Mode Selection in an Aperture-Limited Concentric Maser Interferometer

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The concentric interferometer with a limiting aperture in its mid-plane is analyzed for its mode-selective properties. Two of the lowest-order transverse modes and their losses for the infinite-strip geometry are computed by solving the associated integral equations by the method of successive approximations. The apertured concentric interferometer is found to be more mode-selective than the apertureless concentric interferometer or the Fabry-Perot interferometer with parallel plane mirrors. Computed results indicate that the optimum aperture size for maximum mode selectivity is approximately the size of the major lobe of the diffraction pattern of the dominant mode at the aperture plane. However, the maximum selectivity attainable does not exceed that of the "comparable" confocal system. The latter system is not very practical because it requires either very long resonator lengths or very small mirrors.

I. INTRODUCTION

Interferometer-type resonators that are used in optical masers are inherently multimode devices. The resonant modes that can exist in such devices may be classified as longitudinal and transverse modes. The longitudinal mode order is determined by the number of field variations along the axis of the interferometer, while the transverse mode order is determined by the number of field variations in the plane of the mirrors. For each longitudinal mode order, there exists a set of transverse modes. The number of modes that can partake in the oscillations of an optical maser is dependent on the geometry and the losses of the resonator, the width of the atomic resonance of the active material and the degree of population inversion. Practically, an optical maser will oscillate in several modes simultaneously unless special steps are taken to suppress the unwanted ones. For applications such as optical communication it is desirable from the standpoint of noise, coherence, spectral purity, etc. to suppress all but one mode in a maser oscillator. Therefore, mode-selection schemes are very important if the optical maser is to be a useful source of coherent and monochromatic radiation.

Aside from losses of a random nature such as those due to inhomogeneities in the medium and mirror imperfections, the resonant modes in a maser interferometer also suffer from losses due to diffraction around the mirrors.[1,2] The diffraction loss varies very little with longitudinal mode order, but increases very rapidly with increasing transverse mode order. Thus interferometer-type resonators are inherently mode-selective with respect to transverse modes. By using a long, thin configuration (small mirrors and large mirror separation) it is possible to suppress all but the dominant transverse mode.[3] Also, by operating just above the oscillation threshold[4] or by using a short resonator[5] it is possible to restrict the oscillations to a single longitudinal mode. The output power in these cases is somewhat limited.

Since it is desirable to pump the active medium strongly so as to obtain as much power as possible from a maser oscillator, methods for providing additional mode selection are important. In general, mode selection involves the introduction of loss to the resonator in some prescribed manner. Kleinman and Kisliuk[6] have proposed the use of an additional Fabry-Perot interferometer to discriminate against unwanted longitudinal modes. Kogelnik and Patel[7] obtained essentially a single-frequency output from a gaseous maser using three concave mirrors. Collins and White[8] used two tilted Fabry-Perot etalons within the resonator of a ruby maser to select longitudinal as well as transverse modes. Schemes for suppressing unwanted higher-order transverse modes by employing a limiting aperture in the focal plane of a suitable optical system within the resonator of a ruby maser were tried by Burch,[9] Baker and Peters,[10] and Skinner and Geusic.[11] These latter schemes are essentially equivalent to an interferometer system consisting of a pair of spherical mirrors spaced concentrically and having a limiting aperture in its mid-plane. Since the field distribution over the mid-plane of a concentric system is essentially the far-field pattern (Fourier transform) of the field over the mirrors, the higher-order transverse modes will have a wider lateral spread than the lower-order ones. Therefore, a suitably chosen aperture will introduce very little loss to the dominant mode but will introduce appreciable loss to the higher-order modes, making the apertured system quite mode-selective to transverse orders.

In order to study the mode-selective property of an apertured concentric system, we have set up the appropriate integral equations for the infinite-strip geometry and solved them iteratively on a digital computer for the intensity distributions and the losses of the two lowest-order transverse modes. The computed results indicate that there is an optimum aperture for maximum mode selectivity,* but that this maximum never exceeds that of the "comparable" confocal system. In fact, as the aperture is made smaller and smaller the behavior of the apertured concentric system approaches that of the confocal system. However, the comparable confocal system is not very practical because it requires either very small mirrors or very large mirror spacing.

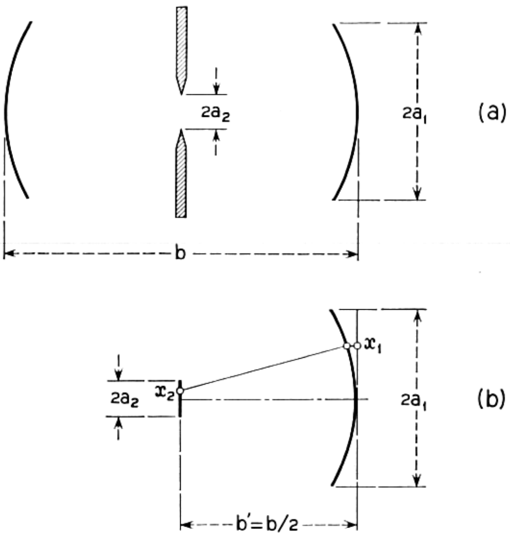

Fig. 1 — Geometry of the aperture-limited concentric maser interferometer. (a) Full-concentric system with aperture. (b) Equivalent half-concentric system.

II. INTEGRAL EQUATIONS OF THE SYSTEM

Since the full-concentric system with a mid-plane aperture is equivalent to a half-concentric system with a plane mirror of the same size as the aperture,[12] it is convenient to use the half-concentric model for the formulation of the integral equations (Fig. 1). Also, it is convenient to use the two-dimensional model of infinite-strip mirrors, since the three-dimensional problem of rectangular mirrors can be reduced to a two-dimensional one.[1] The behavior of systems with circular mirrors is expected to be similar to that of square-mirror systems.

Fig. 1(a) shows the geometry of an apertured concentric interferometer.

* The loss of the higher-order mode relative to the loss of the dominant mode may be regarded as a measure of mode selectivity.

The infinite-strip concentric mirrors are of width $2a_1$ and are separated by b, which is twice the radius of curvature of the mirrors. The aperture is of width $2a_2$ and is in the mid-plane of the interferometer. Fig. 1(b) shows the equivalent half-concentric model which is to be used for the analysis. The integral equations defining the modes can be derived from our previous analysis on interferometers with curved mirrors by setting $g_1 = 0$ and $g_2 = 1$ in equations (11) and (12) of Ref. 12. The resulting equations are

$$\gamma^{(1)}\psi^{(1)}(x_1) = \frac{e^{j\pi/4}}{\sqrt{\lambda b'}} \int_{-a_2}^{a_2} \exp\,[jk(2x_1x_2 - x_2^2)/2b']\psi^{(2)}(x_2)\, dx_2 \quad (1)$$

and

$$\gamma^{(2)}\psi^{(2)}(x_2) = \frac{e^{j\pi/4}}{\sqrt{\lambda b'}} \int_{-a_1}^{a_1} \exp\,[jk(2x_1x_2 - x_2^2)/2b']\psi^{(1)}(x_1)\, dx_1 \quad (2)$$

where the subscripts and superscripts one and two denote the curved mirror and the plane mirror, respectively, as shown in Fig. 1(b). The ψ's are the eigenfunctions that describe the relative field distributions over the mirrors, and the γ's are the corresponding eigenvalues that specify the loss and the phase shift the wave suffers during each transit. The propagation constant k is equal to $2\pi/\lambda$, where λ is the wavelength in the medium. The mirror separation b' is equal to $b/2$.

Equations (1) and (2) are single-transit equations which can be combined to form two round-trip equations. They are

$$\gamma\psi^{(1)}(x_1) = \int_{-a_1}^{a_1} K_1(x_1, \bar{x}_1)\psi^{(1)}(\bar{x}_1)\, d\bar{x}_1 \quad (3)$$

and

$$\gamma\psi^{(2)}(x_2) = \int_{-a_2}^{a_2} K_2(x_2, \bar{x}_2)\psi^{(2)}(\bar{x}_2)\, dx_2 \quad (4)$$

where the kernels K_1 and K_2 are given by

$$K_1(x_1, \bar{x}_1) = \frac{j}{\lambda b'} \int_{-a_2}^{a_2} \exp\,[jk\{x_2(x_1 + \bar{x}_1) - x_2^2\}/b']\, dx_2 \quad (5)$$

$$K_2(x_2, \bar{x}_2) = \frac{j}{\lambda b'} \int_{-a_1}^{a_1} \exp\,[jk\{x_1(x_2 + \bar{x}_2) - (x_2^2 + \bar{x}_2^2)/2\}/b']\, dx_1 \quad (6)$$

and the eigenvalue γ is equal to $\gamma^{(1)}\,\gamma^{(2)}$. Since one round trip in the half-concentric system is equivalent to a single transit in the full-concentric system, (3) may be regarded as the integral equation defining the modes of the aperture-limited concentric interferometer while (4) gives the field distribution across the aperture.

The kernels K_1 and K_2 as defined by (5) and (6) are symmetric; that is, $K_1(x_1, \bar{x}_1) = K_1(\bar{x}_1, x_1)$ and $K_2(x_2, \bar{x}_2) = K_2(\bar{x}_2, x_2)$. Therefore, the eigenfunctions $\psi_n^{(1)}$ and $\psi_n^{(2)}$ corresponding to distinct eigenvalues γ_n are orthogonal[13] in the sense that

$$\int_{-a_1}^{a_1} \psi_n^{(1)}(x_1)\psi_m^{(1)}(x_1)\, dx_1 = 0, \qquad m \neq n \tag{7}$$

$$\int_{-a_2}^{a_2} \psi_n^{(2)}(x_2)\psi_m^{(2)}(x_2)\, dx_2 = 0, \qquad m \neq n. \tag{8}$$

As in the general case of curved mirrors,[12] the eigenfunctions are complex and are orthogonal in the non-Hermitian sense.

The integral equations can be solved numerically using iterative techniques. However, it is possible to extract the asymptotic behavior of the solutions from these equations for very small apertures. Thus for $(a_2/a_1) \ll 1$ and $a_2^2/b\lambda \ll 1$, the terms involving x_2^2 and $\bar{x}_2^2$ can be neglected and the integral equations reduce to those for the asymmetric confocal configuration (with unequal mirrors). It has been shown by Boyd and Kogelnik[14] that the modes and the corresponding losses of an asymmetric confocal system are the same as those of a symmetric system with equal mirrors of width $2a$ where $a = \sqrt{a_1a_2}$. Therefore, the behavior of the aperture-limited concentric system approaches asymptotically that of the confocal system as the aperture is made smaller and smaller. We designate the confocal system with Fresnel number $N_c = a_1a_2/b'\lambda$ as the *comparable* confocal system.

III. COMPUTED RESULTS AND DISCUSSION

The two lowest-order modes and their eigenvalues for an aperture-limited infinite-strip concentric interferometer were computed using the method of successive approximations[1] on an IBM 7090 computer. One hundred increments were used in the numerical integration of (1) and (2). Curves for power loss per transit for the two lowest-order modes and for different values of Fresnel number ($N = a_1^2/b\lambda$) are given in Fig. 2. The abscissa is the half width of the aperture normalized to a_1/N, where both a_1 and N should be regarded as constants. The dashed curves give the loss of the comparable confocal interferometer as functions of its half Fresnel number, which is equal to $a_1a_2/b\lambda$. The losses for the limiting case of infinitely wide aperture are given on the column on the right side of the figure.

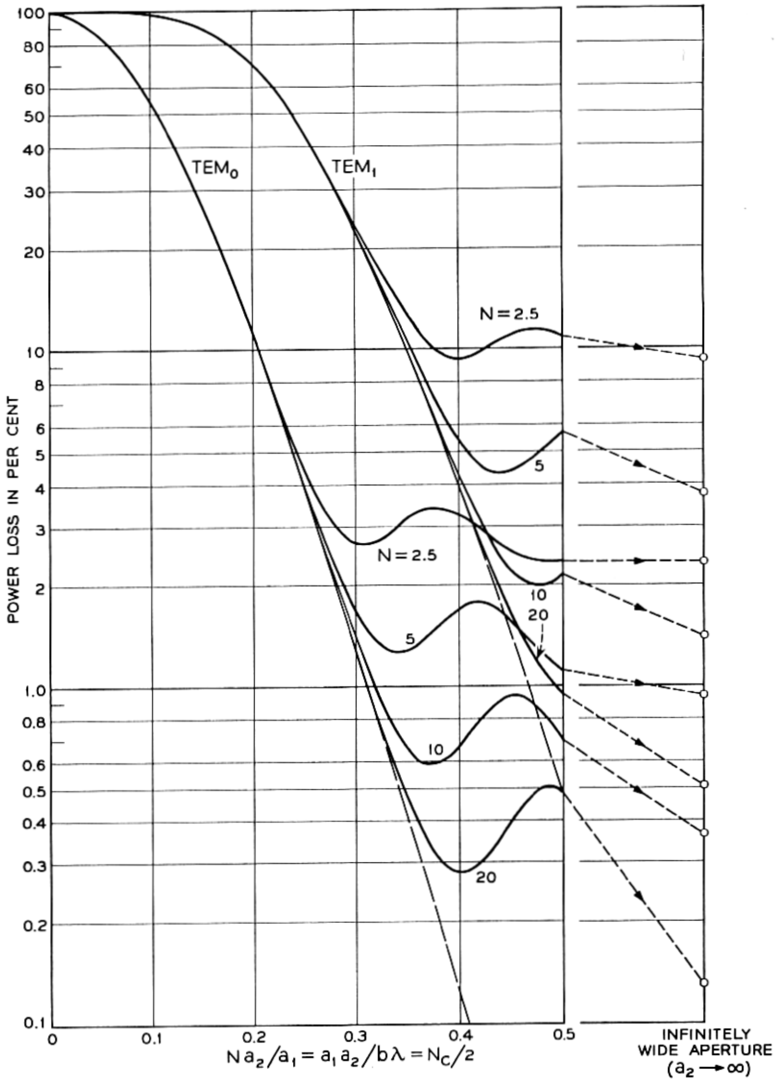

Fig. 2 — Power loss per transit for the two lowest-order modes of an aperture-limited concentric interferometer. The abscissa is the half width of the aperture normalized to a_1/N. (Both a_1 and N should be regarded as constants.) The dashed curves are for the "comparable" confocal interferometer.

Figs. 3 and 4 show the relative amplitude and phase distributions of the field intensity for the two lowest-order modes. Curves A and B are for the aperture-limited concentric interferometer with $N = 10$ and $a_2/a_1 = 0.0325$; A is the field distribution over the mirror while B is the field distribution over the aperture. Curve C is for the comparable confocal interferometer with a Fresnel number (N_c) of 0.65. Curve D is for an apertureless ($a_2 \rightarrow \infty$) concentric interferometer with $N = 10$ and is the same as for the parallel plane configuration except for the reversal of sign in the phase distribution.[1,12]

The ratio of the loss of TEM_1 mode to the loss of TEM_0 mode, which is a measure of mode selectivity, is plotted in Fig. 5 as a function of the normalized aperture half width. The dotted curve represents the same ratio for the comparable confocal system plotted as a function of its half Fresnel number, which is equal to $a_1a_2/b\lambda$. The short slant lines represent segments of the loci of constant loss for the TEM_0 mode. The ratio of the losses for the limiting case of infinitely wide aperture is approximately four* for large N ($N > 1$) and is given on the column on the right side of the figure.

That a suitably chosen limiting aperture placed in the mid-plane of a concentric interferometer should be mode-selective can be surmised by considering the field distribution over its mid-plane. This field distribution, as given by (2), is, except for a quadratic phase factor, the Fourier transform of the mode pattern over the mirrors. It resembles very closely the far-field pattern of the equivalent parallel plane system[1,12] and therefore has a lateral spread which increases with mode order. Consequently, a limiting aperture having the width of the major lobe of the dominant mode would have a small effect on that mode but would increase quite significantly the losses of the higher-order modes. The optimum aperture width (for a given N) corresponding to maximum mode selectivity as shown in Fig. 5 is approximately equal to the major-lobe width. As the aperture is made larger, more and more of the minor lobes are uncovered and they interfere either constructively or destructively at the mirrors, producing the oscillations in the loss curves in Fig. 2 and in the relative loss curves in Fig. 5.

Fig. 5 shows that a suitably chosen aperture can increase the relative loss of the TEM_1 mode several times its apertureless value. The effect of the aperture on other higher-order modes is expected to be even greater. By observing the number of iterations required to produce a

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* For large Fresnel numbers ($N > 1$) the ratio (loss of TEM_n mode)/(loss of TEM_0 mode) is approximately $(1 + n)^2$ for parallel plane or concentric configurations.

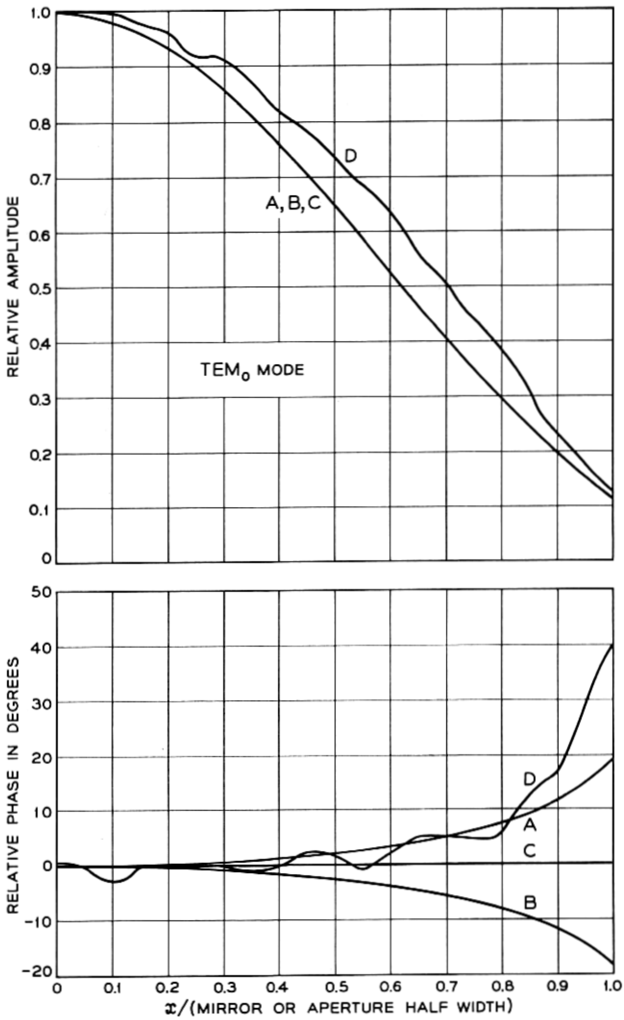

Fig. 3 — Relative amplitude and phase distributions of the lowest-order (TEM_0) mode. Both A and B are for an aperture-limited concentric interferometer with $N = 10$ and $a_2/a_1 = 0.0325$; A is the field distribution over the mirror and B is the field distribution over the aperture. C is for the "comparable" confocal interferometer with a Fresnel number of 0.65. D is for a concentric interferometer with $N = 10$ and $a_2 \rightarrow \infty$.

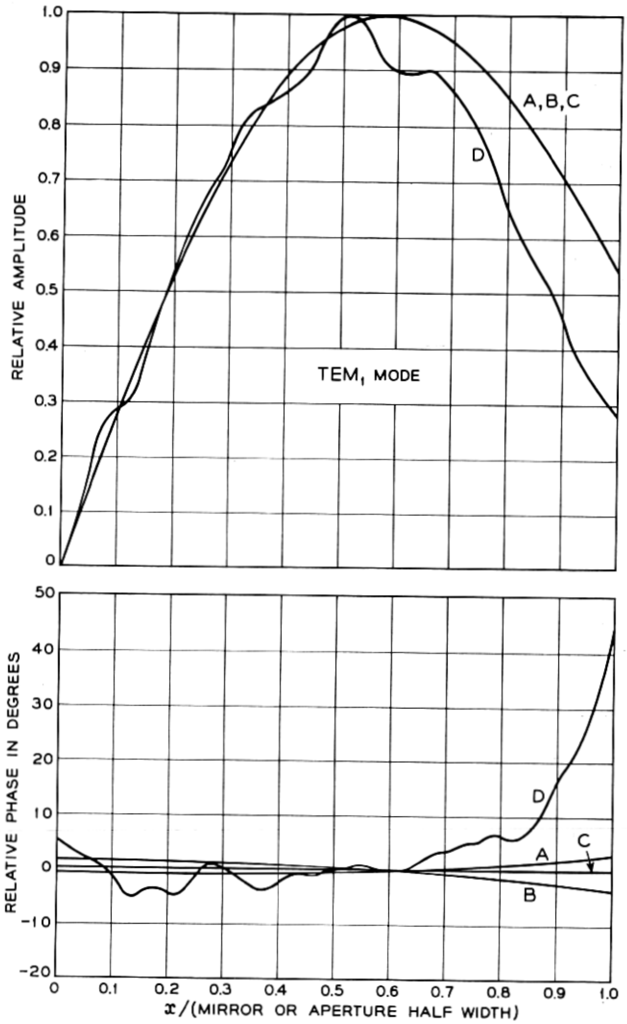

Fig. 4 — Relative amplitude and phase distributions of the second lowest-order (TEM_1) mode. The notation for the curves is the same as for Fig. 3.

steady-state solution from an arbitrary initial trial function, it is possible to infer the relative loss of other higher-order modes. For example, the number of iterations required for the dominant (TEM_0) mode of an apertureless concentric (or parallel plane) system with $N = 10$ is about 800, whereas with an aperture such that $a_2/a_1 = 0.0325$ the number required reduces to about 25, which indicates that the relative losses of the higher-order even-symmetric modes are now very much higher.

The behavior of the apertured concentric system was found analytically to approach that of the confocal system in the limit of very small apertures. The computed results confirm this and show further that the mode selectivity (in terms of relative loss) of the apertured concentric

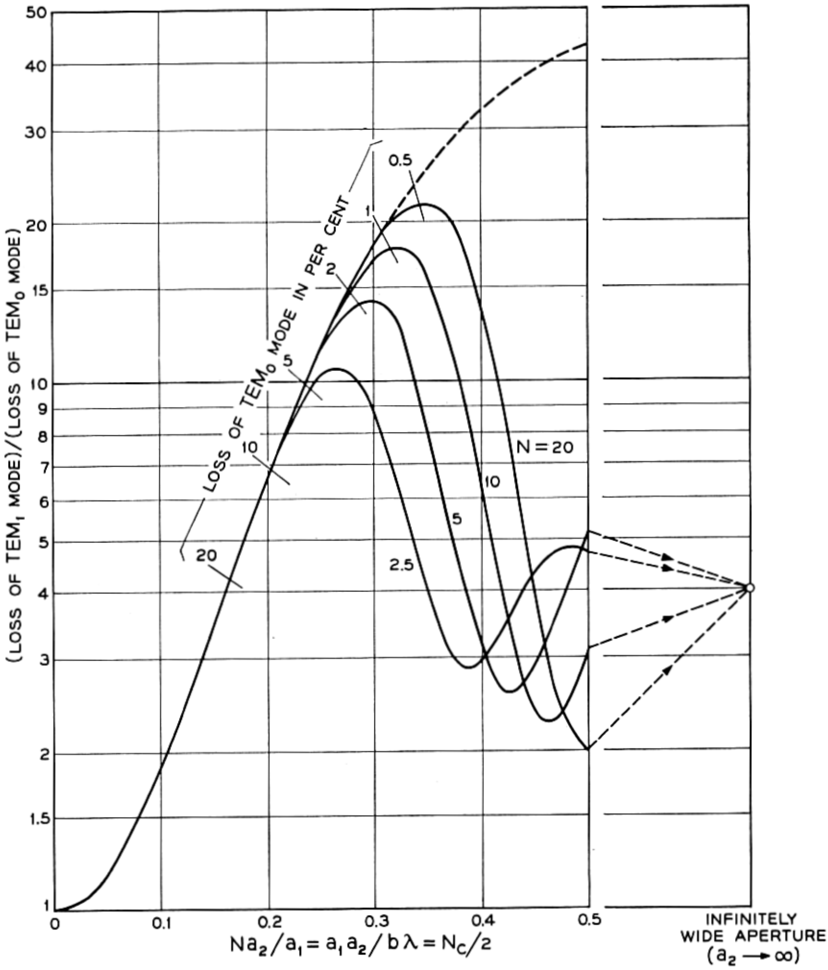

Fig. 5 — Ratio of the losses of the two lowest-order modes versus normalized aperture half width. The dashed curve is the ratio for the "comparable" confocal interferometer. The short slant lines are segments of the loci of constant loss for TEM_0 mode.

system can never be greater than that of the confocal system. However, the effective suppression of unwanted higher-order modes in a maser oscillator requires not only that the relative losses of the higher-order modes be high but also that their absolute losses be greater than the gain of the active medium. To satisfy the second condition a confocal system would have to operate with a very small Fresnel number, whereas a reasonably large Fresnel number can be used for the apertured concentric system. For example, if a maser having a gain of 20 per cent per pass is required to produce a single transverse-mode output, an apertured concentric configuration (with square mirrors) having $N = 20$ and $a_2/a_1 = 0.018$ can be used, but a confocal configuration would need an N_c of about 0.7, which means either very long resonator lengths or very small mirrors. Such configurations are generally undesirable because lengthening the resonator tends to increase the number of longitudinal modes that can oscillate and decreasing the size of the mirrors tends to diminish the power output capability.

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