B. S. T. J. BRIEFS

The Effect of Noise Correlation on Binary Differentially Coherent PSK Communication Systems

By W. M. HUBBARD

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Several authors have calculated the probability of error in binary differentially coherent phase-shift-keyed (DCPSK) communication systems.[1,2,3,4] All of these calculations make the assumption that the sample values of the noise at the sampling instants in adjacent time slots are statistically independent random variables.* Because the noise is band-limited this assumption is not strictly true. The purpose of this note is to justify the assumption for most cases of interest and to point out where the assumption introduces some discrepancy.

Consider a signal of the form

$$S(t) = Q_c(t) \cos \omega t + Q_s(t) \sin \omega t \tag{1}$$

which is corrupted by additive, band-limited, Gaussian noise which can be expressed in the form

$$x(t) = x_c(t) \cos \omega t - x_s(t) \sin \omega t. \tag{2}$$

The properties of such noise are well-known. They are given, for example, in Davenport and Root[5] on page 158 ff. We follow their notation and state their results without proof.

We assume that the signal is processed by an ideal product demodulator.[6] The output of this device is

$$V(t) = \{S(t) + x(t)\}\{S(t - \tau) + x(t - \tau)\}, \tag{3}$$

where τ is the differential delay of the product demodulator. Therefore, we are interested in $x(t)$ at the times t and $t - \tau$. Set

$$\begin{aligned} x_{c1} &= x_c(t) & x_{s1} &= x_s(t) \\ x_{c2} &= x_c(t - \tau) & x_{s2} &= x_s(t - \tau). \end{aligned} \tag{4}$$

The covariance matrix of these variables and its various cofactors are given in Davenport and Root (loc. cit.). The joint probability density

* Noise correlation in other types of systems has been considered, for example, R. R. Anderson, et. al., Differential Detection of Binary FM, B.S.T.J., *44*, January, 1965, pp. 111–159.

function of x_{c1}, x_{s1}, x_{c2}, x_{s2} is given in Davenport and Root[5] in equation 8-101, page 162. From this joint probability density function, one can obtain the probability density function of V_1 where

$$V_1 = A(Q_{c1} + x_{c1}) + B(Q_{s2} - x_{s1}) \tag{5}$$

and

$$\begin{aligned}
V_1 &= V(t) \\
Q_{c1} &= Q_c(t) \qquad Q_{s1} = Q_s(t) \\
Q_{c2} &= Q_c(t - \tau) \qquad Q_{s2} = Q_s(t - \tau) \\
A &= \tfrac{1}{2}[(Q_{c2} + x_{c2}) \cos \omega\tau - (Q_{s2} - x_{s2}) \sin \omega\tau] \\
B &= \tfrac{1}{2}[(Q_{c2} + x_{c2}) \sin \omega\tau + (Q_{s2} - x_{s2}) \cos \omega\tau].
\end{aligned}$$

It is given by

$$p(V_1) = \frac{1}{(2\pi)^{\frac{3}{2}}\sigma^2} \int_{-\infty}^{\infty} dx_{c2} \int_{-\infty}^{\infty} dx_{s2} \frac{1}{\sigma_x} \cdot \exp\left(-\frac{x_{c2}^2 + x_{s2}^2}{2\sigma^2}\right) \exp\left(-\frac{(V_1 - V_0)^2}{2\sigma_x^2}\right), \tag{6}$$

where

$$\sigma_x^2 = \sigma^2(A^2 + B^2)(1 - \psi^2 - \varphi^2)$$

$$\psi = \frac{R_c(\tau)}{\sigma^2}, \qquad \varphi = \frac{R_{cs}(\tau)}{\sigma^2}$$

$$V_0 = A(Q_{c1} + \psi x_{c2} + \varphi x_{s2}) + B(Q_{s2} - \psi x_{s2} + \varphi x_{c2}).$$

Following the method of Bennett and Salz[3] as extended in Ref. 6 one obtains the probability of error

$$\Pi = \frac{1}{8\pi\sigma^2} \int_{-\infty}^{\infty} dx_{c2} \int_{-\infty}^{\infty} dx_{s2} \exp\left(-\frac{x_{c2}^2 + x_{s2}^2}{2\sigma^2}\right) \cdot \left\{\operatorname{erfc} \frac{V_0 + \epsilon}{\sqrt{2}\,\sigma_x} + \operatorname{erfc} \frac{V_0 - \epsilon}{\sqrt{2}\,\sigma_x}\right\}, \tag{7}$$

where 10 Log $\epsilon = S/T =$ ratio (in dB) of expected signal power to minimum signal power required for proper functioning of the regenerator.

In order to proceed further, it is necessary to make some assumptions about the spectral density of the noise. As an illustrative example we assume a raised cosine noise spectrum centered at f_c with full bandwidth

Δ. One then obtains

$$\varphi = 0 \qquad \psi = \frac{\sin \pi \, \Delta\tau}{\pi \, \Delta\tau(1 - (\Delta\tau)^2)}.$$

Ordinarily one is interested, in a DCPSK system, with either a signal that changes phase by an amount 0 or π between sampling points or one which changes phase by $\pi/2$ or $-\pi/2$ between sampling points. In the first case, one chooses $\omega\tau = n\pi$, in the second $\omega\tau = (n + \frac{1}{2})\pi$ in order that the possible signal states be anticorrelated in the product

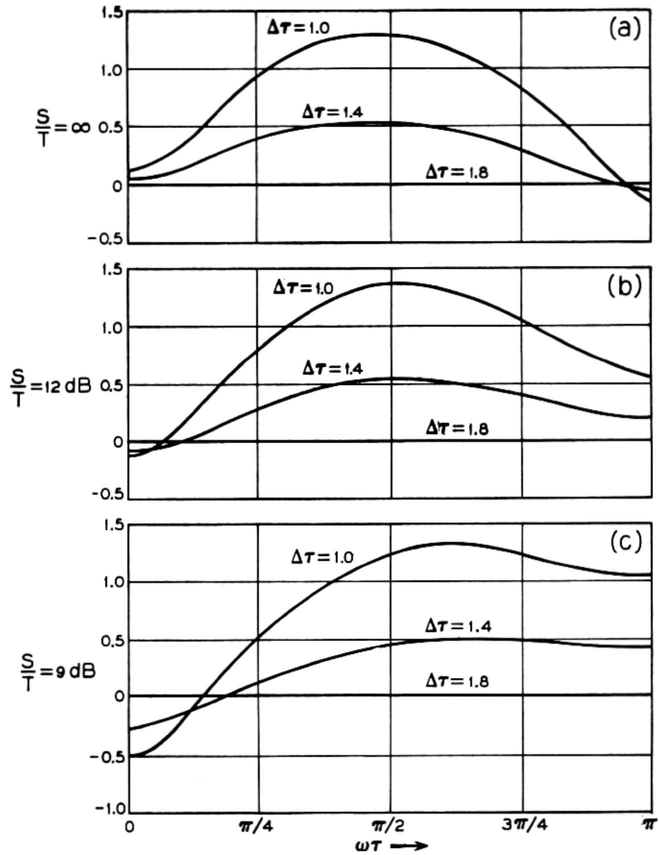

Fig. 1 — (a) Degradation in signal-to-noise ratio for 10^{-9} error probability for $S/T = \infty$. (b) Degradation in signal-to-noise ratio for 10^{-9} error probability for $S/T = 12$ dB. (c) Degradation in signal-to-noise ratio for 10^{-9} error probability for $S/T = 9$ dB.

demodulator. These cases can be specified for the purpose of this calculation as

$$\text{Case I:} \quad Q_{s1} = Q_{s2} = 0 \qquad \omega\tau = n\pi. \tag{8}$$

$$\text{Case II:} \quad Q_{s2} = Q_{c1} = 0 \qquad \omega\tau = (n + \tfrac{1}{2})\pi. \tag{9}$$

If we assume $S(t)$ has constant amplitude then we can write

$$Q_{c1} = Q \cos \theta_1 \qquad Q_{c2} = Q \cos \theta_2$$

$$Q_{s1} = Q \sin \theta_1 \qquad Q_{s2} = Q \sin \theta_2, \; Q \text{ a constant.} \tag{10}$$

Rather than imposing the constraints (8) and (9) we impose a weaker constraint which includes (among other possibilities) the situations specified by Case I and Case II. Namely, in (10), we put

$$\theta_2 = 0, \qquad \theta_1 = \omega\tau. \tag{11}$$

Introducing (10) and (11) into (7) and performing the indicated integration numerically gives the results shown in Fig. 1. This figure shows the change in signal-to-noise ratio, S/N, which is required for a 10^{-9} error rate due to the effects of noise correlation as a function of $\omega\tau$ for various values of S/T and Δ. The graphs are plotted for $\omega\tau \epsilon [0, \pi]$ only since inspection of the integrand reveals that the integral is symmetric about π and periodic in $\omega\tau$ with period 2π.

From this figure one concludes that the error rate is not significantly affected for noise bandwidth greater than about 1.4 times the bit rate, whereas if the noise is limited to bandwidths smaller than this the effect does become appreciable for $\omega\tau = (n + \frac{1}{2})\pi$. For $\omega\tau = n\pi$ the effect is small if n is even regardless of S/T, whereas for n odd, the effect is comparable to that for $(n + \frac{1}{2})\pi$ if S/T is small.

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Excited Level Populations in High Current Density Argon Discharges

By R. C. MILLER, E. F. LABUDA, and C. E. WEBB

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Spontaneous emission intensities of AI and AII in the range 2500 Å to 11,500 Å have been obtained from 2-mm diameter capillaries operated at filling pressures between 0.45 and 5.0 torr and currents up to 10 amperes. Only the AII results at 0.6 torr and 5 amperes are reported

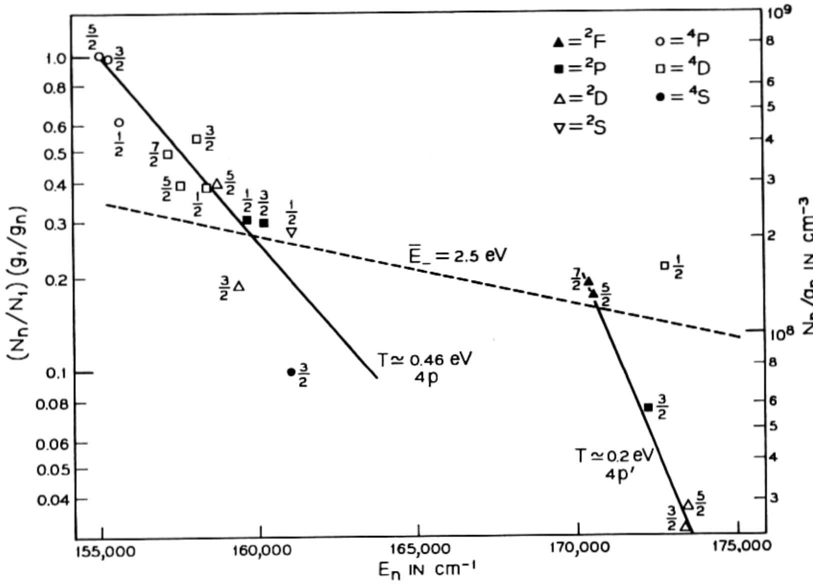

Fig. 1 — Level population in the $4p$ and $4p'$ configurations of AII in a capillary discharge. Capillary diameter = 2 mm, discharge current = 5 amps, filling pressure = 0.6 Torr. The value of electron mean energy $\bar{E}$ was obtained from shielded double probe measurements and the absolute value scale from Ladenburg–Reiche measurements.

here. The spectral sensitivity of the detection system was calibrated with a standard lamp, and the resolution was ≈1 Å. Effects of optical gain and absorption were verified to be negligible for AII.

Relative AII magnetic sublevel populations, N_n/g_n, determined by dividing relative spontaneous emission intensities by measured or estimated A-coefficients[1,2,3,4,5] and by the statistical weight g_n of the emitting level n, are shown in Fig. 1 as functions of excitation energy

E_n above the AII ground state. The results have been normalized to the $4p\,^4P_{5/2}$ population, designated (N_1/g_1), at $E_1 = 155{,}044.07$ cm^{-1}. Hence, the quantity plotted is $(N_n/N_1)(g_1/g_n)$. An approximate absolute value scale of N_n/g_n, obtained from optical absorption measurements,[6] is also shown.* When A-coefficient estimates† existed for several transitions originating from the same level, their self-consistency was checked,‡ and the average value was used.

The $4p$ and $4p'$ populations appear to be grouped along straight lines in the semilogarithmic plot of Fig. 1. Thus, for each of these configurations it is possible to define a "configuration temperature" T (measured in energy units) by the relation

$$(N_n/N_1)(g_1/g_n) \equiv \exp\,[(E_n - E_1)/T].$$

This yields $T \approx 0.45$ and 0.2 eV for the $4p$ and $4p'$ levels, respectively. A line corresponding to $T = 2.5$ eV (the measured value[6] of electron mean energy) has been included for reference.

The existence of an approximately common "temperature", considerably smaller than the prevailing electron "temperature", for all levels within a given configuration (regardless of spin) implies that *intra*-configuration thermal equilibrium is achieved by rearrangement of configurational populations in a time short compared to AII $4p$ and $4p'$ radiative lifetimes, and does not involve charged particle impact.§

Our present belief is that although the excitation energy of AII states is provided by electron impact on the ion, these states can interact rapidly by collisions with the ground and excited states of AI, the final state ion being in the initial AII configuration but not necessarily in the initial level of that configuration. The existence of such collisions might account for part of the large Lorentz broadening of AII line profiles at high current densities.[8] The present experiments yield no information on actual details of these collisions, but there are experimentally documented processes such as formation of molecular ions

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* These measurements show that AII $4s$ metastable populations are in approximate Boltzman equilibrium with the AII ground state at the prevailing electron mean energy.

† The lifetime measurements of Ref. 1 and the calculations of Ref. 2, because of their generally excellent consistency, were adopted in preference to earlier estimates whenever such choice was possible.

‡ This check was possible only for the AII $4p \rightarrow 4s$ calculations of Ref. 2 where, with one exception, the self-consistency was better than 30 percent.

§ This latter restriction is required by the AI and AII spontaneous emission radial profiles,[7] provided that AII excited level populations are derived *ab initio* from electron impact on the AII ground state. The fact that "configuration temperatures" are much lower than the electron mean energy independently suggests that electron impact is not responsible for the rearrangement.

(regarded here as a non-stationary intermediate state) which provide a basis for discussing population rearrangement.

If a resonant nature is supposed for the proposed interaction,* then AII levels are expected to interchange populations rapidly if their excitation energy differences are no larger than the measured 0.1-0.2 eV ion and atom thermal energies.[6,9] The maximum energy differences of 0.1 and 0.2 eV which exist between adjacent levels of the $4p$ and $4p'$ configurations, respectively, imply that the achievement of *intra*-configurational thermal equilibrium is plausible. *Inter*-configuration interaction rates, on the other hand, will be small if the energy gap between them is much larger than $\approx$0.1-0.2 eV (true for the 1.1 eV energy difference between the highest $4p$ and the lowest $4p'$ levels), so that different configurations may very well attain different "temperatures". Also selection rules may reduce or prohibit inter-configuration interactions, core changes, e.g., being difficult to achieve from impact parameter considerations. The existence of an approximately common "temperature" for doublet and quartet $4p$ levels implies that at least some rearrangement cross-sections involving spin change are comparable to those involving no spin change.

From measured discharge parameters[6,10] it can be shown that, if the AI ground state were the sole source of rearrangement collisions, cross-sections $\approx 2 \times 10^{-14}$ cm^2 would provide "thermalization" times shorter than typical $4p$ radiative lifetimes (5×10^{-9} sec). This value is experimentally plausible for resonant collisions between heavy particles,[11] particularly if one of them is in an excited state. Finally, it should be noted that if the above collisions are predominant over radiation, then the determination of cascade contributions to individual levels cannot, in general, be made simply by comparing the total spontaneous emission rates out of and into the level.

Note added in Proof: Where comparison is possible, the absolute level populations of Fig. 1 are in fair agreement with values obtained by Bennett, et al[12] at somewhat lower (pd).

* We do not wish to imply that the suggested collisions must necessarily be treated as a charge-exchange process.

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CW Operation of LSA Oscillator Diodes—44 to 88 GHz

By JOHN A. COPELAND

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Bulk n-GaAs oscillator diodes have been operated on a continuous basis in the LSA (Limited Space-charge Accumulation) mode[1] at frequencies from 44 to 88 GHz. This is the first time a practical solid-state oscillator has operated continuously in this high-frequency range. The reason the LSA diode can produce millimeter wave power at higher frequencies than other solid-state devices such as transistors, tunnel diodes, IMPATT diodes, and Gunn diodes is because it is the first device which is not subject to the "transit-time limitation."

The "transit-time limitation" exists for these other devices because they must be designed so that the time required for a charge carrier to move from the source contact to the drain contact must be shorter than or on the order of one RF cycle. A common principle of all these devices is the bunching of space charge which remains until it drifts into a contact. Since carriers in semiconductors such as silicon, germanium, and gallium arsenide have maximum drift velocities on the order of 10^7 cm/sec, devices for higher frequencies must be designed with proportionally thinner active regions. The power and impedance of such a device both decrease proportionally to the thickness of the active region, so the maximum value of the product of power and impedance decreases as the square of the thickness or as the reciprocal of the square of the frequency, f. The lowest impedance which is practical increases with frequency at microwave frequencies because of skin effect. The result of all these considerations is that the transit-time limitation causes the maximum power of a given device to decrease faster than f^{-2} as f increases.[2,3]

The LSA diode is not subject to the transit-time limitation because it derives its ability to transform dc to ac directly from the negative differential of drift velocity of the individual electrons with respect to electric field[4,5] rather than to the movement of space charge across the device. There are two main requirements for preventing space charge from accumulating and distorting the electric field which would lead to Gunn oscillations at a lower frequency: The ratio of doping to frequency must be within a certain range, which for n-GaAs appears to be 10^4 to 2×10^5 sec/cm^3. Also, the resonant circuit must be properly loaded so that the electric field swings into the positive conductance region below 3000 V/cm for part of each cycle to quench out any space charge which has started to accumulate.[1]

The existence of the LSA mode was first verified on a pulse basis at 1, 10, and 30 GHz, then the experimental effort was directed toward producing a 50-GHz CW oscillator. In interpretation of the results shown in Table I and Fig. 1, it should be kept in mind that these are only preliminary results. In particular, no attempt was made to obtain high pulse powers, the pulse powers reported are from the same devices that operated CW at lower voltages and much lower efficiencies.

Table I gives the results obtained from the first diodes that operated on a continuous basis. All of these devices were made from a wafer of epitaxially grown n-GaAs with a carrier density of about 8×10^{15} cm^{-3}. The active region was 5 microns thick, and the current maximum occurred at 2 volts. Noise due to the diodes could not be detected on the HP-851A spectrum analyzer, indicating the carrier-to-noise ratio was

Table I — LSA Diode Experimental Results

Diode	Voltage (V)	RF Power (mW)	Frequency GHz	Efficiency (%)
Continuous				
D1	3.5	20	84–88	2.0
D2	3.6	15	51	0.7
D3	3.3	20	44–51	0.7
		(40*)		(1.4)
Pulsed				
D1	5.0	50	84	4.0
D2	7.0	400	51	9.0
D3	11.0	500	44–51	3.0
		(700*)		(4.0)

* Holder cooled with dry ice.

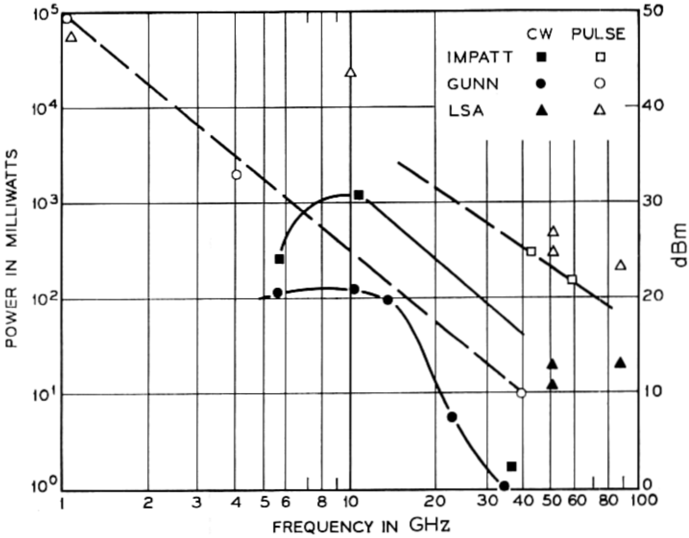

Fig. 1 — Power obtained from three types of solid-state oscillator diodes vs frequency as found in the literature. The LSA points were all obtained at Bell Telephone Laboratories except for the 10-GHz pulse result (Ref. 8).

greater than 90 dB in a one-cycle bandwidth one MHz from the carrier. Similar operation was obtained on a pulse basis at four times higher voltage with material with a 20-micron active region; however, heat problems prohibited CW operation of these samples.

Heating limits the CW efficiency of the diodes by restricting the bias voltage and by causing the carrier velocity as a function of electric field to become less favorable. The heating problem can be alleviated in the future for this device configuration by using a lower carrier density, by using smaller areas, and by making the substrate and liquid-regrowth contacts thinner. Details on the circuit used will be presented in Ref. 6.

In order to fully utilize the LSA mode of oscillation, a radical change in device design is needed. The devices reported on here were originally designed to be used as Gunn diodes at lower frequencies, from 5 to 20 GHz and are thinnest in the direction parallel to the current.[7] In order to increase the power and operate CW at lower frequencies, future LSA oscillator diodes should be long in the direction parallel to the current and thin in a direction perpendicular to the current. The heat will be removed from the sides and the thin dimension perpendicular to the electric field will eliminate skin effect problems.

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