# Scaling Laws for Large Shields in Quasi-Stationary Magnetic Fields

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*The application of the classical scaling laws of electro-magnetic fields to the design of a scaled-down model of, say, a building-sized shield is often difficult, even when using the simplifications permissible with a quasi-stationary magnetic field. The reasons are that (i) the scaled wall thickness often becomes impractically thin and (ii) the required scaling of frequency sometimes reduces the ratio of intrinsic wave-length in air to the enclosure length such that the quasi-stationary field theory no longer applies.*

*In the case of a completely closed shield these limitations can be circumvented by having a model with two distinct geometric scaling factors, one for the wall thickness and one for the overall dimensions. The modified scaling laws governing this type of model are derived.*

## I. INTRODUCTION

Protection of electronic equipment against electromagnetic interference is often achieved by providing a metallic enclosure. Large electronic complexes, such as radar installations and data processing centers may be protected by covering the entire building with a metallic shield. (Some penetrations into this enclosure are usually required for the purpose of air-vents, cable-inlets, access tunnels etc.) The performance of the enclosure is measured by the shielding effectiveness, which is the ratio of the field strength at an exterior location where the field is undisturbed by the shield to the field strength at a point inside the enclosure.

A first approximation of the shielding effectiveness can be obtained analytically.[1] In this case, (*i*) constant permeability is assumed, and (*ii*) the actual shape of the enclosure is replaced by a geometrically simpler shape, such as an infinite cylindrical shell or a spherical shell.

The evaluation of the shielding effectiveness by testing is, for economical reasons, best conducted on scaled down models of the enclosure. The following discussion concerns itself with the constraints on the scaling-factors for distance, time, conductivity, etc., necessary to produce a model either having the same shielding effectiveness as the original, full-scale enclosure or having one of known relation to it. It will be seen that full compliance with the ideal constraints on scaling-factors is rarely possible. However, useful results can be obtained with partial compliance, especially in the case of large enclosures.

## II. IDEAL CONSTRAINTS ON SCALING FACTORS

An ideal, scaled model is a replica of the original configuration with each physical parameter scaled up or down by a fixed ratio. To each point in space and time of the original exists a corresponding point in the model. The ratio of any distance, time, field strength, etc. of the original to its counterpart in the model is called a scaling-factor. If one identifies any parameter or variable of the original with the index "1" and of the model with the index "2" one can write the scaling-factors for distance, time, electric and magnetic field strength, permeability (instantaneous ratio of magnetic flux density to magnetic field strength), dielectric constant, conductivity as $l_2/l_1$, $t_2/t_1$, $E_2/E_1$, $H_2/H_1$, $\mu_2/\mu_1$, $\epsilon_2/\epsilon_1$, $\sigma_2/\sigma_1$. (For instance, $l_1$, represents the distance between two arbitrarily selected points of the original, full-scaled enclosure, whereas, $l_2$ represents the distance between the corresponding points of the model.) Were these scaling-factors selected arbitrarily, the model would not be physically realizable because the electromagnetic field of the model would not satisfy Maxwell's equations. These equations when formulated for the original and for the model contain the constraints required to make the model physically realizable. They also interrelate the scaling-factors for electric and magnetic field strengths. The results* are expressed by (1), (2), and (3)

$$\left(\frac{E_2}{E_1}\right)\cdot\left(\frac{H_1}{H_2}\right) = \left(\frac{l_2}{l_1}\right)\cdot\left(\frac{t_1}{t_2}\right)\cdot\left(\frac{\mu_2}{\mu_1}\right) \tag{1}$$

$$\frac{l_2^2\cdot\sigma_2\cdot\mu_2}{t_2} = \frac{l_1^2\cdot\sigma_1\cdot\mu_1}{t_1} \tag{2}$$

$$\frac{l_2^2\cdot\epsilon_2\cdot\mu_2}{t_2^2} = \frac{l_1^2\cdot\epsilon_1\cdot\mu_1}{t_1^2}. \tag{3}$$

* For the mathematical derivation see either Appendix A or Ref. 2, p. 488.

Introducing the angular frequency, $\omega$, one obtains the following scaling constraints for CW fields:

$$l_2^2\omega_2\sigma_2\mu_2 = l_1^2\omega_1\sigma_1\mu_1 \tag{4}$$

$$l_2^2\omega_2^2\epsilon_2\mu_2 = l_1^2\omega_1^2\epsilon_1\mu_1 \tag{5}$$

## III. INTERPRETATION OF THE MATHEMATICAL RESULTS

First, it should be pointed out that the derivation of (2) and (3) is not based on a field-strength-independent permeability or dielectric constant. Consequently, model tests of ferromagnetic shields of variable permeability will give correct answers, provided the model uses the same steel as the original, is tested at the field strength encountered in the original and satisfies (2) and (3).

Second, the shielding effectiveness, $\eta$, of geometrically similar models changes from model to model only if the expressions (2) and (3) change. In other words, the shielding effectiveness of geometrically similar models is a function of these two dimensionless quantities only, i.e.,

$$\eta = f\left[\left(\frac{l^2\cdot\sigma\cdot\mu}{t}\right), \left(\frac{l^2\cdot\epsilon\cdot\mu}{t^2}\right)\right]. \tag{6}$$

Here, $l$ might be the length of the enclosure, $t$, the pulse duration, etc.

The physical meaning of (4) and (5) becomes clearer if we introduce the skin depth,* $\delta$, of a conductor of constant permeability and the intrinsic wavelength in a pure dielectric, $\lambda_\epsilon$:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} \tag{7}$$

$$\lambda_\epsilon = \frac{2\pi}{\omega\sqrt{\epsilon\mu}}. \tag{8}$$

Substituting these values into (4) and (5) one obtains

$$\frac{l_2}{\delta_2} = \frac{l_1}{\delta_1} \tag{9}$$

$$\frac{l_2}{\lambda_{\epsilon_2}} = \frac{l_1}{\lambda_{\epsilon_1}} \tag{10}$$

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* Here $(2\pi\delta)$ is equal to the intrinsic wavelength in metal, $\lambda_\sigma$.

In other words, the skin depth of the shield material as well as the intrinsic wavelength of the surrounding space have to be scaled by the same ratio as the linear dimensions of the model.

It can now be seen that it usually is not feasible to produce a model which satisfies both (2) and (3). For instance, after the scaling factor $(l_2/l_1)$ has been selected, (2) and (3) can be satisfied only if two other parameters, such as conductivity, $\sigma$, and time, $t$, are properly scaled. Unfortunately, the only scaling factor which usually can be suitably controlled is that of time (duration of an applied pulse, period of an applied ac field). Consequently, only one of the scaling requirements, either (2) or (3) can be readily satisfied. Therefore one has to be content with imperfect models which will be discussed next.

## IV. SEVERAL TYPES OF IMPERFECT MODELS

### 4.1 *The Geometrically Perfect Model in a Quasi-Stationary Magnetic Field*

A model shall be considered geometrically perfect if *all* of its dimensions, the overall dimensions such as width, height, and length as well as the thickness of the shield and the size of its openings are scaled by the same factor.

The quasi-stationary magnetic field is a well known simplifying concept which is applicable whenever the linear dimensions of the configuration are small compared to the intrinsic wavelength of the dielectric medium. It is the magnetic field one obtains mathematically if one assumes the time derivatives of the electric displacement, $\partial/\partial t\,(\epsilon E)$, to be zero.

In this case, as shown in Appendix A, one obtains only one constraint equation for the scaling factors, namely that expressed by either (2), (4), or (9).

In order to obtain an idea of the error caused by this simplification, one may look at a geometrically simple shield, such as a spherical shell, for which analytical solutions are available.[1] According to a graph given in Ref. 1, the magnetic shielding effectiveness at the center of a spherical shell, if calculated on the basis of a quasi-stationary field, is in error by less than 2.6 dB for a wavelength to diameter ratio of 2.8 or higher. The electric shielding effectiveness (electric field outside of the shielded space to that inside) is equal to the magnetic shielding effectiveness at this wave length to diameter ratio and increases rapidly for higher ratios.

Unfortunately, there are two serious shortcomings to this type of model. First, the wall-thickness of the scaled-down enclosure often becomes impractically thin. For instance the original enclosure may have been built with 0.010-inch thick copper. Assuming a geometric scaling factor of 0.1 the model would have to be built of 0.001-inch thick copper. Second, (assuming identical $\sigma$, $\mu$, and $\epsilon$ for model and original) due to the scaling of frequency, as called for by (4), the ratio of the intrinsic wavelength in air to the length of the model becomes proportional to the geometric scaling factor, $(l_2/l_1)$. This is a simple consequence of (4) and (8). Sometimes, this ratio decreases for the model to the point where the quasi-stationary field theory no longer applies.

As shall be shown next, both of these shortcomings can be circumvented in the case of large enclosures without openings by using two geometric scaling-factors, one for the overall dimensions and one for the wall thickness.

### 4.2 *Models with Two Geometric Scaling-Factors, One for the Overall Dimensions and One for the Wall-Thickness of the Enclosure in Quasi-Stationary Magnetic Fields*

In the following, models of large enclosures without openings will be considered. The scaling factor for the overall dimensions is designated as $(L_2/L_1)$ and that for the wall thickness as $(d_2/d_1)$. (The wall thickness does not have to be uniform). Assuming the wall thickness to be very small compared to the overall dimensions of the enclosure, the spaces internal and external to the enclosure of the model remain geometrically similar to those of the original.

With this in mind, it will be shown that the internal and external magnetic fields of this type of model, *individually,* are substantially similar to those of the original if the ratio of shield thickness to skin-depth remains unchanged.

The validity of this statement rests on two simplifying assumptions: namely (*i*) that, the external magnetic field is almost identical to that outside of an enclosure of infinite conductivity (the field component normal to the surface is negligible compared to the tangential one), and (*ii*) that within the shield (see Fig. 1) the rates of change of the tangential magnetic and electric field strength in the direction normal to the surface $(\partial H_z/\partial y)$ and $(\partial E_x/\partial y)$ are much larger than the rate of change of the field strengths normal to the surface in the tangential direction $(\partial H_y/\partial z)$ and $(\partial E_y/\partial x)$; i.e., $\partial H_y/\partial z \ll \partial H_z/\partial y$ and $\partial E_y/\partial x \ll$

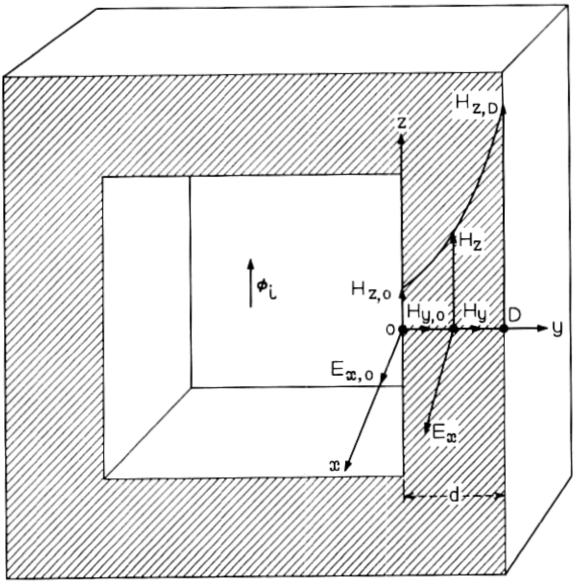


Fig. 1 — Local coordinates of the field in the shield.

$\partial E_x/\partial y$. (The field changes rapidly across the shield but only gradually in the tangential directions.)

From assumption (*ii*) it follows that the tangential magnetic field-strength at some point $D$ on the outside of the shield, $H_{zD}$, is determined by the tangential electric and magnetic field strengths, $H_{z,0}$ and $E_{x,0}$, at the opposing point "0" on the inside surface of the enclosure. Specifically, $H_z$ is governed by the following differential equations:

$$\sigma E_x = \frac{\partial H_z}{\partial y} \tag{11}$$

$$-\frac{\partial}{\partial t}(\mu H_z) = -\frac{\partial E_x}{\partial y} \tag{12}$$

and by the following boundary conditions:

$$\text{At} \quad y = 0: \qquad H_z = H_{z,0}\,; \qquad E_x = E_{x,0} \tag{13}$$

In the case of a sufficiently large enclosure, the boundary conditions can be simplified. Specifically, Appendix B shows that the effect of $H_{z,0}$ on $H_{z,D}$ can be neglected if the following inequalities are satisfied:*

$$\frac{\sqrt{2}\cdot 2\cdot(\mu_r)_{\mathrm{av}}\cdot\delta_{\mathrm{av}}}{D} \ll 1; \qquad d > \frac{\delta_{\mathrm{av}}}{\sqrt{2}} \tag{14}$$

$$\frac{2\cdot(\mu_r)_{\mathrm{av}}\cdot\delta_{\mathrm{av}}^2}{d\cdot D} \ll 1; \qquad d < \frac{\delta_{\mathrm{av}}}{\sqrt{2}}. \tag{15}$$

$(\mu_r)_{\mathrm{av}}$ is the average relative permeability of the shield,† and $\delta_{\mathrm{av}}$ is the skin-depth based on $(\mu_r)_{\mathrm{av}}$. The equivalent diameter, $D$, is four times the cross section of the enclosure divided by its circumference, measured in a plane which is normal to the field and which bisects the enclosure. (For simplicity, one may use for $D$ the smallest major dimension of the enclosure.)

If the above inequalities are satisfied the boundary conditions simplify to

$$y = 0: \qquad H_z = 0; \qquad E_x = E_{x,0}\,. \tag{16}$$

Now, let us put the following question: Provided the internal electric and magnetic fields of model and original are similar to each other, under which condition will the external magnetic fields be similar, too? According to assumption (*i*) the only conditions are (*i*) that the distributions of the tangential magnetic field strength at the outside surface of the shield of the model and of the original are similar to each other and (*ii*) that, of course, the applied external fields are similar to each other. With the internal fields being assumed similar to each other, condition (*i*) is satisfied if the field distributions across the shields are similar, too. In consideration of the simplified field equations (11) and (12) this is the case if, (*i*) the general scaling equation (2) is satisfied with respect to the y-coordinate (see Fig. 1), i.e., if

$$\frac{d_2^2\cdot\sigma_2\cdot\mu_2}{t_2} = \frac{d_1^2\cdot\sigma_1\cdot\mu_1}{t_1} \tag{17}‡$$

and (*ii*) if scaling equation (1) is valid for the boundary values $H_{z,0}$ and $E_{x,0}$ (using the shield parameters, $d$ and $\mu_{\mathrm{shield}}$). For the special

* Note, that, for $d > \delta_{\mathrm{av}}/\sqrt{2}$, if inequality (14) is satisfied, inequality (15) will be satisfied too and for $d < \delta_{\mathrm{av}}/\sqrt{2}$, if inequality (15) is satisfied, inequality (14) will be satisfied too.

† If the shield is several skin depths thick, $(\mu_r)_{\mathrm{av}}$ is the permeability near the inside surface of the enclosure.

‡ For field-strength independent $\mu$ the expression $\sqrt{t/\sigma\mu}$ is proportional to the skin-depth, $\delta$, and (17) becomes $d_2/\delta_2 = d_1/\delta_1$.

case of a large enclosure for which inequalities (14) and (15) are valid, it was shown that $H_{z,0}$ may be assumed equal to zero without substantial effect on the external field. However, with $H_{z,0}$ being zero, scaling equation (1) as applied to the boundary values $H_{z,0}$, $E_{x,0}$ is automatically satisfied since the term $(H_1/H_2)$ becomes (0/0). Consequently, internal and external fields of the model and the original are individually similar to each other if (17) and inequalities (14) and (15) are satisfied.

The relation between the shielding effectiveness of model and original is obtained by using (1). Specifically, $(H_{z,D,1})/(H_{z,D,2})$ and $(E_{x,0,1})/(E_{x,0,2})$ are related to each other as follows:

$$\frac{E_{x,0,2}\cdot H_{z,D,1}}{E_{x,0,1}\cdot H_{z,D,2}} = \frac{d_2\cdot t_1\cdot \mu_2}{d_1\cdot t_2\cdot \mu_1}. \tag{18}$$

Applying, again, (1) to the internal air field one obtains

$$\frac{E_{x,0,2}}{E_{x,0,1}}\frac{H_{z,0,1}}{H_{z,0,2}} = \frac{L_2\cdot t_1}{L_1\cdot t_2}. \tag{19}$$

From (18) and (19) one obtains

$$\frac{H_{z,0,2}}{H_{z,0,1}}\cdot\frac{H_{z,D,1}}{H_{z,D,2}} = \frac{d_2\cdot L_1\cdot \mu_2}{d_1\cdot L_2\cdot \mu_1}. \tag{20}$$

Since the ratio $H_{z,D}/H_{z,0}$ is proportional to the shielding effectiveness one obtains the following relation between the shielding effectiveness of model and original:

$$\eta_1 \approx \left(\frac{L_1\cdot d_2\cdot \mu_2}{L_2\cdot d_1\cdot \mu_1}\right)\cdot \eta_2 \ . \tag{21}$$

Note, that (21) is valid only if the similarity requirement for conductors, as expressed by (17), is satisfied, and if the quasi-stationary field theory is applicable (wavelength in air larger than the linear dimensions of the enclosure). If the scaling factor for the overall dimensions $(L_2/L_1)$ is chosen so that the similarity requirement for air is also satisfied [see (3)] it appears that the restriction to quasi-stationary fields can be dropped. Strictly speaking, this is not so. First, at half wavelengths close to or less than the dimensions of the enclosure, the internal field will no longer be approximately uniform, as this was assumed in Appendix B. This assumption was necessary to show that, for large enclosures, the tangential magnetic field component at the inside surface of the shield is of negligible effect on the magnetic field at the outside surface. However, the higher the frequency the less important becomes the assumption of an approximately uniform magnetic field.

Second, the ratio of magnetic to electric field strength in air along the outside surface of the shield will not satisfy the scaling requirement (1). This latter imperfection, however, will be of minor consequence if the characteristic wave-impedance in the shield is low compared to that of the waves in the external space. In this case, almost complete reflection occurs at the outside wall of the shield, which means that the effect of the tangential electric field strength on the external field is negligible.

All things considered, it is advisable to satisfy for relatively short wavelengths the similarity requirement for air [see (3)].

Appendix C illustrates, with the aid of a numerical example, the above derived scaling laws.

### 4.3 *Shielding Effectiveness of an Enclosure with Uniform Wall*

In Section 4.2 it was shown that the fields internal and external to a model of an enclosure without openings are practically similar to those of the original if the wall of the model is such that the ratio of external tangential magnetic field strength to internal tangential electric field-strength is similar to that of the original. For CW fields, this requirement is satisfied if the inequalities (14) and (15) are satisfied, the shield is uniform along its tangential coordinates (however, it may be nonuniform along the normal coordinate such as in the case of laminated metals) and the permeability and conductivity are constant (field strength independent). Specifically, in the case for which the shield is uniform throughout (including along the normal coordinate), the ratio between tangential magnetic and electric field strength can be given in closed analytical form. According to Appendix D, the external tangential magnetic field strength, $H_{zD}$, is related to the internal tangential electric field strength, $E_{x0}$, (see Fig. 1) as follows:

$$H_{z,D} = \left[\left(\frac{\sigma}{\sqrt{j\omega\mu\sigma}}\right) \sinh\left(\sqrt{j\omega\mu\sigma d^2}\right)\right] E_{x,0} \,. \tag{22}$$

According to the law of induction, $E_{x0}$ is proportional to some representative internal magnetic field strength, $H_i$, to its angular frequency, $\omega$, to a representative linear dimension, $L$, of the shielded space and to its permeability, $\mu_0$ (assuming air). Consequently, one obtains for $H_{zD}$,

$$H_{z,D} = C\left(\frac{\sigma \cdot \omega \cdot L \cdot \mu_0}{\sqrt{j \cdot \omega\mu\sigma}}\right) \sinh\left(\sqrt{j\omega\mu\sigma d^2}\right) \cdot H_i \tag{23}$$

in which $C$ is a constant independent of the scaling factors. The ratio of $H_{z,D}$ to the internal field strength, $H_i$, is proportional to the shielding effectiveness, $\eta$. Consequently,

$$\eta = K\sqrt{\frac{\omega \cdot \sigma \cdot \mu_0^2}{j \cdot \mu}} \cdot L^2 \cdot \sinh\left(\sqrt{j\omega\mu\sigma d^2}\right) \tag{24}$$

in which $K$ is a constant, independent of the scaling factors. It depends only on the geometry of the shielded space, on that of the applied field and on the reference points for external and internal field strengths. $K$ is obtained by measuring $\eta$ on a model of known parameters ($\omega_2$, $L_2$, $\sigma_2$, etc.). Note that, in general, (i.e., if the scaling equation (17) is not satisfied), the scaling factor for $\eta$, i.e., $(\eta_2/\eta_1)$, is frequency dependent. Therefore, the response of the model to a pulsed field is not similar to the response of the original.

For the case that the thickness, $d$, is large compared to the skin depth, $\delta$, one can write (24) in the form

$$\eta \approx \frac{K}{2}\sqrt{\frac{\omega\sigma\mu_0^2}{j\mu}}\, L^2 \exp \sqrt{j\omega\mu\sigma d^2}, \qquad d \gg \delta. \tag{25}$$

For the case in which $d$ is small compared to $\delta$, one can write

$$\eta \approx K(\omega \cdot \mu_0\sigma \cdot d \cdot L), \qquad d \ll \delta. \tag{26}$$

In the latter case $(\eta_2/\eta_1)$ is frequency independent. Therefore, the model can be used to evaluate the effect of pulsed fields as well as CW fields.

## V. SUMMARY

The shielding effectiveness of the scaled model of a metallic enclosure is identical to that of the original if (*i*) the ratio of the wavelength in air to some specified linear dimension (say, the length of the enclosure) remains unchanged, and (*ii*) the ratio of the skin depth of the shielding material to some specified linear dimension remains unchanged. In the case of nonlinear, ferro-magnetic materials, instead of the second requirement, the expression of length$^2\times$ frequency$\times$conductivity$\times$permeability has to be the same for model and original. Given a certain scaling factor for the length the above requirements could be met by properly selecting two other scaling factors, say, those for time and conductivity. Usually, however, only the scaling factor for time can be readily controlled. Consequently, one has to be content with imperfect models which, however, will yield good results over given ranges of frequency.

If one is concerned with the magnetic shielding effectiveness only, the first requirement may be waived, provided the intrinsic wavelength in air is large compared to the linear dimensions of the model. For enclosures of building size this applies to frequencies up to several magacycles. Unfortunately, there are two shortcomings to this type of model: (*i*) The shield thickness of the scaled-down model often becomes impractically thin, and (*ii*) due to the necessary scaling of frequency, the ratio of the intrinsic wavelength in air to the length of the model sometimes decreases to the point where the quasi-stationary field-theory becomes invalid. If the enclosure is free of openings one can use a model with one scaling factor for the overall dimensions and another one for the thickness of the shield, provided the scaling factor for time is selected such that the ratio of skin depth to wall thickness remains unchanged. In this case, a simple formula relates the shielding effectiveness of the original to that of the model. If the enclosure is free of openings, of uniform thickness and of a material of constant permeability (nonferromagnetic metal) and if the applied magnetic field varies sinusoidally with time the dependence of the shielding effectiveness on scaling factors can be established analytically. Consequently, no constraints are put on the selection of the scaling factors of the model.

## VI. ACKNOWLEDGMENT

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## APPENDIX A

### *Derivation of the General Scaling Constraints for Electromagnetic Models*

Maxwell's Equations if applied to the original configuration can be written in integral form as follows.

$$\oint_{a_1} \mathbf{E}_1 \cdot \mathbf{d}l_1 = -\frac{\partial}{\partial t_1} \iint_{A_1} \mu_1 \cdot \mathbf{H}_1 \cdot \mathbf{d}s_1 \tag{27}$$

$$\oint_{a_1} \mathbf{H}_1 \cdot \mathbf{d}l_1 = \iint_{A_1} \sigma_1 \cdot \mathbf{E}_1 \cdot \mathbf{d}s_1 + \frac{\partial}{\partial t_1} \iint_{A_1} \epsilon_1 \cdot \mathbf{E}_1 \cdot \mathbf{d}s_1 \tag{28}$$

($a_1$ is a closed path in the original, or model "1". $A_1$ is an area bounded by $a_1$. The symbol $ds_1$ represents a surface element of $A_1$.) Using the corresponding closed path in the model as well as corresponding length, area and time increments and introducing the scaling factors, Maxwell's equations for the model read as follows:

$$\frac{l_2 \cdot E_2}{l_1 \cdot E_1} \oint_{a_1} \mathbf{E}_1 \cdot \mathbf{d}l_1 = -\frac{l_2^2 \cdot t_1 \cdot \mu_2 \cdot H_2}{l_1^2 \cdot t_2 \cdot \mu_1 \cdot H_1} \cdot \frac{\partial}{\partial t_1} \iint_{A_1} \mu_1 \cdot \mathbf{H}_1 \cdot \mathbf{d}s_1 \tag{29}$$

$$\frac{l_2 \cdot H_2}{l_1 \cdot H_1} \oint_{a_1} \mathbf{H}_1 \cdot \mathbf{d}l_1 = \frac{l_2^2 \cdot \sigma_2 E_2}{l_1^2 \cdot \sigma_1 E_1} \iint_{A_1} \sigma_1 \mathbf{E}_1 \cdot \mathbf{d}s_1 + \frac{l_2^2 \cdot t_1 \cdot \epsilon_2 \cdot E_2}{l_1^2 \cdot t_2 \cdot \epsilon_1 \cdot E_1} \frac{\partial}{\partial t_1} \iint_{A_1} \epsilon_1 \cdot \mathbf{E}_1 \cdot \mathbf{d}s_1 \tag{30}$$

From (27) and (29) one obtains

$$\frac{E_2 \cdot H_1}{E_1 \cdot H_2} = \left(\frac{l_2}{l_1}\right)\left(\frac{t_1}{t_2}\right)\left(\frac{\mu_2}{\mu_1}\right). \tag{31}$$

From (28), (30), and (31) one obtains

$$\left(1 - \frac{l_2^2 \cdot t_1 \cdot \sigma_2 \cdot \mu_2}{l_1^2 \cdot t_2 \cdot \sigma_1 \cdot \mu_1}\right) \iint_{A_1} \sigma_1 \cdot \mathbf{E}_1 \cdot \mathbf{d}s_1 + \left(1 - \frac{l_2^2 \cdot t_1^2 \cdot \epsilon_2 \cdot \mu_2}{l_1^2 \cdot t_2^2 \cdot \epsilon_1 \cdot \mu_1}\right) \cdot \frac{\partial}{\partial t_1} \iint_{A_1} \epsilon_1 \cdot \mathbf{E}_1 \cdot \mathbf{d}s_1 = 0. \tag{32}$$

In general, the ratio $(\iint_{A_1} \sigma_1 \cdot \mathbf{E}_1 \cdot \mathbf{d}s_1)/(\partial/\partial t_1 \iint_{A_1} \epsilon_1 \cdot \mathbf{E}_1 \cdot \mathbf{d}s_1)$ is a function of time. It then follows that (32) can be satisfied only if each of its terms in parenthesis is zero, which leads to the following two scaling equations:

$$\frac{l_2^2 \cdot \sigma_2 \cdot \mu_2}{t_2} = \frac{l_1^2 \cdot \sigma_1 \cdot \mu_1}{t_1} \tag{33*}$$

$$\frac{l_2^2 \cdot \epsilon_2 \cdot \mu_2}{t_2^2} = \frac{l_1^2 \cdot \epsilon_1 \cdot \mu_1}{t_1^2}. \tag{34}$$

In the case of a quasi-stationary magnetic field which assumes the term $\partial(\epsilon\mathbf{E})/\partial t$ to be negligible only (33) is needed to satisfy (32).

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* See Ref. 2, p. 488.

APPENDIX B

*Conditions Under Which the Effect of the Magnetic Field Intensity at the Inside Surface on that at the Outside Surface is Negligible*

Equations (11) and (12) of Section 4.2 can be readily integrated for the special case of constant permeability, $\mu$. The result of this integration with $E$ and $H$ being sinusoidal time functions is

$$H_{z,D} = E_{x,0} \cdot \frac{\sigma}{\sqrt{j\omega\mu\sigma}} \cdot \sinh\left(\frac{\sqrt{2j} \cdot d}{\delta}\right) + H_{z,0} \cdot \cosh\left(\frac{\sqrt{2j} \cdot d}{\delta}\right). \tag{35}$$

For $\sqrt{2} \cdot d > \delta$ the sinh-term is of the same order of magnitude as the cosh-term. Consequently, the effect of $H_{z0}$ on $H_{zD}$ is negligible if

$$\left| E_{x,0} \frac{\sigma}{\sqrt{j\omega\mu\sigma}} \right| \gg | H_{z,0} |; \qquad d > \frac{\delta}{\sqrt{2}}. \tag{36}$$

For $\sqrt{2} \cdot d < \delta$ the sinh-term and the cosh-term shall be approximated by the first terms of their Taylor-series. One obtains

$$H_{z,D} \approx E_{x,0} \frac{\sigma}{\sqrt{j\omega\mu\sigma}} \cdot \sqrt{2 \cdot j} \cdot \frac{d}{\delta} + H_{z,0} \cdot 1 \tag{37}$$

In this case, the effect of $H_{z,0}$ on $H_{z,D}$ is negligible if

$$\left| E_{x,0} \frac{\sigma}{\sqrt{j\omega\mu\sigma}} \cdot \sqrt{2j} \frac{d}{\delta} \right| \gg H_{z,0} ; \qquad d < \frac{\delta}{\sqrt{2}}. \tag{38}$$

If, as in the case of iron, the permeability is variable it appears reasonable to replace in inequalities (36) and (38) the permeability $\mu$ by an average permeability $\mu_{av}$ and the skin depth $\delta$ by an average skin depth, $\delta_{av}$, which is equal to $\sqrt{2/\omega \cdot \mu_{av} \cdot \sigma}$*. The conditions for making the effect of $H_{z0}$ on $H_{zD}$ negligible become then

$$\left| E_{x,0} \frac{\sigma}{\sqrt{j\omega\mu_{av} \cdot \sigma}} \right| \gg | H_{z,0} |; \qquad d > \frac{\delta_{av}}{\sqrt{2}}$$

$$\left| E_{x,0} \frac{\sigma}{\sqrt{j \cdot \omega \cdot \mu_{av} \cdot \sigma}} \cdot \sqrt{2j} \frac{d}{\delta_{av}} \right| \gg | H_{z,0} |; \qquad d < \frac{\delta_{av}}{\sqrt{2}}. \tag{39}$$

The order of magnitude of $(E_{x,0}/H_{z,0})$ for an enclosure of regular

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* In the case that $\delta_{av}$ is small compared to $d$, as this is usual with iron shields one best uses for $\mu_{av}$ the average permeability of that part of the shield that is within a distance $\delta_{av}$ from its inside-surface. This is based on the thought that if $H_z$ is practically independent of $H_{z0}$ at $y = \delta_{av}/\sqrt{2}$ it will remain so for larger values of, $y$, regardless of the permeabilities at larger values of $y$.

shape can be evaluated. The average magnetic field strength normal to the area $A_0$ (see Fig. 2) which passes through point 0 and is normal to the direction of the internal flux will be called $H_{av,0}$. The average electric field strength tangential to the line of length, $l_0$, which is formed by the intersection of plane $A_0$ and the inside surface of the shield will be called $E_{av,0}$. $H_{av,0}$ and $E_{av,0}$ are interrelated as follows:

$$j \cdot \omega \cdot \mu_0 \cdot A_0 \cdot H_{av,0} = E_{av,0} \cdot l_0 \ . \tag{40}$$

If one assumes that the ratio $H_{z,0}/E_{x,0}$ is of the same order of magnitude as $H_{av,0}/E_{av,0}$ one obtains for the order of magnitude of $E_{x,0}/H_{z,0}$

$$(E_{x,0}/H_{z,0})_{\text{order of magnitude}} = \frac{j \cdot \omega \cdot \mu_0 \cdot A_0}{l_0}. \tag{41}$$

If one calls $4(A_0/l_0)$ the equivalent diameter $D$, recalls the expression for $\delta$ and introduces the average relative permeability $(\mu_r)_{av}$ one obtains from expressions (39) and (41)

$$\frac{\sqrt{2} \cdot 2 \cdot (\mu_r)_{av} \cdot \delta_{av}}{D} \ll 1; \qquad d > \frac{\delta_{av}}{\sqrt{2}} \tag{42}$$

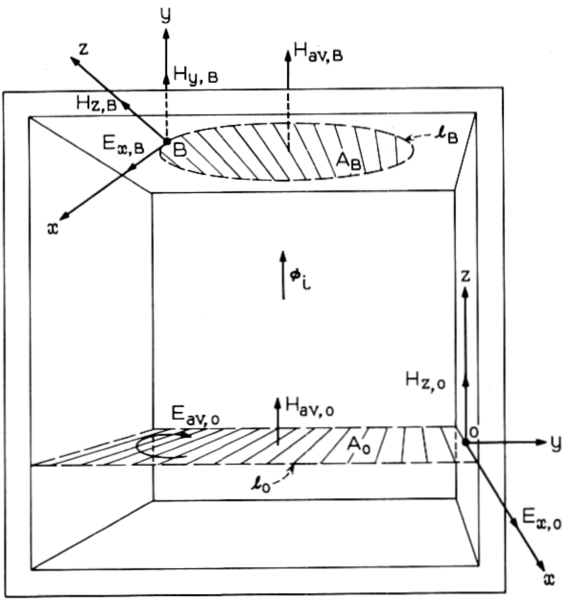


Fig. 2. — Local coordinates at two points of the enclosure.

and

$$\frac{2 \cdot (\mu_r)_{av} \cdot \delta_{av}^2}{d \cdot D} \ll 1; \qquad d < \frac{\delta_{av}}{\sqrt{2}}. \tag{43}$$

If one selects as reference point, point $B$ of Fig. (2) one obtains a relationship as given by (40) between the normal internal magnetic field strength and the tangential, internal electric field strength. Since, with respect to area $A_B$, the direction of the magnetic field is predominantly normal the tangential magnetic field component is less than the normal one. Consequently, as before, the effect of the tangential internal magnetic field strength on the external one is negligible if the inequalities (42) and (43) are satisfied with, $D$, being the equivalent diameter of the area $A_B$. [For symmetry reasons the tangential flux-density approaches zero as the center-line is approached. Consequently, its effect is negligible in this area (small $D$) regardless of inequalities (42) and (43).]

## APPENDIX C

### *Numerical Example for the Scaling Laws of a Model of Two Distinct Geometrical Scaling Factors*

The shielding effectiveness of an enclosure to a magnetic field pulse shall be evaluated by testing a scaled down model. The size of the enclosure is 15 m×15 m×15 m. Its material is sheet steel of thickness $d$ = 0.00317 m(0.125 inch), of average relative permeability $(\mu_r)_{av} = 500$ and of conductivity $\sigma = 1 \times 10^7$ mho/m. The peak excursion of the applied magnetic pulse is 10 oersted and its significant frequency content is within a band from $1 \times 10^4$ to $5 \times 10^6$ Hz.* (The corresponding range of the intrinsic wavelength in air is from 30,000 to 60 m.)

Tentatively the scaling factor for the overall dimensions, $L_2/L_1$, is selected as 0.1. Since the minimum intrinsic wavelength in air is only four times the length of the enclosure it is advisable to satisfy (3) (keeping the ratio of intrinsic wavelength to length of the model unchanged). Accordingly, the time scaling factor $(t_2/t_1)$ becomes 0.1. According to (17) the thickness scaling factor $(d_2/d_1)$ becomes $\sqrt{0.1} =$ 0.316. The shielding effectiveness of the original, $\eta_1$, according to (21),

* It is assumed that the frequency content of the pulse below $10^4$ cps is so small that it will not cause any damage even though the shielding effectiveness of a large enclosure approaches unity at very low frequencies.

becomes

$$\eta_1 = \left(\frac{1}{0.1}\right) \times 0.316 \cdot \eta_2 .$$

Because of the high intensity of the applied pulse the iron-shield will be driven into saturation near its outer surface. Consequently, the model enclosure must be built of the same steel as the original and the peak intensity of the magnetic pulse applied to the model must be equal to that acting on the original.

Finally, one has to test whether the inequalities (14) and (15) are satisfied. Based on a frequency of $1 \times 10^5$ Hz (the lowest significant frequency applied to the model) the average skin-depth is

$$(\delta_{av})_2 = \sqrt{\frac{2}{(2\pi \times 100{,}000) \times (4\pi \times 10^{-7} \times 500) \times (1 \times 10^{7})}}$$

$$= 2.3 \times 10^{-5}\text{m}.$$

With the smallest overall dimension, $D$, being 1.5 m one obtains for the expression of inequality (14)

$$\frac{\sqrt{2} \cdot 2 \times (\mu_r)_{av} \cdot \delta_{av}}{D} = \frac{\sqrt{2} \times 2 \times 500 \times 2.3 \times 10^{-5}}{1.5} = 0.022.$$

One can readily verify that inequality (14) is satisfied for the original as well.

## APPENDIX D

### *Integration of the Differential Equations (11) and (12) of the Field in the Shield for Constant $\mu$ and $\sigma$*

For a CW field, (11) and (12) read as follows:

$$\sigma E_x = \frac{dH_z}{dy} \tag{44}$$

$$j\omega\mu H_z = \frac{dE_x}{dy}. \tag{45}$$

Eliminating $E_x$, one obtains

$$j\omega\mu\sigma H_z - \frac{d^2 H_z}{dy^2} = 0. \tag{46}$$

The general solution of this equation is

$$H_z = C_1 \exp(+\sqrt{j\omega\mu\sigma}\, y) + C_2 \exp(-\sqrt{j\omega\mu\sigma}\, y). \tag{47}$$

With the boundary conditions given by (16) one obtains,

$$H_{z,D} = \left(\frac{\sigma \cdot E_{x,0}}{\sqrt{j\omega\mu\sigma}}\right) \cdot \sinh\left(\sqrt{j\omega\mu\sigma} \cdot d\right) \tag{48}$$

LIST OF SYMBOLS

$C$ = constant factor.
$d$ = thickness of shield.
$E$ = electric field strength.
$H$ = magnetic field strength
$j$ = $\sqrt{-1}$.
$K$ = proportionality factor.
$l$, $L$ = distance proportional to the size of the model.
$t$ = time proportional to a specified time interval of the applied electromagnetic field (for instance duration of a pulse).
$\delta$ = skin depth $\sqrt{2/\omega\mu\sigma}$.
$\epsilon$ = dielectric constant.
$\mu$ = permeability (usually of the shield).
$\mu_0$ = permeability of vacuum, ($4\pi \times 10^{-7}$ Henry/m).
$\mu_r$ = relative permeability
$\sigma$ = conductivity.
$\omega$ = angular frequency.
$\lambda_\epsilon$ = intrinsic wavelength in dielectric $2\pi/\omega\sqrt{\epsilon\mu}$.
$\lambda_\sigma$ = intrinsic wavelength in conductor $2\pi\sqrt{2/\omega\mu\sigma}$.
$\eta$ = shielding effectiveness.
$\Phi$ = magnetic flux.
$(L_2/L_1)$ = linear scaling factor of overall shield dimensions.
$(d_2/d_1)$ = scaling factor of shield thickness.
$(l_2/l_1)$ = linear geometric scaling factor if $(L_2/L_1) \equiv (d_2/d_1)$.

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