THE BELL SYSTEM

TECHNICAL JOURNAL

VOLUME XLVI OCTOBER 1967 NUMBER 8

Convection and Conduction Cooling of Substrates Containing Multiple Heat Sources

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(Manuscript received April 12, 1967)

An analysis is made of the steady-state temperature distribution in a substrate with heat inputs from multiple sources. The problem is of interest in connection with integrated and thin film circuits mounted on ceramic or glass substrates. In these applications, convective heat transfer is present with either conduction along the leads joining the substrate to the heat sink or conduction to one end of the substrate which is heat sinked. A formal three-dimensional solution is obtained which is evaluated for various geometries, thermal conductivities, coefficients of convection and heat-sinking conditions.

I. INTRODUCTION

The remarkable technologies of beam-leaded and thin film integrated circuits have resulted in a new approach to the physical design of electronic circuits.[1] This approach as shown in Fig. 1 consists of bonding beam-leaded integrated circuits to ceramic or glass substrates containing thin film components and conductors. At present, dissipation of the thermal energy generated in these circuits is one of the most severe limitations and affects both device performance and reliability.

The results of this analysis will provide a better understanding of

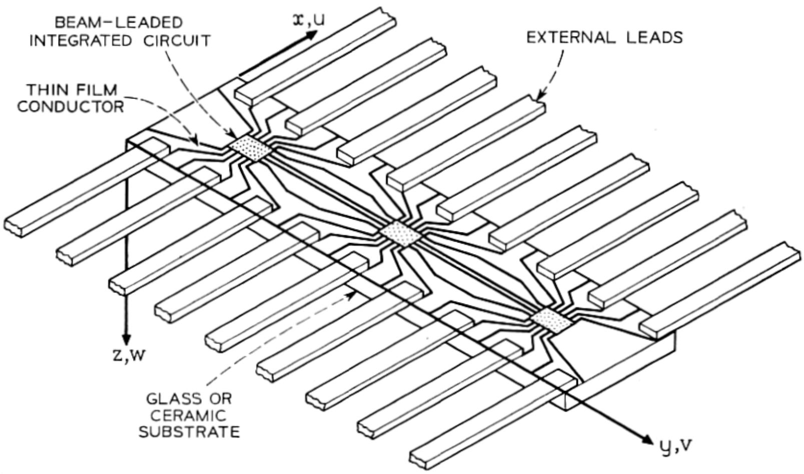

Fig. 1 — Integrated circuit substrate.

the heat transfer phenomena by showing the effects of substrate area, shape, thickness, and thermal conductivity. Included are the effects of the source area, the coefficient of convection and two heat-sinking conditions. The two heat-sinking conditions are: one edge of the substrate is an isothermal boundary, and the leads from the substrate are connected to a heat sink.

Convection is considered on only the two large faces of the substrate since the area of the sides is much smaller. The coefficients of convection are distinguishable for both large faces since some mounting positions will require that these values be different.[2] For instance, the coefficients of convection are quite different for the top and bottom faces of a horizontal plate.

Although the dimensions of most substrates suggest a two-dimensional thin plate model, the very small areas of some heat sources indicate that large thermal gradients in the direction of the normal to the large faces will be present under these sources. This condition is also magnified by the poor thermal conductivity of some substrate materials. Thus, the solution obtained must be three-dimensional to include these effects.

By superposition, multiple sources are considered and the interactions between sources are determined. This provides the necessary information to design the substrate with the desired isolation between temperature sensitive components.

II. MATHEMATICAL MODEL AND BOUNDARY CONDITIONS

The geometry of the problem is shown in Fig. 1 for the case where a substrate is connected to a heat sink through leads. The second heat-sinking condition to be considered is easily visualized if the leads are removed and one edge is considered an isothermal boundary. The temperature in a homogeneous solid of thermal conductivity k satisfies Poisson's equation,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{-Q(x, y, z)}{k}, \tag{1}$$

where Q is the source strength per unit volume. Since there are multiple sources in many of the cases to be considered, it is convenient to use the Green's function approach. It is easily shown[3] that the formal solution can be expressed as

$$\begin{aligned} T(x, y, z) = \int_0^c \int_0^b \int_0^a G(u, v, w \mid x, y, z) Q(u, v, w)\, du\, dv\, dw \\ + k \int_0^b \int_0^a GT_w \Big|_{w=0}^{w=c} - TG_w \Big|_{w=0}^{w=c} du\, dv \\ + k \int_0^c \int_0^a GT_v \Big|_{v=0}^{v=b} - TG_v \Big|_{v=0}^{v=b} du\, dw \\ + k \int_0^c \int_0^b GT_u \Big|_{u=0}^{u=a} - TG_u \Big|_{u=0}^{u=a} dv\, dw, \end{aligned} \tag{2}$$

where a, b, and c are the substrate dimensions in the x, y, and z directions, respectively, and T_u denotes $\partial T/\partial u$. The Green's function $G(x, y, z \mid u, v, w)$ which is symmetric with $G(u, v, w \mid x, y, z)$, satisfies

$$\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} + \frac{\partial^2 G}{\partial z^2} = \frac{-\delta(x - u)\, \delta(y - v)\, \delta(z - w)}{k}. \tag{3}$$

The boundary conditions for $G(x, y, z \mid u, v, w,)$ and $T(x, y, z)$ are

$$\begin{aligned} T_x = G_x &= 0 & x &= a, \\ T_y = G_y &= 0 & y &= 0 \text{ and } b, \\ kT_z = h_1 T \text{ and } kG_z &= h_1 G & z &= 0, \\ -kT_z = h_2 T \text{ and } -kG_z &= h_2 G & z &= c, \end{aligned}$$

and either

$$T = G = 0 \quad \text{or} \quad T_x = G_x = 0, \qquad x = 0. \tag{4}$$

The boundary conditions reference the ambient temperature to zero, and h_1 and h_2 are the coefficients of convection on the two large faces of the substrate. The choice of the last boundary condition depends on whether the substrate under consideration has an isothermal boundary on one end or is supported by leads. If the substrate has leads, the latter boundary condition is used and the heat lost by conduction along the leads is considered by assuming negative sources[3] located at the lead bonding areas. The magnitude of these sources is determined by noting that the temperature difference between the ends of a lead is given by the product of the lead thermal resistance and the amount of heat conducted along that lead.

The case where the leads conduct an appreciable amount of heat is somewhat more difficult and cumbersome than the case of an isothermal boundary at x equals zero or the case where the lead's thermal resistance is of sufficient magnitude to be approximated by an insulated boundary. In what immediately follows, the three cases will be treated separately and generally only one source is considered. The general case of multiple sources follows by superposition and will be treated last.

2.1 *Substrate With High-Resistance Leads*

The solution to this problem is obtained by taking finite cosine transforms[4] of (3) in the x and y directions. The eigenvalues are chosen to satisfy the boundary conditions given by (4). Taking the double cosine transform gives

$$\bar{\bar{G}}_{zz} - (\alpha^2 + \beta^2)\bar{\bar{G}} = \frac{-\delta(z - w) \cos \alpha u \cos \beta v}{k}, \tag{5}$$

where $\alpha = m\pi/a$ $(m = 0, 1, 2, \cdots)$, $\beta = n\pi/b$ $(n = 0, 1, 2, \cdots)$ and $\bar{\bar{G}}$ denotes the transformed dependent variable G. A third transform of (5) is not taken to avoid a triple summation in the final solution. Although the triple summation is no great obstacle, it increases the number of terms needed for numerical evaluation. Thus, (5) is solved for the cases of

$$\alpha^2 + \beta^2 = 0 \quad \text{and} \quad \alpha^2 + \beta^2 \neq 0.$$

If α and β are both zero, by standard methods[5] the solution to (5), satisfying the boundary conditions given by (4) is

$$\begin{aligned} \bar{\bar{G}}(0, 0, z \mid u, v, w) &= \frac{(k/h_2c - w/c + 1)(z + k/h_1)}{k(k/h_1c + k/h_2c + 1)}, \quad z \leqq w, \\ \bar{\bar{G}}(0, 0, z \mid u, v, w) &= \frac{(k/h_2c - z/c + 1)(w + k/h_1)}{k(k/h_1c + k/h_2c + 1)}, \quad z \geqq w. \end{aligned} \tag{6}$$

For α or β not equal to zero, the solution of (5) leads to

$$\bar{\bar{G}}(\alpha, \beta, z \mid u, v, w) = \frac{\cos \alpha u \cos \beta v}{k\gamma} \int_0^z \delta(\zeta - w) \cdot [\phi(\zeta)\psi(z) - \phi(z)\psi(\zeta)]\, d\zeta + A\phi(z) + B\psi(z),$$

where

$$\phi(z) = \sinh \gamma z, \qquad \psi(z) = \cosh \gamma z$$

and

$$\gamma = (\alpha^2 + \beta^2)^{\frac{1}{2}}. \tag{7}$$

Again by standard methods, (7) leads to

$$\begin{aligned}\bar{\bar{G}}(\alpha, \beta, z \mid u, v, w) &= \cos \alpha u \cos \beta v[\phi(z) + (k\gamma/h_1)\psi(z)] \\ &\cdot \frac{\psi(c)[\psi(w) - (h_2/k\gamma)\phi(w)] - \phi(c)[\phi(w) - (h_2/k\gamma)\psi(w)]}{k\gamma[(h_2/k\gamma + k\gamma/h_1)\phi(c) + (1 + h_2/h_1)\psi(c)]}, \quad z \leqq w, \\ \bar{\bar{G}}(\alpha, \beta, z \mid u, v, w) &= \cos \alpha u \cos \beta v[\phi(w) + (k\gamma/h_1)\psi(w)] \\ &\cdot \frac{\psi(c)[\psi(z) - (h_2/k\gamma)\phi(z)] - \phi(c)[\phi(z) - (h_2/k\gamma)\psi(z)]}{k\gamma[(h_2/k\gamma + k\gamma/h_1)\phi(c) + (1 + h_2/h_1)\psi(c)]}, \quad z \geqq w.\end{aligned} \tag{8}$$

The inversion for the double cosine transforms is easily derived[4] and upon substitution into (2) gives

$$\begin{aligned}T(x, y, z) = \int_0^c \int_0^b \int_0^a \Bigg[&\frac{1}{ab} \bar{\bar{G}}(u, v, w \mid 0, 0, z) \\ &+ \frac{2}{ab} \sum_\alpha \bar{\bar{G}}(u, v, w \mid \alpha, 0, z) \cos \alpha x \\ &+ \frac{2}{ab} \sum_\beta \bar{\bar{G}}(u, v, w \mid 0, \beta, z) \cos \beta y \\ &+ \frac{4}{ab} \sum_\alpha \sum_\beta \bar{\bar{G}}(u, v, w \mid \alpha, \beta, z) \cos \alpha x \cos \beta y \Bigg] Q(u, v, w)\, du\, dv\, dw.\end{aligned} \tag{9}$$

It should be noted that the surface integrals of (2) are identically equal to zero because of the boundary conditions and the subsequent choice of eigenvalues. Consequently, these terms do not appear in (9) and the determination of the temperature distribution merely requires the substitution of the appropriate Green's functions from (6) and (8) and the integrations indicated in (9). For the applications cited, the sources are on the z equals zero surface and therefore $Q(x, y, z) =$

$Q(x, y)\ \delta(z)$. Substituting this into (9) gives

$$T(x, y, z) = \int_0^b \int_0^a \left[\frac{1}{ab} \bar{\bar{G}}(u, v, 0 \mid 0, 0, z) + \frac{2}{ab} \sum_\alpha \bar{\bar{G}}(u, v, 0 \mid \alpha, 0, z) \cos \alpha x + \frac{2}{ab} \sum_\beta \bar{\bar{G}}(u, v, 0 \mid 0, \beta, z) \cos \beta y + \frac{4}{ab} \sum_\alpha \sum_\beta \bar{\bar{G}}(u, v, 0 \mid \alpha, \beta, z) \cos \alpha x \cos \beta y \right] Q(u, v)\, du\, dv. \tag{10}$$

Note that in this case $Q(u, v)$ represents real sources since the leads are assumed to be of very high resistance and thereby produce negligible effects.

2.2 *Substrate With Heat Conducting Leads*

The method of solution for this case makes use of the results up to (10). It has been stated that the leads will be treated as negative heat sources and thus by superposition (10) will read

$$T(x, y, z) = \int_0^b \int_0^a \left[\frac{1}{ab} \bar{\bar{G}}(u, v, 0 \mid 0, 0, z) + \frac{2}{ab} \sum_\alpha \bar{\bar{G}}(u, v, 0 \mid \alpha, 0, z) \cos \alpha x + \frac{2}{ab} \sum_\beta \bar{\bar{G}}(u, v, 0 \mid 0, \beta, z) \cos \beta y + \frac{4}{ab} \sum_\alpha \sum_\beta \bar{\bar{G}}(u, v, 0 \mid \alpha, \beta, z) \cos \alpha x \cos \beta y \right] \cdot [Q(u, v) - F(u, v)]\, du\, dv, \tag{11}$$

where $F(u, v)$ represents the heat sources due to leads. It is important to note that both Q and F can represent any arbitrary number of real sources and leads, respectively. This is important since it is quite common for a substrate to have as many as sixteen or eighteen leads. It is obvious at this point that $F(u, v)$ must be specified. The approach used here is to assume a uniform heat source over each lead bond area and determine the magnitudes of these sources by using other available information. The required information is available from (11) since the temperature can be evaluated at each lead location in terms of each source power density. Thus,

$$F(u, v) = f_1 g_1(u, v) + f_2 g_2(u, v) + \cdots f_n g_n(u, v) \tag{12}$$

represents the sources due to each lead location and $f_1, f_2 \cdots f_n$ are unspecified at this time. The functions $g_i(u, v)$ are a combination of step functions that give unity at the lead bond area and zero elsewhere. To determine $f_1, f_2, \cdots f_n$, the condition that the temperature difference between the two ends of a lead is equal to the product of the thermal power and thermal resistance of that lead is used. Equating these two expressions for the temperature at each lead location gives n equations and n unknowns in f_i. In matrix notation this can be expressed as

$$[A + R][F] = [B], \tag{13}$$

where A is an n by n matrix representing the influence coefficients due to the action of the negative sources. R is also an n by n matrix but the only non-zero elements are those where $i = j$. These elements are the thermal resistances of the leads. The column matrix F consists of $f_1, f_2, \cdots f_n$ and represents the unknowns to be determined. The matrix B is a column matrix which represents the effects of the real heat sources. By substitution of $A + R = C$, then (13) becomes

$$CF = B,$$

and, therefore, (14)

$$C^{-1}CF = C^{-1}B,$$

where C^{-1} is the inverse of C. Since $C^{-1}C = I$, where I is the unit matrix, then

$$IF = C^{-1}B \tag{15}$$

gives the desired results.[6] Using these values of $f_1, f_2, \cdots f_n$ in (11) permits the calculation of the temperature at any point in the substrate. In the results reported here, the temperature was always evaluated at the center of the lead bond area and the flux was assumed constant over that area. This is a reasonable approximation since the lead material generally has a much higher thermal conductivity than the substrate. However, the problem can be solved for other functional representations of the flux if there is evidence that these representations are significantly better approximations of the physical situations. Similarly, the reported results for substrates with leads will be for a single heat source although $Q(u, v)$ and the matrix B are not restricted to such situations and can represent any arbitrary number of sources. Thus, the method of obtaining a formal solution for the temperature distribution in a substrate containing multiple sources and leads has been indicated.

2.3 *Substrate With Isotherm on One Boundary*

The solution of the temperature problem for the $x = 0$ boundary being an isotherm is obtained by taking a finite sine transform[4] in the x direction and retaining the cosine transform for the y direction. Applying these transforms to (3) gives

$$\bar{\bar{G}}_{zz} - (\alpha^2 + \beta^2)\bar{\bar{G}} = \frac{-\delta(z - w) \sin \alpha u \cos \beta v}{k}, \tag{16}$$

where $\alpha = (2m - 1)\pi/2a$ $(m = 1, 2, 3, \cdots)$, $\beta = n\pi/b$ $(n = 0, 1, 2, \cdots)$. Comparing (16) and (5) indicates the solution to (5) can be used as the solution to (16) provided $\sin \alpha u$ is substituted to replace $\cos \alpha u$ and α is now given by (16). The solution to (5) was given by (6) for $\alpha^2 + \beta^2 = 0$ and (8) for $\alpha^2 + \beta^2 \neq 0$. Since $\alpha^2 + \beta^2$ is never equal to zero in (16), (6) is not needed and (8) gives the solution provided the indicated changes are made. The inversion formula for these multiple finite transforms gives the temperature distribution as

$$T(x, y, z) = \int_0^b \int_0^a \left[\frac{2}{ab} \sum_\alpha \bar{\bar{G}}(u, v, 0 \mid \alpha, 0, z) \sin \alpha x \right.$$
$$\left. + \frac{4}{ab} \sum_\alpha \sum_\beta \bar{\bar{G}}(u, v, 0 \mid \alpha, \beta, z) \sin \alpha x \cos \beta y \right] Q(u, v)\, du\, dv, \tag{17}$$

where again $G(u, v, w \mid x, y, z) = G(x, y, z \mid u, v, w)$. It should be noted that in (9), (10), (11), and (16) the summations on α and β are only for the non-zero eigenvalues since the $\alpha = 0$ or $\beta = 0$ terms, if they appear, are already indicated in the inversion formulas.

It should be apparent that similar physical situations such as two opposite boundaries being isotherms poses no new or additional problems. For instance if the boundaries $x = 0$ and $x = a$ are isotherms, (17) is valid provided $\alpha = m\pi/a$ $(m = 1, 2, 3, \cdots)$.

III. NUMERICAL RESULTS

In this section results are given for the three cases discussed in Sections 2.1, 2.2, and 2.3. For the results reported, it is assumed that the heat sources have uniform power density. Thus, they are mathematically represented by a combination of step functions. The results are reported in terms of thermal resistance where thermal resistance

(R_t) is defined as the difference in maximum and minimum temperatures based on one watt of power dissipation. Dimensionless variables are used to present the results in their most general form except for several instances where specific results are desired. The dimensionless variables are $\Delta x/a$, x_0/a, c/a, $\Delta y/b$, y_0/b, c/b, $\Delta z/c$, z_0/c, h_1c/k, and h_0c/k, where Δx, Δy, and Δz are the source dimensions and x_0, y_0, and z_0 the coordinates of the source center. For the applications cited, the sources are always plane sources located on one face of the substrate and thus $\Delta z/c = z_0/c = 0$ in (10), (11), and (17) and consequently, in all the results.

Previously, it was suggested that a three-dimensional solution would be necessary to accurately represent the thermal behavior of a low thermal conductivity substrate containing very small heat sources. This is because the temperature gradient in a direction normal to a plane source must increase at the same rate as the source area decreases if the same amount of heat is dissipated. Thus, small sources require large gradients near the source even though the body temperature may be nearly uniform elsewhere as would be expected if the Biot number is small, i.e., $hc/k \ll 1$. The following physical parameters are chosen as a typical example of a substrate with a small centrally located heat source:

$$a/c = 26.0, \qquad b/c = 10.0,$$
$$x_0/a = 0.50, \qquad y_0/b = 0.50,$$
$$\Delta x/a = 0.0123, \qquad \Delta y/b = 0.0320$$
$$h_1c/k = 0.93 \times 10^{-3}, \qquad h_2c/k = 0.93 \times 10^{-3}.$$

Fig. 2 presents the results where the dependent variable is the dimensionless quantity $ckT(x, y, z)$. These results clearly indicate the problem is three dimensional near the source due to spreading resistance and thus a three-dimension solution is required to accurately describe substrates containing small sources.

The next numerical results are for square and rectangular substrates containing one centrally located heat source and leads that conduct a negligible amount of heat. Thus, the leads can be ignored in the analysis and the equations of Section 2.1 are applicable. Obviously, this model always gives an upper bound for the substrate thermal resistance unless the leads are heat sinked at a higher temperature than the atmosphere surrounding the substrate. Except for this unlikely situation, these results give an easy-to-obtain first approximation of the

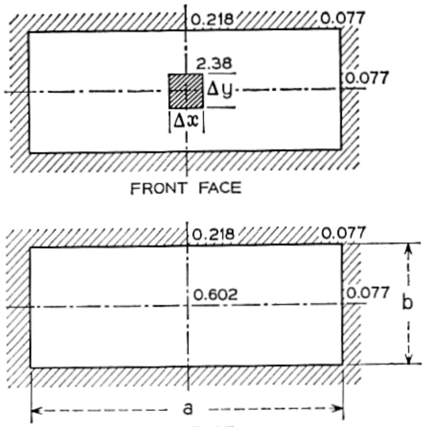

Fig. 2 — Integrated circuit substrate temperature, $ckT(x, y, z)$ as a function of position: $a/c = 26.0$, $b/c = 10.0$, $x_0/a = y_0/b = 0.50$, $\Delta x/a = 0.0123$, $\Delta y/b = 0.0320$, $h_1c/k = h_2c/k = 0.93 \times 10^{-3}$, $P = 1$ watt.

substrate thermal capability and are given in Fig. 3 (a) and (b). Each solid line represents a given substrate whereas each broken line represents a given source. Although the broken lines represent redundant information, they assist in illustrating the effects of various parameters. Thus, Fig. 3(a) and (b) clearly show the effects of heat source and substrate areas.

Results for the case where one end of the substrate is an isothermal boundary are given in Fig. 3(c) and (d). The equation for these results was developed in Section 2.3. By following one of the broken lines, a constant heat source area is being maintained and the effects of changes in the substrate area can be observed. It will be noted that increasing the substrate area provides little benefit since, with a centrally located source, increases in the thermal resistance due to longer conduction paths almost cancel the decreases due to larger convection areas. As with the previous case, it is noted that the square substrate is slightly more efficient than the rectangular one. The Biot number used for the results presented in Fig. 3(a) thru (d) is a typical value for a thin alumina ceramic substrate with free convection from both faces.

Results for a substrate with heat conducting leads are more difficult to generalize since the thermal resistance of the leads, the lead bond areas and locations, and the number of leads are parameters which affect the results. To illustrate the effects of leads, two alumina ceramic substrates containing beam-leaded monolithic integrated circuits are considered. These substrates will be identified as Substrates I and II, and are very similar to the one in Fig. 1 except that both have one heat source. Substrate I is supported by sixteen copper leads, but due to symmetry only one quadrant of the substrate need be considered. Thus, the problem is simplified to a substrate containing one source and four leads. Using the procedure outlined in Section 2.2 gives

$$A = \begin{bmatrix} 608.7 & 500.3 & 476.6 & 466.0 \\ 500.3 & 585.1 & 489.7 & 475.2 \\ 476.6 & 489.7 & 583.7 & 497.1 \\ 466.0 & 475.2 & 497.1 & 602.0 \end{bmatrix}$$

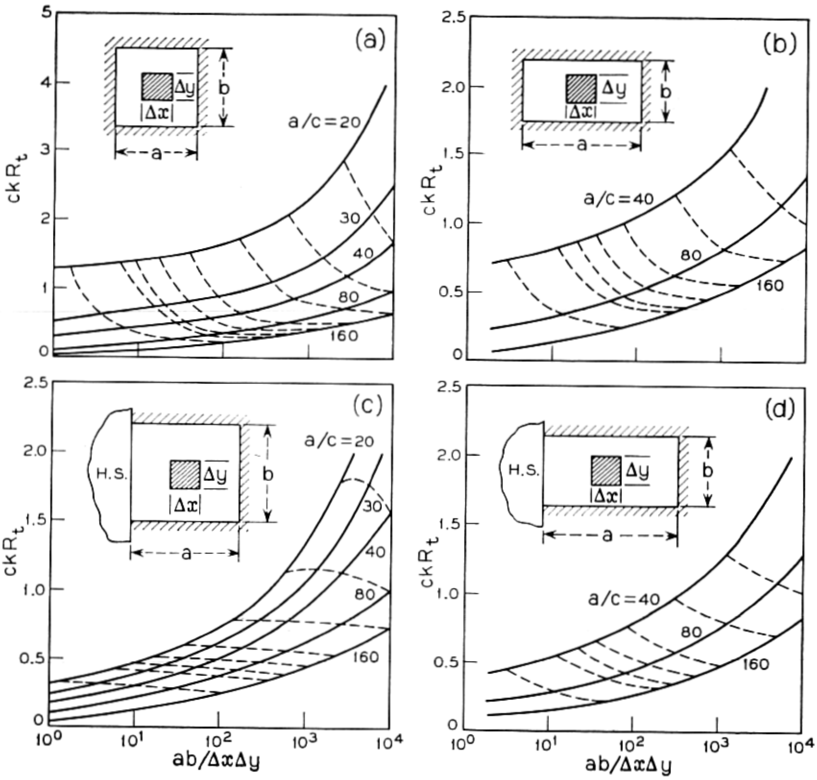

Fig. 3—Substrate thermal resistance: (a) and (c) $a/c = b/c$, $\Delta x/a = \Delta y/b$, $x_0/a = y_0/b = 0.50$, $h_1c/k = h_2c/k = 0.93 \times 10^{-3}$; (b) and (d) $a/c = 2b/c$, $\Delta x/a = \Delta y/2b$, $x_0/a = y_0/2b = 0.50$, $h_1c/k = h_2c/k = 0.93 \times 10^{-3}$.

and

$$B = \begin{bmatrix} 124.2 \\ 121.3 \\ 118.1 \\ 116.1 \end{bmatrix}, \tag{20}$$

where the total output from the heat source has been taken as one-fourth watt since only one-fourth of the real heat source lies in the quadrant being considered. Lead spacing was 0.190 cm (0.075 inch), and each lead bond area was 0.038 cm (0.015 inch) by 0.038 cm. To continue the analysis and determine the f_i's, it is necessary to specify the thermal resistances of the leads as these values are the diagonal elements of the matrix R. To obtain a better understanding of the effects of the leads, it is useful to present the substrate thermal resistance as a function of the lead thermal resistance. These results are given in Fig. 4 and clearly indicate that the substrate thermal resistance can be substantially reduced by heat sinking the leads. Fig. 4 also gives results for 0.107 by 0.107, 0.157 by 0.157, and 0.208 by 0.208 cm square (0.042 by 0.042, 0.062 by 0.062, and 0.082 by 0.082 inches square, respectively) sources to illustrate the effects of source size for substrates with leads. For these physical situations, the change in thermal resistance due to changes in source size is within 1°C/watt of

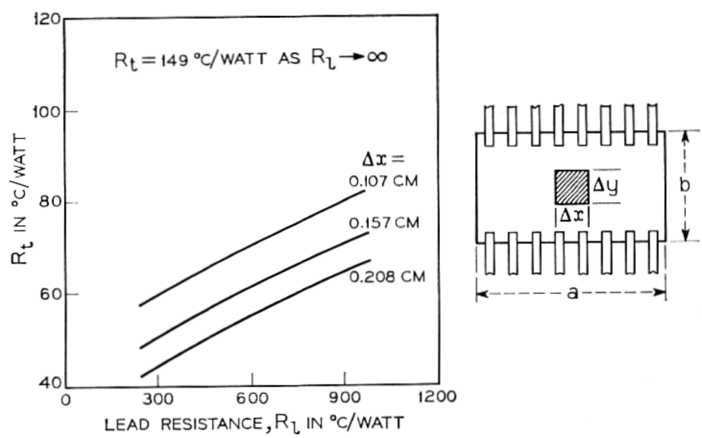

Fig. 4 — Thermal resistance of Substrate I: $a = 1.61$ cm, $b = 0.89$ cm, $c = 0.0635$ cm, $\Delta x = \Delta y$, $k = 0.202$ watt/cm-°C, $h_1 = h_2 = 0.003$ watt/cm²-°C.

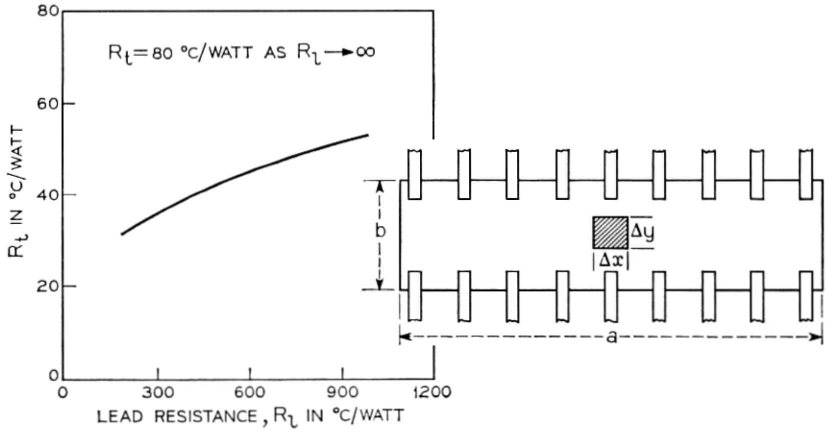

Fig. 5—Thermal resistance of Substrate II: a = 3.30 cm, b = 0.89 cm, c = 0.0635 cm, $\Delta x = \Delta y$, k = 0.202 watt/cm-°C, $h_1 = h_2$ = 0.003 watt/cm²-°C.

the results obtained for substrates without leads. This indicates that for small sources the source area effects are local and the effects of moderate changes in source area can be approximated by the results for substrates without heat conducting leads.

The results for Substrate II are given in Fig. 5. This design is very similar to the previous one except the substrate is approximately twice as long, contains eighteen leads on 0.381 cm (0.150 inch) spacing and each lead bond area is 0.038 cm (0.015 inch) by 0.076 cm (0.030 inch). These results indicate a thermal resistance of 80°C/watt for this substrate as compared to 149°C/watt for the previous one if the effects of the leads are not included. These substrate resistances are reduced to 45 and 62°C/watt, respectively, if the thermal resistance of each lead is 600°C/watt. To illustrate the effects of the leads, Fig. 6 shows the percent reduction in the substrate thermal resistance due to each lead. In this figure the leads are numbered beginning with the lead closest to the heat source. Although results for these two designs do not permit a generalization of lead effects, they do illustrate what effects may be expected for geometries that are reasonably similar.

This significant effect of the leads illustrates one of the disadvantages of glass and glazed ceramic substrates. At present most leads bonded to these substrates have had much higher thermal resistances than those bonded to unglazed ceramics. This is because of the inability to repeatedly bond thick copper leads to glass or glazed substrates with-

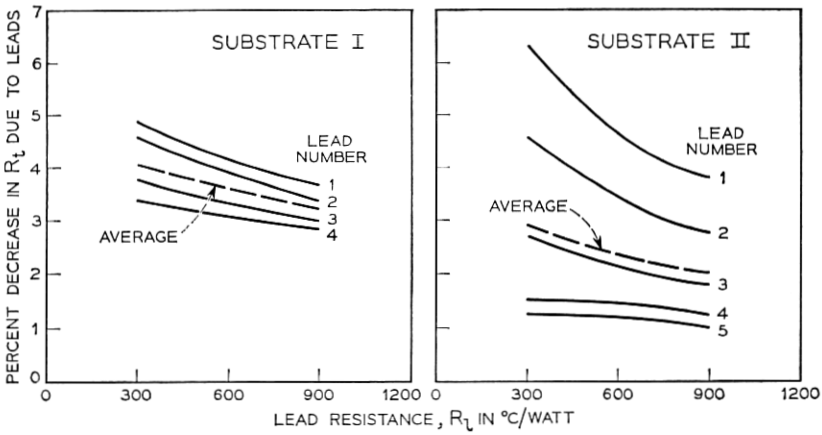

Fig. 6 — Percent decrease in substrate thermal resistance due to leads.

out fractures at the time of bonding or upon subsequent thermal cycling. Thus, the effect of replacing thick copper leads with thin gold ribbon, as is commonly used, can easily result in an additional thermal resistance of 20 to 40°C/watt. These results also suggest that when necessary, ceramic substrates could be glazed in only those areas as required for component performance and thus permit the use of low thermal resistance leads.

A substrate dimension of obvious interest is the thickness. Results considering this parameter are given in Fig. 7(a) and (b) for the indicated rectangular substrates. For the substrate without leads, the change in thermal resistance due to a 40 percent increase in substrate thickness is insignificant whereas for the substrate with an isothermal boundary on one edge a 20 percent improvement in thermal resistance is possible for the small substrate areas. These conclusions are made by comparing the results of Fig. 3(b) and (d) with Fig. 7(a) and (b), where again the Biot number has been chosen to represent a thin alumina ceramic substrate with free convection from both faces.

The coefficient of convection and the substrate thermal conductivity are also parameters that have significant effects on substrate heat transfer characteristics. If the dependent variable is chosen as ckR_t, it is possible to consider the effects of changes in the coefficient of convection and the substrate thermal conductivity by considering various values of the Biot number since these two parameters always appear in this non-dimensional form. However, for the applications cited the thermal conductivities are approximately 0.202 watt/cm-°C for alumina ceramic, 1.35 watt/cm-°C for beryllia ceramic and 0.020 watt/cm-°C for glass while the coefficients of convection range from 0.003 to 0.010 watt/cm²-°C. These coefficients of convection represent the range from free to very moderate forced convection. Since this range is much smaller than the range of thermal conductivities, the results from the evaluation of these two parameters are presented separately. To illustrate the effects of the coefficient of convection, a rectangular substrate without leads is chosen and the results are presented in Fig. 8. As specific examples, the Biot numbers chosen can represent a thin alumina ceramic with free convection in equipment, free convection under laboratory conditions with small substrates and moderate forced convection. These results illustrate the need for adequate air volume and velocity around the substrate since the coefficient of convection[2] is dependent upon both.

All previous results were for Biot numbers in a range applicable to thin alumina ceramic substrates subjected to free or moderate forced convection. The following results will be applicable for beryllia ceramic and glass substrates under the same conditions. Since typical glasses used for thin film substrates have thermal conductivities ten to twenty times smaller than those of alumina ceramics, the results are obviously less favorable. However, if the average temperature term of (10) is considered, it will be noted that this term is most strongly influenced by the reciprocal of the product of the coefficient of con-

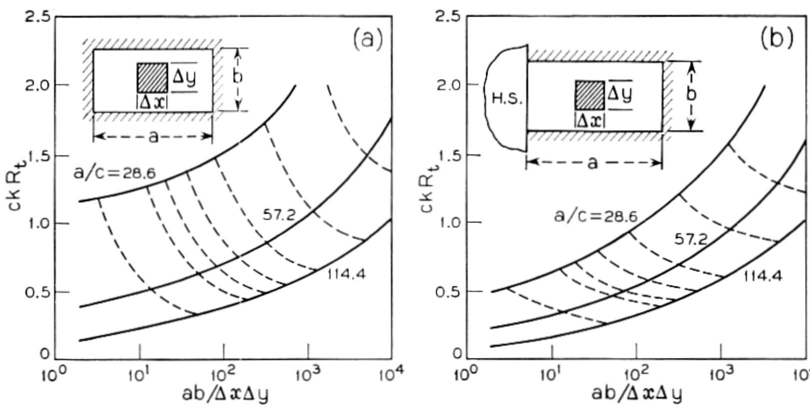

Fig. 7 — Substrate thermal resistance: (a) and (b) $a/c = 2b/c$, $\Delta x/a = \Delta y/2b$, $x_0/a = y_0/b = 0.50$, $h_1c/k = h_2c/k = 1.3 \times 10^{-3}$.

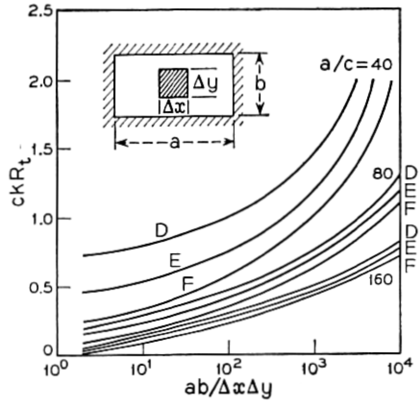

Fig. 8—Substrate thermal resistance for several Biot numbers: $a/c = 2b/c$, $\Delta x/a = \Delta y/2b$, $x_0/a = y_0/b = 0.50$, $h_1c/k = h_2c/k$; (D) $h_1c/k = 0.93 \times 10^{-3}$; (E) $h_1c/k = 1.6 \times 10^{-3}$; (F) $h_1c/k = 3.2 \times 10^{-3}$.

vection and the substrate area. Thus, the substrate thermal conductivity has a minor effect on this term. As the source area-to-substrate area ratio approaches unity, this term is the significant term in the answer since the substrate begins to approach a uniform temperature which is also the average temperature. Thus, the conclusion can be made that if the source area is approximately equal to the substrate

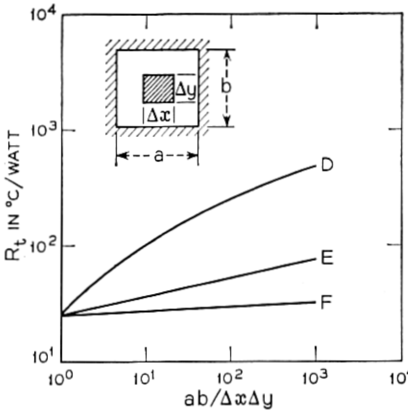

Fig. 9—Thermal resistance of glass, alumina and beryllia ceramic substrates: $a = b = 2.54$ cm, $c = 0.0635$ cm, $\Delta x/a = \Delta y/b$, $x_0/a = y_0/b = 0.50$, $h_1 = h_2 = 0.003$ watt/cm²-°C; (D) $k = 0.020$ watt/cm-°C; (E) $k = 0.202$ watt/cm-°C; (F) $k = 1.35$ watt/cm-°C.

area, the effects of low thermal conductivity substrates will be minimal but if the source area is very small, as is typical with semiconductor sources, the increase in substrate thermal resistance will be most significant. Fig. 9 illustrates these effects for a 2.54 by 2.54 cm (1.0 by 1.0 inch) square substrate.

A direct comparison can be made between alumina and beryllia substrates by comparing the results given in Fig. 10(a) thru (d) with those previously given in Fig. 3(a) thru (d). By choosing several points from the corresponding figures, one can quickly demonstrate the advantage of the high thermal conductivity substrate since the thermal conductivities differ by a factor of 6.7 if the same thickness and coefficient of convection values are used. Specific results giving

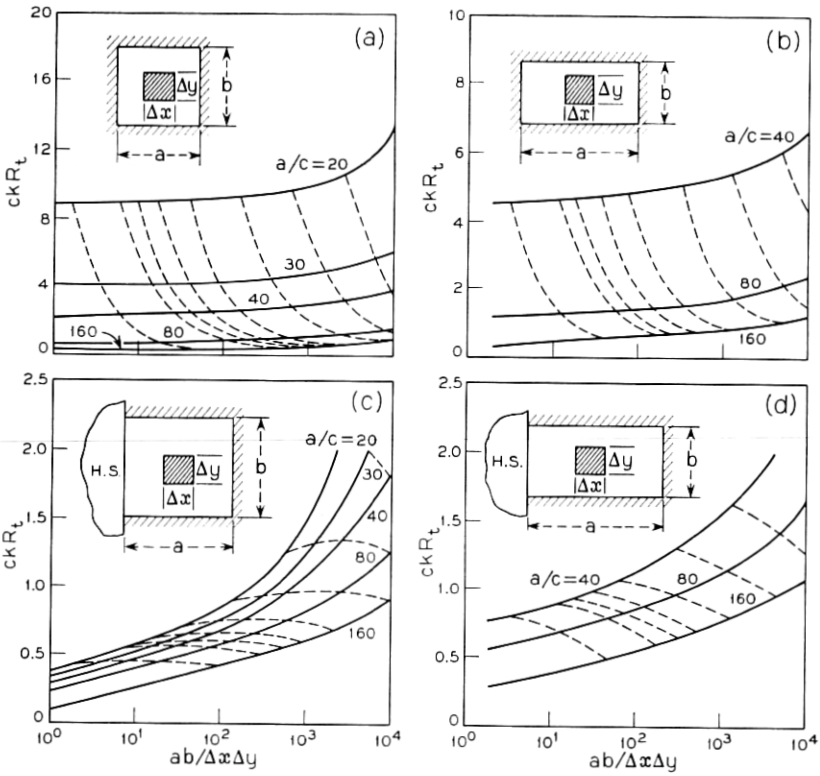

Fig. 10—Substrate thermal resistance: (a) and (c) $a/c = b/c$, $\Delta x/a = \Delta y/b$, $x_0/a = y_0/b = 0.50$, $h_1c/k = h_2c/k = 0.14 \times 10^{-3}$; (b) and (d) $a/c = 2b/c$, $\Delta x/a = \Delta y/2b$, $x_0/a = y_0/b = 0.50$, $h_1c/k = h_2c/k = 0.14 \times 10^{-3}$.

such a direct comparison are given in Fig. 11(a) and (b). These figures demonstrate that for very small sources or for substrates with an isothermal boundary on one edge, the beryllia substrate can make possible approximately a 60°C/watt reduction in the substrate thermal resistance. Since adequate power dissipation is one of the present problems of thin film and integrated circuits, the use of high conductivity substrates should be considered.

The final numerical results illustrate the thermal interaction between sources which must be considered if some components have temperature sensitive parameters. A synchronous clock logic circuit is chosen for this example. One version of this circuit consists of four flat packages appliqued to an alumina substrate and inserted into a socket. The socket design is such that the edge of the substrate which makes contact closely approximates an isothermal boundary. Each flat package contains two or three integrated circuit chips and dissipates approximately one-fourth watt. To complete this analysis the only required additional computation is to calculate the temperatures at other source locations and superimpose these effects. Fig. 12 gives the temperature profile of the substrate based on a one-fourth watt power dissipation from each flat package. It should be emphasized that the temperatures given are referenced to a heat sink and ambient of zero degrees centigrade and that the flat package and integrated circuit chip thermal resistances must be added to the values obtained for

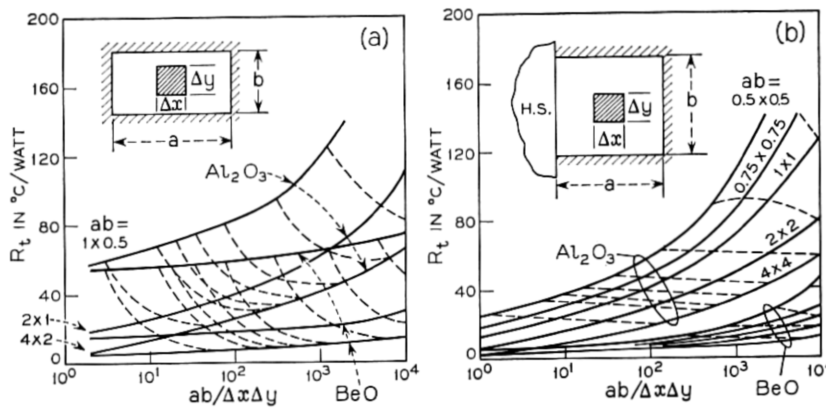

Fig. 11 — Substrate thermal resistance: (a) $a/c = 2b/c$, $\Delta x/a = \Delta y/2b$, $x_0/a = y_0/b = 0.50$, $h_1c/k = h_2c/k = 0.14 \times 10^{-3}$, $c = 0.0635$ cm, $k = 0.202$ watt/cm-°C; (b) $a/c = b/c$, $\Delta x/a = \Delta y/b$, $x_0/a = y_0/b = 0.50$, $h_1c/k = h_2c/k = 0.14 \times 10^{-3}$, $c = 0.0635$ cm, $k = 0.202$ watt/cm-°C.

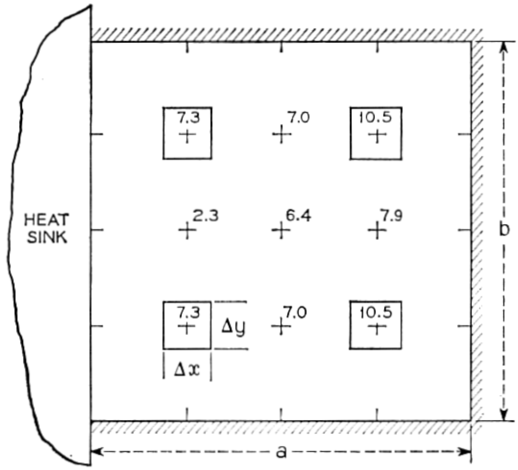

Fig. 12—Temperature profile (°C) per watt for synchronous clock logic circuit: $a/c = b/c = 80$, $\Delta x/a = \Delta y/b = 0.20$, $x_0/a = 0.25$ and 0.75, $y_0/b = 0.25$ and 0.75, $h_1c/k = h_2c/k = 0.63 \times 10^{-3}$, $c = 0.0635$ cm, $k = 0.202$ watt/cm-°C.

the substrate to obtain the overall thermal resistance. A reasonable temperature rise for the flat package and integrated circuit chip is 9.5°C so that the hottest temperature in the circuit should be no greater than 85°C if it is assumed that the maximum ambient temperature is 65°C. Less than a 5°C difference in temperatures due to interaction between the four flat packages is also expected. Thus, the temperature sensitive components of this circuit should track reasonably well. This ability to predict interaction effects between sources is obviously important for circuits with temperature sensitive components.

IV. CONCLUSIONS

The three-dimensional thermal problem of convection from the two large faces of a substrate with heat conducting leads has been solved for a substrate with isothermal edges, insulated edges or combinations of these boundary conditions. Results from this solution show the effects of substrate area, shape, thickness and thermal conductivity. Included are the effects of source area and the coefficient of convection. Two of the more important conclusions concerning heat transfer characteristics of glass and ceramic substrates follow:

(*i*) The coefficient of convection has a significant effect on the thermal resistance of substrates with insulated edges or high resistance leads and becomes the dominant parameter as the heat source area approaches the substrate area.

(*ii*) The thermal conductivity of the substrate is the dominant parameter affecting the thermal resistance of a substrate containing small area heat sources which are typical of beam-leaded integrated circuits.

V. ACKNOWLEDGMENTS

The author is grateful to H. E. Hughes for his helpful discussions and interest in this work.

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