A Quantitative Theory of $1/f$ Type Noise Due to Interface States in Thermally Oxidized Silicon

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A quantitative theory of 1/f type noise is derived from the distribution of trapping times for charges in interface states. The distribution of trapping times has been recently explained quantitatively by means of a random distribution of surface potential caused by a random distribution over the plane of the interface of fixed charges located in the oxide. This model, which agrees with the interface state time constant dispersion measured by the MIS conductance technique, leads to a noise spectrum which is independent of frequency at very low frequencies, tends towards a $1/f^2$ dependence at high frequencies, and has an extended 1/f frequency dependence at intermediate frequencies. The mechanism for time constant dispersion is independent of temperature and silicon resistivity; it depends only on the majority carrier density at the silicon surface, the interface state density, and the density of fixed oxide charges. The dependence of open circuit mean square noise voltage on these parameters and frequency are illustrated for an MOS capacitor.

I. INTRODUCTION

It has long been recognized that states at the Si-SiO_2 interface which exchange charge with the silicon can give rise to $1/f$ type noise. Recently, Sah and Hielscher[1] have shown by experiment that the $1/f$ noise of a metal -SiO_2-silicon (MOS) capacitor is directly related to interface state density and capture conductance over the energy gap. Random capture and emission of carriers by interface states results in fluctuations of trapped charge. In an MOS capacitor, these charge fluctuations cause random changes in admittance constituting noise. These charge fluctuations can be calculated from the dispersion of interface state time constants. A major obstacle to a quantitative theory of $1/f$ type noise arising from interface states has been the lack of an experimentally established mechanism for interface state time constant dispersion. This obstacle has recently been removed. With the MIS conductance technique,[2, 3, 4] accurate small-signal measurements have been made of interface state density and capture conductance over the middle half of the energy gap in the Si-SiO_2 system. A large interface state time constant dispersion was observed in the depletion and accumulation regions. An explanation which quantitatively fits these measurements essentially without any arbitrary adjustable parameters is that the dispersion arises from a random distribution of surface potential over the plane of the interface. The random surface potential distribution is in turn caused primarily by a random distribution of built-in oxide charges and charged interface states over the plane of the interface. The noise measurements of Ref. 1 and the small signal conductance measurements of Ref. 2 through 4 suggest that a quantitative explanation of $1/f$ type noise of an MOS capacitor can be given in terms of the interface state time constant dispersion caused by the random distribution of surface potential and the resulting capture conductance.

It has been reported that low-frequency noise generated at semiconductor surfaces shows a $1/f^n$ spectrum with $n \approx 1$ over many decades of frequency.[5, 6] Various mechanisms have been proposed to explain this, such as slow states in the oxide or at the oxide-air interface or slow time dependent changes in the density of states at the semiconductor surface.[5, 7, 8] Atalla, et al,[6] have shown that surface generated $1/f$ noise extending over many decades of frequency is considerably reduced in magnitude when silicon is thermally oxidized. The noise theory presented here is based on conductance measurements made on thermally oxidized silicon samples prepared as described in Ref. 4. In these samples, oxide thickness is greater than 500 Å. These samples have stable electrical characteristics at room temperature under bias. Also, losses in the oxide layer and bulk silicon are found to be negligible.[2, 4] Thus, they should be free of noise mechanisms other than random emission and capture of carriers by interface states having a time invariant density. The case where interface state transitions dominate the loss as in the measurements of Ref. 4 will be the only case considered here.

This work clearly shows that in thermally oxidized silicon, surface-generated noise arising from random emission and capture of carriers by interface states does not explain a $1/f$ noise spectrum over many decades of frequency. Measurements in which a $1/f$ noise spectrum is found over several decades must involve additional mechanisms as mentioned.

The noise spectrum of a single level state, as is well known,[9, 10] is independent of frequency at low frequencies and has a $1/f^2$ frequency dependence at high frequencies. We shall show that the time constant dispersion found by conductance measurements introduces an intermediate range in the noise spectrum with a $1/f$ type frequency dependence. The resulting open circuit noise voltage appearing across the terminals of an MOS capacitor has been calculated and found to have a large $1/f$ type range and the same dependence on interface state density and capture conductance as in Ref. 1.

The MOS capacitor is the simplest case of interface state $1/f$ type noise to treat quantitatively because there is no dc current flow. The theory for the MOS capacitor will be worked out here in detail. This theory can be extended to explain $1/f$ noise arising from interface states in MOS field effect transistors and oxide passivated bipolar transistors because in these devices, time constant dispersion also will be caused by the random surface potential distribution. This extension will not be made here.

II. THEORY

We shall use the Nyquist formula for the calculation of noise. This is justified by the fact that in the MOS capacitor it is reasonable to assume that the interface states and the silicon are in thermal equilibrium with each other at each bias when no dc leakage current flows through the oxide layer. We shall consider the case where the applied voltage biases the silicon into accumulation or depletion up to within a few kT/q of mid gap. In these regions, majority carrier density is several orders of magnitude greater than minority carrier density at the silicon surface. Neglecting minority transitions will cause little or no error at the frequencies considered in this paper (0.1 Hz to 10^8 Hz) because in these regions of bias there is virtually no recombination-generation through interface states or states in the silicon bulk.[4] Diffusion from the bulk is also negligible. In the MOS capacitor, recombination-generation and diffusion are the only ways the minority carrier band can communicate with an external circuit. Thus, the noise we shall calculate arises primarily by the random capture and emission of majority carriers by the interface states. Fig. 1 shows the noise equivalent circuit for the MOS capacitor at a given bias and angular frequency ω. Using the Nyquist formula, the mean square noise current per cm^2 generated in $G_p(\omega)$ is

$$\langle i_p^2 \rangle = 4kTBG_p(\omega), \tag{1}$$

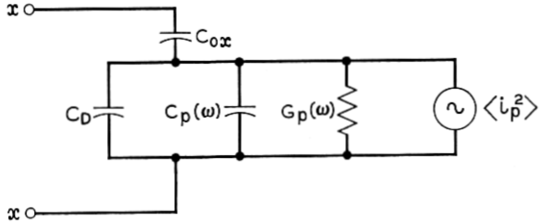

Fig. 1 — Noise equivalent circuit of MOS capacitor. C_{ox} is oxide layer capacitance in farads/cm², C_D is depletion layer capacitance in farads/cm², $C_p(\omega)$ is the interface state capacitance given by (9) in farads/cm², $G_p(\omega)$ is the interface state equivalent parallel conductance given by (8) mhos/cm², and $\langle i_g^2 \rangle$ is the mean square short circuit noise current in amps²/cm².

where $G_p(\omega)$ is the interface state equivalent parallel conductance in mhos/cm², k is Boltzman's constant in Joules/°K, T is the absolute temperature in °K, and B is the bandwidth in Hz. The mean square open circuit noise voltage × cm² appearing across the terminals x-x in Fig. 1 is then

$$\langle v_g^2 \rangle = \frac{4kTBG_p(\omega)}{G_p^2(\omega) + \omega^2[C_D + C_p(\omega)]^2}, \tag{2}$$

where $C_p(\omega)$ is interface state capacitance in farads/cm², and C_D is the depletion layer capacitance in farads/cm².

The problem in evaluating (2) is essentially to find the interface state admittance as a function of bias and frequency. This has been done previously as described in Refs. 3 and 4. This derivation will be briefly outlined here. It is based on a model in which the interface state time constant dispersion required for a $1/f$ type noise spectrum is caused by a random distribution of surface potential. A detailed analysis of this mechanism complete with experimental documentation can be found in Ref. 4.

2.1 *Depletion*

With the silicon surface in depletion or accumulation, it has been shown experimentally (see Ref. 4) that the ohmic loss in the oxide layer and the silicon space-charge region is negligible compared to the ohmic loss arising from transitions between interface states and the majority carrier band. Bulk silicon series resistance and contact resistance can be made negligible in practice[4] or calculated separately. Because this paper is restricted to a discussion of noise due to interface states, these two resistances will be ignored.

A single level interface state is not observed experimentally. Rather, the interface states are observed to be comprised of energy levels so closely spaced in energy that they cannot be distinguished as separate levels. They appear as a continuum over the bandgap of the silicon. The time constant dispersion observed is larger than expected for a continuum. A random distribution of surface potential caused by a random distribution of fixed oxide charges over the plane of the interface is found to quantitatively explain the time constant dispersion measured. To analyze this mechanism, we proceed as follows. Dividing the plane of the interface into a number of squares of equal area, the largest area within which surface potential is uniform is called the characteristic area of the random fixed oxide charge distribution. The admittance of the continuum of levels located in a characteristic area can be obtained by integrating the admittance of a single level over all the levels distributed in energy from the valence band to the conduction band. The resulting total interface state admittance in a characteristic area is[4]

$$Y_{ss} = j\omega \frac{q^2}{kT} \int_{E_v}^{E_c} \frac{N_{ss} f_0(1 - f_0)\, d\psi}{1 + j\omega f_0/c_p p_{so}}, \tag{3}$$

where q is the electronic charge in coulombs, $j = \sqrt{-1}$, N_{ss} is the density of interface states $\text{cm}^{-2} \times eV^{-1}$, f_0 is the Fermi function at a given bias, c_p is the majority carrier capture probability in cm^3/sec, p_{so} is the majority carrier density at the silicon surface in cm^{-3}, and $d\psi$ is a small energy interval in the bandgap in eV. The integrand of (3) is sharply peaked about the Fermi level with a width of about kT/q. Thus, (3) can be easily integrated because both N_{ss} and c_p are experimentally observed to vary only slightly over several kT/q in a range of bandgap energy of about half the gap centered about mid-gap. Making the substitution $f_0(1 - f_0) = (kT/q)(df_0/d\psi)$ transforms (3) into an integral over f_0. Integrating from zero to unity yields

$$Y_{ss} = \frac{qN_{ss}}{2\tau_m} \ln (1 + \omega^2\tau_m^2) + jq \frac{N_{ss}}{\tau_m} \text{arc tan} (\omega\tau_m), \tag{4}$$

where $\tau_m = 1/c_p p_{so}$. Equation (4) was first derived by Lehovec.[11]

Typically N_{ss} is in the range $10^{10}\text{cm}^{-2} \times \text{eV}^{-1}$ to $10^{11}\text{cm}^{-2} \times \text{eV}^{-1}$. This means that the interface states are spacially separated too far apart in the plane of the interface for the wave function of an electron in one center to overlap a neighboring center. Transitions from one center to another, even though the centers are closely spaced in energy, are therefore, highly improbable. Thus, transitions between the majority carrier band and a particular level in the continuum located in energy near the Fermi level are not correlated to transitions between the majority carrier band and other levels nearby in energy.

The total admittance Y_T is obtained by multiplying the admittance contributed by each characteristic area Y_{ss} from (4) by the number of characteristic areas in which the surface potential is between u_s and u_s+du_s and integrating over all the characteristics areas under the field plate. The result is

$$Y_T = \int_{-\infty}^{\infty} Y_{ss}P(u_s)\, du_s \, , \tag{5}$$

where $P(u_s)\,du_s$ is the number of characteristic areas in which the surface potential (in units of kT/q) is between u_s and u_s+du_s and Y_{ss} is given by (4). $P(u_s)$, the probability that the surface potential in a characteristic area is u_s, is obtained from the random distribution of fixed oxide charges.[4] When the mean number of charges in a characteristic area is large, the probability of finding N charges in a characteristic area $P(N)$ is given by the Gaussian approximation of a Poisson distribution. Transforming $P(N)$ to $P(u_s)$ for the case of small fluctuations (see Refs. 3 and 4), we get

$$P(u_s) = (2\pi\sigma^2)^{-\frac{1}{2}} \exp\,[-(u_s - \bar{u}_s)^2/2\sigma^2], \tag{6}$$

where σ is the standard deviation of surface potential and $\bar{u}_s$ is the mean surface potential in units of kT/q at a given bias. The standard deviation of surface potential is

$$\sigma = \frac{(q/kT)W(q\bar{Q}_s/\alpha)^{\frac{1}{2}}}{WC_{ox} + \epsilon_{si}} \, , \tag{7}$$

where W is space-charge width in cm, $\bar{Q}_s$ is the fixed oxide charge density in coul/cm^2, α is the characteristic area in cm^2, C_{ox} is the oxide layer capacitance in farads/cm^2, and ϵ_{si} is the permittivity of the silicon in farads/cm.

Substituting (4) and (6) into (5), we get

$$G_p(\omega) = \tfrac{1}{2}qN_{ss}(2\pi\sigma^2)^{-\frac{1}{2}} \cdot\int_{-\infty}^{\infty} \exp\,[-(u_s - \bar{u}_s)^2/2\sigma^2]\tau_m^{-1} \ln\,(1 + \omega^2\tau_m^2)\, du_s \tag{8}$$

and

$$C_p(\omega) = qN_{ss}(2\pi\sigma^2)^{-\frac{1}{2}} \cdot\int_{-\infty}^{\infty} \exp\,[-(u_s - \bar{u}_s)^2/2\sigma^2](\omega\tau_m)^{-1} \arctan\,(\omega\tau_m)\, du_s \, . \tag{9}$$

For p-type, $\tau_m = 1/c_p p_{so} = (1/c_p N_A) \exp u_s$, where N_A is the acceptor density in the silicon in cm^{-3} and c_p is the hole capture probability.

These integrals can be evaluated numerically using the experimental observation that the density of states and the majority carrier capture probability vary very slowly over several kT of bandgap energy.

The values of $G_p(\omega)$ and $C_p(\omega)$ calculated from (8) and (9) can be used in (2) to obtain the open circuit mean square noise voltage of the MOS capacitor.

To illustrate the noise properties predicted by the statistical model, the spectrum of the trapped charge fluctuations in the interface states is derived from this model. First, the spectrum of charge fluctuations for a single time constant is[9, 10]

$$S_{ss}(\omega) = \frac{4BN_s \tau f_0(1 - f_0)}{1 + \omega^2\tau^2}, \tag{10}$$

where $\tau = f_0 \tau_m$ and N_s is the density of states cm^{-2}.

The noise spectrum for the continuum of states located in a characteristic area is obtained by integrating (10) over bandgap energy in a manner identical to (3). The result is

$$S_{sc}(\omega) = (2kT/q)BN_{ss}(\omega^2\tau_m)^{-1} \ln(1 + \omega^2\tau_m^2). \tag{11}$$

Integrating (11) over all characteristic areas similarly to (5), (8), and (9) yields for the actual spectral distribution

$$S(\omega) = (2kT/q)BN_{ss}(2\pi\sigma^2)^{-\frac{1}{2}} \cdot \int_{-\infty}^{\infty} \exp[-(u_s - \bar{u}_s)^2/2\sigma^2](\omega^2\tau_m)^{-1} \ln(1 + \omega^2\tau_m^2)\, du_s. \tag{12}$$

Equations (8), (9), and (12) have been numerically integrated on an IBM 7094 computer using the trapezoidal rule.

III. DISCUSSION

3.1 *Depletion*

Curve (a) of Fig. 2 shows the noise spectrum for a single level state calculated from (10) with the Fermi level at the trap level. Curve (b) of Fig. 2 shows the noise spectrum for the continuum of levels located in a characteristic area calculated from (11). Both of these curves are normalized to their low-frequency values. Comparing curve (a) to curve (b), it is seen that integration over the continuum of levels results only in minor modifications of the shape of the spectrum. Curve

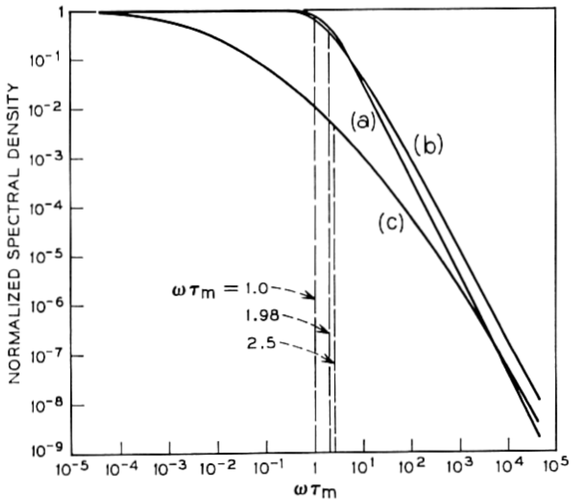

Fig. 2 — Log-log plots of normalized spectral density vs $\omega\tau_m$ for the mean square fluctuations of the number of electrons trapped at the interface states. Curve (a) is for a single time constant calculated from (10). Curve (b) is for a continuum of states calculated from (11). Curve (c) is a plot of (12) using a standard deviation of surface potential of 2.6. All three curves are calculated using a hole density at the silicon surface of 6.4×10^{12} cm^{-3} and a hole capture probability of 2.2×10^{-9} cm^3/sec. The conditions: $\omega\tau_m = 1.0$, 1.98, and 2.5 correspond to the values of $\omega\tau_m$ at which the $G_p(\omega)/\omega$ curve peaks for each case.

(c) of Fig. 2 shows the noise spectrum calculated from (12) using a standard deviation of surface potential of 2.6. This curve is also normalized to its low-frequency value. Curve (c) is seen to be significantly different from curve (a) and curve (b). Fig. 2 shows that the random distribution of surface potential for an experimentally observed standard deviation of 2.6 is the dominant influence on the shape of the noise spectrum. In fact, the random distribution of surface potential will be the dominant influence over the range of standard deviation between 1.8 and 2.6. This is the range found by conductance measurements on several [111] and [100] crystals both n and p type.

In Fig. 3, curve (a) is the noise spectrum calculated from (12) and curve (b) the corresponding $G_p(\omega)/\omega$ vs frequency calculated from (8). For the parameters given in the caption under Fig. 3, $G_p(\omega)/\omega$ goes through a peak of 6 kHz. Fig. 3 shows that:

(*i*) The noise spectrum becomes independent of frequency at low frequencies. For the case considered, this occurs at frequencies much lower than 6 kHz.

(*ii*) The noise spectrum tends towards a $1/f^2$ frequency dependence at high frequencies. This will occur at frequencies much higher than 6 kHz for the case considered.

(*iii*) In the intermediate frequency range where $G_p(\omega)/\omega$ has its highest values, the noise has a $1/f$ type frequency dependence. For the case considered here, this occurs around 6 kHz.

A $1/f$ spectrum is drawn through curve (a) in Fig. 3. To see that the standard deviation determines the frequency range over which a $1/f$ spectrum fits our theory, we transform $P(u_s)$ to $P(\tau_m)$.

$$P(\tau_m) = P(u_s)\, du_s/d\tau_m , \tag{13}$$

where $P(u_s)$ is the probability that the time constant in a characteristic area is τ_m. From the relation $\tau_m = (c_pN_A)^{-1} \exp u_s$ given previously, (13) becomes

$$P(\tau_m) = P(u_s)\tau_m^{-1} . \tag{14}$$

We expand $P(u_s)$ given in (6) in a power series. As long as the condition $(u_s - \bar{u}_s)^2/2\sigma^2 \ll 1$ holds, all terms in the series except the first

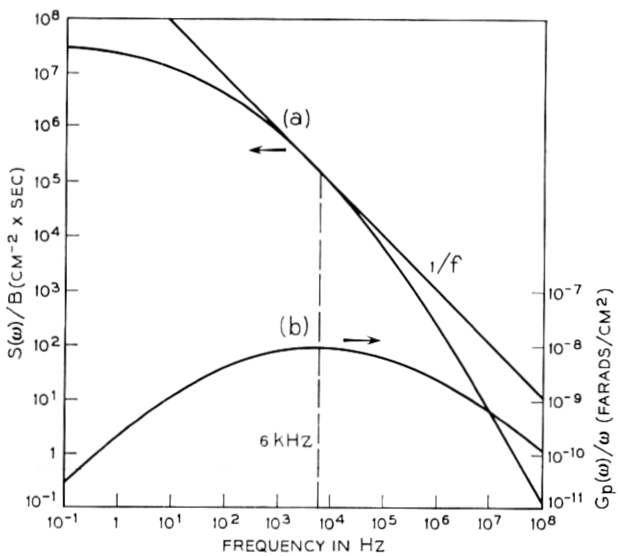

Fig. 3 — Curve (a) is a log-log plot of (12) vs frequency and curve (b) a log-log plot of $G_p(\omega)/\omega$ from (8) vs frequency. Both curves are calculated for a standard deviation of 2.6, a hole density at the surface of 6.5×10^{12} cm^{-3}, a hole capture probability of 2.2×10^{-9} cm^3/sec, an interface state density of 3×10^{11} cm^{-2} $\times$ eV^{-1}, and a temperature of 300°K.

can be dropped. Then, from (14), $P(\tau_m) = (2\pi\sigma^2)^{-1}\tau_m^{-1}$. Superposing spectra of the type $\tau(1 + \omega^2\tau^2)^{-1}$ with a time constant distribution proportional to $1/\tau$ gives a $1/f$ noise spectrum.[12] The integration over the trap levels in one characteristic area results only in a minor change of shape of a $\tau(1 + \omega^2\tau^2)^{-1}$ spectrum as shown in Fig. 2. Essentially only the frequency for $G_p(\omega)/\omega$ maximum shifts to a higher value. Thus, a $1/f$ spectrum will fit our theory over a frequency range determined by the condition $(u_s - \bar{u}_s)^2/2\sigma^2 \ll 1$. The width of the frequency range given by this condition is determined by σ. This can be clarified by an illustrative example. Let us replace $P(u_s)$ with a rectangular distribution of height $(2\pi\sigma^2)^{-\frac{1}{2}}$ and width $(2\pi\sigma^2)^{\frac{1}{2}}$. This distribution gives a $1/f$ spectrum which is within a factor of 2 of curve (*a*) in Fig. 3 over four decades of frequency. The highest and lowest frequencies in this range are given by

$$f_{H,L} = f_p \exp [\pm(\pi/2)^{\frac{1}{2}}\sigma], \tag{15}$$

where f_p is the center frequency of the range and is the frequency at which the corresponding $G_p(\omega)/\omega$ vs log ω curve peaks. This center frequency is proportional to the majority carrier density p_{so} at the silicon surface and is almost independent of σ. For the statistical model with σ between 1.8 and 2.6, $f_p = (2.5/2\pi)\ c_p p_{so}$. Equation (15) shows that the frequency range over which the $1/f$ spectrum is observed depends exponentially on σ.

$C_p(\omega)$ and $G_p(\omega)$ given by (8) and (9) are independent of temperature and silicon resistivity. For a given σ, c_p, and ω, these equations depend only on p_{so} through the variable τ_m. The relation between τ_m and p_{so} is $\tau_m = 1/c_p p_{so}$. Measurements reported in Ref. 4 show that capture probability is independent of temperature. For a wide range of temperature and silicon resistivity, the same value of p_{so} in depletion or accumulation can be obtained just by adjusting field plate bias. Thus, our mechanism for time constant dispersion is independent of temperature and silicon resistivity. Silicon conductivity type is important only because the capture probability for electrons is found to be about ten times larger than for holes.

Fig. 4 shows open circuit mean square noise voltage for two different values of p_{so} or bias calculated from (2) using (8) and (9). Curve (a) is for $p_{so} = 6.4 \times 10^{12}$ cm^{-3} and curve (b) for $p_{so} = 3.5 \times 10^{14}$ cm^{-3} both for $\sigma = 2.6$. Fig. 4 illustrates the bias dependence of the noise voltage vs frequency. The curves in Fig. 4 will be a function of temperature and silicon resistivity as seen from (2).

Fig. 5 shows the influence of standard deviation of surface potential

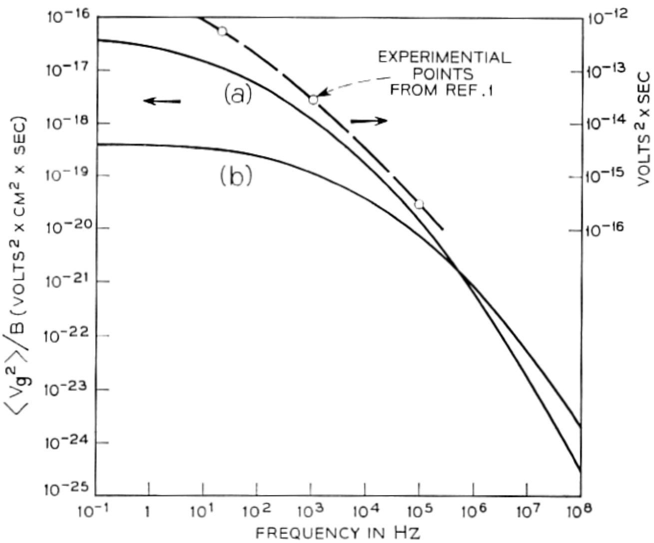

Fig. 4 — Log-log plot of open circuit mean square noise voltage of MOS capacitor vs frequency calculated from (2), (8), and (9). Curve (a) and curve (b) are for hole densities at the silicon surface of 6.4×10^{12} cm^{-3} and 3.5×10^{14} cm^{-3} respectively. For both curves, standard deviation is 2.6, hole capture probability is 2.2×10^{-9} cm^3, acceptor density is 2.1×10^{16} cm^{-3}, interface state density is 3×10^{11} cm$^{-2} \times$ eV^{-1}, and temperature 300°K. Mean surface potential is 8.1 in curve (a) and 4.1 in curve (b). Experimental points are taken from Fig. 1 of Ref. 1 at a gate voltage of —2 volts. Notice similarity of shape to curves (a) and (b).

on the mean square noise voltage vs frequency for a majority carrier density at the silicon surface of 6.4×10^{12} cm^{-3}. Curve (a) is for a standard deviation of 2.6 and curve (b) for a standard deviation of 1.8. These are the largest and smallest values found by conductance measurements on several MOS capacitors.

Fig. 5 shows that:

(*i*) The standard deviation has the greatest influence on the magnitude of the mean square noise voltage at low frequencies.

(*ii*) The range of frequencies over which the mean square noise voltage has a $1/f$ frequency dependence increases with increasing standard deviation of surface potential.

Standard deviation is experimentally observed to be independent of bias over most of the depletion range. It is shown in Refs. 3 and 4 that the relation between characteristic area and space-charge width is

$$\alpha^{\frac{1}{2}} \approx 2W. \tag{16}$$

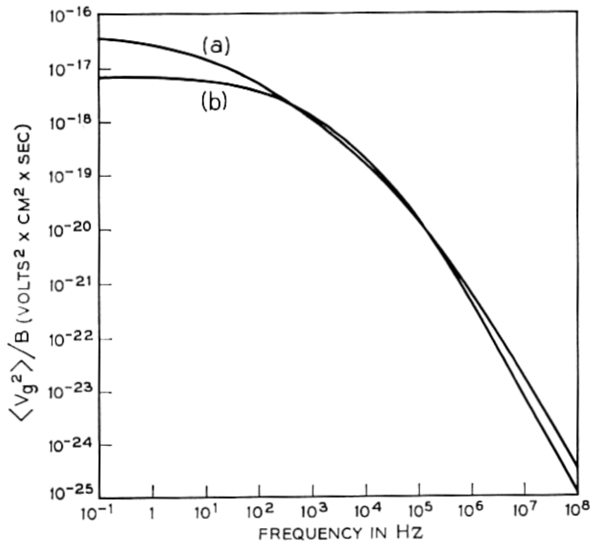

Fig. 5 — Log-log plot of open circuit mean square noise voltage of MOS capacitor vs frequency calculated from (2), (8), and (9). Curve (a) is for a standard deviation of 2.6 and curve (b) for a standard deviation of 1.8. For both curves, hole density at the silicon surface is 6.4×10^{12} cm^{-3}. Hole capture probability, acceptor density, interface state density, and temperature are the same as in Fig. 4. Mean surface potential is 8.1.

Substituting (16) into (7), the fixed charge density causing the random distribution of surface potential can be calculated from the standard deviation. For a standard deviation of 2.6, fixed oxide charge density will be 1×10^{12} cm^{-2} and for a standard deviation of 1.8, fixed oxide charge density will be 5×10^{11} cm^{-2}. A doping density of 2.1×10^{16} cm^{-3} and a mean surface potential of 8.1 have been used in calculating these values of charge density.

It is found experimentally that (8) and (9) are valid over the frequency range from 50 Hz to 500 kHz. The curves in Figs. 3, 4, and 5 cover the frequency range from 10^{-1} to 10^{8} Hz. In extending these curves over a wider frequency range than covered by the conductance measurements, it is assumed that no new important ohmic loss mechanisms arise at the lower and higher frequencies.

Fig. 6 shows open circuit mean square noise voltage calculated from (2) using (8) and (9) as a function of mean surface potential. A frequency of 10 kHz and a standard deviation of 2.6 have been used in this calculation.

In the practical case, the noise voltage curve peaks at the same value of mean surface potential and has the same shape as the equivalent parallel conductance vs $\bar{u}_s$ which would be measured across terminals x-x in Fig. 1.

At a given frequency, Fig. 6 shows that mean square noise voltage decreases at values of mean surface potential near flat bands and saturates in accumulation. A constant density of states with energy has been used in calculating the curve in Fig. 6. Actually, the density of states increases rapidly toward the band edges as shown by Gray and Brown.[13] This means that mean square noise voltage would be greater near flat bands than indicated in Fig. 6. The noise spectrum in this region, however, would have a shape similar to the curves in Fig. 4.

The mean square noise voltage decreases at values of mean surface potential near mid-gap. Because the theory developed here considers only majority carrier transitions, it does not apply without error when the Fermi level is within a few kT/q of mid-gap where both majority and minority carrier transitions become important. For this reason, the curve in Fig. 6 is shown as a dotted extrapolation in this region.

In the region of weak inversion where the Fermi level has moved past mid-gap a few kT/q toward the minority carrier band, the time constant dispersion disappears.[3,4] In this region, the noise spectrum is expected to be similar to curve (a) in Fig. 2 for a single time constant.

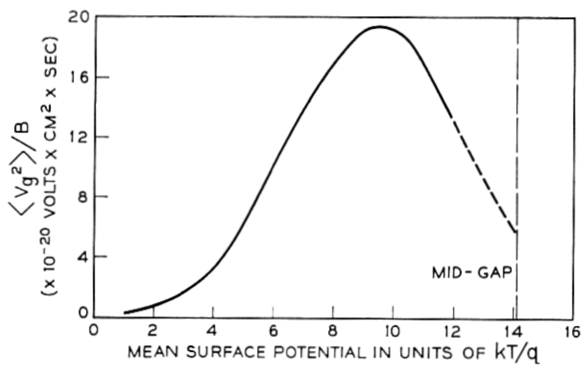

Fig. 6 — Open circuit mean square noise voltage of MOS capacitor vs mean surface potential in units of kT/q. The curve is calculated from (2), (8), and (9) using a frequency of 10 kHz and a standard deviation, acceptor density, hole capture probability, interface state density, and temperature the same as in Fig. 4. Mid-gap is at $\bar{u}_s = 14.1$. Notice similarity in shape to experimental curves in Figs. 1 and 2 of Ref. 1.

IV. SUMMARY AND CONCLUSIONS

A theory of $1/f$ type noise has been presented based on a model for the interface state time constant distribution which quantitatively fits MIS conductance measurements. The noise spectra have been obtained from measured capture conductances $G_p(\omega)$ through the relation $s(\omega) = 4kTB(qw)^{-2} G_p(\omega)$ which is independent of the particular model used. Thus, the noise spectra presented in this paper are based *solely* on *measured* time constant dispersion.

The mean square noise voltage vs frequency in the model discussed in this paper depends on three quantities for a given temperature and silicon resistivity

(*i*) It depends upon the majority carrier density at the silicon surface which in turn is determined by field plate bias.

(*ii*) It depends upon the standard deviation of surface potential which is related to fixed oxide charge density.

(*iii*) It depends upon the density of interface states as seen from (8) and (9) combined with (2).

This theory predicts that $1/f$ type noise due to interface states can be reduced by decreasing the density of states which can exchange charge with the silicon and the density of fixed oxide charge. One consequence is that $1/f$ type noise can be calculated from the electrical properties of the interface obtained by measuring the admittance of an MOS capacitor.

V. ACKNOWLEDGMENTS

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