Power Density Spectrum of the Sum of Two Correlated Intermodulation Noise Contributors in FM Systems

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In the recent literature, two noise contributors in FM systems have been analyzed:(i) intermodulation noise due to transmission deviations, and (ii) AM/PM intermodulation noise. Even though different, these two contributors have the same property of being functions of the baseband signal. Hence, one would expect them to be correlated to some degree.

In this paper, we derive the expression for the power density spectrum for the sum of these two noise contributors. The resulting expression has been programmed on a digital computer. It has been found that, under certain conditions, the correlation can be quite significant. In fact, an example using a representative FM radio relay system shows that the correlation can result in greater than 4 dB error if the two contributors are assumed to be uncorrelated.

I. INTRODUCTION

Two noise contributors in FM systems are: (*i*) intermodulation noise due to transmission deviations, and (*ii*) AM/PM intermodulation noise. The first noise contributor is generated when an FM signal is passed through a linear transmission medium which has transmission deviations. The second noise contributor is generated when an FM signal is passed through such a medium which is followed by an AM/PM conversion device. These two noise sources are different, in general, but have in common the property that they are a function of the baseband signal. Therefore, one would expect that they are correlated to some degree. This would mean that combining the two noise power density spectra together assuming random addition (uncorrelated random variables), i.e., power addition, might not be sufficient in general.

In this paper, we will derive the power density spectrum for the sum of these two noise contributors. We will then examine the results, with the help of a digital computer program, for the conditions under which the two noise contributors are correlated. We will also present an example using a representative FM radio relay system. However, before considering the correlation problem, we will first briefly consider the two noise contributors individually.

II. INTERMODULATION NOISE DUE TO TRANSMISSION DEVIATIONS

Intermodulation noise is produced in FM systems whenever the FM signal is passed through a linear transmission medium which has transmission deviations.* This situation is depicted in Fig. 1 where

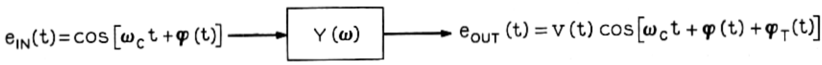

Fig. 1 — Generation of intermodulation noise due to transmission deviations.

the transmission medium, $Y(\omega)$, is represented by power series gain and phase transmission deviations up to fourth order, or

$$Y(\omega + \omega_c) = [1 + g_1\omega + g_2\omega^2 + g_3\omega^3 + g_4\omega^4] \cdot \exp i[b_2\omega^2 + b_3\omega^3 + b_4\omega^4]. \qquad (1)$$

where ω_c = carrier frequency in radians per second. The output signal is phase modulated by the desired signal, $\varphi(t)$, as well as the phase modulating distortion function $\varphi_T(t)$. This distortion function consists of first- (linear), second-, third- and higher-order functions of the input phase modulating function $\varphi(t)$. Because of their significance in FM radio relay systems, we will concentrate on the second- and third-order terms which will generate second- and third-order intermodulation noise once the signal is demodulated. In other words, we will let

$$\varphi_T(t) = \varphi_2(t) + \varphi_3(t), \qquad (2)$$

where $\varphi_2(t)$ and $\varphi_3(t)$ represent the second- and third-order intermodulation noise components produced by the transmission deviations in $Y(\omega)$. In Ref. 1, these components were derived and are given by

* Transmission deviations are defined as any deviation in the gain and phase characteristics from the ideal characteristics of constant gain and linear phase for all frequency components of the FM wave.

$$\varphi_2(t) = \left[-\tfrac{1}{2}\lambda_{2i} + \tfrac{1}{4}l_{2i}\frac{d}{dt} - \tfrac{1}{12}l_{3i}\frac{d^2}{dt^2}\right]\varphi'^2(t) + [\tfrac{1}{24}l_{4i}]\varphi''^2(t) \tag{3}$$

$$\varphi_3(t) = \left[\tfrac{1}{6}\lambda_{3r} - \tfrac{1}{12}l_{1r}\frac{d}{dt}\right]\varphi'^3(t) \tag{4}$$

with the prime (′) notation depicting the derivative with respect to time. The λ and l coefficients can be expressed in terms of the transmission deviation coefficients of the transmission medium by the appropriate equations in Ref. 1. Equations (2), (3), and (4) can be represented as shown in Fig. 2 where

$$H_1(\omega) = [\tfrac{1}{12}l_{3i}\omega^2 - \tfrac{1}{2}\lambda_{2i}] + i[\tfrac{1}{4}l_{2i}\omega] \tag{5}$$

$$H_2(\omega) = \tfrac{1}{24}l_{4i} \tag{6}$$

$$H_3(\omega) = [\tfrac{1}{6}\lambda_{3r}] + i[-\tfrac{1}{12}l_{1r}\omega]. \tag{7}$$

The autocorrelation function of $\varphi_T(t)$ is given by

$$\begin{aligned} R_{\varphi_T}(\tau) &= \overline{\varphi_T(t)\varphi_T(t+\tau)} \\ &= \overline{[\varphi_2(t) + \varphi_3(t)][\varphi_2(t+\tau) + \varphi_3(t+\tau)]} \\ &= \overline{\varphi_2(t)\varphi_2(t+\tau)} + \overline{\varphi_3(t)\varphi_3(t+\tau)} \\ &\quad + \overline{\varphi_3(t)\varphi_2(t+\tau)} + \overline{\varphi_2(t)\varphi_3(t+\tau)} \\ &= R_{\varphi_2}(\tau) + R_{\varphi_3}(\tau) + R_{\varphi_3\varphi_2}(\tau) + R_{\varphi_2\varphi_3}(\tau), \end{aligned} \tag{8}$$

where the bar notation depicts the time average of the function over an infinite interval, and e.g., $R_{\varphi_3\varphi_2}(\tau)$ is the crosscorrelation of $\varphi_3(t)$ and $\varphi_2(t)$. In the Appendix of Ref. 2, it was shown that

$$R_{\varphi_3\varphi_2}(\tau) = R_{\varphi_2\varphi_3}(\tau) = 0 \tag{9}$$

so

$$R_{\varphi_T}(\tau) = R_{\varphi_2}(\tau) + R_{\varphi_3}(\tau). \tag{10}$$

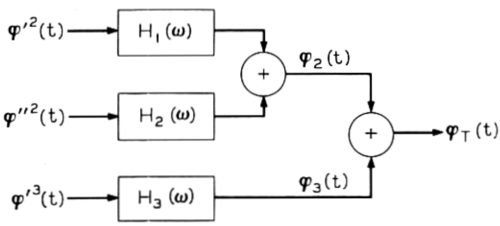

Fig. 2 — Block diagram of total intermodulation noise due to transmission deviations.

Taking the Fourier Transform of (10) gives

$$S_{\varphi_T}(\omega) = S_{\varphi_2}(\omega) + S_{\varphi_3}(\omega), \tag{11}$$

where $S_{\varphi_T}(\omega)$ is the intermodulation noise power density spectrum due to transmission deviations. We see from (11) that the second- and third-order noise contributions are additive on a power basis since they are not crosscorrelated. It can be shown that[1]

$$\begin{aligned} S_{\varphi_T}(\omega) = 2 \mid H_1(\omega) \mid^2 \mathfrak{F}[R^2_{\varphi'}(\tau)] + 2 \mid H_2(\omega) \mid^2 \mathfrak{F}[R^2_{\varphi''}(\tau)] \\ + 2[H_1(-\omega)H_2(\omega) + H_2(-\omega)H_1(\omega)]\mathfrak{F}[R^2_{\varphi'\varphi''}(\tau)] \\ + 6 \mid H_3(\omega) \mid^2 \mathfrak{F}[R^3_{\varphi'}(\tau)], \end{aligned} \tag{12}$$

where $\mathfrak{F}$ denotes the Fourier Transform.

III. AM/PM INTERMODULATION NOISE

We see in Fig. 1 that the output signal from the transmission medium is both envelope and phase modulated. If the transmission medium shown in Fig. 1 were followed by an AM/PM converter*, i.e., a device that converts envelope variations at its input to phase perturbations at its output, then the signal at the converter output will possess an added phase modulating distortion function along with that shown in Fig. 1. This situation is depicted in Fig. 3. The added

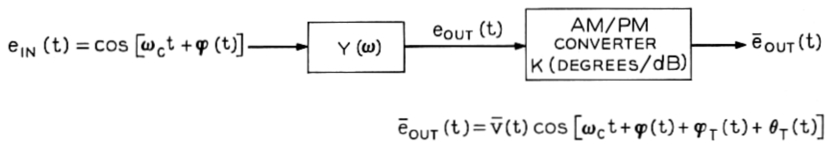

Fig. 3 — Generation of AM/PM intermodulation noise.

distortion term, $\theta_T(t)$, is similar in format to $\varphi_T(t)$ and analogously we will concentrate on the second- and third-order AM/PM intermodulation noise components. That is,

$$\theta_T(t) = \theta_2(t) + \theta_3(t), \tag{13}$$

where $\theta_2(t)$ and $\theta_3(t)$ are the second- and third-order AM/PM intermodulation noise components. These components were derived in Ref.

* The characterization of the AM/PM converter is discussed on pp. 1750-51 in Ref. 2.

2 and are given by

$$\theta_2(t) = k\left[-\frac{1}{2}\lambda_{2r} + \tfrac{1}{4}l_{2r}\frac{d}{dt} - \tfrac{1}{12}l_{3r}\frac{d^2}{dt^2}\right]\varphi'^2(t) + k[\tfrac{1}{24}l_{4r}]\varphi''^2(t) \tag{14}$$

$$\theta_3(t) = k\left[-\tfrac{1}{6}\lambda_{3i} + \tfrac{1}{12}l_{1i}\frac{d}{dt}\right]\varphi'^3(t), \tag{15}$$

where the λ and l coefficients can be determined from Ref. 2, and k is the AM/PM conversion parameter which is defined as the phase modulation index in radians divided by the amplitude modulation index. Equations (13), (14), and (15) can be represented as shown in Fig. 4 where

$$G_1(\omega) = [k\tfrac{1}{12}l_{3r}\omega^2 - k\tfrac{1}{2}\lambda_{2r}] + i[k\tfrac{1}{4}l_{2r}\omega] \tag{16}$$

$$G_2(\omega) = k\tfrac{1}{24}l_{4r} \tag{17}$$

$$G_3(\omega) = [-k\tfrac{1}{6}\lambda_{3i}] + i[k\tfrac{1}{12}l_{1i}\omega]. \tag{18}$$

Since $\theta_2(t)$ and $\theta_3(t)$ are uncorrelated, we can write

$$R_{\theta_T}(\tau) = R_{\theta_2}(\tau) + R_{\theta_3}(\tau) \tag{19}$$

or

$$S_{\theta_T}(\omega) = S_{\theta_2}(\omega) + S_{\theta_3}(\omega), \tag{20}$$

where $S_{\theta_T}(\omega)$ is the AM/PM intermodulation noise power density spectrum. It can be shown that[2]

$$\begin{aligned} S_{\theta_T}(\omega) = {} & 2\,|\,G_1(\omega)\,|^2\,\mathfrak{F}[R^2_{\varphi'}(\tau)] + 2\,|\,G_2(\omega)\,|^2\,\mathfrak{F}[R^2_{\varphi''}(\tau)] \\ & + 2[G_1(-\omega)G_2(\omega) + G_2(-\omega)G_1(\omega)]\mathfrak{F}[R^2_{\varphi'\varphi''}(\tau)] \\ & + 6\,|\,G_3(\omega)\,|^2\,\mathfrak{F}[R^3_{\varphi'}(\tau)]. \end{aligned} \tag{21}$$

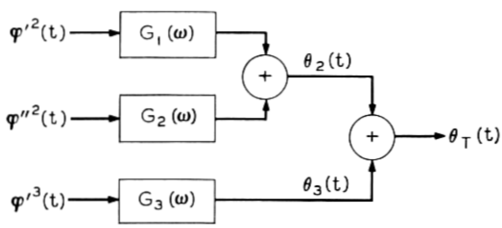

Fig. 4 — Block diagram of total AM/PM intermodulation noise.

IV. POWER DENSITY SPECTRUM OF THE SUM OF TWO NOISE CONTRIBUTORS

The previous two sections presented basic system models which were used to describe the two noise sources under study. In this section, we will treat the more general case whereby the signal to be demodulated, after passing through a transmission system, will be given by

$$e(t) \propto \cos [\omega_c t + \varphi(t) + \underbrace{\varphi_T(t) + \theta_T(t)}_{\text{composite distortion}}].$$

The power density spectrum for each noise contributor has been derived in Refs. 1 and 2, as previously discussed. However, adding these two spectra together on a power basis, i.e., assuming that they are uncorrelated, may yield a result which is grossly in error. In order to determine the degree to which $\varphi_T(t)$ and $\theta_T(t)$ are correlated, we must derive the crosscorrelation function and examine its effect.

To examine the effects of crosscorrelation we combine Figs. 2 and 4 as shown in Fig. 5. The autocorrelation function of the sum of the two noise contributors, $\varphi_T(t)$ and $\theta_T(t)$, is

$$\begin{aligned} R_{\varphi+\theta}(\tau) &= \overline{[\varphi_T(t) + \theta_T(t)][\varphi_T(t+\tau) + \theta_T(t+\tau)]} \\ &= R_{\varphi_T}(\tau) + R_{\theta_T}(\tau) + \overline{\theta_T(t)\varphi_T(t+\tau)} + \overline{\varphi_T(t)\theta_T(t+\tau)}. \end{aligned} \tag{22}$$

The first two terms are given by (10) and (19), respectively. Sub-

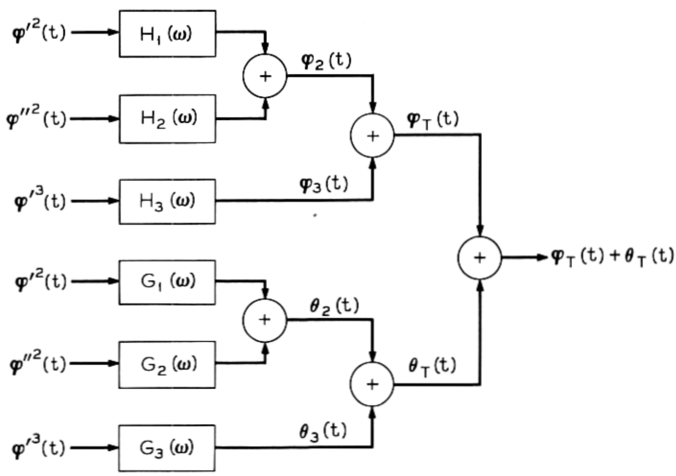

Fig. 5 — Block diagram of composite intermodulation noise.

stituting (2) and (13) into (22) gives

$$R_{\varphi+\theta}(\tau) = R_{\varphi_T}(\tau) + R_{\theta_T}(\tau) + \overline{\theta_2(t)\varphi_2(t+\tau)} + \overline{\theta_3(t)\varphi_3(t+\tau)} + \overline{\theta_2(t)\varphi_3(t+\tau)} + \overline{\theta_3(t)\varphi_2(t+\tau)} + \overline{\varphi_2(t)\theta_2(t+\tau)} + \overline{\varphi_3(t)\theta_3(t+\tau)} + \overline{\varphi_2(t)\theta_3(t+\tau)} + \overline{\varphi_3(t)\theta_2(t+\tau)} \tag{23}$$

which can be written

$$R_{\varphi+\theta}(\tau) = R_{\varphi_T}(\tau) + R_{\theta_T}(\tau) + R_{\theta_2\varphi_2}(\tau) + R_{\theta_3\varphi_3}(\tau) + R_{\theta_2\varphi_3}(\tau) + R_{\theta_3\varphi_2}(\tau) + R_{\varphi_2\theta_2}(\tau) + R_{\varphi_3\theta_3}(\tau) + R_{\varphi_2\theta_3}(\tau) + R_{\varphi_3\theta_2}(\tau). \tag{24}$$

Using the same approach as in the Appendix of Ref. 2, it can be shown that

$$R_{\theta_2\varphi_3}(\tau) = R_{\theta_3\varphi_2}(\tau) = R_{\varphi_2\theta_3}(\tau) = R_{\varphi_3\theta_2}(\tau) = 0 \tag{25}$$

so

$$R_{\varphi+\theta}(\tau) = R_{\varphi_T}(\tau) + R_{\theta_T}(\tau) + R_{\theta_2\varphi_2}(\tau) + R_{\theta_3\varphi_3}(\tau) + R_{\varphi_2\theta_2}(\tau) + R_{\varphi_3\theta_3}(\tau). \tag{26}$$

Hence,

$$S_{\varphi+\theta}(\omega) = S_{\varphi_T}(\omega) + S_{\theta_T}(\omega) + S_{\theta_2\varphi_2}(\omega) + S_{\theta_3\varphi_3}(\omega) + S_{\varphi_2\theta_2}(\omega) + S_{\varphi_3\theta_3}(\omega). \tag{27}$$

4.1 *Consideration of $S_{\theta_3\varphi_3}(\omega)$ and $S_{\varphi_3\theta_3}(\omega)$*

To examine $S_{\theta_3\varphi_3}(\omega)$, the cross-power density spectrum of $\theta_3(t)$ and $\varphi_3(t)$, we reduce Fig. 5 to the block diagram shown in Fig. 6. Using the relationship for the crosscorrelation of linearly transformed random functions, we have

$$S_{\theta_3\varphi_3}(\omega) = G_3(-\omega)H_3(\omega)S_{\varphi'^3}(\omega). \tag{28}$$

It can be shown that[1]

$$R_{\varphi'^3}(\tau) = 6R^3_{\varphi'}(\tau) + 9R^2_{\varphi'}(0)R_{\varphi'}(\tau). \tag{29}$$

The term $9R^2_{\varphi'}(0)\ R_{\varphi'}(\tau)$ is merely a scaled power density spectrum of the input baseband signal and hence can be neglected since it does not contribute to the intermodulation noise distortion. Therefore,

$$S_{\theta_3\varphi_3}(\omega) = 6G_3(-\omega)H_3(\omega)\mathfrak{F}[R^3_{\varphi'}(\tau)]. \tag{30}$$

By inspection, we can write

$$S_{\varphi_3\theta_3}(\omega) = 6H_3(-\omega)G_3(\omega)\mathfrak{F}[R^3_{\varphi'}(\tau)]. \tag{31}$$

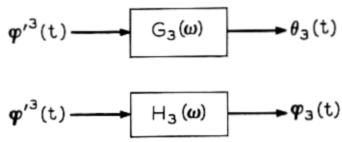

Fig. 6 — Third-order noise correlation.

4.2 *Consideration of* $S_{\theta_2\varphi_2}(\omega)$ *and* $S_{\varphi_2\theta_2}(\omega)$

To examine $S_{\theta_2\varphi_2}(\omega)$, we reduce Fig. 5 to the block diagram shown in Fig. 7. Now

$$\begin{aligned} R_{\theta_2\varphi_2}(\tau) &= \overline{\theta_2(t)\varphi_2(t+\tau)} \\ &= \overline{[x(t)+y(t)][u(t+\tau)+v(t+\tau)]} \\ &= R_{xu}(\tau)+R_{yv}(\tau)+R_{xv}(\tau)+R_{yu}(\tau). \end{aligned} \tag{32}$$

Taking the Fourier Transform of (32) and referring to Fig. 7, we can directly write

$$\begin{aligned} S_{\theta_2\varphi_2}(\omega) &= G_1(-\omega)H_1(\omega)S_{\varphi'^2}(\omega)+G_2(-\omega)H_2(\omega)S_{\varphi''^2}(\omega) \\ &\quad + G_1(-\omega)H_2(\omega)S_{\varphi'^2\varphi''^2}(\omega)+G_2(-\omega)H_1(\omega)S_{\varphi''^2\varphi'^2}(\omega). \end{aligned} \tag{33}$$

Now it can be shown that[1,2]

$$S_{\varphi'^2}(\omega) = 2\mathfrak{F}[R^2_{\varphi'}(\tau)] \tag{34}$$

$$S_{\varphi''^2}(\omega) = 2\mathfrak{F}[R^2_{\varphi''}(\tau)] \tag{35}$$

$$S_{\varphi'^2\varphi''^2}(\omega) = 2\mathfrak{F}[R^2_{\varphi'\varphi''}(\tau)] = S_{\varphi''^2\varphi'^2}(\omega) \tag{36}$$

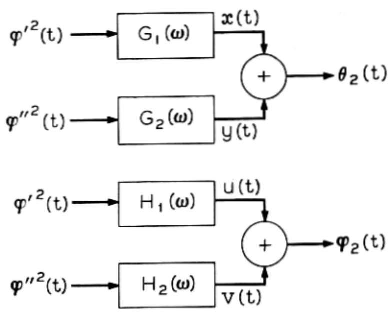

Fig. 7 — Second-order noise correlation.

neglecting the dc components. Hence,

$$S_{\theta_2\varphi_2}(\omega) = 2G_1(-\omega)H_1(\omega)\mathfrak{F}[R^2_{\varphi'}(\tau)] + 2G_2(-\omega)H_2(\omega)\mathfrak{F}[R^2_{\varphi''}(\tau)] + 2[G_1(-\omega)H_2(\omega) + G_2(-\omega)H_1(\omega)]\mathfrak{F}[R^2_{\varphi'\varphi''}(\tau)]. \quad (37)$$

Similarly,

$$S_{\varphi_2\theta_2}(\omega) = 2H_1(-\omega)G_1(\omega)\mathfrak{F}[R^2_{\varphi'}(\tau)] + 2H_2(-\omega)G_2(\omega)\mathfrak{F}[R^2_{\varphi''}(\tau)] + 2[H_2(-\omega)G_1(\omega) + H_1(-\omega)G_2(\omega)]\mathfrak{F}[R^2_{\varphi'\varphi''}(\tau)]. \quad (38)$$

Substituting (12), (21), (30), (31), (37), and (38) in (27) gives

$$\begin{aligned} S_{\varphi+\theta}(\omega) &= 2\{\mid H_1(\omega)\mid^2 + \mid G_1(\omega)\mid^2 \\ &+ G_1(-\omega)H_1(\omega) + H_1(-\omega)G_1(\omega)\}\mathfrak{F}[R^2_{\varphi'}(\tau)] \\ &+ 2\{\mid H_2(\omega)\mid^2 + \mid G_2(\omega)\mid^2 + G_2(-\omega)H_2(\omega) + H_2(-\omega)G_2(\omega)\}\mathfrak{F}[R^2_{\varphi''}(\tau)] \\ &+ 2\{H_1(-\omega)H_2(\omega) + H_2(-\omega)H_1(\omega) + G_1(-\omega)G_2(\omega) + G_2(-\omega)G_1(\omega) \\ &+ G_1(-\omega)H_2(\omega) + G_2(-\omega)H_1(\omega) + H_2(-\omega)G_1(\omega) \\ &+ H_1(-\omega)G_2(\omega)\}\mathfrak{F}[R^2_{\varphi'\varphi''}(\tau)] + 6\{\mid H_3(\omega)\mid^2 + \mid G_3(\omega)\mid^2 \\ &+ G_3(-\omega)H_3(\omega) + H_3(-\omega)G_3(\omega)\}\mathfrak{F}[R^3_{\varphi'}(\tau)]. \end{aligned} \quad (39)$$

This expression, (39), gives the baseband power density spectrum for the sum of two intermodulation noise sources: (*i*) intermodulation noise due to transmission deviations, and (*ii*) AM/PM intermodulation noise. The effect of the crosscorrelation relationship shows up as cross products of the defining transfer functions for each noise source as would be expected. The second and third order distortions of the summed noise spectra are additive as was the case for the two individual noise contributors.

4.3 *Signal-To-Noise Ratio*

The signal characterization is the same as in Refs. 1 and 2, or

$$S_{\varphi'}(\omega) = P_0[a_0 + a_2f^2 + a_4f^4 + a_6f^6], \qquad \mid f\mid \leqq f_b \quad (40)$$

= pre-emphasized multichannel baseband signal power density spectrum at the input to an FM modulator.

The constant P_0 is given by

$$P_0 = \frac{(2\pi\sigma)^2}{2f_b\left(a_0 + \frac{a_2f_b^2}{3} + \frac{a_4f_b^4}{5} + \frac{a_6f_b^6}{7}\right)} \text{ (rad/sec)}^2/\text{Hz}, \quad (41)$$

where σ = rms frequency deviation, in Hertz, due to the multichannel baseband signal; the a's are the pre-emphasis coefficients; and f_b is the top baseband frequency, in Hertz. Hence, the signal-to-noise ratio is

$$10 \log \frac{S_{\varphi'}(\omega)}{\omega^2 S_{\varphi+\theta}(\omega)}, \tag{42}$$

where the signal and noise are defined by (40) and (39), respectively.

V. SOME BASIC TRENDS

The power density spectrum for the correlated sum given by (39) does not readily lend itself to any generalized remarks as to the conditions under which correlation exists and to what degree. In order to derive some useful information on the subject, (39) was programmed on a digital computer. The computer input consisted of the fundamental system parameters, e.g., peak frequency deviation, number of message channels in the baseband, etc., as well as the transmission deviation values to be used for "AM/PM intermodulation noise" and those to be used for "intermodulation noise due to transmission deviations. Note that the transmission medium may be different for the two cases. For example, we could have the case where the "AM/PM intermodulation noise" is created by a quartic gain transmission deviation prior to an AM/PM converter, and the "intermodulation noise due to transmission deviations" is caused by a linear delay distortion. The transmission functions for the two cases would be

$$Y_I(\omega + \omega_c) = \exp ib_2\omega^2, \qquad Y_{AM/PM}(\omega + \omega_c) = 1 + g_4\omega^4. \tag{43}$$

As a clarifying point, it should be remembered that the AM/PM theory of Ref. 2, and associated computer program, are set up so that we only obtain the "AM/PM intermodulation noise" due to quartic gain, and not the "intermodulation noise due to transmission deviations" caused by a quartic gain transmission deviation.

Both of the noise sources discussed in Sections II and III have transfer functions associated of the basic form given in (1). Each transfer function can have seven transmission deviations, so the problem of permuting all possible combinations to see which are correlated becomes unreasonably cumbersome. However, a potentially useful test is to evaluate the correlation between $\varphi_T(t)$ and $\theta_T(t)$ when $Y_I(\omega)$ and $Y_{AM/PM}(\omega)$ each have only one transmission deviation for each computer run. There are 49 possible combinations, one of which is given by (43). Any results one obtains will depend on the inputs used, and because of the format of (39), one cannot make one run using normalized results and then scale all future runs according to some predetermined rules. For that reason, the results to be given can only be used to indicate trends.

The 49 runs previously mentioned yielded eight significantly correlated combinations. The evaluation of the results was made by adding the individual power density spectra $S_{\varphi_T}(\omega)$ and $S_{\theta_T}(\omega)$ on a power basis and comparing the results with those obtained from $S_{\varphi+\theta}(\omega)$, i.e., the power density for the correlated sum. Only top channel noise was used in the comparison. A combination was considered to be significantly correlated when the power sum and correlated sum differed by more than a few tenths of a dB. The amount that they differed depended on the inputs, but for the values used,* some cases had the correlated sum up to 3 dB *above* the power sum, in the top channel, and some combinations caused the correlated sum to be as much as 15 dB *below* the power sum, in the top channel.

The eight significantly correlated combinations are shown in Table I.

TABLE I—CORRELATED COMBINATIONS

	Transmission deviation in		Conditions under which the correlated sum is higher than the power sum (assuming k is positive)
	$Y_I(\omega)$	$Y_{AM/PM}(\omega)$	
1.	g_3	b_3	g_3 or b_3 is negative
2.	b_2	g_1	b_2 is negative
3.	b_2	g_2	b_2 and g_2 positive *or* b_2 and g_2 negative
4.	b_2	g_4	b_2 and g_4 positive *or* b_2 and g_4 negative
5.	b_3	g_3	b_3 and g_3 positive *or* b_3 and g_3 negative
6.	b_4	g_1	b_4 is negative
7.	b_4	g_2	b_4 and g_2 positive *or* b_4 and g_2 negative
8.	b_4	g_4	b_4 and g_4 positive *or* b_4 and g_4 negative

We see from Table I that the sign of the transmission deviations determine if the correlation is positive or negative, i.e., whether the

* Same values as those used in Section 3.2.2 of Ref. 2: all gain transmission deviations have 1 dB distortion, relative to the carrier, at 10 MHz away from the carrier; all delay transmission deviations have 1 nanosecond distortion, relative to the carrier, a 10 MHz away from the carrier.

actual noise power is larger or smaller than the power one would obtain from power addition of the two individual contributors. Also, we see that all eight cases have the same format, i.e., if one transfer function has a delay transmission deviation, the other transfer function has a gain transmission deviation and vice versa.

Referring back to (39), we see that all of the correlation terms have the same format, i.e., a $G_i(\pm\omega)H_j(\pm\omega)$ product. Since these terms have k as a component, the sign of k will play a role in determining if we have positive or negative correlation.

The results of Table I give us an idea of when to expect significant correlation, that is, when power addition should not be used. These results should prove useful for the more complex problems which are confronted in practice.

VI. SYSTEM EXAMPLE

In Section 3.3.4 of Ref. 2, a representative FM radio relay system's repeater characteristics were used in deriving the noise responses shown in Fig. 11 of Ref. 2 and reproduced here as Fig. 8. The radio system carried 1200 message channels, had a peak frequency deviation of 4 MHz and a top baseband frequency of 5.772 MHz, and had an rms frequency deviation of 0.771 MHz. The system was pre-emphasized by the function shown in Fig. 5 of Ref. 2. The basic repeater was

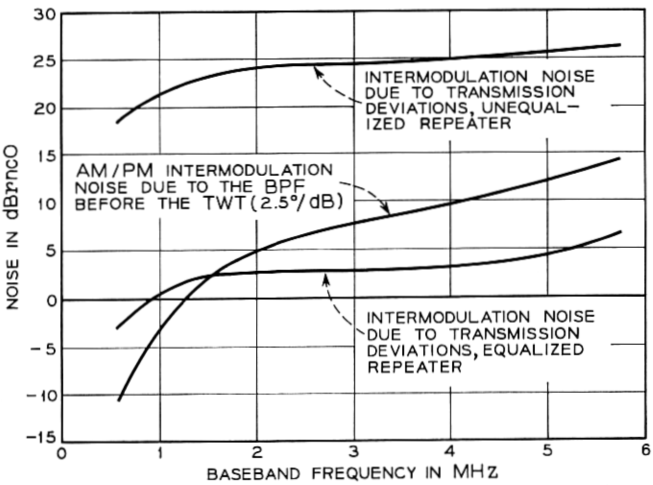

Fig. 8 — Representative radio system noise responses.

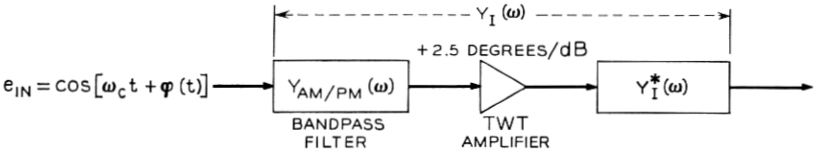

Fig. 9 — Repeater model.

gain and delay equalized and, from an analytical point of view, could be represented as shown in Fig. 9. The transmission characteristics for the equalized repeater, $Y_I(\omega)$, and for the bandpass filter, $Y_{AM/PM}(\omega)$, are shown in Fig. 10 of Ref. 2 and reproduced here as Fig. 10. The associated least squares fitted transmission deviations are as follows

Transmission deviations	$Y_I(\omega + \omega_c)$	$Y_{AM/PM}(\omega + \omega_c)$
g_1	-9.67×10^{-11}	3.81×10^{-11}
g_2	7.09×10^{-18}	9.17×10^{-19}
g_3	1.17×10^{-25}	9.04×10^{-27}
g_4	-2.57×10^{-33}	-1.97×10^{-33}
b_2	6.50×10^{-18}	-4.82×10^{-18}
b_3	5.58×10^{-26}	8.09×10^{-25}
b_4	-3.16×10^{-33}	-3.35×10^{-34}

The power sum of the AM/PM intermodulation noise and the intermodulation noise due to the equalized repeater's transmission deviations (both are shown in Fig. 8) is shown in Fig. 11. Also shown is the power density spectrum which includes the effects of correlation between the two contributors. We see for this example that the actual noise is 4.5 dB *lower* in the top channel than that obtained from power addition alone.

Another interesting result is the curve shown in Fig. 12. This figure shows how the sign and magnitude of the AM/PM conversion factor can affect the noise response of a given system.

In Table I, eight correlated combinations were given. Of these eight possibilities, the first combination is probably responsible for the correlated sum being smaller than the power sum as shown in Fig. 11. There are three reasons for this observation: (*i*) the parabolic delay in $Y_{AM/PM}(\omega)$ is quite large, and the cubic gain in $Y_I(\omega)$ is a significant

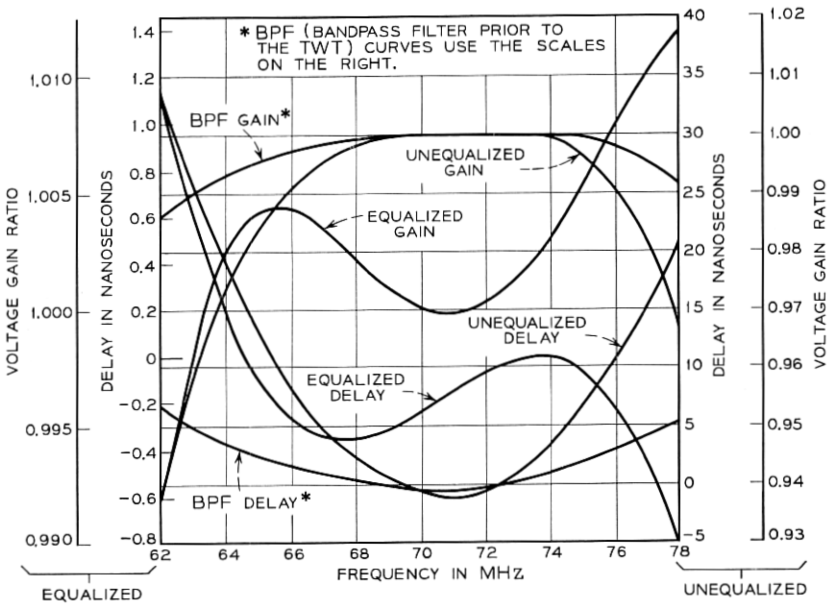

Fig. 10 — Gain and delay characteristics.

part of the equalized repeater gain shape; (*ii*) this combination causes negative correlation when the transmission deviations are both positive and the AM/PM conversion factor is positive; and (*iii*) a negative value for K causes positive correlation,* for this combination, which is consistent with the results shown in Fig. 12. Hence, the trends given in Table I can be useful for more complicated problems and may serve as a tool for optimizing a system's noise performance.

VII. CONCLUSIONS

In this paper, we have studied the correlation which exists between two noise contributors in FM systems: (*i*) intermodulation noise due to transmission deviations, and (*ii*) AM/PM intermodulation noise. The first contributor is generated when an FM signal is passed through a linear transmission medium which has transmission deviations. We denoted this medium by $Y_I(\omega)$. The second contributor is generated when an FM signal is passed through a similar medium, denoted by $Y_{AM/PM}(\omega)$, which is followed by an AM/PM conversion device. These

* When the transmission deviations are both positive.

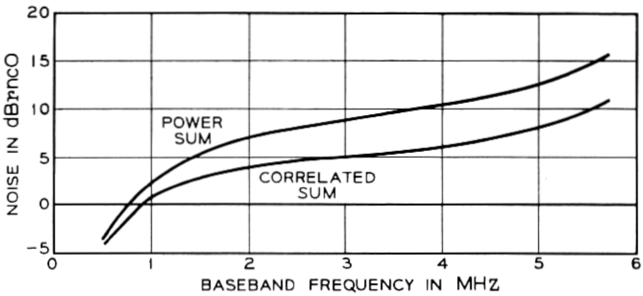

Fig. 11 — Comparison of power sum and correlated sum for a representative radio system.

two media may or may not possess the same transmission deviations in practice.

Even though these two contributors are different, they do possess the same property of being a function of the baseband signal. Therefore, one would expect that they would be correlated to some degree. This would mean that combining the two noise power density spectra assuming random addition (uncorrelated random variables), i.e., power addition, might not be sufficient in general.

To study the amount and character of this correlation, the power density spectrum was derived for the sum of the two noise contributors. The resulting equation was programmed on a digital computer and evaluated. Because of the format of the equation, no generalized results could be obtained. However, it was found that certain conditions exist under which the correlation can be significant. These conditions, even though not all engrossing, should prove useful in the complex problems which occur in practice.

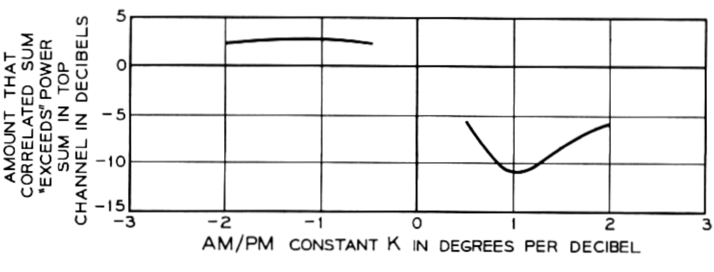

Fig. 12 — Affect of sign and magnitude of AM/PM constant on correlated sum for a representative radio system.

A representative FM radio relay system was examined, and it was found that the power density spectrum for the correlated sum of the two noise contributors was substantially different than the power addition of the individual noise spectra. In the top channel, the correlated noise power was about 4.5 dB lower than the noise resulting from a power sum.

It was also shown that a simple change in sign of the AM/PM conversion factor k, or certain transmission deviations, can cause the correlated noise to be substantially higher or lower than the power sum of the individual spectra.

VIII. ACKNOWLEDGMENT

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