Command Guidance of *Telstar* Launch Vehicle

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The Telstar I satellite was launched into orbit by a three-stage Delta launch vehicle guided by the Bell Telephone Laboratories command guidance system. The Delta program is a National Aeronautics and Space Administration sponsored series of missile flights designed to place various scientific payloads into orbit around the earth. This paper discusses the theory of the guidance equations employed by the command guidance system in the Delta program.

I. INTRODUCTION

The Telstar I satellite was launched into orbit by a three-stage Delta vehicle on July 10, 1962, at the Atlantic Missile Range. The Bell Telephone Laboratories command guidance system, developed for the Air Force, was employed to guide the Delta vehicle. The guidance system is shown in Fig. 1. The missile-borne equipment, housed in the second stage of the Delta vehicle, serves as a radar beacon to provide return pulses to the tracking radar and as the receiving portion of the command data link between the ground and the missile. The tracking radar functions as the transmitting portion of the data link and also serves as a sensor to determine the slant range, azimuth angle, and elevation angle of the missile during its flight. The precision tracking radar and the missile-borne guidance package were manufactured by the Western Electric Co. The guidance computer was designed and manufactured by the Univac Division of the Sperry Rand Corporation.

The three-stage Delta missile, designed by the Douglas Aircraft Company, consists of two liquid propellant stages and a solid propellant third stage. The powered flight portion of the Telstar satellite trajectory is shown in Fig. 2. The guidance system transmits corrective pitch and yaw steering commands during first- and second-stage powered flight. Second-stage engine cutoff is ordered by the guidance system when the

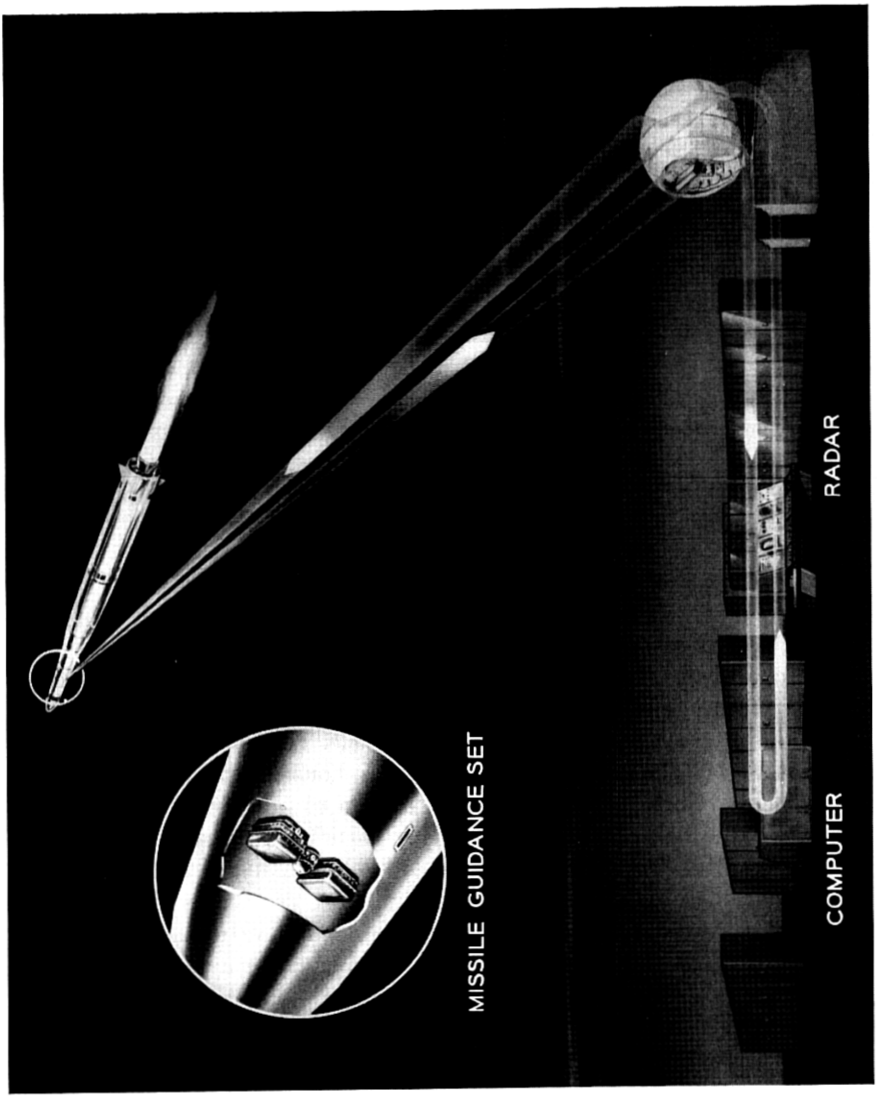

Fig. 1 — Guidance system.

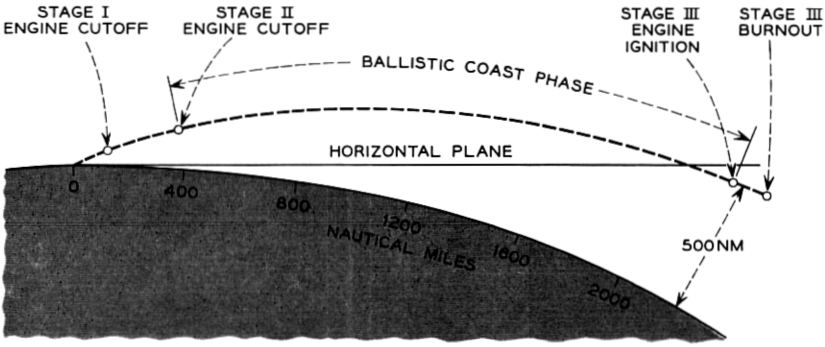

Fig. 2 — Telstar I trajectory.

position and velocity of the missile are such that the addition of the third stage velocity impulse at the end of the ballistic coast phase would yield the desired orbit. The unguided third stage is spin-stabilized to maintain attitude control. For the Telstar satellite trajectory, the third-stage velocity impulse was added at the perigee of the final orbit after a ballistic coast phase of approximately 600 seconds between second-stage cutoff and third-stage ignition. The Telstar satellite was separated from the third stage 120 seconds after third-stage burnout.

The guidance system steering and cutoff commands during the first and second stage ascent are calculated in the guidance computer, using the radar tracking data of the missile's position as the basic input information. The computer is programmed with a set of guidance equations that process the radar data and compute the desired commands to the missile.

This paper contains a description of the theory and design of the guidance equations used in the Telstar satellite flight. Guidance concepts are presented from the point of view of orbital mechanics and control, followed by a description of first- and second-stage guidance. The last section summarizes the results achieved in the Telstar satellite flight.

II. GUIDANCE CONCEPTS

The Keplerian motion of an earth satellite is completely defined by the specification of the vector position and velocity at an epoch. The satellite coordinates at insertion into orbit can be expressed in terms of the spherical coordinate system of Fig. 3 as follows: V_3, γ_3, and β_3 are are the magnitude, the elevation angle above the local horizontal, and the azimuth from North, respectively, of the velocity vector; R_3, λ_3,

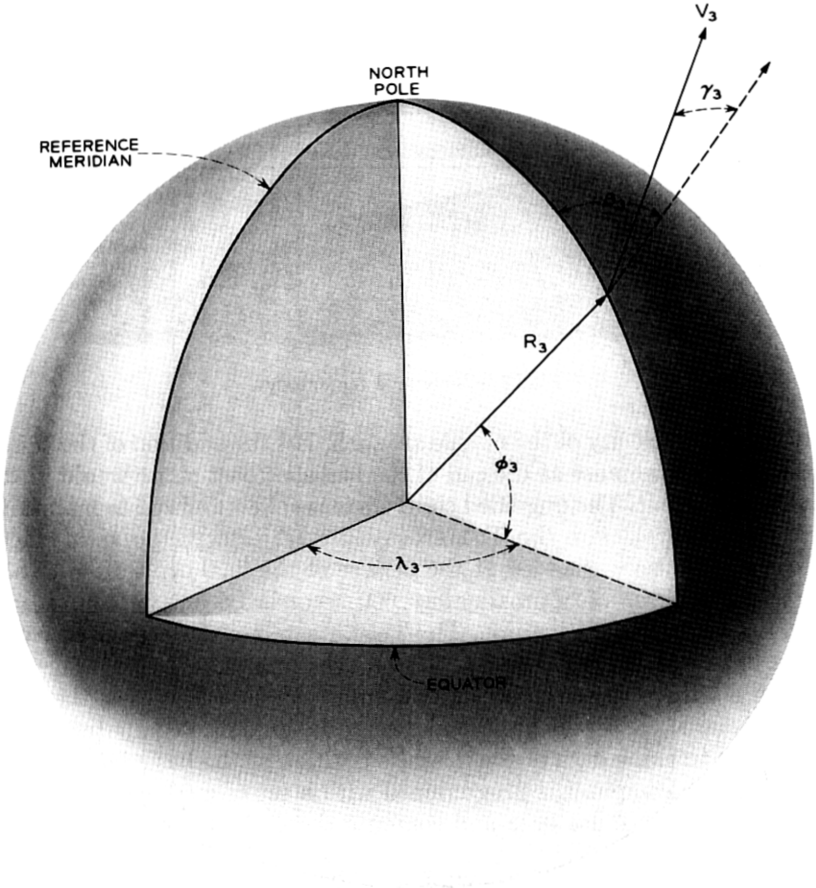

Fig. 3 — Insertion coordinates.

and φ_3 are the radial distance from the earth's center, the longitude, and geocentric latitude, respectively. The elements of the ellipse, i.e., apogee and perigee distances, are determined by V_3, R_3, and γ_3. The orientation of the ellipse relative to the earth in terms of inclination, argument of perigee, and longitude of the ascending node depends, in general, on all the insertion coordinates. It would be necessary to control all six coordinates of the satellite as well as the time of insertion to achieve a specified orbit in inertial space. For earth satellites the requirements on the insertion time are usually not stringent and are largely determined by launch time variations. The problem of guidance thus consists of achieving a specified set of six insertion coordinates.

In the case of the Telstar satellite trajectory, the unguided third stage ignites at a predetermined time on the transfer ellipse following the completion of guidance at second-stage cutoff. Since the characteristics of the third stage are known, fixed relations exist between the insertion coordinates and the coordinates of the missile at second-stage cutoff. The transfer ellipse is defined by the position and velocity vectors, $\mathbf{R}_2$ and $\mathbf{V}_2$, respectively, of the missile at second-stage cutoff. The direction of the velocity impulse added by the third stage is determined by the attitude of the missile's roll axis (axis of thrust application) at second stage cutoff. If we define $\boldsymbol{\varrho}_2$ as a unit vector lying along the roll axis of the missile, the eight independent coordinates of the missile at second-stage cutoff uniquely determine the six insertion coordinates. That is,

$$\{\mathbf{V}_2, \mathbf{R}_2, \boldsymbol{\varrho}_2\} \rightarrow \{V_3, \gamma_3, \beta_3, R_3, \lambda_3, \varphi_3\}. \tag{1}$$

Also the six insertion coordinates specify any six of the cutoff coordinates in terms of the remaining two.

During the first and second stages, guidance of the missile is limited to steering in the pitch and yaw planes and cutting off the second-stage rocket engine. The missile is constrained to fly a predetermined trajectory by pitch and yaw steering during first and second stage. This trajectory, if followed exactly, would yield the coordinates at second-stage cutoff that would produce the desired orbit. However, propulsion system variations, deviations in the attitude control system, and radar noise cause dispersions in the cutoff coordinates from the expected values.

As discussed in Section IV, $|\mathbf{V}_2|$ and $\boldsymbol{\varrho}_2$ can be controlled directly at cutoff to provide direct control over three of the insertion coordinates. Control of $|\mathbf{V}_2|$ by cutoff of the rocket engine provides the most sensitive control of V_3. The two coordinates defining $\boldsymbol{\varrho}_2$ are controlled by pitch and yaw steering. Steering could be based on deviations of position, velocity, or attitude coordinates from the desired reference trajectory. The selection of the coordinates to be controlled by steering is determined by the steering system design, which is based on minimizing the errors in the insertion coordinates. As demonstrated in Section 4.2, control of the pitch and yaw attitude angle, i.e., control of $\boldsymbol{\varrho}_2$, affords the best control over insertion coordinates.

For the Telstar satellite trajectory, $|\mathbf{V}_2|$ and $\boldsymbol{\varrho}_2$ were used to constrain V_3, γ_3, and β_3 at insertion to provide the desired apogee altitude and inclination. The other orbital elements were effectively controlled by steering the missile to the desired reference trajectory during the first-and second-stage powered flight. The equations relating $|\mathbf{V}_2|$ and $\boldsymbol{\rho}_2$ to the desired orbital conditions, and the equations for pitch and yaw steering derived from the reference trajectory, were programmed into the guidance computer. The targeting task for the Telstar satellite mission was to determine the numerical coefficients for the equations used in first- and second-stage guidance.

III. FIRST-STAGE GUIDANCE

The missile's position as tracked by the radar in slant range, azimuth, and elevation angle is converted to the earth-fixed Cartesian frame shown in Fig. 4. The angle A_0 is selected such that the Y-Z plane is approximately parallel to the pitch plane of the missile and the X axis lies in the yaw plane. The angle E_0 is determined by passing the Y axis through the expected position of the missile at second-stage cutoff.

From approximately 90 seconds after lift-off, pitch and yaw steering orders are transmitted to the missile. Yaw steering is based upon deviations in the $\dot{X}$ velocity from a reference polynomial in $\dot{Y}$. The polynomial is selected to match the desired $\dot{X}$ component of velocity, which is a function of the launch azimuth, the pitch and yaw programmed

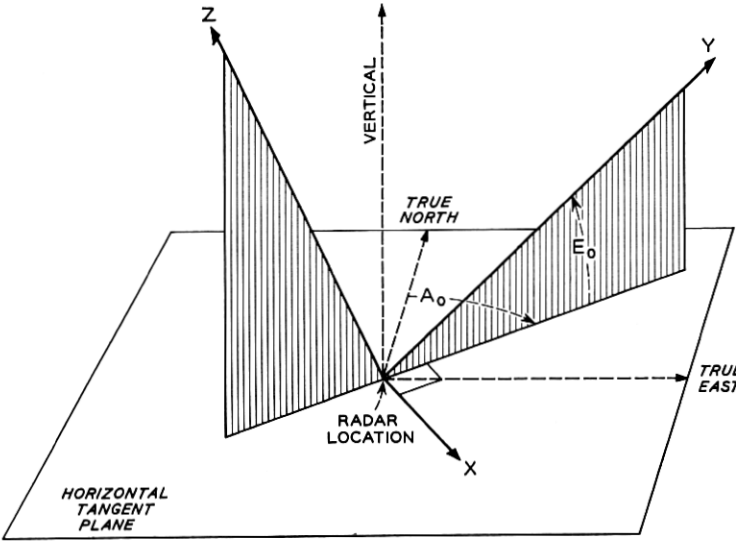

Fig. 4 — Computational coordinate system (X axis is in the horizontal plane; Y axis is at azimuth angle A_0 from true north and at elevation angle E_0 above horizontal plane; Z axis is perpendicular to X and Y axes, completing the right-hand Cartesian coordinate system).

rates in the missile, and the performance of the propulsion system. In a similar manner, pitch steering commands are based on deviations of $\dot{Z}$ from the reference velocity. The steering orders sent to the missile are related to the error signals in a way that balances loop response to missile autopilot errors, propulsion system dispersions, and radar tracking noise. The steering commands are transmitted to the missile via the radar data link at the pulse repetition rate of the radar.

IV. SECOND-STAGE GUIDANCE

4.1 *Second-Stage Cutoff*

In this section the methods used for determining the required coordinates at cutoff will be derived. Since the Telstar satellite was inserted into orbit at perigee, control of apogee distance (r_a) and inclination (i) required that the following relations be satisfied.

$$V_3 = \sqrt{\frac{2K/R_3}{1 + R_3/r_a}} \tag{2a}$$

$$\gamma_3 = 0 \tag{2b}$$

$$\beta_3 = \sin^{-1} (\cos i/\cos \varphi_3). \tag{2c}$$

As discussed in Section II, we can write

$$\alpha_K = f_K (\mathbf{V}_2, \mathbf{R}_2, \boldsymbol{\rho}_2), \qquad K = 1 \text{ to } 6 \tag{3}$$

where α_K represents any one of the six insertion coordinates. The six vector functions of (3) depend only on the transfer ellipse and the characteristics of the third stage. If $\mathbf{V}_2$, $\mathbf{R}_2$, and $\boldsymbol{\rho}_2$ are expressed in terms of the coordinate system of Fig. 4, (3) can be expanded in a Taylor series about the expected cutoff coordinates. Linearity studies on the variations in the cutoff coordinates indicate that only the first-order terms in the expansion are significant. Equation (3) simplifies to the form

$$\alpha_K - \alpha_{K_0} = \sum_{i=1}^{8} \frac{\partial \alpha_K}{\partial C_i} (C_i - C_{i_0}) \tag{4}$$

where C_i for $i = 1$ to 8 represents the 8 cutoff coordinates. The partial derivatives in (4) are obtained by perturbing cutoff coordinates and integrating numerically in a digital computer through third-stage burnout to determine the incremental changes in the insertion coordinates. Equation (2) can also be approximated by the first-order terms of a

Taylor expansion and combined with (4), giving three linear equations which can be concisely expressed by the single vector equation below.

$$A_1(\mathbf{V}_2 - \mathbf{V}_{2_0}) + A_2(\mathbf{R} - \mathbf{R}_{2_0}) + A_3(\boldsymbol{\varrho}_2 - \boldsymbol{\varrho}_{2_0}) = 0. \qquad (5)$$

A_1, A_2, and A_3 are 3×3 matrices whose (constant) elements are defined by the partial derivatives used in (4) and the expansion of (2). Equation (5) can be solved for any three of the cutoff coordinates as a function of the remaining five. The three coordinates selected are $|\mathbf{V}_2|$, the magnitude of the velocity vector at second-stage cutoff, and the unit vector $\boldsymbol{\varrho}_2$ as defined by the pitch and yaw Euler angles, θ_2 and ψ_2. The missile is cut off when the measured $|\mathbf{V}_2|$ satisfies (5). The attitude constraints on $\boldsymbol{\varrho}_2$ are met by comparing the values of θ_2 and ψ_2 required for the solution of (5) with measured values and commanding the missile to turn by the differences.

4.2 *Steering System Design*

It was shown in the previous section that if orbital elements are to be controlled, the relationships of (5) must be satisfied at second-stage cutoff. Design of the steering system is based not on minimizing the dispersions of the individual variables, $\mathbf{R}_2$, $\mathbf{V}_2$, and $\boldsymbol{\varrho}_2$ at the end of second stage, but rather on minimizing the orbital errors which are caused by errors in these variables.

The pitch and yaw steering system design consists of finding a steering transfer function which minimizes insertion errors due to radar tracking noise and missile dispersions. Missile dispersions include propulsion system variations and errors in the attitude control system of the missile.

4.2.1 *Steering System*

A block diagram of the pitch steering system is shown in Fig. 5. A similar diagram could be drawn for yaw steering.

The computer, operating on the radar data, obtains its measure of the vehicle position in the Z direction, Z_c. After steering has started, the measured trajectory variables are compared with a reference trajectory and corrective pitch turning rates, $\dot{\theta}_g$, are sent to the missile. The pitch rate programmed in the missile, $\dot{\theta}_p$, and the ordered turning rates are the inputs to the autopilot's reference integrating gyro. The function of the autopilot is to align the direction of the acceleration vector, θ, with the desired attitude, as indicated by the gyro output, θ_r.

If the missile's roll axis is aligned with the Y axis of Fig. 4, a small

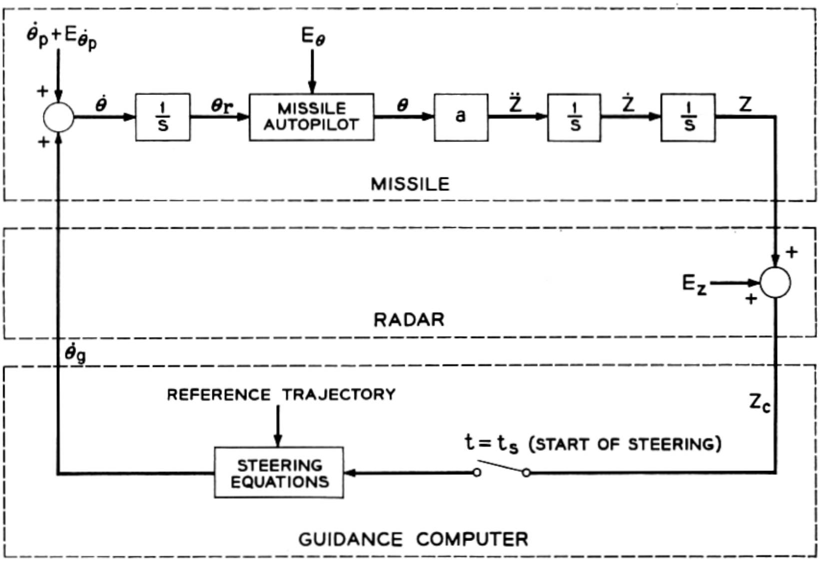

Fig. 5 — Pitch steering system (a is the missile thrust acceleration; $\dot{\theta}_p$ is the programmed turning rate; $E_{\dot{\theta}_p}$ is the error in the programmed rate; E_θ is the error in the thrust vector direction; E_Z is the radar error in measuring the position Z; Z_c is the computer's measurement of Z; and $\dot{\theta}_g$ is the guidance ordered turning rate).

change in pitch attitude, θ, multiplied by the thrust acceleration, a, is the change in Z-direction acceleration, $\ddot{Z}$. Two integrations, represented by their Laplace notation in Fig. 5, then give the vehicle position, Z.

Some of the missile error sources are gyro input errors, propulsion system errors, and misalignments between the direction of the vehicle acceleration, θ, and the reference attitude, θ_r. The gyro input errors are caused by errors in the programmed rate and by electrical or mechanical unbalances. Because the reference trajectory is determined by the programmed turning rates and the expected performance of the propulsion system, propulsion system variations can be replaced in the block diagram by equivalent programmed rate errors. All of the attitude errors, including the integrated rate errors, are caused by slowly varying disturbances.

The radar noise error in Z, E_Z, is the product of the elevation angle error and the distance from the radar to the missile. The radar noise power spectrum has most of its energy at frequencies higher than those encountered in the missile attitude disturbances—a fact important in the steering loop optimization.

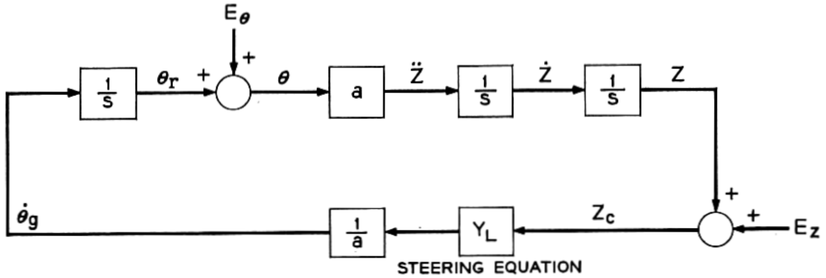

Fig. 6 — Simplified steering loop.

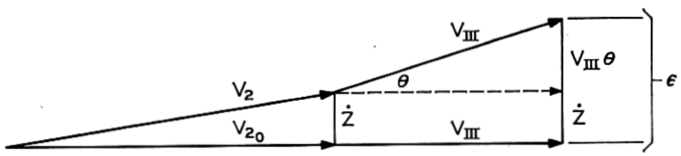

Fig. 7 — Velocity errors.

4.2.2 *Steering Loop Optimization*

In Fig. 6, the programmed pitch rate and the reference trajectory it describes have been removed from the steering loop. All of the missile dispersions are combined in a single attitude error source, E_θ. With the reference trajectory removed, θ, Z, $\dot{Z}$, and $\ddot{Z}$ represent dispersions about the reference values. The quantity to be minimized by steering may be a linear combination of some of the above variables, as will now be shown.

Fig. 7 shows the effect of θ and $\dot{Z}$ on the total insertion velocity vector of the Delta missile with its unguided third stage (assuming θ is small) to be

$$\epsilon = \dot{Z} + V_{\text{III}}\theta. \tag{6}$$

The 0 subscript in Fig. 7 indicates reference values, and V_{III} is the magnitude of the velocity increment of the third stage. In complex frequency notation

$$\epsilon(s) = \frac{a}{s}\left(1 + \frac{s}{\omega_2}\right)\theta(s) = \theta(s)\ Y_{\epsilon/\theta} \tag{7}$$

where $\omega_2 = a/V_{\text{III}}$, and in general $Y_{a/b}$ is the transfer function from b to a. The total error in terms of the attitude and noise errors can be expressed as

$$\epsilon(s) = Y_{\epsilon/E_Z}\,E_Z + \left(Y_{\epsilon/\theta} + \frac{aY_{\epsilon/E_Z}}{s^2}\right)E_\theta. \tag{8}$$

The total variance of the error, σ_ϵ^2, is the integral over all frequencies of the power spectrum of the error signal. The optimization problem is to determine the transfer function, Y_{ϵ/E_Z}, (which in turn determines the steering equation, Y_L, Fig. 6) that minimizes σ_ϵ^2. The fact that the spectral density of the attitude errors is concentrated at very low frequencies while the radar error power contains higher frequency components suggests that the error may be minimized by a frequency-selective steering equation.

The attitude errors can be approximated by a Markov power spectrum, having a low cutoff frequency,

$$P_{E_\theta}(\omega) = \frac{2\sigma_{E_\theta}^2 \omega_\theta}{\omega^2 + \omega_\theta^2}, \tag{9}$$

where ω_θ is very small. Using techniques described by Bode and Shannon[1] for minimizing the mean squared error, the optimum steering equation is:

$$Y_L = \frac{-s\left[1 + \left(2T + \dfrac{1}{\omega_2}\right)s\right]}{2T^2 + \dfrac{2T}{\omega_2} + \left(T^3 + \dfrac{2T^2}{\omega_2}\right)s + \dfrac{T^3}{\omega_2}s^2} \tag{10}$$

where

$$T = \left(\frac{\sigma_{E_Z}^2}{a^2 \sigma_{E_\theta}^2 \omega_\theta \omega_E}\right)^{1/6}. \tag{11}$$

For the Delta second stage, $1/\omega_2$ equals V_{III}/a. If typical numbers for V_{III}, a, and T are inserted, then $1/\omega_2 \gg T$, which leads to the approximation:

$$Y_{L_{II}} = \frac{-s^2}{2T\left(1 + Ts + \dfrac{T^2}{2}s^2\right)}. \tag{12}$$

The two differentiations of the position data indicated in (12), together with the division by a, Fig. 6, convert the position data to attitude data which is in turn smoothed to give the ordered turning rates.

In the guidance equations the pitch and yaw attitude of the missile is determined by operating on the position data of the missile. The total acceleration of the missile is computed by taking second differences of the position data expressed in the coordinate system of Fig. 4. Gravity, Coriolis and centripetal accelerations are subtracted from the total acceleration to obtain the thrust component of acceleration. Since the thrust acceleration vector is aligned with the missile's roll axis and the missile is roll stabilized, the pitch and yaw attitude angles can be computed. These are compared with the desired values to obtain pitch and yaw attitude errors. The desired attitude angles are computed from polynomials in time designed to match the programmed rates built into the missile's attitude control system.

In this paper, all transfer functions have been given in continuous form (as functions of s), although the transfer functions are actually converted to digital form (functions of $z = e^{sT}$) before being programmed into the digital computer. This conversion does not significantly change any of the relations given here, because the frequency ranges important in the above equations are much less than half of the sampling frequency.

4.2.3 *Accuracy Improvement by Final Value Control*

Equation (12) is the optimum steering equation for minimizing the mean squared error. However, the injection errors are functions of the error at one instant of time, second-stage cutoff. Examination of the system's response suggests means of improving the steering design.

The slowly varying attitude disturbances represented by a Markov distribution may also be represented by initial attitude and velocity errors and a constant gyro drift. The system's response to these errors, derived from (12), is shown in Fig. 8 as a function of time normalized with respect to the steering parameter T.

Since the Delta design is not restricted to a continuous control system, velocity errors at second-stage cutoff in excess of those obtained with the optimum continuous design can be allowed. These velocity dispersions can be measured and used to aim the third stage in accordance with (5). The velocity errors shown in Fig. 8 are either constant or linearly increasing with time and can be measured by filtering the position data. The steady-state attitude error response to gyro drift is a constant. If gyro drift is a major attitude error source, continuous closed loop control of attitude may also be relaxed, and the constant attitude error can be measured with a polynomial filter.[2] The errors can be corrected just prior to cutoff by turning the missile. The reason for relaxing the attitude and velocity dynamic control [making T greater than the "optimum" value in (11)] is that doing so decreases the attitude error caused by the effect of radar noise on the steering system.

The outputs of the attitude and velocity filters used to measure the dynamic residuals also have errors caused by the tracking noise. These decrease as the filter length, T_F, is increased. Since the attitude error signals involve second derivatives of tracking data, the attitude filters require long smoothing times. However, if the filters are started too

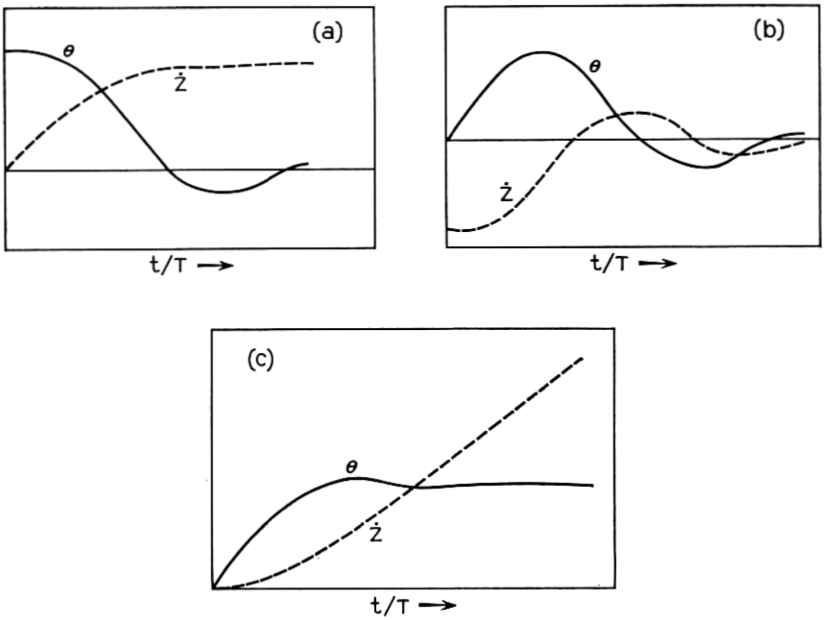

Fig. 8 — Dynamic steering response: (a) response to initial attitude error; (b) response to initial velocity error; and (c) response to gyro drift.

early in the second stage, they are subject to increased dynamic error because the steering system has not yet settled to the steady state (Fig. 8). Thus, optimum filter design is dependent on the steering parameter T.

For the Delta missile, the critical design criterion was that of attitude control. That is, for a system with optimum attitude control, the optimum velocity filter has negligible error, and the measured dispersions can be used as a basis for an attitude correction (aiming the third stage).

For given dynamic error sources and filter length, the total attitude error, after correction of the measured error, is the combination of the filter's dynamic error and the noise error of the filter plus the steering noise error, Fig. 9. The steering system's noise error is a function of T and decreases as T increases. The filter's noise error is independent of T but decreases as the filter length, T_F, increases. The filter's dynamic error is a function of both T_F and the transient response of the steering system as determined by T. Because the steering and filter noise errors are highly correlated, they are added together directly and root sum squared with the filter's dynamic error. T and T_F are chosen from Fig. 9 to minimize the total attitude error.

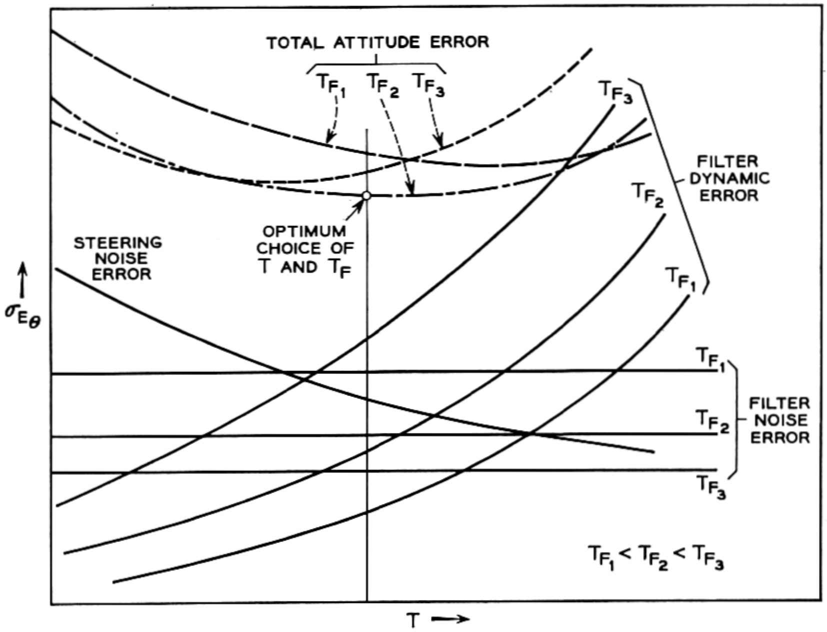

Fig. 9 — Final attitude errors as functions of T and T_f .

In summary, the Delta second-stage guidance equations give closed-loop control of missile attitude. Relatively large dynamic errors in velocity and attitude are permitted in order to decrease noise errors, because the dynamic errors can be measured and corrected. An attitude order to correct the measured attitude error and compensate for velocity and position dispersions in accordance with (5) is sent to the missile shortly before second-stage cutoff.

V. *TELSTAR* I SATELLITE FLIGHT RESULTS

The steering history of the first and second stage of the Telstar I satellite flight is given in Figs. 10 and 11. The maximum steering orders during first-stage guidance were 3.8 degrees in pitch and 1.6 degrees in yaw. Steering in second-stage guidance reached a peak of 0.25 degree in pitch and 1 degree in yaw. Corrective commands of 0.35 and 0.13 degree were issued in pitch and yaw, respectively, just prior to second-stage cutoff. The second-stage engine was cut off at 269.6 seconds from liftoff, as compared to a preflight value based on average propulsion performance of 273.3 seconds. The orbital results as determined by NASA are given in Table I with the preflight reference values.

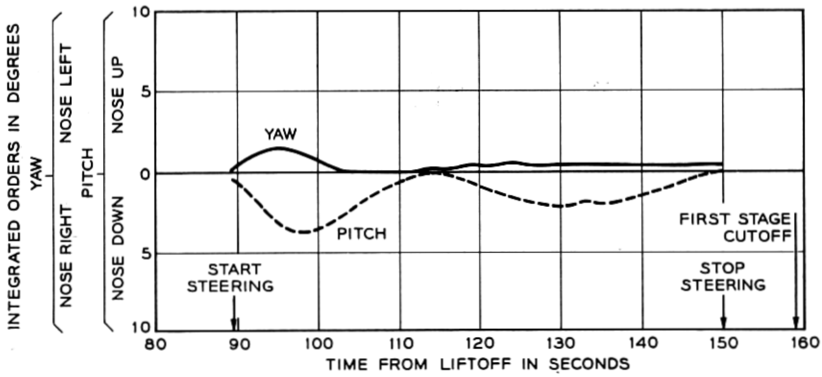

Fig. 10 — First-stage steering.

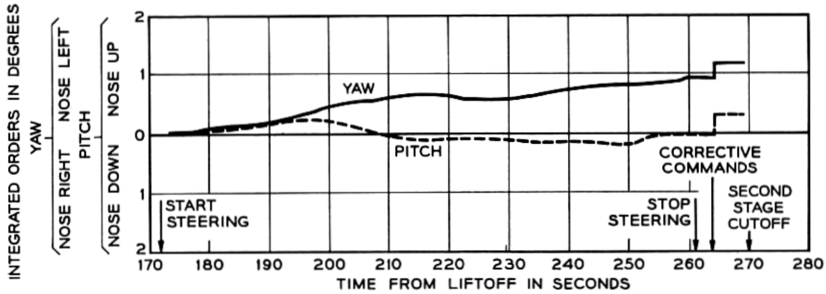

Fig. 11 — Second-stage steering.

TABLE I — ORBITAL RESULTS

	Preflight Reference	NASA Minitrack
Altitude		
Apogee (nm)	3000	3044
Perigee (nm)	500	515
Eccentricity	0.2407	0.242
Period (minutes)	156.47	157.82
Inclination (degrees)	45	44.79
Argument of perigee (degrees)	166.6	165
Right ascension of ascending node (degrees)	203.6	204

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