Modulation of Laser Beams by Atmospheric Turbulence – Depth of Modulation*

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We have studied the fluctuations produced in a laser beam by atmospheric turbulence over transmission paths up to 2400 feet long as a function of size of receiving aperture, range, and atmospheric conditions. The depth of modulation decreases rapidly with increasing size of receiving aperture for apertures smaller than the direct beam. It does not go to zero, however, but rather levels off at an approximately constant, finite value for apertures larger than the direct beam.

When all of the direct beam is collected, the depth of modulation varies approximately with the $\frac{3}{2}$ power of range from about 100 to 2400 feet, the largest range used. At ranges less than about 100 feet, however, the dependence is consistently much less than $\frac{3}{2}$. These results are independent of weather conditions, of time of day, of local conditions along the path, of whether the transmitter is inside or outside a building, of a twofold change in diameter of the launched beam, of whether the range is a straight pass or is multiply-folded, and of mirror separation in the multiply-folded arrangement. The $\frac{3}{2}$ dependence is consistent with near-field scattering theory and leads to an estimate for the lower bound of the effective scale size of turbulence of 5 centimeters.

The depth of modulation, however, depends sensitively on atmospheric conditions; in a time of the order of seconds the value can change as much as an order of magnitude. We have systematically measured depth of modulation of the direct beam simultaneously with wind velocity and variability, temperature gradients, and time of day. No simple dependence on these variables was found.

I. INTRODUCTION

From the point of view of communications, one of the serious effects of the atmosphere on propagation of laser beams is the fluctuations in

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the received signal caused by variations in the dielectric constant of the air. An important measure of the fluctuations is their power spectrum. Hogg[1] first measured these spectra and obtained an exponential distribution with a baseband width of the order of a few hundred cycles. Hogg used a multimode 6328 Å laser and a range of 2.6 km. The receiver, 5 cm in diameter, was located in the center of the received beam, which was about 25 cm in diameter. Hogg observed that an increase in the angular beamwidth of the source caused an increase in spectral width.

Hinchman and Buck[2] measured the low-frequency fluctuations in a 6328 Å laser beam at distances of 9 and 90 miles. Their receiver, 3 inches in diameter, collected a very small fraction of the total beam power. They observed a very large depth of modulation. The spectral density of the fluctuations was found to decrease with increasing frequency up to 50 Hz, the highest frequency measured.

Subramanian and Collinson[3] propagated a single-mode, diffraction-limited 6328 Å beam and examined the dependence of the spectrum on a variety of parameters. The transmitted beam diameter was changed from 1 to 38 mm, beam divergence was adjusted by focusing the telescope employed, ranges of 120 and 360 meters were used, and the receiver aperture was varied from much smaller to much larger than the received beam size. The spectrum, which had an exponential distribution (in agreement with Hogg), was independent of these variables within experimental error. The width of spectrum, however, was sensitive to atmospheric conditions. In general, the spectrum became wider as refractive gradients along the path became larger. For example, temperature gradients (caused by the sun) and pressure gradients (caused by turbulent wind) systematically gave broader spectra. The total width of the spectrum (above detector noise) varied over a total range of 60 to 1000 Hz, with typical value of a few hundred hertz. Thus, the spectral width was about the same as Hogg's, although the distance was about an order of magnitude less.

Buck[4] took additional data at more distances on the same 90-mile range, the smallest distance being 550 meters. Although the analytic shape of the power spectra was not given, it is nevertheless significant that the spectra shown all reached cutoff at about 200 Hz, and he stated that there was no systematic variation in the spectra when the detector aperture or path length was changed. Buck commented that, with a very large aperture, a noiseless dc signal is obtained, but this was not found by Subramanian and Collinson.[3] Thus, the three sets of observers at three locations have found a characteristic width of about a few hundred hertz, rather independent of the experimental arrangement, and, in particular, apparently insensitive to changes in distance from 120 meters to 145 km.

Another important measure of the fluctuations in the received signal is depth of modulation, or the ratio of the rms power in the fluctuations to the average beam power. While spectral width may be independent of the experimental parameters, one expects the depth of modulation to show a strong dependence, especially on distance. The present work was undertaken to establish the nature of the dependence of modulation depth on such variables as distance, receiver aperture, and atmospheric conditions.

II. EXPERIMENTAL ARRANGEMENT

Since beam diameter will vary with distance (due to diffraction as well as atmospheric refraction) and since modulation depth presumably will depend on receiver aperture and on distance, the experiment must be arranged to allow adequate separation of these variables. For changes in distance to be meaningful, the receiver must bear some appropriate, uniform relation to the beam size regardless of distance. The simplest approach is to use apertures always larger than the direct beam, since it is known[3] that the fluctuations do not then disappear.

In order always to collect substantially all of the beam, the distance used should not be too large. Otherwise, the beam will be large and a receiver of an inconvenient size will be needed. With horizontal paths near the ground, it is commonly observed that atmospheric refraction produces angular spreading of the order of 10^{-4} radian, so paths miles long would imply beams feet in diameter. Moreover, it is desirable to be able to change the distance essentially continuously, and ranges where this can be done beyond a few hundred feet are not easily obtained.

Such short distances imply a very low level of atmospheric modulation with some values of modulation depth lower than 0.1 percent. This means, in turn, that the amplitude of the noise of the laser must not exceed about 0.01 percent. (In all cases, percent modulation is defined as 100 times the ratio of the rms power of the fluctuations to the average power). The laser used was designed[5] for high intrinsic frequency stability, and, when properly operated, it has the necessary amplitude stability as well. The RF power supply must be well regulated, and dc (rather than 60 Hz) power must be used on the filaments of the power supplies. At the frequencies of interest, acoustically-coupled noise is substantial in the average laboratory and must be reduced.

An easy and highly effective method of isolation is to seal the entire laser (at one atmosphere) in a gas-tight container. This was done with a glass bottled fitted with an optical-quality window at one end and a polished flange at the other end. The flange made an O-ring seal to a Lucite plate. (It is clear that pressure fluctuations cause considerable amplitude noise, since enclosures which do not form vacuum quality seals are not as effective.) It is helpful to place a felt pad between the laser and the bottle and Isomode (corrugated rubber) pads between the floor and the legs of the table on which the laser and bottle rest. With this arrangement, the amplitude modulation of the laser could be maintained at or below 0.005 percent.

The laser used[5] was also single-mode and RF-excited in order to further ensure that measurements did not include any spurious noise. Amplitude fluctuations can appear in the output of multimode lasers as a result of mode competition and self-beating effects. Hodara has calculated[6] and measured[7] the excess noise caused by mode-beating in multimode lasers. He observed that many of the reported discrepancies in measurements of laser noise probably arose because multimode lasers sometimes were used.

While our method of mounting the transmitter is not massive, nevertheless no detectable variation in beam pointing occurred during an experiment, and the arrangement has the advantage that it could be readily modified. As will be seen, the variety of necessary experiments required flexibility in both the transmitter and the receiver. The transmitted beam emerged from the room through a selected Lucite window. Ordinary plate glass caused noticeable refraction of the beam, but some parts of new panels of $\frac{1}{4}$-inch thick Lucite produced no measurable distortion of the beam. Many "A-B" experiments were made, and no difference could be found between modulation results with a Lucite pane and the results with an open sash.

The range was located on the flat roof of a two story building at the Whippany location of Bell Telephone Laboratories. The roof surface was asphalt and gravel, but usually most of the path was over a wooden catwalk whose surface was 3 inches above the roof. The beam path was 3 feet above the roof and ran 30° east of north to a total available range of 330 feet.

The receiver took a variety of forms, and it will be best to give each experimental arrangement with the corresponding results. However, many of the details of a representative setup can be seen in Fig. 1. The equipment cabinet on casters gave the mobility which was required for rapid change of propagation distance. Atop the cabinet is a section of triangular tower used when a long focal length, large aperture lens provided the receiving aperture. In the arrangement shown, a diffraction limited, 6-inch diameter, 8-foot F.L. lens was mounted in the right end of the tower section. The detector, an RCA 7265 photomultiplier, was equipped with a 3 Å wide-interference filter. The detector housing appears in the left end of the tower section.

Just to the left of the tower section is a bank of 6-inch diameter, $\frac{1}{4}$-wave flat mirrors supported by a bench which crosses the catwalk. Such mirrors were used to fold the beam and provide transmission paths up to 2400 feet long. The bench rested directly on the roof, and this provided adequate stability of the multiply-folded optical path.

On the bench in the foreground can be seen an RCA vacuum tube voltmeter, used to measure the dc voltage across the photomultiplier load, and a Hewlett-Packard 403A ac voltmeter, used to measure the rms ac voltage. The bandwidth of the 403A is 1 Hz to 1 MHz. To the best of our knowledge, this is the only ac meter that has such a low frequency response. Such response is important since the fluctuations are exponentially weighted toward the low end of the few-hundred hertz band. Since instantaneous voltage output from the photomulti-

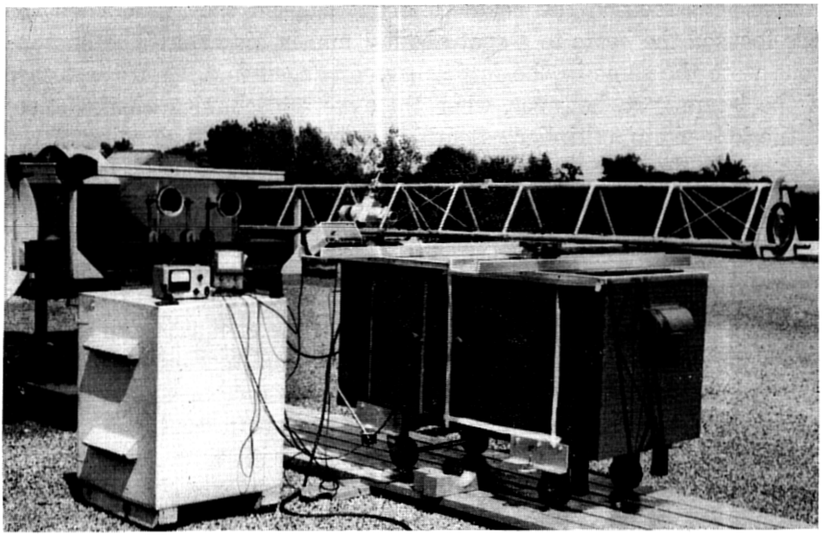

Fig. 1 — Receiving station.

plier is proportional to the instantaneous incident optical power, it follows that percent modulation is given by 100 times the ratio of the ac voltage to the dc voltage.

Since the fluctuations do not disappear[3] when all of the direct beam is collected, it is necessary to show that possible sources of spurious noise are negligible. The laser, for example, was checked periodically to insure that transmitter fluctuations were no larger than about 0.005 percent. Another possibility was that the fluctuations resulted from the product of the time-varying intensity profile of the beam and the uneven sensitivity profile of the photocathode. Also, the noise might have resulted from the dancing of a small beam (order of a millimeter diameter) over the sensitivity profile of the photomultiplier. Two experiments were made to assess these possibilities.

First, a photovoltaic cell (Hoffman 110C) which was one cm square was interchanged in "A-B" fashion with the photomultiplier. When the cell was used, the one-cm beam was focused to about one millimeter in diameter, and all of it therefore was collected by the cell. The spatial variation of sensitivity of the cell was orders of magnitude smaller than that of the photomultiplier, and the measured fluctuations were the same as with the photomultiplier, within experimental error.

In the second check, a comparison was made between the fluctuations obtained when a 1-cm diameter beam fell directly on the photomultiplier, when a 4-inch diameter, 60-inch focal length diffraction limited lens focused the beam to about a $\frac{1}{2}$-cm spot, and when the lens focused the beam to a spot about $\frac{1}{2}$ mm in diameter. The fluctuations were the same in the former two arrangements. In the last case of the $\frac{1}{2}$-mm spot, however, when there were mechanical disturbances of the lens-photomultiplier assembly large enough that dancing of the spot on the detector was visible, the level of fluctuations was measurably higher. Since the photocathode had a sensitivity structure with a scale size of the order of a few millimeters, it seems clear that the excess noise was caused by the random scanning of the small spot over the spatially varying sensitivity of the detector. As a result, in all of the work which employed a collecting lens, the spot was deliberately defocussed to about $\frac{1}{2}$ to 1 cm in diameter, and mechanical disturbances of the receiver were avoided.

The first and dominating result of any measurement of depth of modulation is that the value changes steadily with time. Another way of stating this is that the fluctuation spectrum extends well below 1 Hz, the low-frequency limit of the ac meter, and the ac value changes steadily. The consequence is that this temporal change in modulation is unavoidably combined with the dependence of modulation on the measured variables (such as distance) when the data are taken at different times. Thus, when distance is changed by moving the receiver, the value obtained changes because of variations both in distance and in time.

The seriousness of this depended on the degree of temporal instability of the modulation. Typically, the measured value of percent modulation varied by a factor of about two or perhaps three in a period of a few minutes. This was tolerable, but it meant that for a determination of distance or aperture dependence to be meaningful it was necessary to average the results for a large number of runs. For this reason it was important to arrange the experiment so that successive readings could be taken quickly. In general, it was possible to move the receiver and make a reading in about two or three minutes. Thus, a run involving five points took about 10 minutes. On many occasions, however, the temporal instability of modulation was severe, and the value might change by an order of magnitude within a few minutes. This is comparable with the total change produced by the variations in distance and aperture, and useful data could not then be obtained. Most of our results were taken at night, a few hours after the sun had gone down. During this period, changes in weather conditions were relatively small. Besides, the alignment of the receiver could be made very rapidly at night, hence a number of runs could be taken in quick succession.

An alternative which would circumvent this problem would be to divide the beam with beam splitters into receivers at each distance or aperture value. Modulation then would be measured simultaneously at all the receivers. However, this would have required more extensive facilities than were readily available.

III. RESULTS—APERTURE DEPENDENCE

It should once again be emphasized that while the fluctuation spectrum may be insensitive to size of receiving aperture, one expects the depth of modulation to depend rather critically on it. In particular, if the fluctuations are produced entirely by variations in power collected by a finite aperture, one might expect the depth of modulation to decrease as larger apertures are employed. With a large enough receiver, the fluctuations should then go to zero.

Depth of modulation was measured with a beam which appeared to the dark-adapted eye to be about $\frac{1}{2}$ to $\frac{3}{4}$ inch in diameter and with

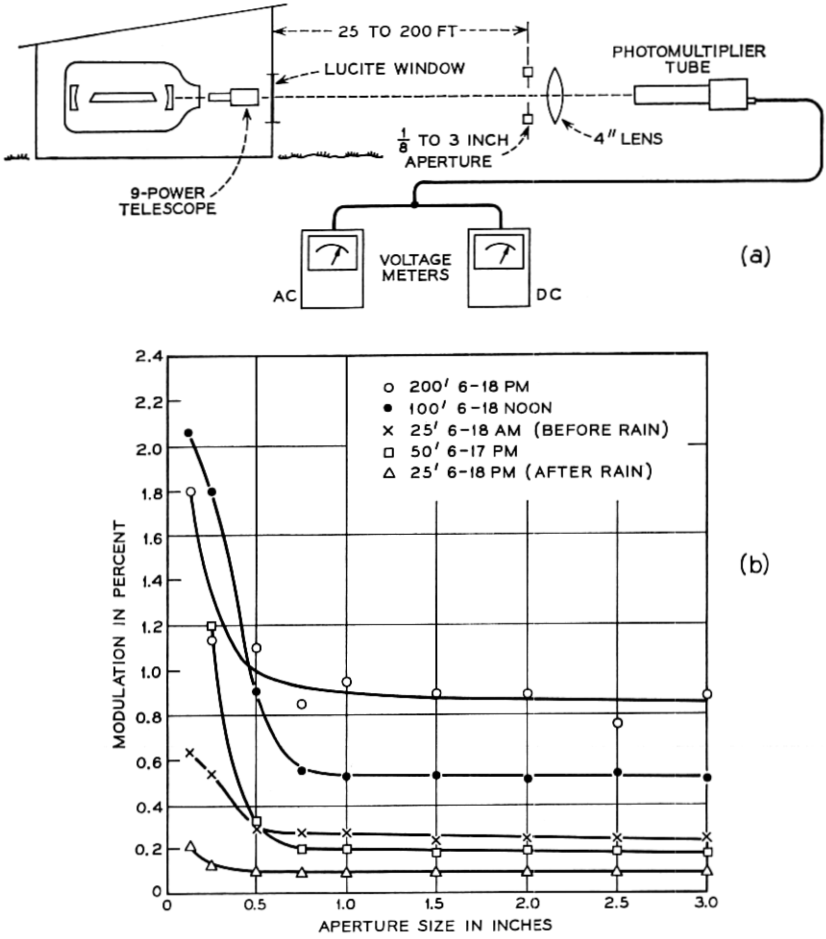

Fig. 2—Aperture dependence: (a) schematic, (b) data.

receiving apertures from $\frac{1}{8}$ to 3 inches in diameter. Fig. 2(a) shows the schematic of the arrangement. Aperture dependence was thus measured at various distances. At each distance, several successive data runs were taken and averaged. The results are shown in Fig. 2(b).

There are two curves for the 25-foot distance, one with considerably higher percentage modulation than the other. The former was taken before a rain storm and the latter immediately after the rain. Although the rain appeared to have a decided effect in this case, firm conclusions should not be drawn since this was a single observation.

(Nature did not present us with more than one such opportunity.) Dependence on the atmospheric conditions will be discussed more fully below.

IV. RESULTS—RANGE DEPENDENCE

The dependence of depth of modulation on range was measured, using apertures large enough to collect all of the direct beam. The aperture dependence [See Fig. 2(b)] with small apertures is sufficiently strong that a meaningful range dependence would be difficult, or impossible, to obtain using apertures smaller than the beam. Generally, the aperture used was at least twice the size of the direct beam as it appeared at night (i.e., to a dark-adapted eye). Thus, all data for range dependence were obtained in the region in which the value is sensibly independent of aperture size.

The schematic of the first experiment is the same as given in Fig. 2(a) except that a 20-power telescope was used instead of the 9-power one. The averages of all the data are plotted in Fig. 3 on log-log coordinates. (The bars are averages of the mean deviations in the data for each night. The deviations are indicative of the unavoidable uncertainty caused by changing atmospheric conditions).

The results from 100 to 300 feet suggest that depth of modulation

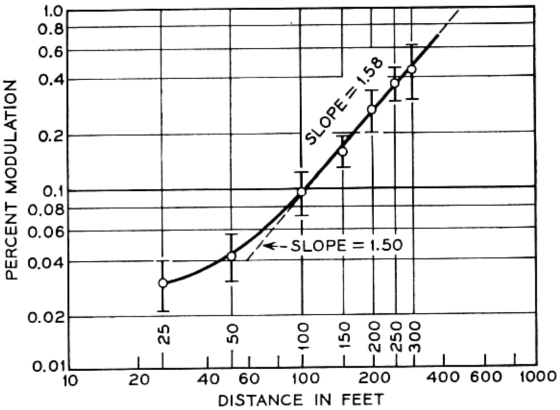

Fig. 3 — Range dependence—single pass—(25–300 ft);

—night—4 runs,
5/20/65 night—8 runs,
5/26/65 night—6 runs,
5/28/65 night—6 runs,
6/24/65 night—2 runs.

goes approximately with the 3/2 power of distance. The data at 25 and 50 feet seem to indicate an end effect not intrinsic in the atmosphere. It suggests that the source was noisy, producing spuriously large values at small ranges. To examine this possibility, the data were replotted on linear-linear coordinates, and the curve was extrapolated to the y-intercept. This yielded an apparent zero-distance modulation of 0.024 percent. This is about five times the laser noise of 0.005 percent measured inside a quiet room, so that the laser cannot be the dominating cause of the change in curve shape at short distances.

Another possible explanation is that local conditions affected the atmosphere differently along the beam. Such variations could have been caused by a row of four large exhaust blowers along a line about 30 feet east of the range. On the night of June 24, 1965, we arranged to turn off all the blowers. Two runs were taken, and the results showed the same distance dependence as with the blowers on, so closely, in fact, that the data were simply included in the final curve of Fig. 3.

Another possibility is that local conditions affected the atmosphere differently near the transmitter room than far away from it. For example, the room itself may well have changed the turbulence of the wind. We therefore set up the transmitter outside the room and 30 feet away from the room (which is 8 by 14 feet). If local condi-

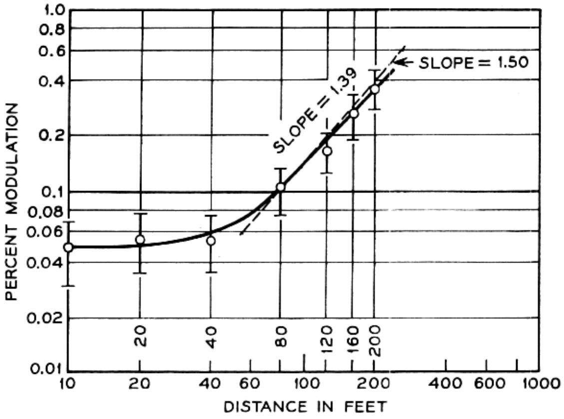

Fig. 4—Range dependence: single pass (10–200 ft) with transmitter located outside the building;

8/3/65—day—6 runs,
8/3/65—night—6 runs.

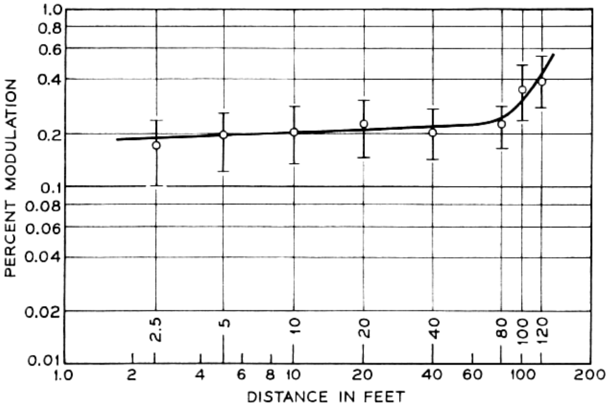

Fig. 5 — Range dependence: single pass (0–120 ft) with transmitter located outside the building and telescope changed to 9 power;

7/20/65—night—4 runs,
7/21/65 day—4 runs,
7/21/65 night—4 runs,
7/22/65 day—5 runs.

tions were the cause, the entire curve should displace to the left on the abscissa by 30 feet.

The schematic of this arrangement is identical to that shown in Fig. 2(a) except that the laser is located outside the building with a 20-power telescope. The results are given in Fig. 4. The curve did not shift along the abscissa and shows generally the same behavior at the same distance from the transmitter. Whatever the cause of the near-distance behavior of the modulation, it seems to have moved 30 feet with the transmitter.

Because of the surprising behavior of the depth of modulation at short ranges, more measurements were made, still with the transmitter outside, but now adding some very short distances. The 20-power telescope was replaced with one of 9 power, giving a reduction in diameter of transmitted beam from about 2 to about 1 cm. This was done to examine what effect beam divergence might have on the distance at which the knee of the curve appears. Data were taken during both day and night. The daytime data were found to be not significantly different from the night data. All values were averaged and are plotted in Fig. 5. It appears that the relatively large values of modulation depth which yield a small slope at short distances persist at distances as small as $2\frac{1}{2}$ feet from the transmitter. (The laser noise level of 0.005 percent is measured at distances of this order, but in a laboratory where the air turbulence is low.)

There appears in Fig. 5 to have been a significant change from the distance dependence displayed in Figs. 3 and 4. The knee of the curve seems to have moved to larger distances such that the 80-foot value now is aligned with the dependence at short distances. Since the change in beam size was an obvious potential explanation, and since the principal experimental difficulty still was the large temporal variation in modulation, the following experiment was conducted. The laser was mounted outside and 30 feet away from the transmitter room, and the beam was split into two beams of approximately equal intensity. One beam was transmitted with the 20-power telescope, the other with the 9-power telescope. The beams were parallel and 20 inches apart. At each distance, the modulation was measured on both beams before changing distance. Three runs were made on one night. This did not give enough data to define smooth curve shapes, but it was enough to show that the curve shapes were nearly identical for the two beams. The change in Fig. 5 from Figs. 3 and 4, therefore, cannot be attributed to the change in telescopes.

Having explored the behavior of modulation at short distances, we now turned to distances larger than 300 feet and in particular to the question whether the 3/2 power dependence of modulation depth on distance would continue at large distances. Fig. 3 suggests that modulation depth at 300 feet may be a little lower than a 3/2 power dependence would imply. Results for three of the five nights summarized in Fig. 3 showed a rather pronounced reduction in the value expected at 300 feet by extrapolation from shorter distances.

In order to obtain much longer paths in the available space, we now folded the beam back and forth over the range, as shown in Fig. 6(a). The laser again was mounted in the transmitter building, and the 20-power telescope was used in order to reduce beam spreading by diffraction over these longer distances. At the maximum distance of 2100 feet, the spot generally was about two to three inches in diameter. The mirrors used were always substantially larger than the beam. In Fig. 6(a), the first two mirrors were four inches square, and the last four were six inches in diameter. All the mirrors were flat to a quarter-wave and front-aluminized. Fig. 1 is a photograph of the receiving end of this arrangement. Modulation now was measured at distances of 300, 900, 1500, and 2100 feet by moving the receiver laterally in 18-inch increments, placing it in the four appropriate positions to intercept the beam.

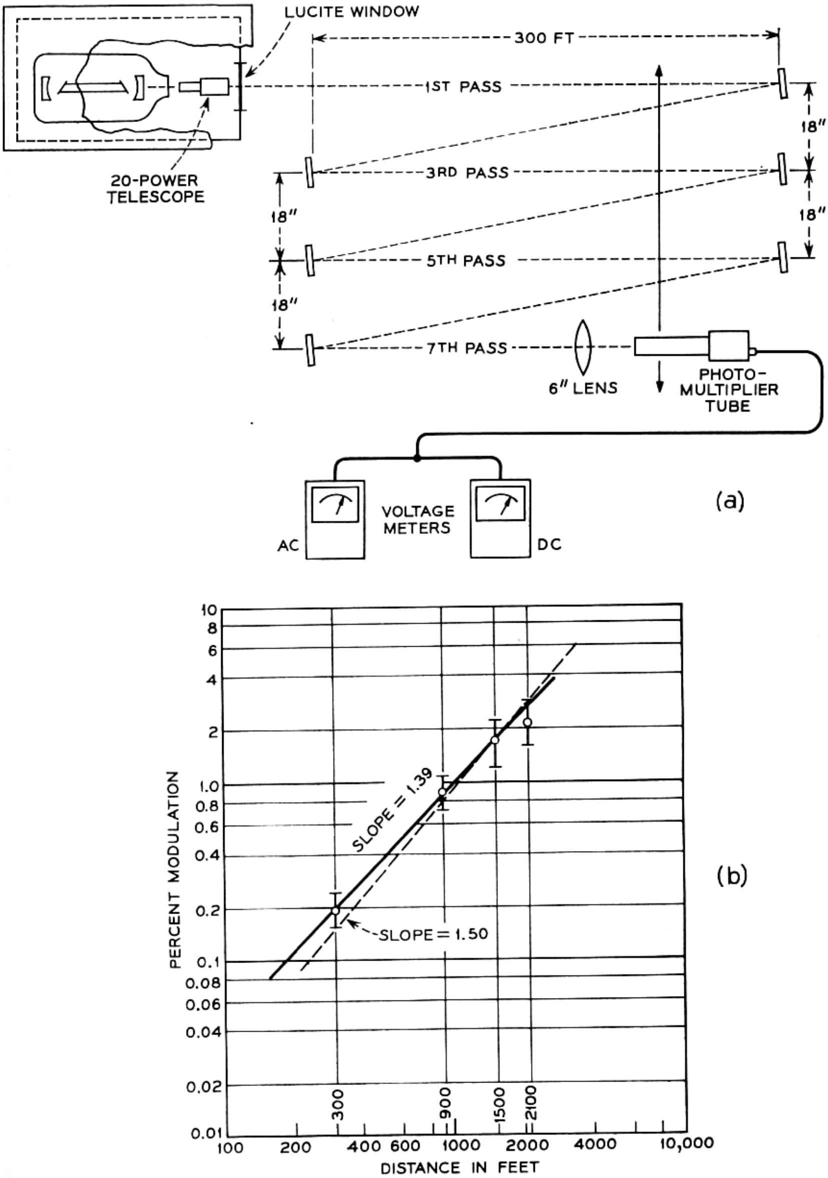

Fig. 6 — Range dependence: multiple pass (300–2100 ft);
(a) schematic,
(b) data 8/6/65 night—8 runs,
8/8/65 night—8 runs,
8/12/65 night—5 runs,
8/17/65 night—10 runs.

On some nights, beam dancing was severe enough to carry the beam off the final mirrors periodically, so that data could not then be taken. In the early part of most summer nights there was a slow drift of the beam upward as the air cooled increasingly below the temperature of the roof and the strength of the inverted "prism" increased with time (the situation is different during winter nights, see Fig. 11). A representative speed of vertical beam movement at 2100 feet was about two inches per hour. This required us not only to align the mirror system for each night's work but to realign each night every 20 to 30 minutes. Mechanical stability of the mirrors was good. When atmospheric conditions were stable enough there was no noticeable movement of the beam, so that building vibrations appeared to have no important effect.

The averages of the data were plotted in Fig. 6(b). It is seen at once that there is no significant change in slope beyond 300 feet. The points at 300, 900, and 1500 feet yield a well-defined straight line of slope 1.39, in good agreement with results below 300 feet. The value at 2100 feet seemed distinctly low, and it was not used in fitting the data. This is reminiscent of the behavior of the last point in Fig. 3 which led to the present measurements. It was not possible to determine any systematic cause of error in the measurements at the largest distance. The fact that there the beam is largest, and most difficult to collect, would explain a modulation which is too large, not too small.

Once again an increase in the distance seemed necessary, and this was accomplished by adding one more 300-foot pass to the folded system and moving the receiver to the transmitter end. This is shown in Fig. 7(a). Here the first three mirrors were four inches square, and the last four were six inches in diameter. The data, averaged in the usual way, are presented in Fig. 7(b). The slope of 1.46 again agrees with previous values, within experimental error. The value at 2400 feet shows that there is no decrease in slope beyond 2100 feet. It would seem that the 3/2 power dependence of modulation depth on distance continues to at least 2400 feet. It would be instructive to continue these measurements at larger distances, but, for this, larger optics would be required than were available.

The folded-path method of obtaining large ranges is both convenient when space is limited and desirable when readings must be made quickly at positions which would be widely spaced in a straight path. However, the question remains whether this is in every sense equivalent to the unfolded, straight path. For the distances considered here,

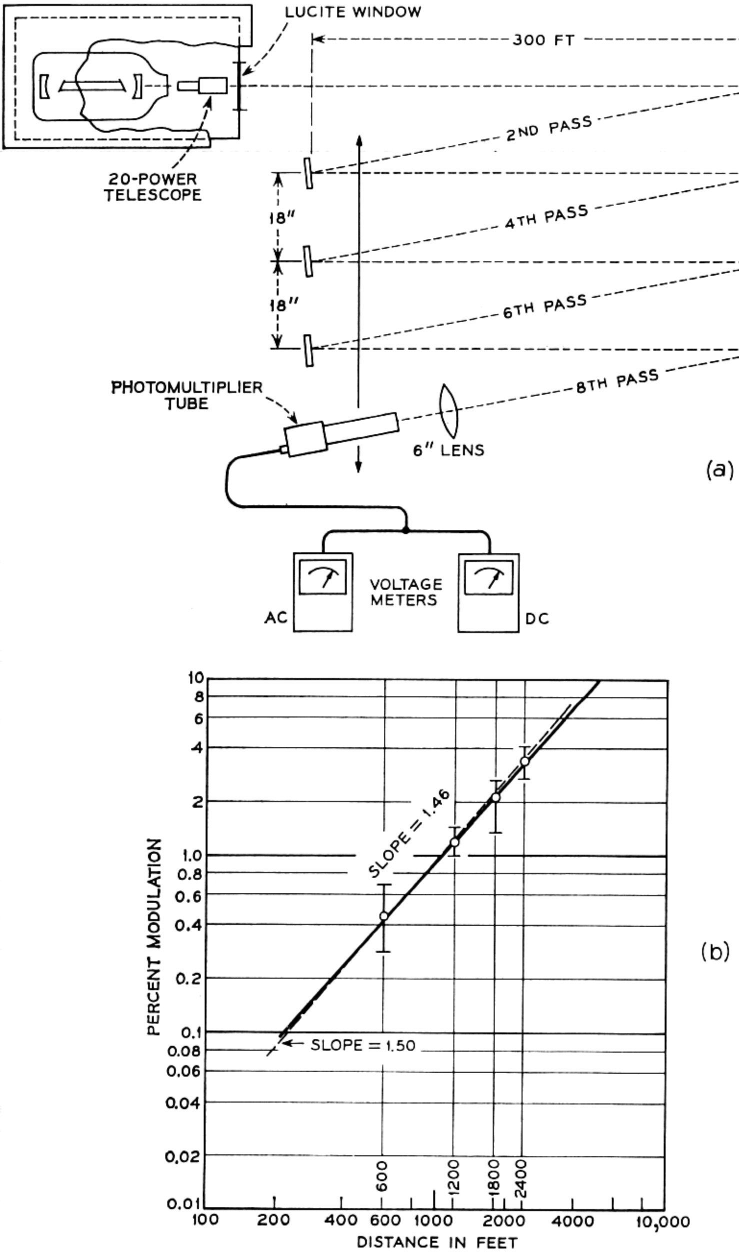

Fig. 7 — Range dependence: multiple pass (600–2400 ft);
(a) schematic,
(b) data 8/18/65 night—8 runs,
8/19/65 night—7 runs,
8/23/65 night—8 runs.

it takes only about 0.3 microsecond for the light to travel each lap and thus a total of 2.4 microseconds for 8 laps of 2400 feet. The characteristic time of turbulence is of the order of milliseconds, and consequently the turbulence can be considered frozen during the time required for the light to travel the 2400-foot folded path. Under the circumstances, not only are there possible correlations between the closely adjoining segments of the folded path but, since the beam travels oppositely along the successive segments, there may be an actual reduction in the net effect of atmospheric turbulence.

To determine this, we now set up a mutiply-folded system of mirrors, in all respects the same as the 2100-foot system in Fig. 6(a) except that now the segment length was reduced from 300 feet to 38 feet. This gave a total, folded distance of 270 feet so that comparison could be made with the results for a straight 300-foot path which were given in Fig. 3. The new arrangement is shown in Fig. 8(a). The results appear in Fig. 8(b). The functional dependence is sensibly unchanged from that in Fig. 3. This therefore not only vindicates the folded-path method used for large distances, but it further confirms the short-distance behavior of the modulation.

Although this experiment provided no evidence of correlation effects, it seemed worthwhile to make a further attempt to detect any correlation effects that may affect depth of modulation. The folded path, therefore, was rearranged as shown in Fig. 9(a). The telescope was focused on the receiver to give the smallest possible spot, which was about one cm in diameter. The first, second, fifth and sixth mirrors were two inches in diameter and quarter-wave flat. They were mounted so that no frame of the mirror or other obstruction extended in front of the six-inch diameter third and fourth mirrors. It was thus possible to position the spots on each bank of mirrors just two inches apart between centers. Hence, the average separation of successive segments of the path was only one inch. However, this made it impossible to intercept the beam with the photomultiplier, as was done before, without obstructing a previous segment of the beam. Consequently, the beam was reflected to the photomultiplier with an elliptical mirror (of the telescope-diagonal type). This mirror was then displaced laterally to intercept the beam. In this experiment, the positioning of the mirror was simple enough that a run could be completed in about 2 minutes, so that many runs were quickly made and exceptionally good averaging should result.

The average values are plotted in Fig. 9(b). Once again, the now familiar features of the curve appear. It is interesting to note that, in

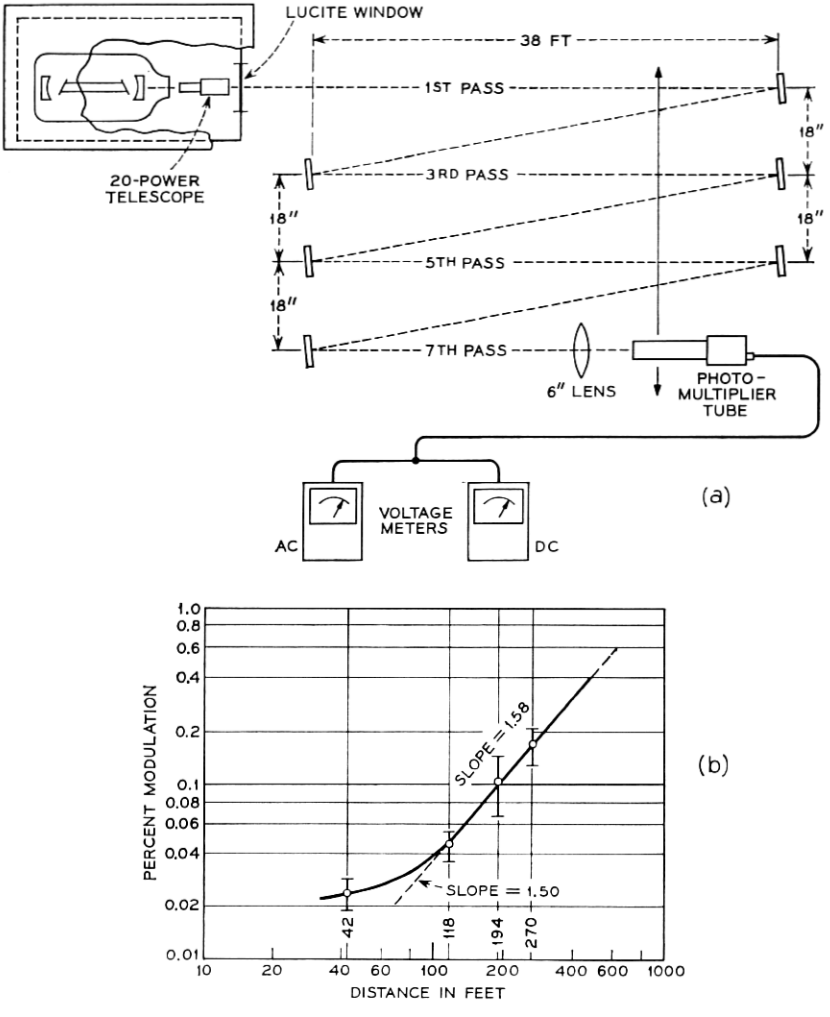

Fig. 8 — Range dependence: multiple pass (42–270 ft);
(a) schematic,
(b) data 10/7/65 day—5 runs,
10/13/65 night—10 runs,
10/14/65 night—7 runs.

the previous experiments, the slope beyond 100 feet has centered around 3/2 and that, in the present experiment in which especially good averaging against changing conditions was expected, the slope was 3/2 as closely as such curve-drawing will allow. The present results again give no evidence of correlation effects.

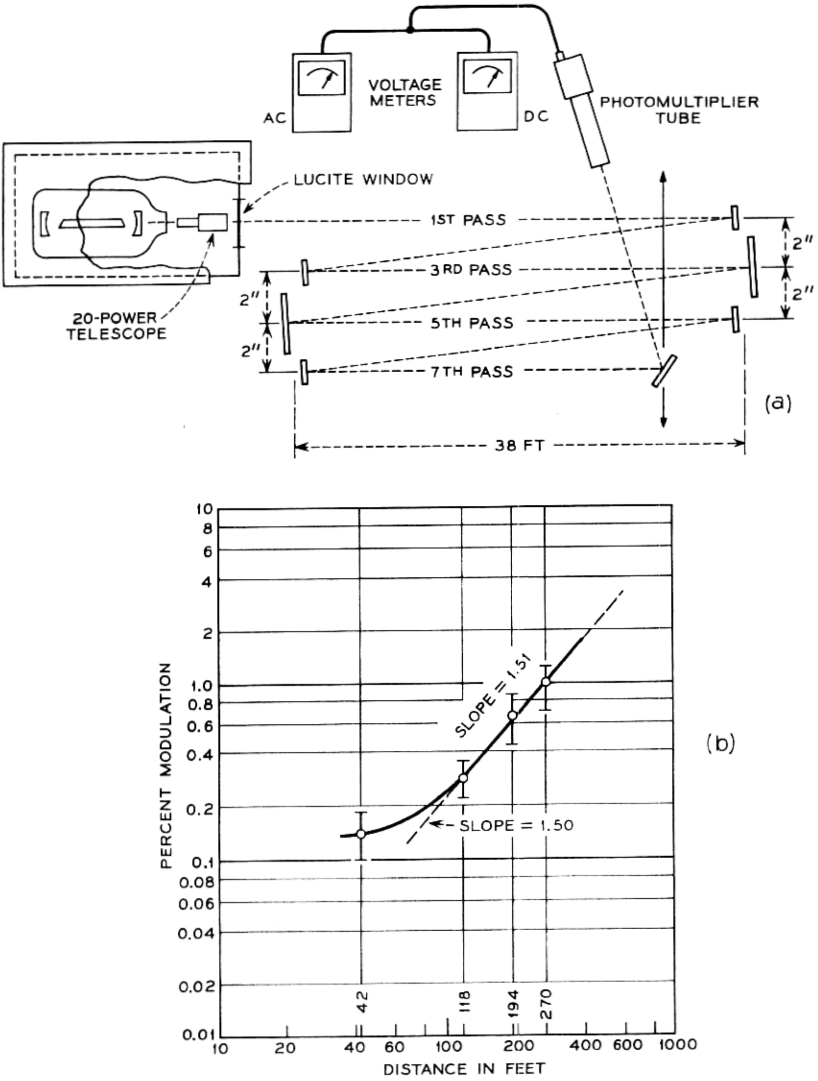

Fig. 9 — Range dependence: multiple pass (42–270 ft) with mirror separation of 2″;
(a) schematic,
(b) data 8/27/65 night—7 runs,
8/30/65 night—11 runs,
9/2/65 night—13 runs.

V. RESULTS—DEPENDENCE ON ATMOSPHERIC CONDITIONS

It was remarked before that the first and dominating result of any measurement of depth of modulation is that the value changes steadily with changing atmospheric conditions. This persists during, and interferes with, all attempts to measure the dependence of modulation on any other parameter. Hence, a great deal of qualitative observation of the effects of conditions was inescapably made during the measurements of aperture and range dependence. And this extensive experience taught simply that the depth of modulation was unpredictable. There never appeared any simple correlation between modulation and qualitative observables such as wind speed or direction, sun conditions, or time of day. The most promising lead which developed was the observation already cited in the aperature work that the depth of modulation was markedly smaller over a 25-foot range after a short heavy rain than before [Fig. 2(b)]. The rain, of course, would have both cooled the roof (which had been heated by the sun) and reduced dust in the air. Since no connection previously was apparent between modulation and temperature conditions, it then was thought that the modulation might be affected strongly by particulate scattering.

Indeed, at night the beam was always well decorated by forward scattering from what appeared to be dust and haze particles. Normally the beam could be seen with the eye placed within about 10^{-2} radian of the forward direction. This was the case with clear or hazy conditions. Only with fog could the beam be seen substantially more than 10^{-2} radian away from the forward direction. Under no conditions, including light fog, could backscattering be detected by eye. The result with the multi-folded system was that, when standing at one end of the range and looking toward the other, only those segments in which the beam was approaching could be seen. This is shown by the photograph of Fig. 10, which was taken looking toward the transmitter. Speed of the film was ASA 125. Exposure time was 15 minutes, so the beams appear well filled-in. There seem to be four separate, parallel beams from four separate sources. There is no trace of the diagonal connections of the returning beams. It seems clear that back scattering was orders of magnitude weaker than forward scattering. The right-hand beam came from the laser, and light scattered at the source caused the saturation of the film.

When measuring depth of modulation, therefore, the level of forward scattering out of the beam was readily observed. Attempts were made to correlate modulation with scattering, and no qualitatively

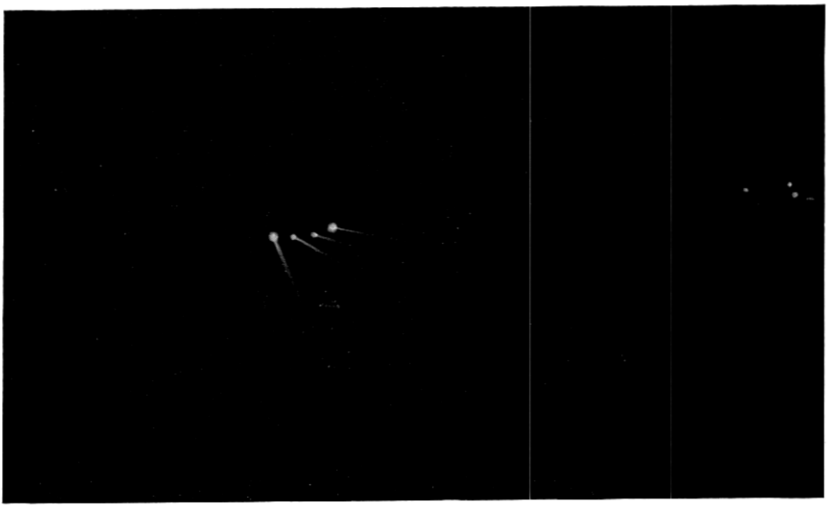

Fig. 10 — Photograph of multiple-pass laser beam illustrating the lack of back scattering.

apparent relation could be found. The level of scattering did vary in a pronounced way as the density of particles varied, but modulation did not seem to change correspondingly. With short ranges, such as 25 feet, cigarette smoke was deliberately blown into and along several feet of the beam causing brief and strong decoration of the beam but having no noticeable effect on modulation.

Having thus found no dependence by casual observation, we arranged to measure modulation while also systematically noting wind speed, direction, and variability, temperature of the roof, and temperature of the air at beam level. The arrangement was that shown in Fig. 3 except the distance was 138 feet. Temperatures were measured at the middle of path. Readings were taken every half hour continuously for 24 hours so that any diurnal variation would be detected. In particular, one might expect a change in modulation depth around sunrise and sunset. (Reliable observations of "good-seeing" at sunrise and sunset date at least from 1878 when Michelson measured the velocity of light and found that he could not work at any other time due to excessive "boiling" of the image of his slit.[8])

The 24-hour period began at 5:45 p.m. on November 3, 1965. A broad variety of wind conditions was obtained both at night and during the day, ranging from dead calm to a period which was violently gusty in the late morning. The sky was clear at the beginning, becoming

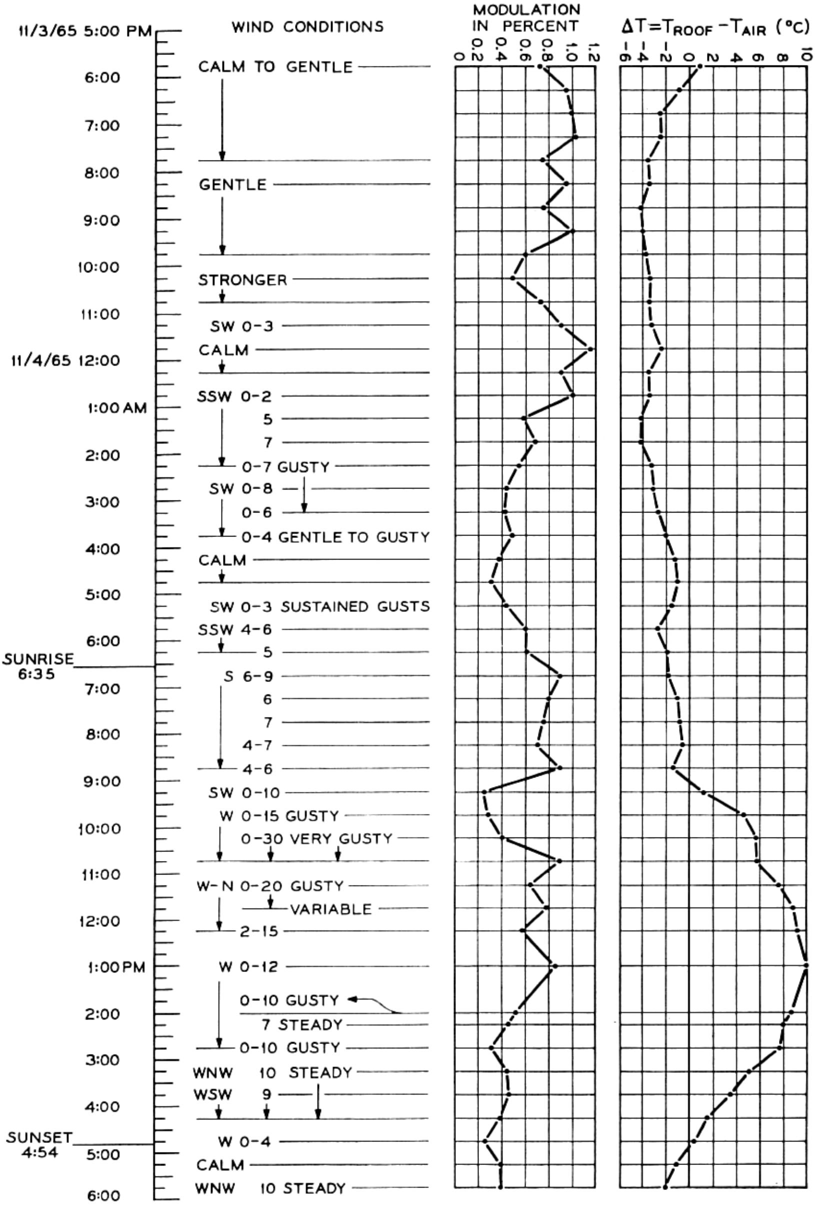

Fig. 11 — Dependence on weather conditions.

gradually overcast until no stars could be seen by 3:45 a.m. on November 4, 1965. This cleared to only a hazy sky by 8:00 a.m., and the daytime then remained clear except for occasional clouds.

The results are assembled in Fig. 11. The difference between roof and air temperature is recorded since one expects temperature gradients to be more significant than temperature level. Wind conditions are given below depth of modulation. It is apparent at once that there was no simple correlation between modulation and temperature difference or time of day. Any distinct reduction or other effect on modulation at sunrise or sunset is conspicuously absent.

The negative results of the previously qualitative observations therefore have been extended by these more quantitative results. It is still clear that modulation depth depends upon atmospheric conditions in a sensitive way, but the measurements so far have not revealed the nature of the dependence.

VI. THEORETICAL BACKGROUND: DISTANCE DEPENDENCE

The amplitude and phase fluctuations of an electromagnetic signal that has propagated through a random medium have been theoretically analyzed using different approaches. Of the more familiar ones are the following: the ray theory, the first Born approximation, and Rytov's method. The results obtained by all three methods agree, though they may hold true only for certain regions of distance.[9] Of the above three approaches, Rytov's method lends itself to a generalized treatment that holds good for both short (Fresnel zone) and long (Fraunhofer zone) distance regions of the scatterer. It should be noted that in the following discussion the Fresnel and Fraunhofer regions are with respect to the scattering medium and not with respect to the transmitter aperature.

The physical configuration for which the calculations are made is shown in Fig. 12. An infinite plane wave is incident upon a semi-infinite ($-\infty < x < +\infty; -\infty < y < +\infty; z \geqq 0$) inhomogeneous and random medium which is assumed to be quasistatic. A point detector is located at $x = L$ which not only sees the unscattered direct wave but also waves scattered from within a scattering volume that is in the form of a cone. This cone has its vertex at the receiver and has an aperture angle of the order of $1/ka$, where k is the propagation constant ($= 2\pi/\lambda$) and a is the scale size of the tubulence. The justification of using this configuration for our measurements will be given at the end of this section.

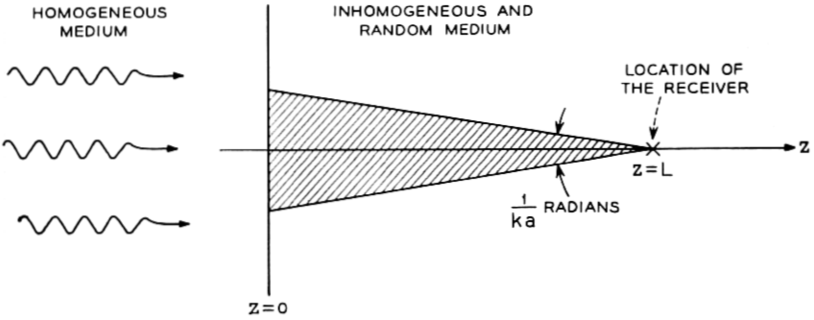

Fig. 12 — Physical configuration for calculation of amplitude fluctuations.

The functional dependence of the amplitude fluctuations will depend on whether the receiver is located in the Fresnel or Fraunhofer zone. The extent of these zones are determined by a dimensionless parameter D, called the wave parameter. It is given by

$$D = \frac{4L}{ka^2}. \tag{1}$$

$D \ll 1$ for the Fresnel region and $\gg 1$ for the Fraunhofer region. The dependence of the mean square fluctuation of the amplitude $\overline{B^2}$ on this distance L is given by[10]

$$\begin{aligned} \text{For} \quad D \ll 1 \quad & \overline{B^2} \alpha L^3 \\ D \gg 1 \quad & \overline{B^2} \alpha L. \end{aligned} \tag{2}$$

In other words, the root mean square value of the fluctuations will have a functional dependence on distance given by the distance raised to the power 3/2 and 1/2 for the Fresnel and Fraunhofer zones, respectively.

The quantity measured in the experiment is the percentage modulation given by 100 times the ratio of the rms voltage to the average value. This ratio is the same as the ratio of the rms value of the fluctuating light power to the average light power expressed in percentage. For values of ac-to-dc power ratio very small, it can be shown that

$$\frac{V_{\text{rms}}}{V_{\text{av}}} = \frac{P_{\text{rms}}}{P_{\text{av}}} = 2\frac{E_{\text{rms}}}{E_{\text{av}}}, \tag{3}$$

where V's refer to the voltages across the photomultiplier load, P's to the light power and E's the electric field intensities in the light radiation field. From (2) and (3), we see that the ratio of the V_{rms} to V_{av}

should vary as $L^{3/2}$ in the Fresnel zone and as $L^{1/2}$ in the Fraunhofer zone. Even though the configuration used for the theoretical calculation does not seem to represent our experimental arrangement, its validity can be justified by the following argument. The 3/2 power law, which is the one that will be used to compare with our results, can also be derived using ray theory in which fluctuations in an infinitely thin ray of light are considered. The fluctuations in this case can be measured using a point detector. In our case, the ray is of finite diameter (that corresponding to the laser beam diameter), and consequently the detector is also of finite dimension to insure collection of the entire ray.

VII. DISCUSSION

The modulation of a single-mode, single-frequency laser beam at 6328A° by atmospheric turbulence was investigated by varying the propagation distance as the parameter. Unlike the spectral width of modulation, the depth of modulation does depend on distance and varies as 3/2 power of the distance. In making the measurement, it was ensured that all of the direct beam was collected by the receiver. The range of the propagation distance was extended from 0 to 2400 feet. The empirically obtained 3/2 power law agreees well with the theoretical result obtained using Rytov's method, provided the propagation distance is within the Fresnel zone of the scatterer. This assumption leads to an estimation of the scale size of the atmospheric turbulence. The effective scale size in (1) is estimated to be larger than 5 cm in diameter for $L \geqq 2400$ feet and $D = 0.1$.

An interesting feature of the dependence of depth of modulation on distance is its large value at distances lower than about 100 feet. This produces a functional dependence on distance which is other than 3/2 power. We have also noticed that the spectral width which is characteristicaly a few hundred hertz and whose exponential decay with frequency is otherwise independent of distance undergo changes at these short distances. The amplitudes of the low-frequency components decrease more rapidly than those of the high-frequency components as the distance is made shorter. We believe that the short distance variation of the depth of modulation and the spectral width are related and are caused possibly by the same phenomenon. This is under further study.

In comparing the absolute value of depth of modulation with those obtained by others, it has to be borne in mind that we collect all of the direct beam in contrast to previous work in which only part of the direct beam was collected. Consequently, the magnitude of the depth of modulation measured by us is much less than that of others. For example, Edwards and Steen[12] have observed with a zirconium arc source for a 300-meter path a depth of modulation as high as 80 percent, whereas our value for the same distance is of the order of 1 percent or less. Another measurement by Portman, et al[13] with a partially collected beam at 500 meters yielded a peak to peak percentage modulation of 150 percent.

Although the main goal of this work was the functional dependence of the depth of modulation on distance, two other observations were made which are worth noting. Contrary to the behavior of the spectral width, which depends systematically on weather conditions, the depth of modulation has no clear-cut dependence on the atmospheric conditions which were measured so far. It seems modulation is a sensitive but obscure function of atmospheric conditions. The atmospheric variables which need to be measured evidently are relatively fine. Besides many qualitative observations, the quantitative results of a 24-hour run were a demonstration of this.

Also, we have not been able to observe any back scattering of the laser radiation even under severe weather conditions. Most of our experiments were conducted during night time. Even on very dark nights, the dark adapted eye (of several observers) could not detect any trace of back scattering. This is true in conditions of clear atmosphere with various amounts of particulate matter, haze, fog, and under severe rain storms. From these qualitative observations, we are led to estimate that the back scattering is orders of magnitude lower than the narrow angle forward scattering—a value considerably lower than the 2 percent obtained by Carrier and Nugent.[14] This observation is surprising also in view of the various reports of atmospheric back scattering observed with optical radar systems (e.g., Collis and Ligda[15]).

VIII. ACKNOWLEDGMENT

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