The Fabry-Perot Electrooptic Modulator

by E. I. GORDON and J. D. RIGDEN

(Manuscript received October 10, 1962)

The Fabry-Perot modulator, consisting of Fabry-Perot etalon plates separated by an electrooptic material such as KDP, is analyzed in detail. Time-dependent perturbation theory is used to describe the coupling of the axial modes by spatial and time varying perturbations in the dielectric constant. The perturbations are produced by the applied microwave modulating field. It is shown that the correct choice of the spatial variation of the microwave modulating field is essential to achieve efficient modulation and the choice is equivalent to matching the phase velocities of the microwaves and the light.

Power requirements, heating, and bandwidth are discussed and a comparison is made to the traveling-wave modulator described by Kaminow.

Calculations indicate that bandwidths of several hundred megacycles, centered at any microwave frequency, can be obtained with the expenditure of several watts of modulating power.

I. INTRODUCTION

Light modulators, operating at microwave frequencies, have been receiving considerable attention. Recently, Kaminow,[1] using the Pockel's effect in KDP, produced usable amounts of modulation at an X-band frequency. The modulator was operated on a pulse basis because of the large power dissipated per unit volume in the KDP. One solution for the problem of heat dissipation as discussed by Kaminow[1] and others,[2] requires careful matching of the phase velocity of the microwaves to the light velocity in the electrooptic material, and calls for the construction of a long modulator. The resulting large coherence volume decreases the power dissipation to a value low enough to allow continuous operation.

The mechanical difficulties, as well as the transmission loss associated with long rods, among other problems, has made the realization of such low power modulators impractical. On the other hand, the advent of sources of monochromatic light energy, such as the optical maser, provides alternatives not available with conventional light sources.

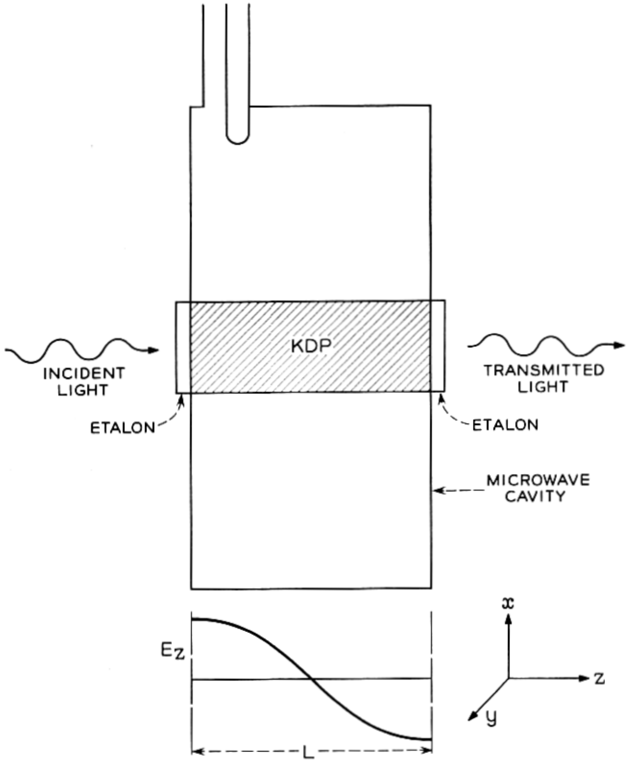

Fig. 1 — Schematic of Fabry-Perot modulator.

We refer, in particular, to resonance or cavity techniques analogous to those employed at microwave frequencies. Here, the coherence volume of the material is made effectively larger by virtue of the multiple reflections within the resonant cavity. The particular technique which we wish to describe is the use of a Fabry-Perot etalon (FPE) in which is included an electrooptic material, as shown in Fig. 1. Although the bandwidth over which modulation can be accomplished is limited by the use of an optical cavity, a bandwidth of several hundred megacycles centered at any microwave frequency should be readily attainable.

Before proceeding with an analysis of the modulator it is worthwhile to consider some simple points of view which indicate the correctness of this approach. The relative transmission I_t through a lossless FPE can be written[3]

$$I_t = [1 + 4\Gamma^2(1 - \Gamma^2)^{-2} \sin^2 \varphi]^{-1} \tag{1}$$

in which Γ^2 is the power reflectivity of the reflectors and φ is the phase shift for a single pass. The phase shift, for normal incidence, can be written $\varphi = 2\pi f L\mu/c + \theta$, in which f is the optical frequency, L is the etalon spacing, c is the vacuum light velocity, μ is the index of refraction and θ is the phase shift upon reflection.[3] It should be possible to construct a modulator in which L, θ or μ is varied, thereby varying φ and modulating the transmitted intensity. We shall be concerned with variations in μ through the electrooptic effect. In particular we will be interested only in phase shifts associated with volume effects rather than the surface effects associated with variations in reflectivity. The latter are many orders of magnitude less significant.

In the absence of modulation, the transmission characteristic as a function of frequency is given by (1) and is shown in Fig. 2. The transmission peaks occur at frequencies $f_a = ac/2L\mu$ for which $\varphi = a\pi$. The integer a is the number of half wavelengths in the cavity. The frequency difference between adjacent peaks is $c/2L\mu$. For the purpose of discussion assume that the frequency of the incident light corresponds to one of the transmission peaks. Assume also that the index of refraction has a varying component that takes the form of a cosinusoidal standing wave as shown in Fig. 3(a), with a frequency $\omega_m/2\pi = c/2L\mu$ and wavelength $2L$. The variation is equivalent to oppositely traveling waves with phase velocity equal to the light velocity, c/μ. With a peak variation $\delta\mu \cos \omega_m t$, the amplitude of each wave is $\frac{1}{2}\delta\mu$. Under these circumstances, the perturbed phase shift travels in synchronism with the light producing a cumulative phase shift,

$$\delta\varphi = \pi f_a L c^{-1} \delta\mu \cos \omega_m t = \tfrac{1}{2} a\pi(\delta\mu/\mu) \cos \omega_m t.$$

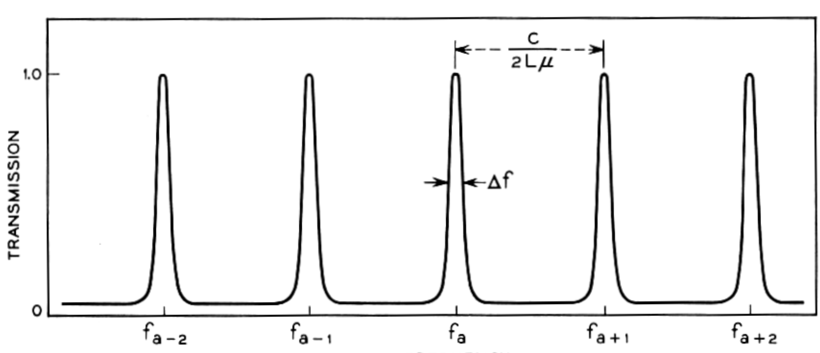

Fig. 2 — Transmission vs frequency for a Fabry-Perot etalon.

Since $\delta\mu/\mu$ is of order 10^{-6} for practical cases, and $a\pi$ is of order 10^5, it follows that $\delta\varphi < 10^{-1}$ and (1) can be written

$$I_t \approx [1 + \Gamma^2(1 - \Gamma^2)^{-2}(a\pi\delta\mu/\mu)^2 \cos^2 \omega_m t]^{-1}. \tag{2}$$

Choosing the reflectivity $\Gamma^2 = 0.9$ indicates that the coefficient of $\cos^2 \omega_m t$ can easily achieve the value unity, leading to large intensity modulation. Note that the modulation rate is twice the modulating frequency, which is consistent with the fact that the transmitted intensity is an even function of the induced phase shift.

From (1) the separation of half-power frequencies or pass band of the FPE, shown as Δf in Fig. 1, is given by

$$\Delta f \approx f_a(1 - \Gamma^2)/a\pi\Gamma. \tag{3}$$

For a one centimeter long etalon, with $\Gamma^2 = 0.9$, and $c/2L\mu = f_a/a = 10^{10}$, $\Delta f \approx 300\ mc/s$. One wonders whether this represents a limiting rate at which the light can be modulated, since the pass band Δf represents the maximum rate at which the stored energy in the cavity can be varied. When the perturbation in dielectric constant is uniform across the etalon, it will be shown that the maximum possible modulation rate is of order $\Delta f/2$. However, when the phase velocities are matched as described above, there is no limiting modulation rate.

We can obtain an intuitive appreciation of the lack of a limiting modulation rate by considering Fig. 3. The perturbed part of the dielectric constant ϵ, shown as $\delta\epsilon$ in Fig. 3(a), is depicted as a standing wave. This standing wave corresponds to the microwave modulating electric field which is assumed to have a field component normal to the reflectors. This component produces the variation $\delta\epsilon$. The optical field, excited by the incident light, is shown as the standing wave, E_a in Fig. 3(b). In addition to the displacement current, $\epsilon\partial E_a/\partial t$, there will be a perturbed displacement current, $\partial\delta\epsilon E_a/\partial t$, shown in Fig. 3(c). It can be seen that the perturbed displacement current has the appropriate spatial distribution to excite the modes $E_{a\pm1}$ shown in Fig. 3(d) and 3(e).

The microwave modulating frequency $f_m = \omega_m/2\pi$ is chosen to have the value $c/2L\mu$, so that the sum and difference frequencies appearing in the time dependence of the perturbed part of the displacement current correspond to the frequencies, $f_{a\pm1}$. As a result, components of the perturbed part of the displacement current have the proper spatial variation and frequency to excite the sideband modes. The transmitted light will then have upper and lower sideband frequencies which result from the oscillating sideband modes.

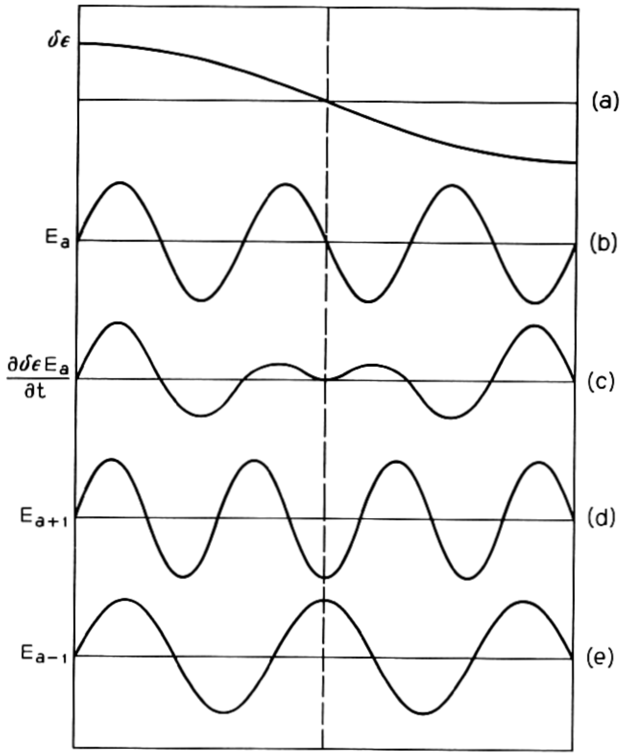

Fig. 3 — Mode coupling resulting from perturbations in the dielectric constant.

From the point of view of coupling of modes, there is no inherent limitation to the rate at which modulation can be accomplished, except that afforded by the electrooptic effect itself. However, the range over which the microwave modulating frequency can be varied must correspond to the bandwidth of the FPE modes, otherwise the sideband modes cannot be excited strongly.

The conditions under which modulation is possible can be summarized by the equations

$$\begin{aligned} f_{a\pm m} &= f_a \pm f_m \\ \beta_{a\pm m} &= \beta_a \pm \beta_m \end{aligned} \tag{4}$$

in which β_n is the propagation constant of the nth mode, in this case f_m and β_m are also the microwave frequency and propagation constant. Equation (4) is recognizable as the Tien $\omega - \beta^4$ relation for reactive frequency mixing. Equation (4) also implies that the microwave phase velocity equals the light phase velocity, assuming negligible dispersion.

It is not clear yet that the variation in transmitted intensity given by (2), which follows from relatively simple considerations, is consistent with the coupled mode picture. The next sections will be devoted to an analysis of the modulator using time-dependent perturbation theory to determine the transmitted field amplitudes and phases. The variation in transmitted intensity will be computed and shown to be consistent with (2). Other modes of operation will be described which enhance the modulation efficiency and which can yield linear intensity modulation at the microwave frequency. Power requirements and heating will be considered for an X-band modulator for light at 6328 Å.† Some of the optical requirements of the Fabry-Perot cavity will be discussed with respect to available maser beams. Finally a comparison will be made to the traveling-wave modulator described by Kaminow.

II. THEORY

The analytical description will be similar to that used in time-dependent perturbation theory. The optical fields within the FPE will be expanded in a set of eigenfunctions appropriate to perfectly reflecting boundaries. It will be seen that the expansion coefficients are time dependent and satisfy "harmonic-oscillator-like" linear differential equations which are coupled by the perturbed part of the dielectric constant. The mixing or coupling terms involve integrals over the coupled modes and the perturbed part of the dielectric constant.

The microwave fields will not explicitly enter into the calculation; rather they will serve only to produce the perturbed part of the dielectric constant. Although the dielectric constant in electrooptic materials often exhibits tensor properties we will suppress this feature since the notation would be burdened without any real gain in generality.

The basis of the calculation will be the normal mode formulation given by Slater.[6] In the absence of the perturbation in the dielectric constant the electric and magnetic fields within the FPE are divergence-free. Thus, we will use only solenoidal eigenfunctions which are defined in the volume V appropriate to the FPE by the equations

$$k_a \mathbf{E}_a(\mathbf{r}) = \operatorname{curl} \mathbf{H}_a(\mathbf{r}), \qquad k_a \mathbf{H}_a(\mathbf{r}) = \operatorname{curl} \mathbf{E}_a(\mathbf{r}) \tag{5}$$

in which k_a is the eigenvalue of the ath mode. A consistent set of boundary conditions on $\mathbf{E}_a$ and $\mathbf{H}_a$ are taken to be

† As the modulation efficiency increases with decreasing wavelength, we have used the wavelength of the presently shortest wavelength maser.[5]

$$\begin{aligned} \mathbf{n} \times \mathbf{E}_a &= 0, \quad \mathbf{n}\cdot\mathbf{H}_a = 0 \quad \text{on} \quad S_s \\ \mathbf{n} \times \mathbf{H}_a &= 0, \quad \mathbf{n}\cdot\mathbf{E}_a = 0 \quad \text{on} \quad S_o \end{aligned} \tag{6}$$

in which $\mathbf{n}$ is a unit vector normal to the outer surface of the volume V. The surface S_s corresponds to the short circuit boundary while S_o is the open circuit boundary; together they represent the total surface of the volume V. Under these conditions, Slater shows that the functions $\mathbf{E}_a$ and $\mathbf{H}_a$ represent an orthogonal normalized set satisfying the equations

$$\nabla^2\mathbf{E}_a + k_a^2\mathbf{E}_a = 0 \quad \text{and} \quad \nabla^2\mathbf{H}_a + k_a^2\mathbf{H}_a = 0.$$

The orthogonality relations take the form

$$\int_V \mathbf{E}_a\cdot\mathbf{E}_b dV = \int_V \mathbf{H}_a\cdot\mathbf{H}_b dV = \delta_{ab}$$

The starting point for the analysis is the Maxwell equations

$$\begin{aligned} \text{curl } \mathbf{E} &= -\mu_0 \partial\mathbf{H}/\partial t \\ \text{curl } \mathbf{H} &= \partial\epsilon'\mathbf{E}/\partial t + \sigma\mathbf{E} \end{aligned} \tag{7}$$

in which σ is the volume conductivity associated with the transmission loss of the electrooptic material, μ_0 is the vacuum permeability and ϵ' is the permittivity. As shown by Slater the various terms in (7) have expansions of the form

$$\begin{aligned} \mathbf{E}(\mathbf{r}, t) &= \sum_a e_a(t)\mathbf{E}_a(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}, t) &= \sum_a h_a(t)\mathbf{H}_a(\mathbf{r}) \\ \text{curl } \mathbf{E} &= \sum_a \mathbf{H}_a \left[k_a e_a + \int_{S_s} (\mathbf{n} \times \mathbf{E})\cdot\mathbf{H}_a dS \right] \\ \text{curl } \mathbf{H} &= \sum_a \mathbf{E}_a \left[k_a h_a + \int_{S_o} (\mathbf{n} \times \mathbf{H})\cdot\mathbf{E}_a dS \right] \end{aligned} \tag{8}$$

in which dS is an element of the surface. The mode amplitudes e_a and h_a are defined by

$$\begin{aligned} e_a(t) &= \int_V \mathbf{E}(\mathbf{r}, t)\cdot\mathbf{E}_a dV \\ h_a(t) &= \int_V \mathbf{H}(\mathbf{r}, t)\cdot\mathbf{H}_a dV \end{aligned} \tag{9}$$

in which dV is a volume element. Substituting the relations in (8) into

(7) and using the orthogonality and normality of the functions $\mathbf{E}_a$ and $\mathbf{H}_a$ yields equations for the coefficients of the form

$$\mu_0 \partial h_a/\partial t + k_a e_a = -\int_{S_s} (\mathbf{n} \times \mathbf{E}) \cdot \mathbf{H}_a dS \tag{10}$$

and

$$\epsilon \partial e_a/\partial t + \sigma e_a - k_a h_a - \int_{S_0} (\mathbf{n} \times \mathbf{H}) \cdot \mathbf{E}_a dS + \partial \sum_b \langle a \mid \delta\epsilon \mid b \rangle e_b/\partial t = 0 \tag{11}$$

in which we have written $\epsilon' = \epsilon + \delta\epsilon$, ϵ being the unperturbed part of the dielectric constant, and we have used the notation

$$\langle a \mid f(\mathbf{r}) \mid b \rangle = \int_V \mathbf{E}_a \cdot \mathbf{E}_b f(\mathbf{r}) dV. \tag{12}$$

Iterating (10) and (11) yields

$$c'^{-2}\partial^2 e_a/\partial t^2 + k_a^2 e_a + \mu_0 \sigma \partial e_a/\partial t + c'^{-2}\partial^2 \sum_b \langle a \mid \delta\epsilon/\epsilon \mid b \rangle e_b/\partial t^2 = \mu_0 \partial \left(\int_{S_0} (\mathbf{n} \times \mathbf{H}) \cdot \mathbf{E}_a dS \right) \Big/ \partial t - k_a \int_{S_s} (\mathbf{n} \times \mathbf{E}) \cdot \mathbf{H}_a dS \tag{13}$$

in which $c' = (\mu_0 \epsilon)^{-\frac{1}{2}}$ is the unperturbed velocity of light in the dielectric. A similar redundant equation obtains for h_a. The surface integrals contain two types of field terms: (i) external optical fields incident on the cavity as they appear on the inside boundary wall of the reflector, which serve to excite the FPE modes, and (ii) internal fields which account for energy lost by radiation, in particular the transmitted beam.

In the following treatment of the modulator we will assume that the reflecting surfaces of the etalon can be characterized by a reflection coefficient for field amplitude Γ and that the surface is a "short," i.e., a surface S_s. The fields will be assumed to be parallel to the etalon which is of sufficient lateral extent that the axial modes of the FPE are essentially TEM waves with no diffraction loss. The dielectric modulation is assumed to be uniform across the cross section. Later we will consider nonuniform excitation of the dielectric in the finite cross section, and it will be seen that the lowest-order axial modes can be coupled to the off-axis higher-order modes.

Under these assumptions, the electric and magnetic field on the inside face of the reflectors constituting the surface S_s can be shown to be related by

$$\mathbf{n} \times \mathbf{E} = c'\mu_0 \mathbf{H}(1 - \Gamma)/(1 + \Gamma) \tag{14}$$

neglecting the extremely small variation in Γ produced by the variation of the dielectric constant within the etalon. First we consider the loss terms in the cavity. For the right-hand side of (13) we may write, using (14) and (8)

$$\begin{aligned} -k_a \int_{S_s} (\mathbf{n} \times \mathbf{E}) \cdot \mathbf{H}_a dS \\ = -k_a c' \mu_0 (1 - \Gamma)(1 + \Gamma)^{-1} \sum_b h_b \int_{S_s} \mathbf{H}_b \cdot \mathbf{H}_a dS \end{aligned} \tag{15}$$

there being no surface S_o over which $\mathbf{n} \times \mathbf{H} \cdot \mathbf{E}_a$ is nonzero. Substituting (11) into (15) to eliminate the term in h_b yields

$$\begin{aligned} -k_a \int_{S_s} (\mathbf{n} \times \mathbf{E}) \cdot \mathbf{H}_a dS = -k_a c'^{-1} (1 - \Gamma)(1 + \Gamma)^{-1} \\ \times \sum_b k_b^{-1} \left[\partial e_b/\partial t + \sigma e_b/\epsilon + \partial \sum_d \langle b \mid \delta\epsilon/\epsilon \mid d \rangle e_d/\partial t \right] \\ \int_{S_s} \mathbf{H}_b \cdot \mathbf{H}_a dS. \end{aligned} \tag{16}$$

For the axial modes of the FPE, the appropriate normalized eigenfunctions are given by

$$\begin{aligned} H_a &= (2/AL)^{\frac{1}{2}} \cos k_a z \\ E_a &= (2/AL)^{\frac{1}{2}} \sin k_a z \end{aligned} \tag{17}$$

in which A is the cross-sectional area, L the length of the modulator, and $k_a = \pi a/L$. The integer a is the number of half wavelengths within the etalon. Remembering that there are two faces of the etalon composing the surface S_s, we note that the integral

$$\int_{S_s} \mathbf{H}_b \cdot \mathbf{H}_a dS = 4/L \tag{18}$$

when b differs from a by an even number, while the integral has the value zero when b differs from a by an odd number. The implication is that the surface loss terms couple only modes of the same longitudinal symmetry. The integral is zero when H_b is one of the off-axis FPE modes because of the orthogonality. Defining

$$Q_a = \tfrac{1}{4}\pi a(1 + \Gamma)^2/(1 - \Gamma^2) \tag{19}$$

(16) becomes

$$\begin{aligned} -k_a \int_{S_s} (\mathbf{n} \times \mathbf{E})\cdot\mathbf{H}_a dS \\ = -k_a c'^{-1} \sum_b{}' Q_b^{-1} \left[\partial e_b/\partial t + \sigma e_b/\epsilon + \partial \sum_d \langle b \mid \delta\epsilon/\epsilon \mid d\rangle e_d/\partial t\right] \end{aligned} \tag{20}$$

in which the prime implies that only those values of b which differ from a by an even number are to be included in the sum. The third term in the square bracket of (20) is of order $\delta\epsilon/\epsilon$ compared to first. The second term is of order $\sigma/\epsilon\omega_a \sim 10^{-6}$ compared to the first. Since the first term is as small as any other term in (13) because of the factor $Q_b^{-1} \sim 10^{-6}$, we can safely neglect the other terms. In the absence of internal losses the term Q_a is known as the quality factor of the cavity. It defines the resolution of the FPE as an interferometer and defines the bandwidth by the relation $\Delta f \approx f_a/Q_a$ which differs from the bandwidth defined in (3) by the factor $4\Gamma/(1 + \Gamma)^2 \approx 1$.†

Considering the incident wave amplitude as transformed to the inside surface of the reflector it can be shown in similar fashion‡ that

$$-k_a \int_{S_s} (\mathbf{n} \times \mathbf{E})\cdot\mathbf{H}_a dS = -\tfrac{1}{2}k_a c'^{-1} Q_a^{-1} \partial e/\partial t \tag{21}$$

in which e is proportional to the magnetic field of the incident wave on the surface of the reflector. The factor $\tfrac{1}{2}$ appears because the incident fields appear only on one face. Thus (13) becomes

$$\begin{aligned} \partial^2 e_a/\partial t^2 + \omega_a^2 e_a + \omega_a Q_{ad}^{-1} \partial e_a/\partial t \\ + \omega_a \sum_b{}' Q_b^{-1} \partial e_b/\partial t + \partial^2 \sum_b \langle a \mid \delta\epsilon/\epsilon \mid b\rangle\, e_b/\partial t^2 \\ = -\tfrac{1}{2}\omega_a Q_a^{-1} \partial e(\omega t)/\partial t \end{aligned} \tag{22}$$

in which $\omega_a = k_a c'$ is the resonant frequency of mode a and $Q_{ad} = \omega_a \epsilon/\sigma$ is the dielectric quality factor or inverse loss tangent. An alternate expression can be written $Q_{ad} \approx \omega_a \ell/c'$ in which ℓ is the distance for which the energy decays to e^{-1}.

Next, we evaluate the quantities $\langle a \mid (\delta\epsilon/\epsilon) \mid b\rangle$. Since the spatial variation of $\delta\epsilon/\epsilon$ can take the form of $\sin m\pi z/L$ or $\cos m\pi z/L$ in which m is some integer, which corresponds to the appropriate microwave

† The factor $4\Gamma/(1 + \Gamma)^2$ differs from unity by less than one per cent for $\Gamma^2 > 0.7$.

‡ The mirror acts as a transformer with turns ratio $(1 + \Gamma)^{\frac{1}{2}}/(1 - \Gamma)^{\frac{1}{2}}$.

field variation, we evaluate two types of coupling terms:

$$\left\langle a \left| \begin{matrix} \sin \pi mz/L \\ \cos \pi mz/L \end{matrix} \right| b \right\rangle = (2/L) \int_0^L \left(\sin \pi az/L \begin{matrix} \sin \pi mz/L \\ \cos \pi mz/L \end{matrix} \sin \pi bz/L \right) dz \tag{23}$$

in which we have used (17). Performing the integration yields

$$\langle a \mid (\sin \pi mz/L) \mid b \rangle = \frac{1}{2}\left[\frac{[1 - \cos (a - b + m)\pi]}{(a - b + m)\pi} + \frac{[1 - \cos (b - a + m)\pi]}{(b - a + m)\pi} + \frac{[1 - \cos (a + b - m)\pi]}{(a + b - m)\pi} - \frac{[1 - \cos (a + b + m)\pi]}{(a + b + m)\pi} \right]. \tag{24}$$

Terms like $(a + b \pm m)^{-1}$ will be of order 10^{-5} and the last two terms can be neglected. When $a - b \pm m = 0$ or an even integer the remaining terms vanish. When $a - b \pm m$ is an odd integer the terms are finite, yielding

$$\langle a \mid \sin \frac{m\pi z}{L} \mid b \rangle = -\frac{2m/\pi}{(a - b)^2 - m^2}, \qquad a - b \pm m = \text{odd integer} \tag{25}$$

the largest values corresponding to $a = b$, $m = 1$ with a value $2\pi^{-1}$. Likewise,

$$\langle a \mid \cos \frac{m\pi z}{L} \mid b \rangle = \frac{1}{2}\left[-\frac{\sin (a + b - m)\pi}{(a + b - m)\pi} + \frac{\sin (a - b - m)\pi}{(a - b - m)\pi} + \frac{\sin (a - b + m)\pi}{(a - b + m)\pi} - \frac{\sin (a + b + m)\pi}{(a + b + m)\pi} \right] = \tfrac{1}{2} \quad \text{for} \quad |a - b| = m. \tag{26}$$

The sine and cosine cases are mutually exclusive. Many modes are coupled by the sine variation since for a given value of a and m there are multiple values of b satisfying (25). There are only two values of b satisfying (26) for the cosine variation. Note that for $m = 0$, corresponding to uniform perturbation of the dielectric, the only coupling is for $a = b$. That is, the modulation can occur only within the mode. This justifies our earlier statement that the modulation rate is restricted to the bandwidth of the FPE mode for uniform excitation.

The cosine distribution yields the largest coupling terms except for the case when $a = b$ and $m = 1$. The case $m = 1$ implies that the sine variation has a space average part and it is this part which produces the coupling for $a = b$. This is borne out by the fact that for $a = b$, m must be odd and the strength of the coupling decreases with increasing m. Notice that even though the modes of interest are standing waves these results correspond precisely to that predicted by the β condition of (4). The factor of $\frac{1}{2}$ for the cosine distribution arises because only half the amplitude of the standing wave is effective for a given traveling-wave component. The factor $2\pi^{-1}$ for the sine variation can be shown to arise from the fact that although the β condition is not precisely satisfied, the interaction with each traveling-wave component is down by π^{-1}.

In the next sections, we apply these results to calculate the amplitude of the modulation sidebands, their bandwidth, and the intensity modulation of the light.

III. AMPLITUDE OF THE MODULATION SIDEBANDS

Assuming that we have used the appropriate cos $m\pi z/L$ distribution for that component of the microwave field which varies ϵ', since we want large coupling for $a \neq b$, we can write (22) as

$$\begin{aligned}\frac{\partial^2}{\partial t^2} e_a + \omega_a^{\,2} e_a + \omega_a Q_{ad}^{\;-1} \frac{\partial e_a}{\partial t} + \frac{1}{2}\frac{\partial^2}{\partial t^2}\frac{\delta\epsilon(\omega_m t)}{\epsilon}\,[e_{a+m} + e_{a-m}] \\ + \omega_a \sum_b{}' Q_b^{-1}\frac{\partial e_b}{\partial t} = -\tfrac{1}{2}\omega_a Q_a^{\;-1}\frac{\partial e(\omega t)}{\partial t}\end{aligned} \qquad (27)$$

which holds for all values of a.† We note that modes e_a and $e_{a\pm m}$ are coupled together. Since the Q of the modulator is so extremely high, it is approximately correct to say that any driving terms at frequencies outside the pass band of a given mode have negligible effect on that mode. Later we will see that for very strong modulation (a case of little practical interest) the high-Q approximation leaves a little to be desired. Hence, to good approximation, we may write for any of the modes a,

$$\sum_b{}' Q_b^{\;-1}\partial e_b/\partial t \simeq Q_a^{\;-1}\partial e_a/\partial t.$$

All the neglected terms are at frequencies well outside the passband of mode a. With these definitions, the quantity $1/Q_{aL} = 1/Q_a + 1/Q_{ad}$

† It is also possible to have $| a - b | = 1$, $m = 2$ yielding a coupling term $4/3\pi$ which is only slightly less than $\frac{1}{2}$. We will not consider this case which is considerably more complicated.

is properly called the loaded Q of the resonator. The bandpass in the absence of mode coupling is defined by $\Delta\omega = \omega_a/Q_{aL}$. The quantity $2Q_a$ is also properly called the external Q.

We will assume that the carrier frequency corresponds to the resonant frequency of the ath mode, $\omega = k_a c' = \omega_a$. There is no loss of generality in this assumption since the carrier frequency would normally be fixed. The microwave modulating frequency, on the other hand, must vary over some frequency range. Ideally the microwave frequency has the value $mc'/2L$, since under this circumstance the mixing terms, represented by the fourth term of (27), vary at rates corresponding to the resonant frequencies of the coupled modes. However the microwave frequency must be able to vary over some frequency range. We define the microwave angular frequency as

$$\omega_m = (m\pi c'/L) + \omega_a\delta/Q_{aL} \tag{28}$$

and the parameter δ measures the excursion from center band frequency. In view of the fact that $\omega_a/Q_{aL} \simeq \Delta\omega_a$, the half power frequencies are given approximately by $\delta = \pm\frac{1}{2}$. The $\pm n$th sideband frequencies are given by

$$\omega_a \pm n\omega_m = \omega_a \pm nm\pi c'/L \pm n\delta\omega_a/Q_{aL} = \omega_{a\pm nm} \pm n\delta\omega_a/Q_{aL}. \tag{29}$$

The pertinent mode amplitudes are defined by

$$\begin{aligned} e_{a\pm nm} &= E_{a\pm nm} \exp i(\omega_a \pm n\omega_m)t + \text{complex conjugate} \\ \delta\epsilon/\epsilon &= [\delta\epsilon/\epsilon] \exp i\omega_m t + \text{complex conjugate} \\ e &= E \exp i\omega_a t + \text{complex conjugate} \end{aligned} \tag{30}$$

which when substituted into (27) yield

$$-[\chi^* E_{a+m} + \chi E_{a-m}] + iE_a = -\tfrac{1}{2}iEQ_{aL}/Q_a \tag{31}$$

$$2n\delta E_{a-nm} - [\chi^* E_{a-(n-1)m} + \chi E_{a-(n+1)m}] + iE_{a-nm} = 0 \tag{32}$$

$$-2n\delta E_{a+mn} - [\chi^* E_{a+(n+1)m} + \chi E_{a+(n-1)m}] + iE_{a+nm} = 0. \tag{33}$$

The coefficients are complex quantities and the asterisk denotes complex conjugate. The modulus of the coefficients equals half the peak value and the argument represents the phase of the related real variable. The quantity

$$\chi = \tfrac{1}{2}Q_{aL}[\delta\epsilon/\epsilon] \tag{34}$$

is a "modulation index" whose modulus equals the product of Q_{aL} and one quarter of the peak variation of $\delta\epsilon/\epsilon$. The argument of χ equals the phase of the microwave field. Note that we have set all the terms $Q_{a\pm nm}$ equal to Q_a. Reference to (19) indicates that the variation in Q_a from one mode to the next is of order $a^{-1} \cong 10^{-5}$ and thus the subscript is superfluous. Likewise, all other terms of order $\pi c'/2L\omega_a = a^{-1}$ have been neglected.

It is helpful to observe that (33) is redundant. If we define

$$E_{a+nm} = (-1)^n E_{a-nm}{}^* \tag{35}$$

and substitute into (33), we obtain (32).

Defining the normalized field amplitude

$$g_n = (i/\chi^*)^n E_{a-nm}/(\tfrac{1}{2} E Q_{aL}/Q_a) \tag{36}$$

and substituting into (31) and (32) yields

$$2 \mid \chi \mid^2 R_e g_1 + g_0 = -1 \tag{37}$$

$$(1 - i2n\delta) g_n - g_{n-1} + \mid \chi \mid^2 g_{n+1} = 0 \tag{38}$$

which is a three term recursion relation with nonconstant coefficients. Placing the question of the microwave bandwidth aside momentarily we may assume that the microwave frequency is at its appropriate center band value corresponding to $\delta = 0$ in (38). Under this condition solutions for the recursion formula, (38), of the form $g_n = b^n$, yield as the defining equation for b, $\mid \chi \mid^2 b^2 + b - 1 = 0$. The general solution may be written $g_n = A b_+{}^n + B b_-{}^n$ with

$$b_\pm = \left[-\frac{1}{2 \mid \chi \mid^2} \pm \sqrt{\left(\frac{1}{2 \mid \chi \mid^2} \right)^2 + \frac{1}{\mid \chi \mid^2}} \right]. \tag{39}$$

We note three things about g_n: (i) it is real, since g_0 is real and all the coefficients in the recursion formula are real; (ii) it is a function of $\mid \chi \mid^2$ and (iii), since for $n > 0$, $(E_{a-nm}/E) \to 0$ as the modulation index $\mid \chi \mid \to 0$, so must $g_n \mid \chi \mid^n \to 0$ as follows from (36).

The solution b_- has the dependence $b_-{}^n \sim \mid \chi \mid^{-2n}$ and therefore does not have the correct behavior as $\mid \chi \mid \to 0$. Thus, we choose the constant $B = 0$. The constant A is evaluated by substituting $g_n = A b_+{}^n$ into (37), yielding $A = -(1 + 4 \mid \chi \mid^2)^{-\frac{1}{2}}$, so that we may write

$$g_n = -(\tfrac{1}{2})^n \frac{1}{\mid \chi \mid^{2n}} [\sqrt{1 + 4 \mid \chi \mid^2} - 1]^n / \sqrt{1 + 4 \mid \chi \mid^2} \tag{40}$$

and from (36)

$$E_{a-nm}/\tfrac{1}{2}E = -(i2\chi)^{-n} (Q_{aL}/Q_a) [\sqrt{1 + 4 \mid \chi \mid^2} - 1]^n \div \sqrt{1 + 4 \mid \chi \mid^2}. \tag{41}$$

The same expression with $\chi \rightarrow \chi^*$ applies for $E_{a+nm}/\frac{1}{2}E$. We note that the amplitude of the carrier wave, $n = 0$, is given by

$$E_a/\tfrac{1}{2}E = -(Q_{aL}/Q_a)[1 + 4 \mid \chi \mid^2]^{-\frac{1}{2}} \tag{42}$$

and is reduced in amplitude from its value of $-Q_{aL}/Q_a$ in the no modulation limit $\mid \chi \mid = 0$. The value $E_a = -\frac{1}{2}E$ in the no modulation case, with $Q_{aL} = Q_a$ corresponding to $Q_{ad} = \infty$, is eminently reasonable. Remembering that the surface is a "short" the implication is that the magnetic field of mode a of the resonator exactly cancels out the half of the total carrier magnetic field E on the inner reflecting surface which is associated with the reflected wave of amplitude $E/2$. The incident wave also has amplitude $E/2$, yielding a total field E. Likewise the magnetic field on the opposite reflecting surface is $\pm E/2$. Thus, the incident carrier field $E/2$, which measures the incident energy, correctly accounts for the energy leaving through the opposite face, and we have 100 per cent transmission through the etalon and no reflection. In microwave parlance, the resonator is matched to the incident wave. This always occurs when the external Q equals twice the loaded Q. In the presence of internal loss, $Q_{ad} < \infty$, the resonator cannot be matched if the etalon plates have equal reflectivity. In the presence of the modulation, the carrier mode is loaded by the sidebands even without internal loss and the FPE is no longer matched to the carrier; consequently carrier energy is reflected as well as transmitted. This shows up as a reduction in transmitted power as well as imperfect cancellation of the reflected wave since $\mid E_a \mid < \mid E/2 \mid$.

The relative amplitude of sideband to carrier leaving the opposite face of the etalon is given by

$$(E_{a-nm}/E_a) = (i2\chi)^{-n}[\sqrt{1 + 4 \mid \chi \mid^2} - 1]^n. \tag{43}$$

In the limit of small $\mid \chi \mid$

$$(E_{a-nm}/E_a)_{\mid \chi \mid \to 0} = (\chi^*/i)^n \tag{44}$$

and the amplitude is down by $\mid \chi \mid^n$. The power in the first sideband is proportional to $\mid \chi \mid^2$ which is proportional to the microwave modulating power. For large $\mid \chi \mid$

$$(E_{a-nm}/E_a)_{\mid \chi \mid \to \infty} = (\mid \chi \mid / i\chi)^n = \exp - in(\pi/2 + \text{Arg}\, \chi) \tag{45}$$

so that the modulation approaches 100 per cent. As we may note from (41), the absolute amplitude of all of the modes decreases with increasing $\mid \chi \mid$ as $(1 + 4 \mid \chi \mid^2)^{-\frac{1}{2}}$. Thus we might expect maximum sideband energy for some finite value of χ. Maximizing E_{a-nm}/E shows that for the nth sideband the value of $|\chi|$ for which the energy is a maximum is given by the positive real solution of

$$16|\chi|^4 - 4n^2|\chi|^2 - n^2 = 0.$$

For the first sideband the maximum occurs at

$$|\chi| = \tfrac{1}{2}[\tfrac{1}{2}(1 + \sqrt{5})]^{\frac{1}{2}} = 0.636.$$

The transmitted light beam contains 23 per cent of its energy in the first sideband, and in the absence of internal loss the transmitted carrier energy is reduced to 38 per cent of the original carrier energy. As will be seen later, this modulation can be accomplished with only a few watts of microwave power over a bandwidth of several hundred megacycles at X-band.

The microwave bandwidth over which modulation can be accomplished is determined from a solution of (38) with $\delta \neq 0$. The value of δ for which the energy in the first sideband falls to half its value at center band is determined by a solution of $|g_1(\delta)|^2/|g_1(0)|^2 = \frac{1}{2}$. The details of this calculation are too lengthy to be included here. The calculation was performed by first noting that when the first sideband mode is driven near its band edge the second sideband mode is driven well off resonance. As a result, the values of $g_n(\delta)$ decrease rapidly as $\delta \rightarrow \frac{1}{2}$. It follows from (38) that setting $g_n(\delta) = 0$ is equivalent to neglecting terms of order $(2n)^{-2} \ll 1$. We assumed that $g_3 = 0$ and solved the three relations that follow from (37) and (38). The result can best be expressed in the form

$$\delta = \pm\tfrac{1}{2}(1 + 4|\chi|^2 + \cdots) \tag{46}$$

which indicates that δ is increased over its zero modulation value of $\pm\frac{1}{2}$. This result is understandable if we consider that the modulation process tends to load the carrier mode and decrease its amplitude. This process is more effective when the sidebands are excited strongly. The sideband amplitude is proportional to the carrier amplitude; thus the center of the sideband modes decreases more rapidly than the sides for a given modulation index yielding, in effect, a broader modulation bandwidth. From another point of view we may say that the coupling of a given mode to adjacent modes increases the rate at which energy can be dissipated in the given mode. This decreases the effective loaded Q and increases the bandwidth.

For radiation at 6328 Å, $f_a \simeq 5 \times 10^{14}$ cps, $f_m = 10^{10}$ cps, it follows that $a = 5 \times 10^4$ and for $\Gamma^2 = 0.9$ we have $Q_a \simeq 1.5 \times 10^6$. Thus the modulation bandwidth is at least 3×10^8 cps. For large values of $| \chi |$ the actual bandwidth may be considerably larger but the modulation will not be linear. In any case the high-Q approximation, which is fundamental in this analysis, is probably no longer valid and all conclusions for large $| \chi |$ must be considered as only qualitative.

IV. EFFECT OF NONUNIFORM DIELECTRIC EXCITATION

So far we have assumed that the dielectric constant variation induced by the microwave field depends only on z and t. However, it is necessary to match the microwave phase velocity to the light phase velocity. Since the dielectric constant at the microwave frequency is, in general, considerably larger than it is at the light frequency, the microwave field cannot propagate as a plane wave with no transverse variation. The transverse variation could probably be avoided by making the diameter of the Fabry-Perot cavity very small so that the microwave fields are only partially slowed by the dielectric, but this is not usually possible. As a consequence the microwave field must have a variation of amplitude across the Fabry-Perot cavity. The resultant transverse variation of the perturbed part of the dielectric constant will cause coupling of the FPE modes.

In general we must describe the ath FPE mode in terms of a series of transversely varying eigenfunctions. Thus

$$\mathbf{E}_a = \sum_{lm} A_{lm} \mathbf{R}_l(\mathbf{k}_{lm} \cdot \mathbf{r}) \sin \pi a z / L \tag{47}$$

in which A_{lm} is a constant determined by the geometry. The frequency of the lmth mode is

$$f_{alm} = (c'/2\pi)[(\pi a/L)^2 + k_{lm}^2]^{\frac{1}{2}} = f_a[1 + k_{lm}^2/(\pi a/L)^2]^{\frac{1}{2}} \tag{48}$$

so that each of the modes characterized by the number of half wavelengths equal to a has a different resonant frequency. However, k_{lm} is of order $m\pi/D$ in which D is some characteristic transverse dimension of the FPE. The ratio $k_{lm}/(\pi a/L)$ is of order (m/a) (L/D). Since L/D will be of order ten or less and m/a will be order 10^{-5} for the lower-order transverse variations, we see that the frequencies f_{alm} differ from the fundamental frequency by about five parts in 10^9. For 6328 Å radiation this amounts to about three megacycles, well within the passband of the FPE.

The microwave modulating signal designed to mix adjacent FPE modes will not produce mixing of different transverse modes with the same a number but it can produce transverse mixing between adjacent modes. The degree of mixing will depend on integrals of the sort

$$\int \mathbf{R}_l(\mathbf{k}_{lm}\cdot\mathbf{r})\cdot(\delta\epsilon(\mathbf{r})/\epsilon)\mathbf{R}_{l'}(\mathbf{k}_{l'm'}\cdot\mathbf{r})dS \tag{49}$$

evaluated over the cross section of the FPE. The functions $\mathbf{R}_l(\mathbf{k}_{lm}\cdot\mathbf{r})$ are assumed to be suitably normalized and orthogonal. It is apparent that when $\delta\epsilon/\epsilon$ has only a slow variation over the cross section then both l' and m' cannot differ much from l and m. Thus the distribution of fields over the output face of the FPE will not differ much from that associated with the incident carrier wave and the angular spread of the output carrier and the modulation sidebands will not be seriously deteriorated. In addition the amplitude of the modulation index will not be drastically reduced. Thus it appears that even though the microwave field may vary from a maximum value at the center of the FPE to a value approaching zero at its transverse boundaries, no serious effects will result. Very rapid transverse variations of the microwave field, on the other hand, could cause difficulties.

V. AMPLITUDE MODULATION

In the traveling-wave modulator proposed by Kaminow[1] and the FPE modulator described here, the fundamental effect is a phase modulation. By suitable use of polarizers and birefringent plates, the phase modulation of the traveling-wave modulator can convert linearly polarized light into elliptic polarization and hence into amplitude or intensity modulation. It is therefore of interest to look into the intensity modulation properties of the FPE modulator. We require an expression for the total electric field associated with the transmitted light E_t. Using (28), this is given by

$$\begin{aligned} E_t = E_a \exp i\omega_a t\,[1 + \sum_{n=1}^{\infty} ([E_{a-nm}/E_a] \exp - in\omega_m t \\ + [E_{a+nm}/E_a] \exp in\omega_m t)] \end{aligned} \tag{50}$$

in which E_a is assumed to be polarized parallel to one of the privileged axes which we shall denote as the x-axis of Fig. 1. Combining (35) and (41), (50) becomes for the x-component of the FPE field,

$$\begin{aligned} E_{tx} &= E_{ax} \exp i\omega_a t\,[1 + \sum_{n=1}^{\infty} (y^n + [-y^*]^n)] \\ &= E_{ax} \exp i\omega_a t\,[(1-y)^{-1} + (1+y^*)^{-1} - 1] \\ &= E_{ax} \exp i\omega_a t\, F(y) \end{aligned} \tag{51}$$

in which

$$y = [E_{a-m}/E_a] \exp - i\omega_m t \tag{52}$$
$$= (i2\chi)^{-1} [\sqrt{1 + 4 |\chi|^2} - 1] \exp - i\omega_m t,$$
$$F(y) = [(1 - y)^{-1} + (1 + y^*)^{-1} - 1]$$

and we have used the identity $\sum_{n=1}^{\infty} y^n = (1 - y)^{-1} - 1$. Since the privileged axes are driven push-pull, the expression for the transmitted electric field when the incident light is polarized parallel to the other privileged axis is identical, except that χ must be replaced by $-\chi$, and thus y must be replaced by $-y$. Therefore, we may write

$$E_{ty} = E_{ay} \exp i\omega_a t\, F(y)^*. \tag{53}$$

Since the incident field has the value $E/2$ when the light is polarized along one of the privileged axes, the relative intensity of the transmitted light is given by $I_t = |2E_{tx}/E|^2$, and we can write, using Eqs. (42), (51) and (52)

$$I_t = |F(y)|^2 (Q_{aL}/Q_a)^2/[1 + 4 |\chi|^2]. \tag{54}$$

It can be shown after some manipulation that

$$F(y) = \sqrt{1 + 4 |\chi|^2}\, \frac{[1 - i2 |\chi| \cos(\omega_m t + \varphi)]}{[1 + 4 |\chi|^2 \cos^2(\omega_m t + \varphi)]} \tag{55}$$

in which φ is the argument of χ which corresponds to the phase of the microwave field. Substituting (55) in (54) yields

$$I_t = (Q_{aL}/Q_a)^2/[1 + 4 |\chi|^2 \cos^2(\omega_m t + \varphi)] \tag{56}$$

for the intensity modulation. For comparison with (2), which is the intensity modulation derived from the Fabry-Perot equation, we assume that the internal loss is negligible so that $Q_{aL} = Q_a$. Using (34) and (19), and writing $\delta\epsilon/\epsilon = 2\delta\mu/\mu$, it follows that

$$2 |\chi| = \frac{\pi a}{4} \frac{(1 + \Gamma)^2}{1 - \Gamma^2} \frac{\delta\mu}{\mu}. \tag{57}$$

We note that the equivalent term in (2) is $\Gamma(\pi a \delta\mu/\mu)/(1 - \Gamma^2)$, which differs by a factor $4\Gamma/(1 + \Gamma)^2 \approx 1$ from $2 |\chi|$. The negligible difference from unity can be attributed to the high-Q approximation. Thus the relatively naive considerations which lead to (2) have been shown to be precisely correct.

Other, more efficient, types of intensity modulation can be achieved through the use of polarizers. Suppose that the incident light is polarized at 45° to the privileged axis, and that the modulator is followed by an analyzer which is set parallel to the polarizer. Under these circumstances $E_{ax} = E_{ay}$ and

$$E_t = (E_{tx} + E_{ty})/\sqrt{2} = \tfrac{1}{2}E_a \exp i\omega_a t\, [F(y) + F(y)^*].$$

Thus we may write

$$\begin{aligned} I_t &= [R_e F(y)]^2 (Q_{aL}/Q_a)^2/[1 + 4\,|\,\chi\,|^2] \\ &= (Q_{aL}/Q_a)^2/[1 + |\,\chi\,|^2 \cos^2 (\omega_m t + \varphi)]^2 \end{aligned} \tag{58}$$

which we note differs from (55) in that the denominator is squared. As a result the depth of modulation is roughly twice that attained without the use of polarizer and analyzer. The extra efficiency arises from the fact that the cavity also enhances the induced birefringence and this effect is brought out by the use of the polarizers.

When the analyzer is set at 90° to the polarizer

$$E_t = (E_{tx} - E_{ty})/\sqrt{2} = \tfrac{1}{2}E_a \exp i\omega_a t\, [F(y) - F(y)^*]$$

and

$$\begin{aligned} I_t &= [I_m F(y)]^2 (Q_{aL}/Q_a)/[1 + |\,\chi\,|^2] \\ &= (Q_{aL}/Q_a)^2 4\,|\,\chi\,|^2 \cos^2 (\omega_m t + \varphi)/[1 + 4\,|\,\chi\,|^2 \cos^2 (\omega_m t + \varphi)]^2 \end{aligned} \tag{59}$$

and we note that the light is 100 per cent intensity modulated. The peak transmitted intensity occurs for $|\,\chi\,| \cos (\omega_m t + \varphi) = 0.5$.

So far, the intensity modulation has been at twice the microwave frequency. It is possible to modulate at the microwave frequency by introducing a quarter-wave plate between the polarizer and analyzer, with the polarizers either parallel or crossed. Under these circumstances,

$$E_t = (E_{tx} \pm iE_{ty})/\sqrt{2} = \tfrac{1}{2}E_a \exp i\omega_a t\, [F(y) \pm iF(y)^*],$$

and

$$\begin{aligned} I_t &= \tfrac{1}{2}[R_e F(y) \pm I_m F(y)]^2 (Q_{aL}/Q_a)^2/[1 + 4\,|\,\chi\,|^2] \\ &= \tfrac{1}{2}(Q_{aL}/Q_a)^2\, [1 \mp 2\,|\,\chi\,| \cos (\omega_m t + \varphi)]^2/ \\ &\qquad [1 + 4\,|\,\chi\,|^2 \cos^2 (\omega_m t + \varphi)]^2 \end{aligned} \tag{60}$$

which has a component at the microwave frequency. The introduction of a quarter-wave plate provides a bias, or steady retardation, which allows linear modulation.

VI. MODULATION POWER

The microwave power required for modulation is a function of the percentage modulation. For the sake of discussion we will assume the modulation index is 0.636, corresponding to 23 per cent modulation or maximum energy in the first sideband as described earlier. In terms of the peak variation of the index of refraction μ, the modulation index is given by $\chi = \frac{1}{2}Q_{aL}\delta\mu/\mu$, as follows from (34). For light polarized perpendicular to the z-axis of KDP[7] and parallel to one of the privileged axes

$$\delta\mu = \pm\tfrac{1}{2}\mu^3 r_{63} E_p \tag{61}$$

in which E_p is the peak microwave electric field parallel to the z-axis and r_{63} is the electrooptic constant. The peak electric field can be related to the microwave power absorbed in the KDP by

$$P \approx \tfrac{1}{4}\sigma E_p^2 V \tag{62}$$

in which P is the absorbed power, V is the volume of KDP and σ is the microwave conductivity given by

$$\sigma = \omega\epsilon/Q_d \tag{63}$$

in which ω is the angular microwave frequency, ϵ is the microwave dielectric constant and Q_d is the dielectric Q or inverse loss tangent. It has been assumed that the energy loss associated with microwave electric fields transverse to the z-axis is negligible, a condition which is easily satisfied. The factor $\frac{1}{4}$ arises from a time and space average of the microwave standing wave pattern. Combining (61) to (63) yields

$$P \approx (\omega\epsilon/Q_d)(2\chi/Q_{aL}\mu^2 r_{63})^2 V. \tag{64}$$

At a frequency of 10^{10} cps (X-band) the dielectric constant $\epsilon \approx 20\epsilon_0$, in which ϵ_0 is the dielectric constant of free space, and $Q_d \approx 150$.† For $f_a = 5 \times 10^{14}$ cps, $r_{63} \approx 10^{-11}$ meters/volt, $\Gamma^2 = 0.9$, $Q_{aL} \approx 10^6$ and $\mu \approx 1.5$, (64) yields a power requirement of 240 watts/cc of KDP for $|\chi| = 0.636$. For modulation in the first sideband we must have $c'/2L = 10^{10}$ or $L = 1$ cm. For modulation powers of order ten watts the cross-sectional area of the KDP must be under 0.04 cm^2, corresponding to a diameter of about 0.2 cm.

The temperature rise ΔT from surface to center of a uniformly heated rod which is long compared to its diameter can be written

† Measured at room temperature.

$$\Delta T \approx \tfrac{1}{4} P R^2 / K V \tag{65}$$

in which R is the radius, K is the thermal heat conductivity and P/V is defined by (64). For KDP, $K \approx 10^{-2}$ watts/cm°K yielding a temperature rise of about 60°K for a diameter of 0.2 cm. Clearly the diameter of the rod should be made as small as possible.

Another alternative is that the length of the FPE can be doubled. By modulating in the second sideband, the same X-band modulation can be accomplished with one-half the power and one-quarter the power dissipation per unit volume. The modulation bandwidth would be halved.

The choice of $|\chi| = 0.636$ was motivated by the fact that this yields maximum energy in the first sideband. For superheterodyne systems this is a desirable objective. For envelope detection, amplitude modulation at the modulating frequency is desirable. In this case the combination of polarizer analyzer and quarter-wave plate is required yielding the modulation described by (60). For this case $|\chi| \approx 0.1$ would be quite adequate, and the heat dissipation, temperature rise and required power would be reduced by a factor of four.

VII. OPTICAL CONSIDERATIONS

It has been supposed that the light beam to be modulated is perfectly collimated, thereby allowing the energy to be completely coupled into the fundamental FPE mode. Since no light beam is perfectly collimated, we would like to examine this assumption and to determine what limitations are placed on the optical cavity.

When a diverging beam of monochromatic light is incident on the FPE, the transmitted beam will form the typical Fabry-Perot fringes[3] at infinity. The central fringe corresponds to one of the axial FPE modes. Thus we can be assured that the incident energy will excite only the axial mode if the angular spread of the beam is less than the angular spread of the central fringe. The phase difference between successive averaging rays moving at an angle θ within the FPE is given by[3]

$$2\varphi = 4\pi f L \mu c^{-1} \cos\theta.$$

Substituting into (1) indicates that the transmitted intensity will fall to half its $\theta = 0$ value for a bright central fringe at an angle $\theta_{\frac{1}{2}}$ given by $\theta_{\frac{1}{2}} \approx [c(1 - \Gamma^2)/2\pi f_a L \mu \Gamma]^{\frac{1}{2}} \approx Q_a^{-\frac{1}{2}}$. Thus the angular spread of the incident beam must be less than $2\mu Q_a^{-\frac{1}{2}}$. Taking the internal losses of the FPE into account, we would find the somewhat larger angle $2\mu Q_{aL}^{-\frac{1}{2}}$. For the FPE cavity discussed earlier with $Q_{aL} \approx 10^6$, $2\mu Q_{aL}^{-\frac{1}{2}} \approx 3 \times 10^{-3}$

corresponding to 10 minutes of arc. For a beam whose angular divergence is diffraction-limited, the required aperture of the FPE is

$$D \approx 1.2cQ_{aL}^{\frac{1}{2}}/2\mu f_a$$

or about 0.25 mm for the example given above. When "walkoff" of light rays incident at the angle $\mu\theta_{\frac{1}{2}}$ is considered, the required aperture is increased by approximately $2\theta_{\frac{1}{2}}L/(1 - \Gamma^2) = 0.15$ mm. Since KDP rods with a diameter much less than about 1 mm do not seem practical for the present, no constraints will be imposed by any diffraction limited beam such as that from an optical maser.

One important constraint imposed on the optical maser is that it oscillate in only one longitudinal mode or, if not that, that the modes be separated by at least the frequency Δf but less than $c/2L\mu$.

In a manner similar to that by which the half angle $\theta_{\frac{1}{2}}$ was found it can be seen that the quantity μL must be controlled such that

$$\Delta(\mu L)/\mu L < (2Q_a)^{-1} \approx 3 \times 10^{-7}.$$

Changes in temperature of the cavity will tend to change the product μL. Our best guess, in view of the lack of data, is that $\Delta\mu L$ per degree centigrade is of order 2×10^{-6} for one centimeter length. Thus, the temperature must be controlled to better than 0.1°C. Likewise, the etalon should be flat and parallel to within 3×10^{-7} cm.

VIII. COMPARISON TO TRAVELING-WAVE MODULATORS

In traveling-wave reactive modulators the light makes one traversal of the electrooptic material. The transmitted electric field amplitude, for light polarized along one of the privileged axes, is given by

$$E_t = E_0 \exp(i\gamma \sin \omega_m t) \tag{66}$$

in which $\gamma = (\omega L/c')(\delta\mu/\mu)$ and E_0 is the amplitude of the incident light wave at frequency ω. The comparison to the FPE modulator is made by relating the amplitudes of the various sidebands. Thus we may write for E_t

$$E_t = E_0 \sum_{n=-\infty}^{+\infty} J_n(\gamma) \exp in\omega_m t \tag{67}$$

in which J_n is the Bessel function of order n.† The amplitude of the nth sideband compared to the carrier is

† We have used the identity $\exp iz \sin\theta = \sum_{n=-\infty}^{+\infty} J_n(z) \exp in\theta$.

$$E_n/E_0 = J_n(\gamma)/J_0(\gamma) \approx \gamma^n/2^n n! \quad \text{for} \quad \gamma \ll 1 \tag{68}$$

which should be compared to (44). We note that for first sideband modulation $\gamma/2$ should be compared to $|\chi|$. Thus we compare

$$\tfrac{1}{2}(\omega L/c')(\delta\mu/\mu)$$

and $\frac{1}{8}[(1 + \Gamma)^2/(1 - \Gamma^2)](\omega L/c')(\delta\mu/\mu)$. This indicates that the FPE modulation is more efficient by the factor $\frac{1}{4}(1 + \Gamma)^2/(1 - \Gamma^2) \approx \Gamma(1 - \Gamma^2)^{-1}$. For $\Gamma^2 = 0.9$ and equal modulation lengths this is a factor of ten in $\delta\mu/\mu$, requiring one hundred times the power for equal modulation. The advantage of multiple reflection can be compensated somewhat by increasing the interaction length of the traveling-wave modulator. For example, increasing L by $\Gamma(1 - \Gamma^2)^{-1}$ equates $|\chi|$ and $\gamma/2$ for equal $\delta\mu/\mu$ but requires $\Gamma(1 - \Gamma)^{-1}$ times as much power, since for equal $\delta\mu/\mu$ the power dissipation per unit length must be the same. Increasing L by $\Gamma^2(1 - \Gamma^2)^{-2} \approx 100$ and decreasing $\delta\mu/\mu$ by $(1 - \Gamma^2)^{-1}$ again equates $\gamma/2$ and $|\chi|$. In this case the power requirement for either modulator is the same. The potentially greater bandwidth of the traveling-wave device in this case is probably obviated by the difficulty of matching microwave and optical phase velocities closely because of the long interaction length. The power dissipation per unit volume is of course, lower by $\Gamma^{-2}(1 - \Gamma^2)^{+2}$ in the traveling-wave device and this is an extremely important advantage. However, the difficulties associated with fabricating a long rod of electrooptic material appear to be formidable at the moment. In addition, the optical loss of the KDP, as has been recently pointed out by Kaminow,[8] is not completely negligible. A rod sufficient to achieve the last mentioned advantages would be about one hundred centimeters long at X-band modulation frequencies. Since the energy decay distance is about 35 centimeters,† the light intensity would fall to $\approx e^{-3}$ of its initial value, i.e., about 13 db down.

In order to determine the transmission loss through the FPE modulator we may consider the ratio

$$(Q_{aL}/Q_a)^2 = [1 + Q_a/Q_{ad}]^{-2}.$$

Since

$$Q_a/Q_{ad} \approx L/\ell(1 - \Gamma^2) \approx \tfrac{2}{3}$$

the transmitted intensity is decreased by a factor slightly less than three, which is comparable to that of the intermediate length modulator.

† A rough measurement at 6328 A.

corresponding to 10 minutes of arc. For a beam whose angular divergence is diffraction-limited, the required aperture of the FPE is

$$D \approx 1.2cQ_{aL}^{\frac{1}{2}}/2\mu f_a$$

or about 0.25 mm for the example given above. When "walkoff" of light rays incident at the angle $\mu\theta_{\frac{1}{2}}$ is considered, the required aperture is increased by approximately $2\theta_{\frac{1}{2}}L/(1 - \Gamma^2) = 0.15$ mm. Since KDP rods with a diameter much less than about 1 mm do not seem practical for the present, no constraints will be imposed by any diffraction limited beam such as that from an optical maser.

One important constraint imposed on the optical maser is that it oscillate in only one longitudinal mode or, if not that, that the modes be separated by at least the frequency Δf but less than $c/2L\mu$.

In a manner similar to that by which the half angle $\theta_{\frac{1}{2}}$ was found it can be seen that the quantity μL must be controlled such that

$$\Delta(\mu L)/\mu L < (2Q_a)^{-1} \approx 3 \times 10^{-7}.$$

Changes in temperature of the cavity will tend to change the product μL. Our best guess, in view of the lack of data, is that $\Delta\mu L$ per degree centigrade is of order 2×10^{-6} for one centimeter length. Thus, the temperature must be controlled to better than 0.1°C. Likewise, the etalon should be flat and parallel to within 3×10^{-7} cm.

VIII. COMPARISON TO TRAVELING-WAVE MODULATORS

In traveling-wave reactive modulators the light makes one traversal of the electrooptic material. The transmitted electric field amplitude, for light polarized along one of the privileged axes, is given by

$$E_t = E_0 \exp (i\gamma \sin \omega_m t) \tag{66}$$

in which $\gamma = (\omega L/c')(\delta\mu/\mu)$ and E_0 is the amplitude of the incident light wave at frequency ω. The comparison to the FPE modulator is made by relating the amplitudes of the various sidebands. Thus we may write for E_t

$$E_t = E_0 \sum_{n=-\infty}^{+\infty} J_n(\gamma) \exp in\omega_m t \tag{67}$$

in which J_n is the Bessel function of order n.† The amplitude of the nth sideband compared to the carrier is

† We have used the identity $\exp iz \sin \theta = \sum_{n=-\infty}^{+\infty} J_n(z) \exp in\theta$.

Thus a traveling-wave modulator with a length about $\Gamma(1 - \Gamma^2)^{-1}$ times that of the FPE modulator is comparable with respect to heat dissipation and transmission loss but requires $\Gamma(1 - \Gamma^2)^{-1}$ times the modulation power. An increase in length by another factor $\Gamma(1 - \Gamma^2)^{-1}$ is somewhat drastic, since the transmission loss increases too rapidly.

IX. CONCLUSION

The modulation properties of an electrooptic material placed in a Fabry-Perot etalon have been analyzed in detail. The main conclusion of the analysis is that the FPE modulator offers considerable advantage over the traveling-wave modulator with respect to modulation power when the traveling-wave modulator is not extremely long.

The bandwidth of the modulator is limited by the passband of the FPE, which for practical purposes is several hundred megacycles. In general this bandwidth is compatible with the bandwidth of the microwave modulating cavity (without wasting modulation power) since the dielectric Q of the material is 150, yielding cavity Q's of order 75 and a bandwidth of approximately 130 mc/s. When the electrooptic material does not fill the entire cavity or when additional microwave bandwidth is required, additional loss must be coupled into the cavity. Likewise multimesh broadbanding techniques can be useful.

The requirement of a stable monochromatic source of light for the modulator represents no real disadvantage, since we suppose that the modulator would be used in a communication system which needs such a source anyway.

A major requirement for the successful operation of the FPE modulator is the choice of the correct microwave field configuration which, in effect, matches the microwave phase velocity to that of the light in the electrooptic medium.

REFERENCES

1. Kaminow, I. P., Phys. Rev. Letters, **6,** May 15, 1961, p. 528.
2. Pershan, P. S., and Bloembergen, N., *Advances in Quantum Electronics*, Ed. J. R. Singer, New York, Columbia Univ. Press, 1961, p. 187.
3. Jenkins, F. A., and White, H. E., *Fundamentals of Optics*, New York, McGraw-Hill, 1950, p. 258.
4. Tien, P. K., J. Appl. Phys., **29,** 1958, p. 1357.
5. White, A. D., and Ridgen, J. D., Proc. I.R.E., **50,** July, 1962, p. 1697.
6. Slater, J. C., *Microwave Electronics*, Princeton, D. Van Nostrand, p. 57.
7. Seitz, F., and Turnbull, D., *Solid State Physics*, Vol. 4, New York, Academic Press, p. 90.
8. Kaminow, I. P., private communication.