squared error. It is possible to use the automatic equalizer simply as an automatic network synthesizer.

The distinction between these two cases (channel equalization and network synthesis) is made in Fig. 4. Fig. 4(a) shows the conventional application of the equalizer wherein the equalizer strives to first determine and then synthesize the function

$$C(\omega) \cong 1/X(\omega), \tag{9}$$

where $X(\omega)$ is the frequency response function of the distorting channel. If the equalizer could perfectly synthesize $1/X(\omega)$ (plus an arbitrary flat time delay) the distortion would be completely removed. The use of the equalizer for network synthesis is shown in Fig. 4(b). Here, the transversal filter with complex frequency response $C(\omega)$ strives to approximate $A(\omega)$ directly so that the quantity

$$E = \int_{-\infty}^{\infty} | A(\omega) - C(\omega) |^2 \, d\omega \tag{10}$$

is minimized. As in the case of channel equalization the error can be given a frequency sensitive weighting, $W(\omega)$.

So far, the discussion has centered upon an equalizer of the transversal filter type (as in Fig. 3). This is by no means the only possibility, and a more general equalizer/synthesizer is shown in Fig. 5. The common ground shared by the schemes (as shown in Fig. 3 and 5) is that both rely upon the sum of weighted responses. The parallel net-

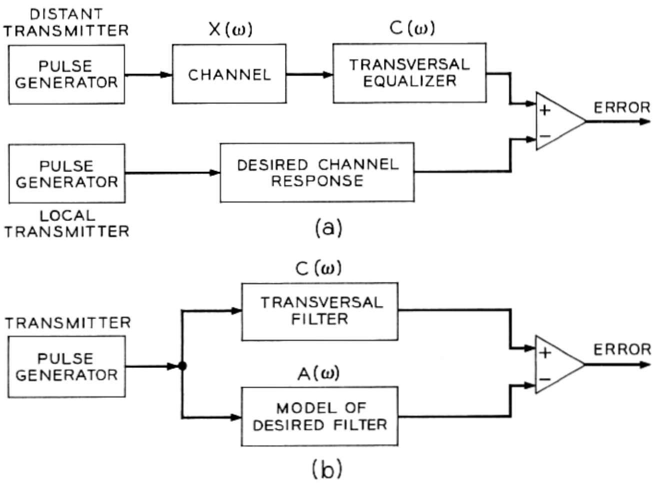



Fig. 4 — (a) Channel equalization. (b) Filter syntheses.

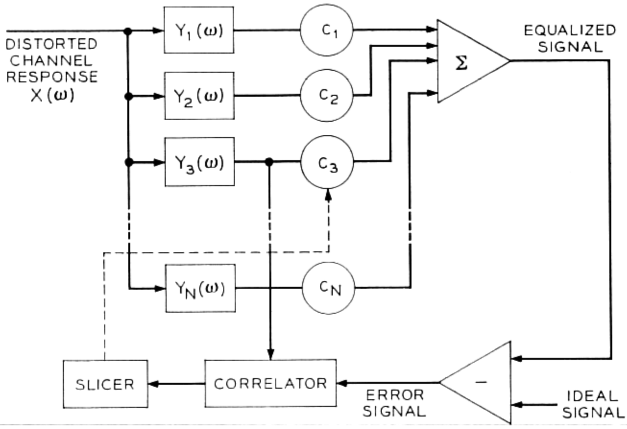



Fig. 5 — Generalized mean-square equalizer/synthesizer.

work layout of Fig. 5 has greater flexibility than the series network layout of Fig. 3. The series layout has the advantage that much of the filtering necessary for one particular response is performed by the preceding networks.

For practical reasons only it is required that the responses of the various networks shown in Fig. 5 be linearly independent; it is desirable (but by no means necessary) to have network responses orthogonal to each other so as to minimize the interaction between the setting of the various weighting coefficients. The desirable orthogonality results when (11) is satisfied.

$$0 = \int_{-\infty}^{\infty} | X(\omega) |^2 \, Y_i(\omega) Y_j(\omega) \, d\omega, \qquad i \neq j. \tag{11}$$

In (11) the $Y_i(\omega)$ are the transfer functions of the various networks and $X(\omega)$ the Fourier transform of their common input. A discussion of various sets of such orthogonal networks may be found in Lee.[13]

If $X(\omega)$ is constant from dc to $f_1$ Hz and if the taps on a delay line are spaced at $1/2f_1$ second intervals, the desired (but again not necessary) orthogonality is obtained. In the case of the equalization of a communication channel, orthogonality can not usually be obtained. Here $X(\omega)$ is affected by the amplitude response of the distorting channel and this of course is unknown, *a priori*.

An application closely related to network synthesis is that of a relatively new technique for echo suppression: echo cancellation. This problem most commonly occurs in long-haul voice communication. Here the possibility of an improperly terminated hybrid makes undesirable returned echoes probable. These echoes are generally dispersed in time by the transmission medium. Previous techniques have introduced attenuation into the echo path. The recently developed technique uses, instead, principles identical to those developed here to generate a replica of the echo. The actual echo and its replica are then added together in such a fashion that they cancel. This can be achieved automatically and adaptively as discussed in Refs. 14 through 17.

### 2.4 *Performance in the Presence of Noise*

In the network synthesis problem, the environment is largely under the control of the designer and as a result noise represents a negligible problem. This is not the case for equalization, where noise is definitely to be reckoned with. Noise effectively alters the equalized channel's frequency characteristic. It will be shown in what follows that the change in the frequency characteristic is a desirable one, i.e., the total mean-square error is minimized.

Noise also increases the settling time in a very complicated fashion. However, in the implementation discussed, this increase is very small and for that reason will not be further discussed here.

#### 2.4.1 *The Mean-Square Criterion in a Noisy Environment*

In the process of equalizing a communication channel to approach the desired flat amplitude and linear phase-frequency responses, care must be taken that the noise in the channel is not increased to harmful levels. Ideally, when noise is present the equalizer should minimize the average total error consisting of both the component resulting from the imperfect channel frequency characteristic and the component resulting from noise. If the spectral weighting function $W(\omega)$ is chosen properly, the equalizer described here attains this objective. The noise in the channel is assumed to be the same during and after equalization. It will be shown that the proper choice (in the sense above) is a $W(\omega)$ function which makes the equalizer test signal's power spectrum duplicate the information signal's power spectrum.

Consistent with the notation used previously, let the channel [with impulse response $x(t)$] be used to transmit information $w(t)$. (The square of the amplitude frequency response of the error weighting filter is thus picked to be identical with the power spectral density of the transmitted signal.) The received noise $\eta(t)$ will be taken as a sample from a stationary random process. Thus, the received signal, $y(t)$, is given by

$$y(t) = w(t) * x(t) + \eta(t). \tag{12}$$

The error criterion $E_n$ is again taken as the average mean-square error between the equalized received signal $h(t)$ and the transmitted signal passed through the ideal, noiseless channel $G(\omega)$.

$$E_n = \langle [h(t) - w(t) * g(t)]^2 \rangle. \tag{13}$$

The brackets $\langle\ \rangle$ denote a time average. The equalized signal $h(t)$ is given by

$$h(t) = \sum_{n=-N}^{N} c_n[\eta(t - n\tau) + w(t) * x(t - n\tau)] \tag{14}$$

using the transversal filter equalizer of Fig. 3. As before, the partial derivatives of the distortion are computed with respect to the various tap gains $c_j$.

$$\frac{\partial E_n}{\partial c_j} = 2\langle [h(t) - w(t) * g(t)][\eta(t - j\tau) + w(t) * x(t - j\tau)] \rangle. \tag{15}$$

When this relation is compared with Fig. 3, it is seen that the expected value of the output signal of the cross-correlator is given precisely by (15). Thus, the equalizer does minimize the total expected mean-squared error in the presence of noise. Again, this is true provided that the test signal used for purposes of equalization has a spectral density identical to that of the signal to be transmitted over the equalized channel.

Often the power spectrum of the information transmission signal is not known beforehand. In this instance a flat weighting can be used. Examples of the effect of various weighting functions are given in Section IV.

## III. IMPLEMENTATION

This section is devoted to the description of an implementation of a general-purpose automatic equalizer. The discussion of the implementation will be broken down into three parts: The automatic transversal filter itself, carrier recovery, and timing recovery. The first part is of general interest and pertinent to both the problems of automatic equalization and network synthesis; the second and third parts are peculiar to the equalization process. The need for carrier recovery arises because of the incidental modulation which can occur in some transmission channels (notably those involving carrier facilities). Timing is needed to ensure the proper synchronization of the desired signal generated at the equalizer with the signal received from the distant transmitter.

Throughout this discussion, reference will be made to Fig. 6.

### 3.1 *Implementation of the Automatic Transversal Filter*

Many sets of functions could have been used as the basis functions for the equalizer. Only one set, the set of functions generated at regu-

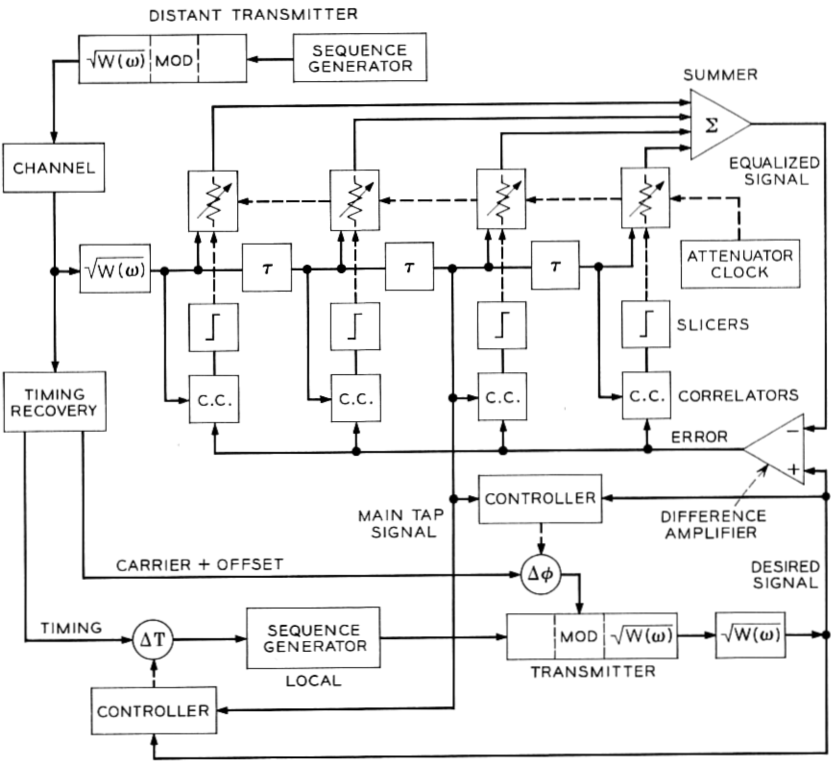



Fig. 6 — Equalizer block diagram.

larly spaced taps on a delay line, will be discussed here. There are two reasons for this, the first being that the transversal filter has been found to be a reasonably efficient means for the removal of distortion and the second being that considerable experience with the use of tapped delay lines is available.[1]

In the selection of an appropriate delay line, three parameters must be established: bandwidth, tap spacing, and number of taps. Because the equalization is carried out at passband, it is clear that the usable bandwidth of the delay line must be at least coincident with the channel's bandwidth. Often, as in a telephone voice channel, the passband extends sufficiently close to dc that it is reasonable to use a line which provides delay from dc to the upper frequency limit of the passband.

The tap spacing has already been touched on in Section 2.1. For the case just mentioned, the tap spacing $\tau$ was chosen equal to the reciprocal of twice the upper band-edge frequency, thus making the tap spacing slightly smaller than the Nyquist interval. The alternative in this case would be separating the taps by the Nyquist interval and providing additional, frequency-independent phase shifting networks at the various taps. This is equivalent to translating the passband into a comparable low-pass channel, equalizing, and retranslating to passband.

The number of taps necessary depends on the nature and degree of the dispersion (or distortion) likely to be found in the channel and on the precision of the equalization desired. A very rough approximation can be obtained from paired-echo theory.[18] This estimate equates the necessary number of taps to four times the number of cycles in the highest frequency Fourier series component needed to represent the distorting frequency characteristic function. The accurate determination of the necessary number of taps can be made only by case-by-case calculation. Examples showing the effect of a varying number of taps will be given later.

The attenuators associated with each tap on the delay line are capable of providing both positive and negative weights to the tap signals. The attenuators are controlled by digital counters composed of a number of binary memory elements. These are connected in such a fashion that the total count can be increased or decreased by one at any time and are therefore given the name up-down counters. All the attenuators are changed at the same time by a common clock. The outputs of the binary elements control the solid-state switching of constant-resistance ladder networks. A full count corresponds to a normalized tap weight of $+1$, a zero count to $-1$, and a half-full count to 0. Each attenuator is thus a kind of granular potentiometer with constant increments. The number of increments is determined by the number of binary elements and for $K$ elements is equal to $2^K$.

The number of steps in the attenuator and the relative ranges of the attenuator determine an upper bound on the accuracy of the equalizer. Taking into account the required polarity information and assuming the attenuator settings to be off by half an increment, the accuracy to which an attenuator may be set is $1/(2)^K$. If there are $(2N+1)$ taps, then the maximum signal-to-noise ratio (considering the residual distortion as noise) attainable is

$$(\mathrm{S/N})_{\mathrm{RES}} = 10 \log_{10} \frac{(2)^{2K}}{(2N+1)} \mathrm{dB} \tag{16}$$

or

$$(\mathrm{S/N})_{\mathrm{RES}} \cong 6K - 10 \log_{10} (2N+1) \mathrm{dB}. \tag{17}$$

In the implemented equalizer of 19 taps and 10-bit attenuator-counters this residual signal-to-noise ratio is about 54 dB. The relations above assume the ranges of all attenuators to be the same. Often the characteristics of the channels to be equalized permit the ranges of the various attenuators to be tapered as one moves from the center towards the ends of the delay line. This would make the above estimate somewhat pessimistic.

The settings of the attenuators are controlled by the cross-correlators whose inputs are the error and delay line tap signals. The multiplying function necessary in measuring cross-correlation is accomplished through the use of a switched modulator driven by a pulse-width modulated signal. The output so obtained is directly proportional to the normalized cross-correlation coefficient and the magnitudes of the two input signals. This particular scheme was selected from the many available because first, it is capable of handling the very large dynamic ranges of the two input signals and second, it determines the true cross-correlation, thereby guaranteeing convergence for all reasonable input signals.

The measurement of cross-correlation also requires integration in time, in fact, integration over the infinite interval. This is, of course, simply too long to wait. A simple resistor-capacitor low-pass filter provides a suitable approximation to real integration.

The outputs of the low-pass filters in the correlators are sliced about the zero level. The polarity of the output signals from the slicers determines whether the corresponding counter is incremented or decremented when a repetitive clock pulse occurs.* (The repetition rate of this clock pulse will be discussed later.) When the equalization reaches equilibrium, the clock pulses are removed and the attenuator weightings are retained permanently by the binary memory elements.

An equalizer consisting of the elements just described is shown in Fig. 7. The tapped delay lines are shown clustered in the top left-hand corner. The remaining cards in the top row are the resistive-ladder attenuators. There are 20 attenuators, 19 associated with the 19 delay-line taps and the remaining unit serving as part of the automatic gain control loop which regulates the signal level on the delay line. The two rows below the attenuators serve only as lamp indicators for the attenuator settings. The two rows below the lamps contain the binary memory elements and associated logic. The bottom row of cards consists of the cross-correlators. This equalizer was constructed for voiceband use; the delay line has a usable bandwidth in excess of 3,000 Hz, and the tap spacing is 150 microseconds. Examples of its performance will be given in a subsequent section.

### 3.1.1 *Settling Time*

The settling time (the time required for the equalizer to reach equilibrium) is determined in large measure by the time-constant of the low-pass filter in the correlators and the frequency of the clock which controls the counters.

Nothing has been said to this point about the nature of the test signal used to determine the equalizer settings. The test signal is a passband signal obtained by modulating a smoothed pseudo-random sequence† into the passband frequency range. The pseudo-random sequence[19] was used because this facilitates the generation of identical signals at the transmitter and receiver. The smoothing is done in accordance with the error spectrum weighting filter $W(\omega)$; the modulation is necessary because of the likelihood of frequency offset on carrier transmission facilities. These subjects will be treated in greater detail later.

The pseudo-random sequence has a periodic auto-correlation func-

* The magnitude of the cross-correlation coefficients can be used to control the rate of change of the attenuator settings as in Refs. 10, 15, and 16.

† The pseudo-random sequence is a repetitive sequence of binary digits chosen in a random manner. The sequence can be generated by a binary shift register.

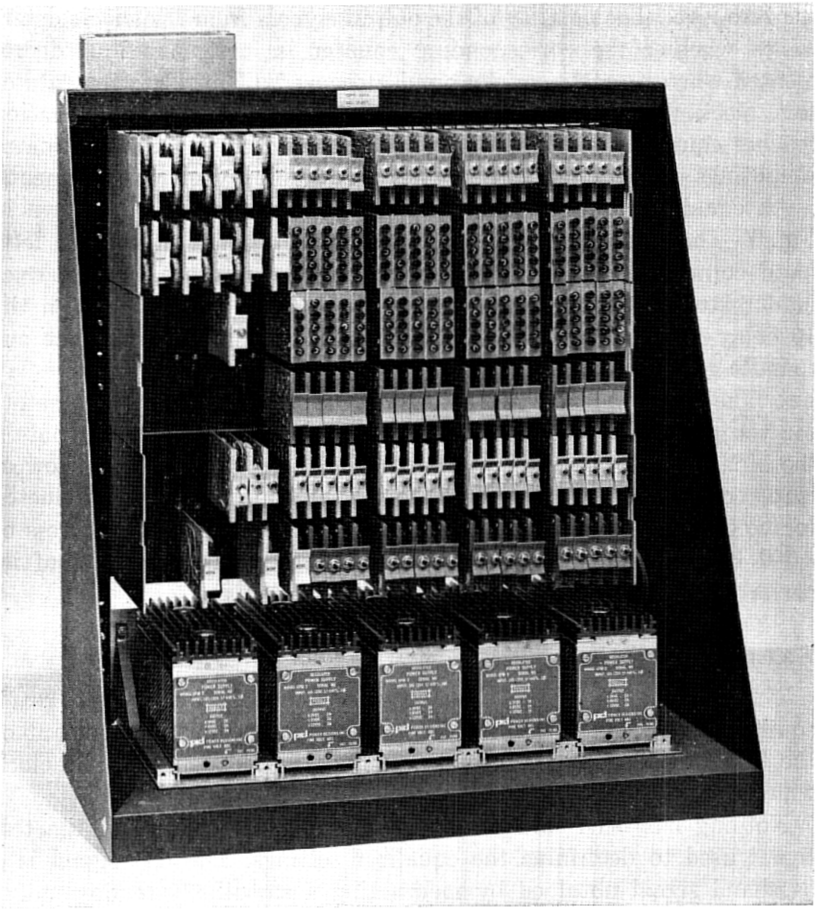


Fig. 7 — Photograph of 19-tap equalizer.

tion. The sequence generator must be designed in such a manner that the period of this auto-correlation function is larger than the length of the expected dispersion in the channel. If this were not the case, the correlator would react to properties of the test signal, rather than to properties of the channel. It is then necessary to integrate, in the correlator, for a length of time corresponding to several periods of the pseudo-random sequence. In the implemented equalizer, the integration is performed in a simple resistor-capacitor low-pass network; the RC-product was established at about four times the pseudo-random sequence period. The polarity of the output of the correlator must be sampled at a still slower rate so that time is provided for the integrators to reach the steady-state after a change in the attenuator settings. Again, several time constants must be allowed for this to take place. The ratio of the repetition rate of the pseudo-random sequence to the sampling rate of the correlators is about 20 for this particular implementation.

Thus, in general, the length of the dispersion of the channel in time (i.e., the length of the significantly nonzero portion of the channel's impulse response) determines the repetition rate of the pseudo-random sequence and, in turn, this repetition rate determines the rate at which the attenuators are adjusted. The settling time for the equalizer can then be calculated by dividing the number of steps the attenuator must change by the rate of change.

As an example consider a voice channel wherein most of the dispersion is confined to a five millisecond interval. If the repetition rate of the pseudo-random sequence is established at 10 milliseconds, then in accordance with the above comments the clock rate for the pulse controlling the attenuators should be 20 times slower or about 5 Hz. In an equalizer using 10-bit attenuators and starting from the reset condition of zero attenuator weights, the longest travel of an attenuator would be some 500 increments. It would take 100 seconds to traverse the full range. This is a rather long time to wait, even for an equalizer used in such a manner that it is divorced from the communicating modems. There is, fortunately, an easy remedy and this involves letting the attenuators run rapidly to their approximate values and then slowly to their exact values. This dual-mode operation of the attenuator clock can decrease the settling time by a considerable factor.

The settling time for this particular implementation is 10 seconds. This is achieved by running the attenuator clock at a high rate for a fixed initial period and then by continuing operation at a slower rate.

### 3.2 *Carrier Recovery*

The implementation was designed for use on all voice-frequency channels, including carrier channels. The nature of carrier channels is such that the channel may introduce a slight frequency shift. If such a frequency shift were not compensated, the output of the correlators would be modulated by the shift frequency, ruling out the possibility of satisfactory operation. There are two equivalent means of dealing with this frequency shift. The first is to remove the frequency shift from the received channel signal and the second is to alter the modulating frequency of the generator of the comparison or desired signal as indicated ("carrier + offset") in Fig. 6. In either case, the frequency offset generated by the channel must be detected; the latter scheme was selected here.

A technique suggested by F. K. Becker[20] was used to recover both modulating frequency and the frequency necessary to drive the random sequence generator. In this approach, two pilot tones are added to the transmitted signal, one each at the upper and lower edges of the band of interest. These tones can then be combined in such a fashion that the transmitted carrier plus frequency offset can be recovered. At the same time, by combining the two pilot tones in another manner, the sequence generator clock can be recovered. In the case of the equalizer shown in Fig. 7, the modulating carrier frequency (2400 Hz) plus carrier offset ($\delta$ Hz) is obtained from the two pilot tones at 600 Hz and 3000 Hz as indicated by (18).

$$(2400 + \delta) = (3000 + \delta) - \frac{(3000 + \delta) - (600 + \delta)}{4}. \qquad (18)$$

Once the proper modulating frequency is obtained, it remains to establish the proper phase. This is achieved by transmitting energy at the carrier frequency. The phase of the carrier generated at the equalizer is adjusted until it agrees with the received carrier phase as it appears at the output of the "main" equalizer tap. This is achieved through the use of a cross-correlator. After the proper phase has been established, the variable phase shift element is locked.

### 3.3 *Timing Recovery*

In conjunction with the discussion of settling time, it was stated that the test signal is derived from a pseudo-random sequence generator. It is necessary to synchronize the remote and local generators (which are identical) so that near-optimum use is made of the transversal filter.

Like the modulating carrier, the clock frequency required to drive the sequence generator at the equalizer is derived from the two pilot tones. In the implemented equalizer shown in Fig. 7, the clock frequency of 2400 Hz is obtained via the relation

$$(2400) = (3000 + \delta) - (600 + \delta). \qquad (19)$$

In addition to obtaining the proper phase for this clock, it is necessary to synchronize the random 63-bit sequences. These ends are attained in a sequence of two steps.

It is known that the autocorrelation function $R_{pp}(\nu)$ of a pseudo-random sequence of the variety used here has a shape like that of Fig. 8(a). (In the equalizer, the $W(\omega)$ weighting function causes a smoother function to be generated for the autocorrelation function of the desired signal.) The timing recovery circuitry is built upon this fact.

What in essence is needed is an estimate of the arrival time $T$ of the received signal $x(t)$. Knowing this, the desired signal $g(t)$ can be properly synchronized. A maximum likelihood estimate of $T$ is developed using a correlation detector.[21] Under the assumption that the noise is Gaussian, white, and additive, it can be shown[21] that the maximum likelihood estimate of $T$ can be found by adjusting $T$ so that

$$q(T) \equiv \int_0^{t_1} g(t + T)x(t)\,dt \tag{20}$$

is a maximum. Because of the noise component in $x(t)$ there will be some ambiguity in deciding exactly where the maximum of $q(T)$ is, but this ambiguity can be reduced by increasing the length of the observation time $t_1$. In fact, when $t_1$ is very large $q(T)$ approaches the $R_{pp}(T)$ shown in Fig. 8(a), assuming no spectral weighting, band limiting, or channel distortion.

It is known that the effect of linear distortion in a bandlimited channel can be represented in terms of pairs of echoes of the impulse response in the time domain.[18] An estimate of $T$ is obtained for the distorted $x_d(t)$ just as it was in the distortion-free case but because of

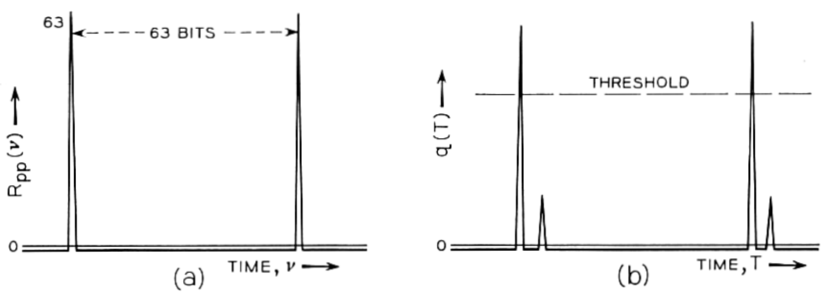



Fig. 8 — Synchronization waveforms. (a) Autocorrelation function of pseudo-random word generator (63-bit length). (b) $q(T)$ function for pseudo-random word generator in presence of linear distortion.

the distortion only an approximation to the maximum likelihood estimate is obtained. Again

$$q(T) = \int_0^{t_1} g(t + T)x_d(t)\, dt \tag{21}$$

is maximized. As in the distortion-free case the ambiguity in $q$ resulting from noise can be made vanishingly small by making $t_1$ very large. However, the distortion echoes do contribute systematically to $q(T)$ and an increase in the observation time does not diminish their contribution. If the distortion were such that a single echo were introduced by the channel, the $q(T)$ function might have the appearance of Fig. 8(b), again ignoring the effects of band-limiting and smoothing by the filter $W(\omega)$. It can be seen, then, that linear distortion makes the search for the absolute maximum of $q(T)$ more difficult by introducing greater undulations in the $q(T)$ function. Because of the complexity of the $q(T)$ function, the search for its absolute maximum is made in two successive modes.

In the first mode, gross synchronization is attained. This means that the timing of the desired waveform sequence is shifted until it is roughly lined up with the received signal as it appears at the "main" tap. This coarse alignment is obtained by cross-correlating the two signals just mentioned and comparing the result with a fixed threshold. Until the output of the cross-correlator, $q(T)$, reaches the threshold, the phase of the timing signal is continuously increased (over an interval which may be as large as 63 symbol periods in the case of the 63-bit sequence). When the threshold is reached the phase is locked. The threshold is determined empirically so that only the one large spike (corresponding to the undistorted pulse) penetrates the threshold. Thus, in the first mode the proper "spike" of $q(T)$ is found; in mode 2 the maximum of this spike is found.

The maximum of $q(T)$ can be found by partial differentiation of (21) with respect to $T$ and setting the result equal to zero.

$$\frac{\partial q(T)}{\partial T} = 0 = \int_0^{t_1} g'(t + T)x_d(t)\, dt, \tag{22}$$

where $g'(t)$ is the time derivative of $g(t)$. This approach could not be used from the start because $q(T)$ can be assumed to be a convex function only over a small region. The operation indicated in (22) is achieved through yet another cross-correlator.

A few words are in order about what has been called the "main" tap—that tap which is used for both carrier and timing reference. The main tap would normally be the center tap on the delay line. It turns out empirically, however, that most distorting echoes lag the undistorted impulse. Hence, lower residual distortion is obtained by shifting the main or reference tap to a position about two-thirds down the delay line.

## IV. PERFORMANCE

### 4.1 *Computer Simulations*

In order to determine the theoretical performance of this equalization technique, a computer simulation was made. Fig. 9 shows results for a voiceband channel. The unequalized channel characteristic was taken as a typical Direct-Distance-Dialed connection as given in Alexander, Gryb, and Nast.[22] The amplitude characteristic has a 15 dB/octave falloff starting at 240 Hz, is flat from 240 to 1100 Hz, has a linear logarithmic slope to 7.6-dB loss at 3000 Hz and an 80 dB/octave loss commencing at 3000 Hz. The delay characteristic is parabolic, centered at 1500 Hz, with a maximum delay of 1 millisecond at 0 and 3000 Hz. In the simulation, the error spectral weighting function $W(\omega)$ used was of raised cosine shape, symmetric about a peak at 1650 Hz and zero at 300 and 3000 Hz. The tap spacing was established at 150 microseconds. In Fig. 9 the amplitude and delay frequency-response curves for both unequalized and equalized channels are shown. Three cases are shown, those of 9, 13, and 25 taps.

A simulation was also made for a baseband channel with group bandwidth.‡ Only the amplitude frequency responses are shown because the delay distortion was not significant in this particular case and remained essentially invariant throughout the equalization process. Both uniform and nonuniform spectral weightings were investigated. In the cases where a nonuniform spectral weighting $W(\omega)$ was used, $W(\omega)$ was selected as a half-cosine rolloff shape, essentially flat to 12.5 kHz, and then falling to zero at 37.5 kHz as a cosine. Energy at very low frequencies was given small weighting by a simple high-pass filter with 2-kHz corner frequency. Fig. 10 displays the amplitude characteristics on both linear and logarithmic frequency scales as the number of taps is increased from 13 to 51, all with the half-cosine rolloff weighting. Performance improves with the number of taps but

‡ A "group" is twelve voice channels with a bandwidth of about $12 \times 4$ or 48 kHz.

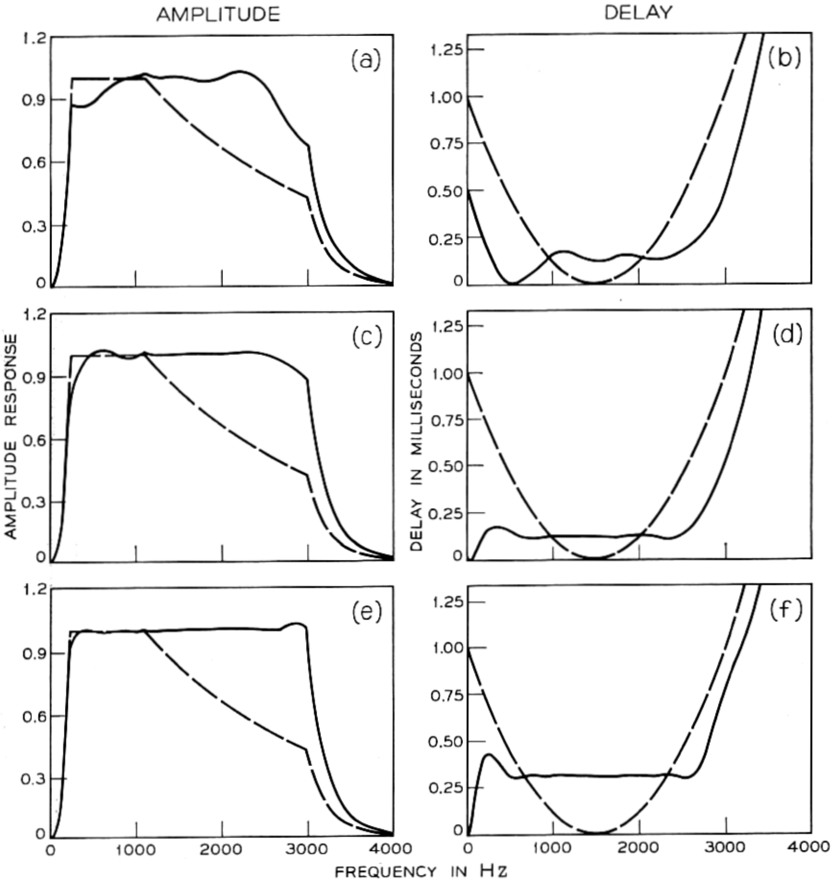



Fig. 9 — Simulated voiceband performance (a) Amplitude characteristics, 9 taps, raised cosine weighting. (b) Delay characteristics, 9 taps, raised cosine weighting (c) Amplitude characteristics, 13 taps, raised cosine weighting. (d) Delay characteristics, 13 taps, raised cosine weighting. (e) Amplitude characteristics, 25 taps, raised cosine weighting. (f) Delay characteristics, 25 taps, raised cosine weighting.

the change is rather subtle compared with the voiceband case. Fig. 11 illustrates the effects of error weighting (weighted and unweighted) and the effect of signal-to-noise ratio for the case of white noise. Note that in the case of very small noise, the equalized channel characteristic may behave erratically in the region where the error has very little weight (i.e., hear 37.5 kHz). The addition of noise or the use of a more uniform weighting prohibits this behavior. Note also that the use of weighting forces the best equalization to occur in the region of highest weight, i.e., 2 to 25 kHz.

It may also be seen that in the case of abnormally high noise the equalizer minimizes the total error energy consisting of both distortion and noise as predicted in Section 2.3.

### 4.2 *Measurements on Real Facilities*

Measurements have also been made with the experimental equipment operating over real facilities. Fig. 12 shows the equalization of an actual L-carrier looped facility from Holmdel, N. J. to Chicago and return. Both the unequalized and equalized delay and amplitude frequency responses are shown. The results are those for thirteen taps with a raised cosine error spectral weighting function with zero weight at 600 and 3000 Hz. Fig. 13 shows the binary eyes resulting from

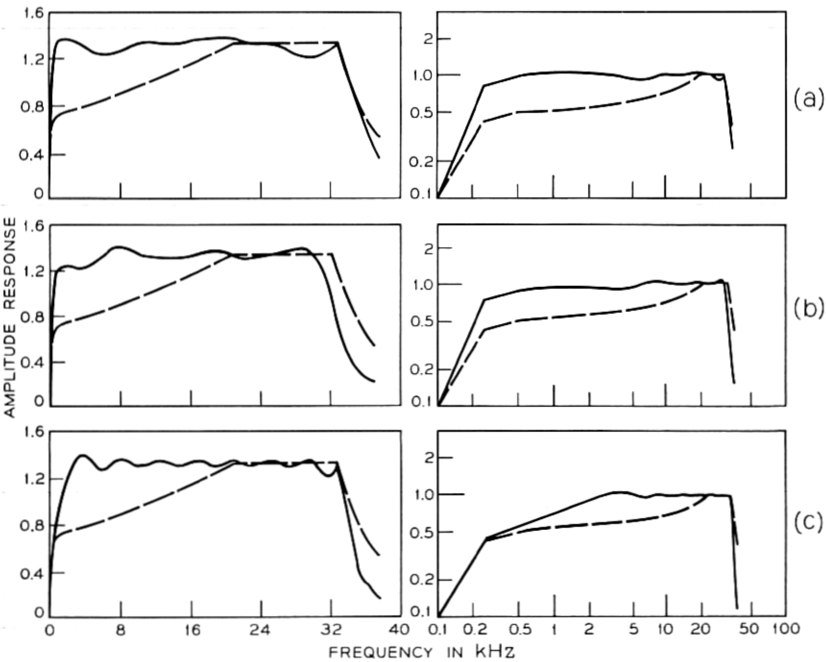



Fig. 10 — Effect of number of taps.—group band, weighted error, S/N=25dB. (a) 13 taps. (b) 25 taps. (c) 51 taps.

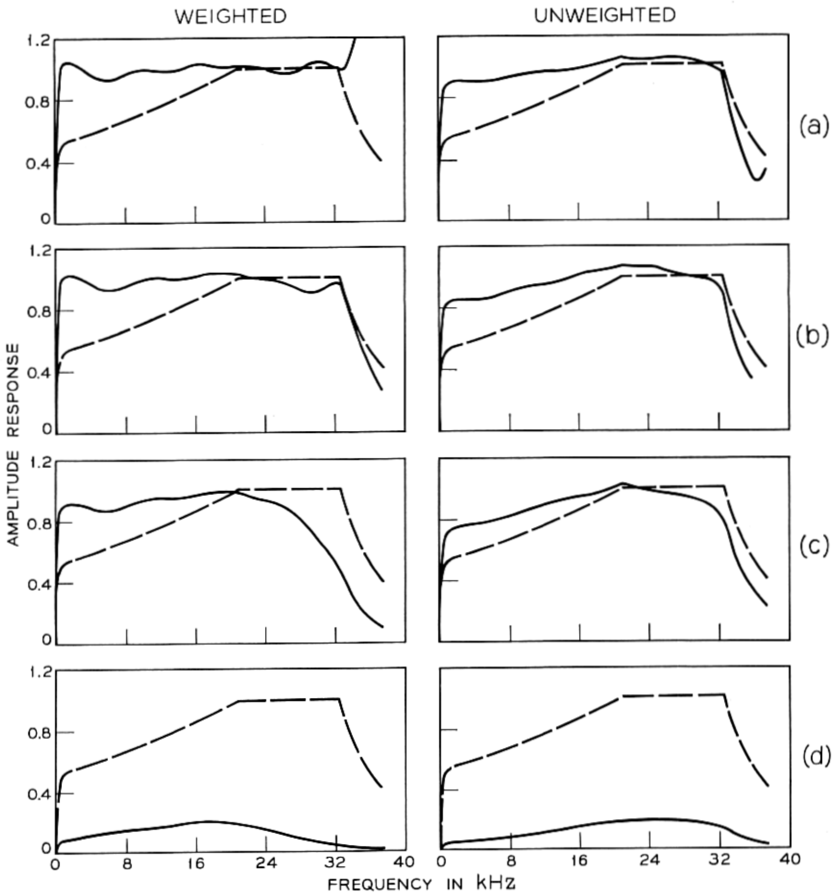



Fig. 11 — Effect of signal-to-noise ratio and error weighting. Group band, 15 taps (a) S/N=95dB. (b) S/N=25dB. (c) S/N=10dB. (d) S/N=−10dB.

transmission of an FM data signal on a DDD looped facility from Holmdel to Denver and return. A 19-tap equalizer was used for the rms-equalized case. As an example of asynchronous transmission over similar facilities, Fig. 14 shows facsimile transmission, equalized and unequalized. Fig. 14 was obtained using the Bell System DATA-PHONE* Data Set 602A which contains an FM modem.

The nineteen-tap equalizer shown in Fig. 7 was used to equalize a

* Service mark of the Bell System.

looped facility from Holmdel, N. J. to Omaha and return. The results of this test are shown in Fig. 15 with the requirements for a schedule 4B line. The weighting function is identical to that used for the equalization obtained in Fig. 12. As can be seen, the 4B requirements are met by the equalized channel except at the band edges.

### 4.3 *Filter Synthesis*

As an example of automatic filter synthesis, the curves in Fig. 16 were obtained. The system configuration is that of Fig. 4(b). The desired or model amplitude response is shown for comparison with 5-, 9-, and 19-tap approximations to it.

## V. CONCLUSION

Automatic Equalization is a powerful tool for increasing the efficiency of communication channels. The implementation described is of general utility and need not be married to a particular modem. It functions conveniently in the passband and is especially suited to the equalization of a large number of communication channels terminating at a common location where the adjustment circuitry can be shared. In addition, the principles of the techniques seem applicable to a wide class of problems.

Much work remains to be done before generalized automatic equali-

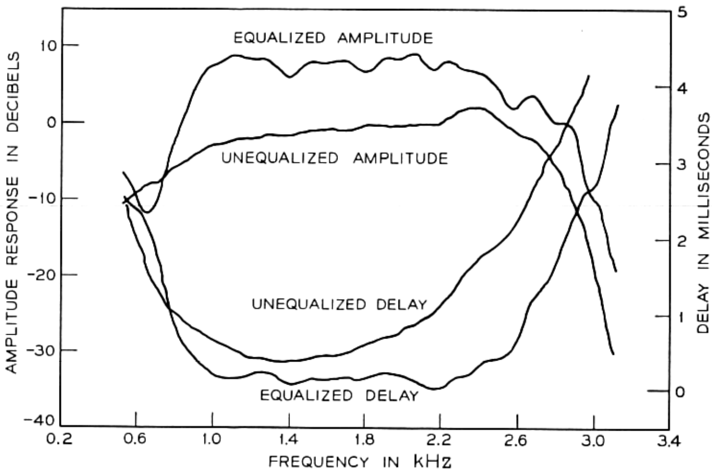



Fig. 12 — Actual performance—frequency response.

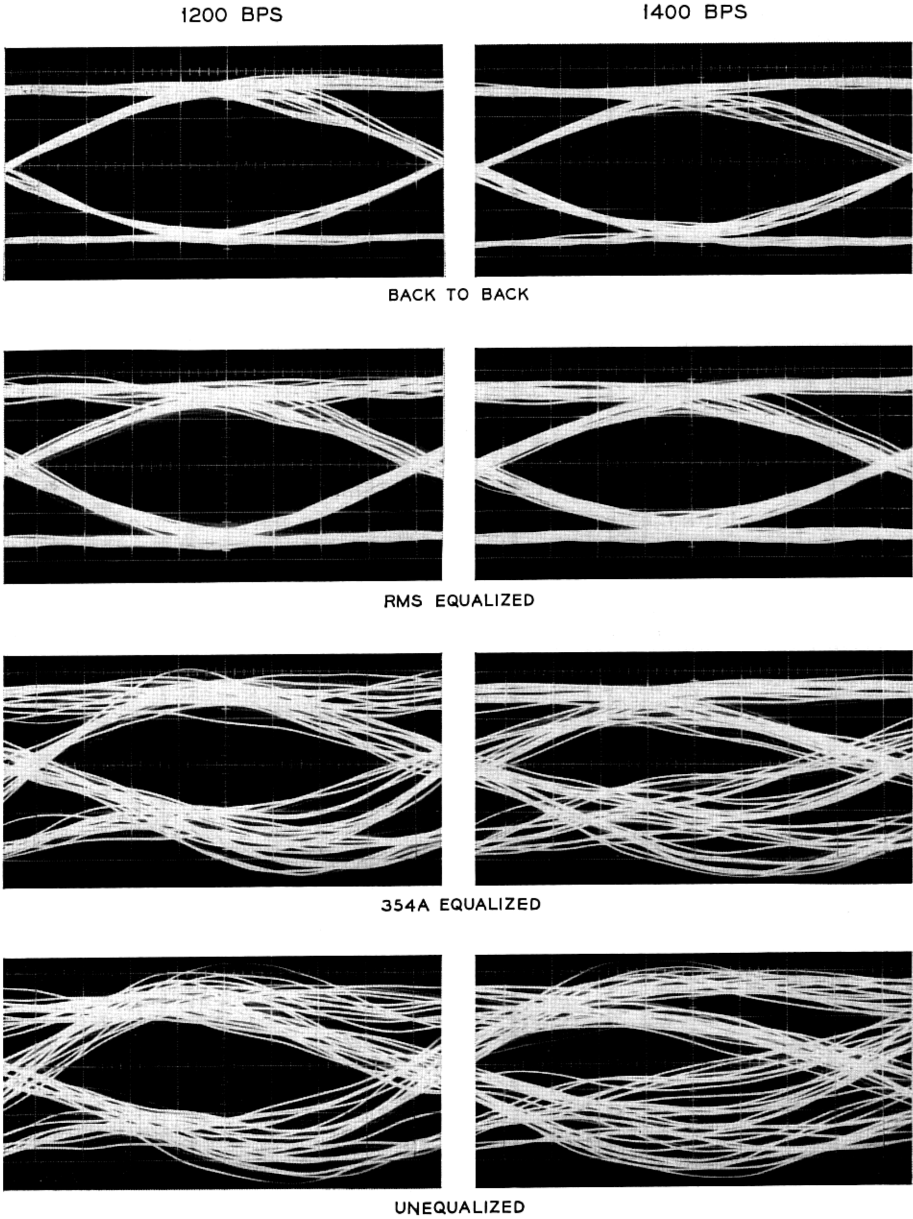
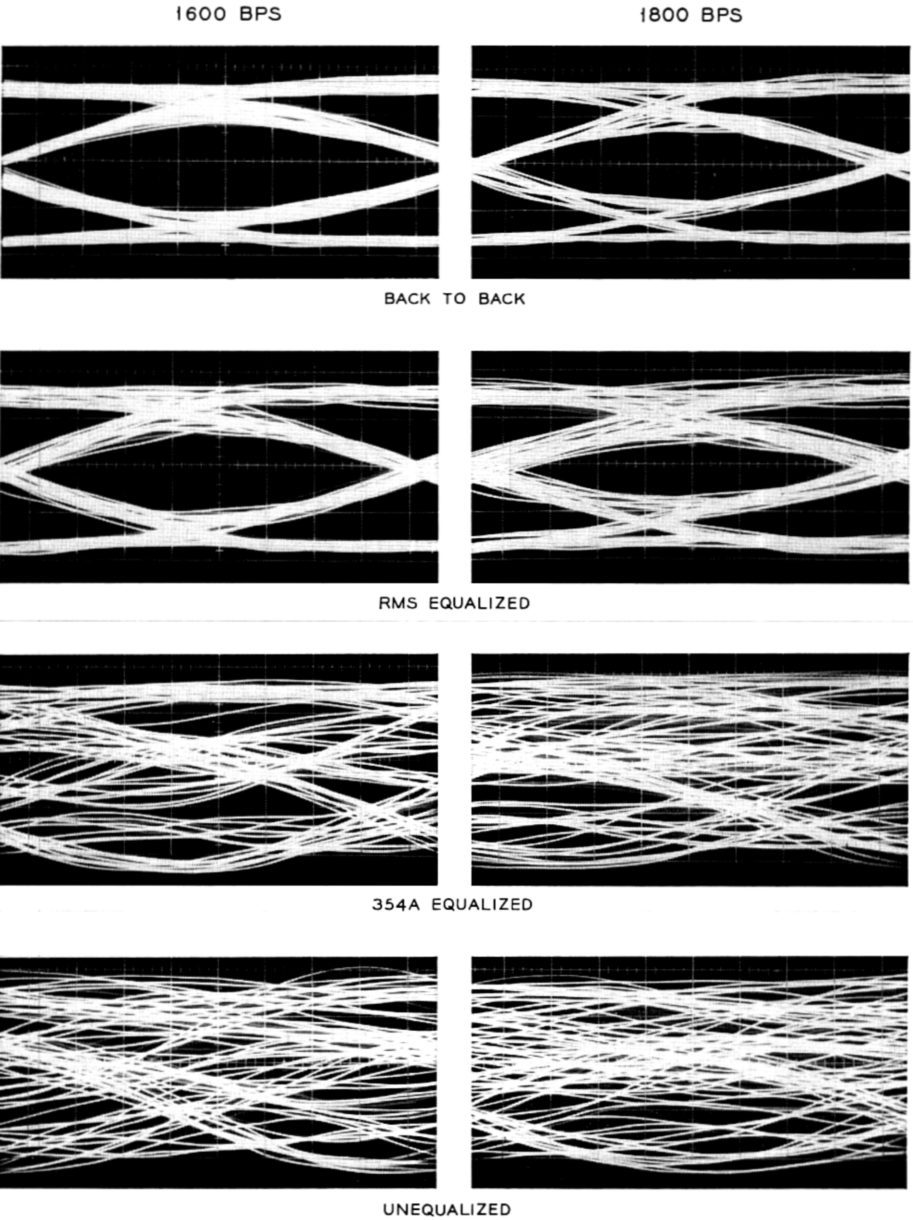



Fig. 13 — Actual performance—eye patterns for the Bell System Data Set 202D. —unequalized, partially equalized using a fixed compromise equalizer, automatically equalized, back-to-back operation.


1600 BPS
1800 BPS
BACK TO BACK
RMS EQUALIZED
354A EQUALIZED
UNEQUALIZED

zation becomes practical. In addition, the equalizer described here does nothing for the problem of nonlinear distortion or for the time-varying channel. However, the results obtained thus far are encouraging.

The authors gratefully acknowledge the encouragement and contribution of their colleagues, especially F. K. Becker, L. N. Holzman, and E. Port.

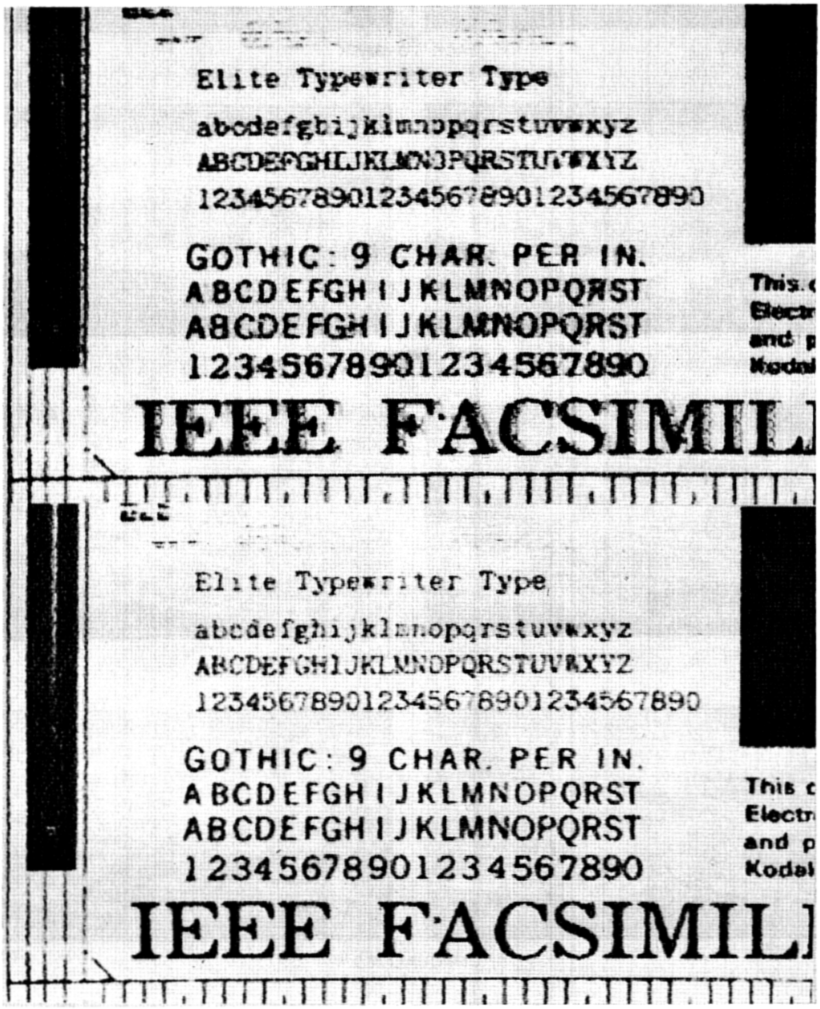



Fig. 14 — Actual performance—facsimile transmission.

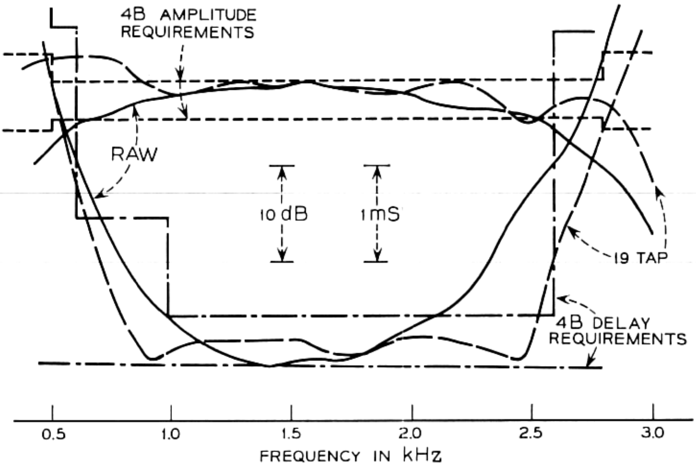



Fig. 15 — Actual performance—frequency response—Omaha loop.

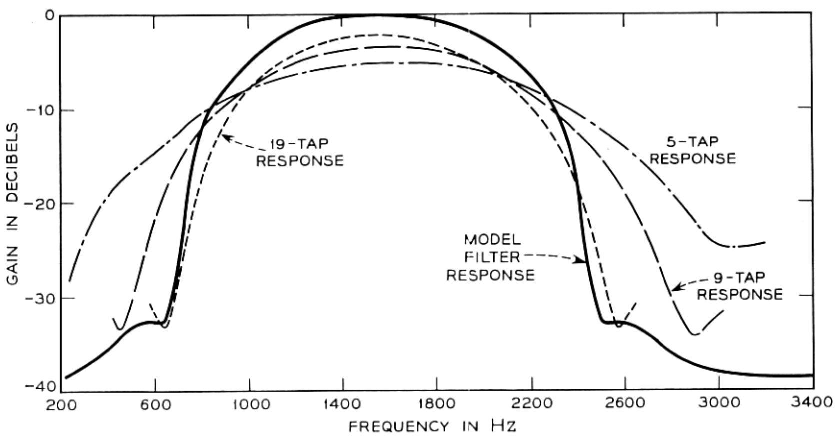



Fig. 16 — Actual performance—filter synthesis.

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