Theory of a Frequency-Synthesizing Network

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The theoretical basis for designing frequency-combining and selecting circuits is developed. By the introduction of "sideband algebra" and of a frequency symbolic network, the new method offers formal design procedures in place of intuitive ones. This leads directly to finding optimal solutions for frequency-adding or frequency-subtracting problems without limitations as to the relative frequency ratios. The derivation of typical frequency-synthesizing circuits, such as "slave" oscillators and digital frequency selection systems, is discussed, and examples of practical solutions are given.

I. THE SYMBOLIC FREQUENCY-COMBINING NETWORK

1.1 *Introduction*

The frequency accuracy of ac signal sources plays an increasingly important role in various technical and physical fields of precision measurements.[1] A primary reason for this perhaps is a numerical property associated with signal frequencies that is rather unique among quantitative physical phenomena: with the use of relatively simple nonlinear elements[2] and electric wave filters,[3] signal frequencies, like numerical quantities, can be both added to or subtracted from each other and multiplied or divided by an integer.[4,5] The digital character of these operations permits the devising of systems that provide extremely high frequency accuracies and almost unlimited resolution capabilities.[6,7]

In many of these techniques related to, and based on, signal-frequency generation, adjustability, readability or interpolation, the combining of two signal frequencies to obtain a single frequency signal in the form of their sum or difference is one of the most basic operations. When the ratio between the two frequencies to be thus combined is low, the method is quite straightforward, since the desired result can be arrived at by using a nonlinear element (modulator) and an appropriate electrical wave filter. However, when the ratio between the two frequencies increases, the problem of separating the desired sideband from the undesired products becomes progressively difficult and, eventually, impractical.

Another class of frequency-combining problems where a satisfactory solution is particularly difficult to find consists of cases when two signal frequency sources must track each other by a constant, relatively small frequency interval. A classic solution to this problem uses a "slave" oscillator system that employs both mechanical and electrical servomechanism control elements.[8] The servomechanisms perform satisfactorily when the "master" frequency is not subject to rapid frequency variations and random drifts, or when tracking accuracy requirements are not critical. With a high rate of these variations, however, the inherent mechanical and electrical inertia of these systems produces unacceptable tracking errors. Thus, a solution based on a noninertial system is necessary when high-accuracy requirements must be met.

The theory presented here leads to a general method of designing frequency-combining systems having no limitations as to the relative frequency ratios and employing only standard circuit elements such as modulators, filters and oscillators. This method can also be extended to variable frequency sources and to the design of "slave" oscillators and signal frequency systems covering extensive ranges that may be adjustable by decade or other digital steps.

1.2 *Sideband Algebra of Frequency-Combining Circuits*

Operations with numbers representing or standing for signal frequencies are restricted in certain ways by the very physical nature of the frequency-combining means and methods. In order to facilitate the process of logical deductions leading to the solutions of frequency-combining problems, in the following discussion these restrictions are codified and then imposed upon the algebraic operations that are directly related to the combining of signal frequencies. The rules thus evolved are called *sideband algebra.*

1.2.1 *An Elemental Frequency-Combining Scheme*

As is generally known, two signal frequency sources, f_1 and f_2, applied to a nonlinear element (modulator) produce a spectrum of modulation products of the general form $pf_2 \pm qf_1$, where p and q are integers and "$+$" or "$-$" indicates the upper or the lower sideband spectra, respectively.[9] For the present considerations, the most important among these products are the upper $(f_2 + f_1)$ and the lower $(f_2 - f_1)$ single-frequency sideband products and the carrier frequency (the higher of the two primary frequencies). In order to obtain the desired combination of the two frequencies, all the other modulation products need to be suppressed to the desired degree. In the circuits under consideration, this suppression is accomplished by an electrical wave filter.

An elemental frequency-combining scheme is shown in Fig. 1(a). Two signal-frequency sources, f_1 and f_2, are connected to a nonlinear element, or modulator, M, while the electrical wave filter, F, passes the desired sum or difference frequency, f_c, and suppresses to a desirable degree the other modulation products, in particular the other sideband product and the carrier frequency f_2. This operation may be expressed by the formula

$$f_2 +/- f_1 = f_c \tag{1}$$

where the symbol $+/-$ stands for "either the upper or the lower sideband product".

It can be noted that the frequency differences between a desired modulation product, f_c, and the nearest unwanted modulation products are equal to f_1, or the lower of the two frequencies to be combined. Therefore, the lower the relative value of the f_1 frequency (or the higher the ratio between f_2 and f_1 frequencies), the more difficult are the frequency-discrimination requirements that must be met by the filter design. Thus, if n denotes the maximum frequency ratio that can be accommodated efficiently within a practical filter design under consideration, then two numerical quantities representing frequencies f_1 and f_2 can be directly added to or subtracted from each other only if

$$\frac{f_2}{f_1} \leqq n, \tag{2}$$

where f_1 is the lower of the two frequencies.

In Fig. 1(a) all three frequencies involved in the elemental frequency-

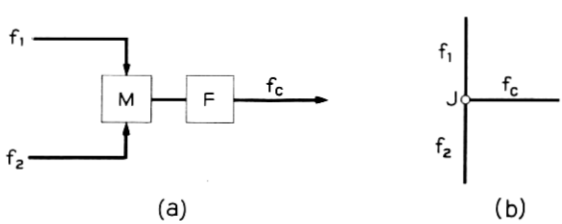


Fig. 1 — Elemental frequency-combining scheme.

combining scheme are known, and also it is predetermined which of these three frequencies, available at the filter output, is the result of combining the other two. It should be noted, however, that, with the same three frequencies involved, two other modulator-filter arrangements are possible, where either of the two frequencies shown in Fig. 1(a) as the "input" frequencies can be obtained as a combination of the two other frequencies. To contain all of these possibilities, a symbolic frequency-combining diagram, as shown in Fig. 1(b), is introduced. Here the sequential modulator-filter arrangement of Fig. 1(a) is replaced by an asequential arrangement of a "junction point," J, at which the three radial lines, each representing signal frequency, converge. In order that the symbolic frequency-combining diagram have a physical basis and be translatable, when needed, into a practical arrangement as shown on Fig. 1(a), the following self-explanatory rules will apply:

i. The highest of the three frequencies converging at the junction point is equal to the sum of the two other frequencies.

ii. If the highest of the three frequencies is the derived output frequency, then the given junction point performs summation (selection of the upper sideband frequency combination); if the highest frequency is one of the two original input frequencies, then the given junction point performs subtraction (selection of the lower sideband frequency combination).

1.2.2 *The Frequency-Ratio Index*, k

Let us assume that the ratio between two frequencies to be combined, f_k and f_0, is considerably larger than an acceptable n ratio, as defined in (2). In this case, the ratio f_k/f_0 can be presented as equal to, or smaller than, the product of a minimum number, k, of individual n ratios, so that:

$$\frac{f_k}{f_0} \leqq n_1 n_2 \cdots n_k ,$$

where $n_1, n_2, \cdots, n_k$ represent individual frequency ratios, as defined by (2) for a sequence of frequency levels between the f_0 and f_k frequencies. The integer k will thus represent the lowest number of realizable modulation levels that must intervene between the f_0 and f_k frequencies in order to keep the frequency-combining system within the requirements of a practical filter design. If we assume that n' represents the geometric mean of the $n_1, n_2, \cdots, n_k$ ratios, or

$$n' = \sqrt[k]{n_1 n_2 \cdots n_k},$$

then

$$\frac{f_k}{f_0} \leqq (n')^k. \tag{3}$$

The k integer, called the *frequency ratio index*, is a basic design parameter of the frequency-combining systems described here.

Thus, to make possible a transition from the low frequency, f_0, to the high frequency, f_k, a number of intermediate frequency sources, f_a, must be introduced. In an arrangement such as shown in Fig. 2, let f_0 and f_k represent two high-ratio frequencies to be combined. Similarly, $J_1, J_2, \cdots, J_k$ junction points represent modulator-filter stages, while $f_{a1}, f_{a2}, \cdots, f_{a(k-1)}$ represent intermediate frequencies. Inspection of the arrangement shown in Fig. 2 indicates that the number, m_a, of these intermediate frequency sources is:

$$m_a = k - 1. \tag{4}$$

It should be noted that the straightforward modulation scheme, as shown in Fig. 2, provides a single-sideband combination, not only of the desired f_0 and f_k frequencies, but also of all the intermediate frequencies. This scheme, therefore, while illustrating a physically realizable transi-

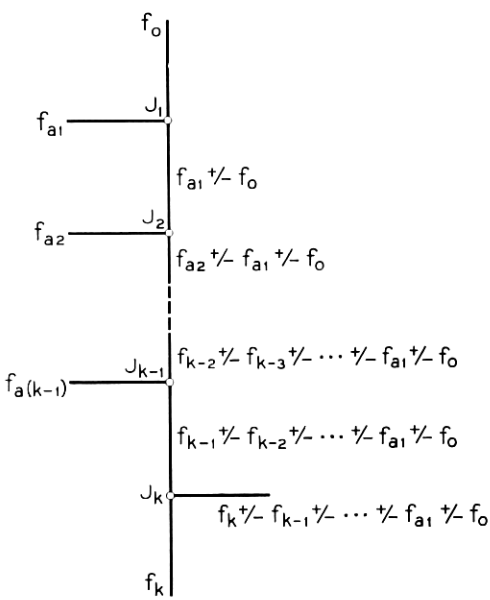


Fig. 2 — Straightforward modulation scheme.

tion between two high-ratio frequency levels, does not provide a solution of the problem of obtaining the sum or difference of the two original high-ratio frequencies alone.

1.2.3 *Algebraic Signs of Frequency Combining*

Denotations of frequency, expressing, in general, a number of events per unit of time, usually have no algebraic "positive" or "negative" signs ascribed. In the present considerations, however, we are not only concerned with frequencies as nominal "numbers of events," but also with their inherent variations with time, generally known as *drifts*. The individual and nonreproducible character of these variations, as well as the necessity of eliminating some of them along with the associated frequency components, leads eventually to certain algebraic methods of manipulating signal frequencies. These manipulations may be facilitated if, within the meanings and conditions described below, algebraic signs are assigned to the symbols representing frequencies within the frame of the sideband algebra.

It can be observed that the instantaneous value of the combined drift of a frequency that is derived as an upper sideband combination of two frequencies is equal to the sum of the instantaneous values of the individual drifts of the component frequencies. On the other hand, the instantaneous value of the drift of a lower sideband frequency combination is equal to the difference of the individual drifts. Thus, the physical meaning of selecting the upper or the lower sideband combinations can be translated into algebraic concepts associated with positive or negative signs, respectively. In relation to these operations, the following rules concerning the algebraic frequency signs are proposed:

i. It is assumed that the frequency of any primary signal source (such as an oscillator, for instance), before being combined within the system, has a positive sign.

ii. When combining two single frequencies (each of which may be the result of some antecedent combining operations), the higher of the two frequencies (with its original component frequencies, if any) does not change its sign; the lower of the two frequencies (with its original component frequencies, if any) retains its sign in the upper sideband combination, but reverses it in the lower sideband combination.

This completes a set of rules instrumental for the derivation of the frequency-combining diagram as shown below. By application of the above rules, it will be proved also that this diagram provides a general solution for the problem of adding or subtracting two frequencies independently of their ratio.

1.3 *The General Frequency-Combining Network*

Let us consider a symbolic network such as the one shown in Fig. 3. This network consists of a number of junction points and branches that link the adjacent points. As explained previously, each of the junction points represents symbolically a realizable modulator-filter ensemble, while each of the branches represents a single signal frequency. Besides the frequencies represented by bilateral network branches (such as f_1, f_2, f_{01}, etc.), there are three more frequencies (f_0, f_k and f_{0k}), each represented by a line unilaterally connected to one of the three corners of the network. The first two of these frequencies are the originally-to-be-combined frequencies, f_k and f_0, respectively (marked by incoming arrows), while the third is the output frequency from the modulation network (marked by an outgoing arrow). Each of the three frequencies that converge at any junction-point must meet the requirements of (1) and, in addition, combinations of two of these frequencies must meet the requirements of (2).

Each of the bilateral network branches may represent either an "auxiliary" frequency applied simultaneously to two modulator-filter

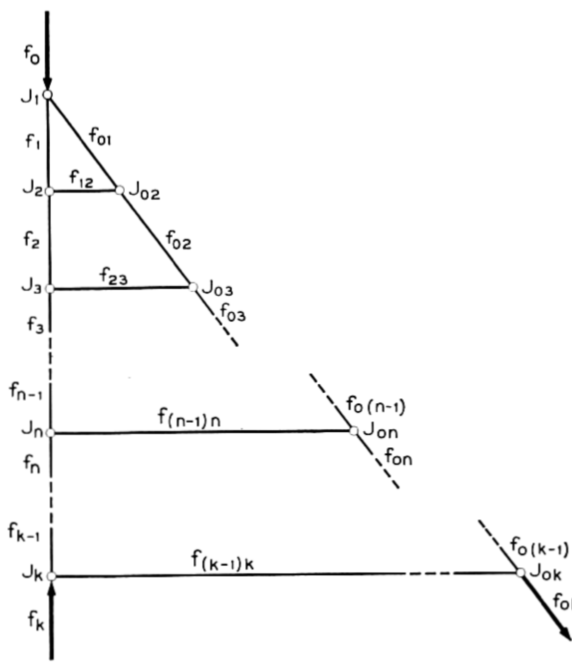


Fig. 3 — Symbolic frequency-combining network.

ensembles from an extraneous signal source, or it may represent a derived frequency that is a single sideband combination of one or both of the original frequencies and of other auxiliary frequencies. This method of connecting bilaterally all auxiliary frequencies and their combinations between a pair of adjacent junction points is significant, since it permits eventual cancellation of all auxiliary component frequencies and their drifts from the final network output. Conversely, the two original frequencies, f_0 and f_k, each connected only to a single junction point (J_1 and J_k, respectively) and thus represented by unilateral lines, are the sole "surviving" components whose single sideband combination is present at the eventual output from the frequency-combining system under discussion.

1.3.1 *Modulation Frequency Levels and Frequency-Network Meshes*

It has been previously established that the number of modulation levels intervening between the low (f_0) and the high (f_k) ratio frequencies to be combined is equal to k [see (3)]. As shown in Fig. 3, at the lowest ("first") frequency level there is only one modulator-filter ensemble, which is shown in the symbolic diagram as junction point J_1, to which point the lower frequency to be combined, f_0, is applied. At each of the following modulation levels, however, there are two junction points (J_2–J_{02}, J_3–J_{03}, etc.). Eventually, to one of the two junction points at the highest frequency level, the higher of the two frequencies to be combined, f_k, is applied, while the above-mentioned output frequency f_{0k} is taken from the other junction point, J_{0k}.

The total number of junction points can be found from Fig. 3 as:

$$m_j = 2k - 1. \tag{5}$$

The described configuration of junction points and interconnecting links forms a certain number of meshes within the frequency-combining network. The first of these meshes contains but three branches, while each of the succeeding meshes has four branches. Any two adjacent meshes are coupled with each other by two common junction points and by the common branch linking these points.

On the basis of the foregoing considerations, it can be found that the conditions sufficient for determining all frequencies within any mesh, are as follows:

i. The out-of-mesh frequencies, applied unilaterally to all mesh corners except one, must represent input frequencies of known values.

ii. Within any given mesh one branch must represent a known extraneously introduced "auxiliary" frequency that is simultaneously (i.e. bilaterally) applied to two adjacent junction points of the mesh.

iii. A branch that is common to two meshes may represent an auxiliary frequency for either one of two meshes, but not for both of them.

Application of these rules will be illustrated in the following discussion.

1.3.2 *Frequency Relationship Within the Symbolic Network*

Following the above rules, we stipulate that, within each mesh of the network shown in Fig. 3, one branch represents an auxiliary frequency assigned to this mesh. The number of the auxiliary frequencies within the network will be thus equal to the number of meshes, or:

$$m_m = k - 1. \tag{4'}$$

It should be noted that this is the same number of auxiliary frequencies as is found in (4) in relation to Fig. 2.

For the sake of simplicity, let us assume that all auxiliary frequencies, f_a, are located in the series-connected network branches marked in Fig. 3 as $f_1, f_2, \cdots, f_{k-1}$. If $n_1, n_2, \cdots, n_k$ denote permissible frequency ratios as defined in (2) for $J_1, J_2, \cdots, J_k$ junction points, respectively, then

$$\frac{f_1}{f_0} = n_1; \quad \frac{f_2}{f_1} = n_2; \quad \cdots; \quad \frac{f_k}{f_{k-1}} = n_k. \tag{2.1}$$

Each of the third frequencies $f_{12}, f_{23}, \cdots, f_{(k-1)k}$ converging at these junction points will have values complementary to two other frequencies. Thus,

$$\text{for } J_2: \quad f_{12} = f_2 +/- f_1; \tag{1.1}$$

$$\text{for } J_3: \quad f_{23} = f_3 +/- f_2; \tag{1.2}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$\text{for } J_k: \quad f_{(k-1)k} = f_k +/- f_{k-1}. \tag{1.k}$$

Similarly, the third frequency converging at the J_1 junction point can be found as

$$f_{01} = f_1 +/- f_0. \tag{1.1.1}$$

Examination of (1.1) and (1.1.1) shows that both frequencies f_{12} and f_{01} contain the common frequency component, f_1. From (1.1), (2.1) and

(1.1.1) it can be found that

$$\frac{f_{12}}{f_{01}} = n_1 \left(\frac{n_2 \pm 1}{n_1 \pm 1} \right).$$

In cases of practical filter design, values of $n_1, n_2, \cdots, n_k$ can be assumed to be considerably larger than one. Thus,

$$\frac{f_{12}}{f_{01}} \cong n_2 . \tag{6}$$

From (6) it can be concluded that frequencies f_{12} and f_{01} can be combined at the junction point J_{02} by using a filter of a design similar to the filter at J_2. By doing so, we obtain:

$$f_{02} = f_{12} +/- f_{01} = (f_2 +/- f_1) +/- (f_1 +/- f_0).$$

Examination of this formula shows that, by proper selection of upper and lower sideband combinations at J_1, J_2 and J_{02} junction points, we can eliminate the auxiliary frequency f_1 from the combination frequency f_{02} at J_{02} junction. To achieve this, the necessary condition is that the sign, as eventually chosen for the f_1 frequency in (1.1) be opposite to the sign of this frequency in (1.1.1) when the upper sideband at J_{02} junction is selected, or that these signs be the same when the lower sideband at this junction is selected. For instance, if we select $f_{12} = f_2 - f_1$ and $f_{01} = f_1 +/- f_0$, then, for the upper sideband:

$$f_{02} = f_{12} + f_{01} = (f_2 - f_1) + (f_1 +/- f_0) = f_2 +/- f_0 . \tag{7}$$

Or if we select $f_{12} = f_2 + f_1$, then, for the lower sideband at J_{02}:

$$f_{02} = f_{12} - f_{01} = (f_2 + f_1) - (f_1 +/- f_0) = f_2 -/+ f_0 . \tag{7'}$$

It should be noted that in both cases either of the sideband combinations of f_2 and f_0 frequencies is available.

In a similar manner it can be proved that the frequencies

$$f_{03}, f_{04}, \cdots, f_{0(k-1)}$$

can be obtained as single-sideband combinations of the original low frequency f_0 and of the successive frequencies $f_3, f_4, \cdots, f_{k-1}$, respectively. Eventually, from the next to the last stage $(k - 1)$ we obtain

$$f_{0(k-1)} = f_{(k-1)} +/- f_0 .$$

At the last modulation level, from the J_k junction, we obtain

$$f_{(k-1)k} = f_k +/- f_{(k-1)}.$$

By combining these last two frequencies at the J_{0k} junction point, we obtain the final output frequency f_{0k} from the modulation system as

$$f_{0k} = f_{(k-1)k} +/- f_{0(k-1)} = [f_k +/- f_{(k-1)}] +/- [f_{(k-1)} +/- f_0],$$

from which:

$$f_{0k} = f_k +/- f_0 . \tag{8}$$

Thus, the output from the junction J_{0k} will provide the desired single sideband combination of the original frequencies f_k and f_0, with all auxiliary frequency components canceled out.

From the above considerations it is evident that the network shown on Fig. 3 can be extended by an addition of any required number of four-branch meshes. Consequently, the solution it offers has no inherent limitation as to the ratio between two frequencies to be combined. It can be concluded, therefore, that this network represents a general solution to the problem of adding or subtracting two high-ratio ac signal frequencies by means of sequential single-sideband frequency combining.*

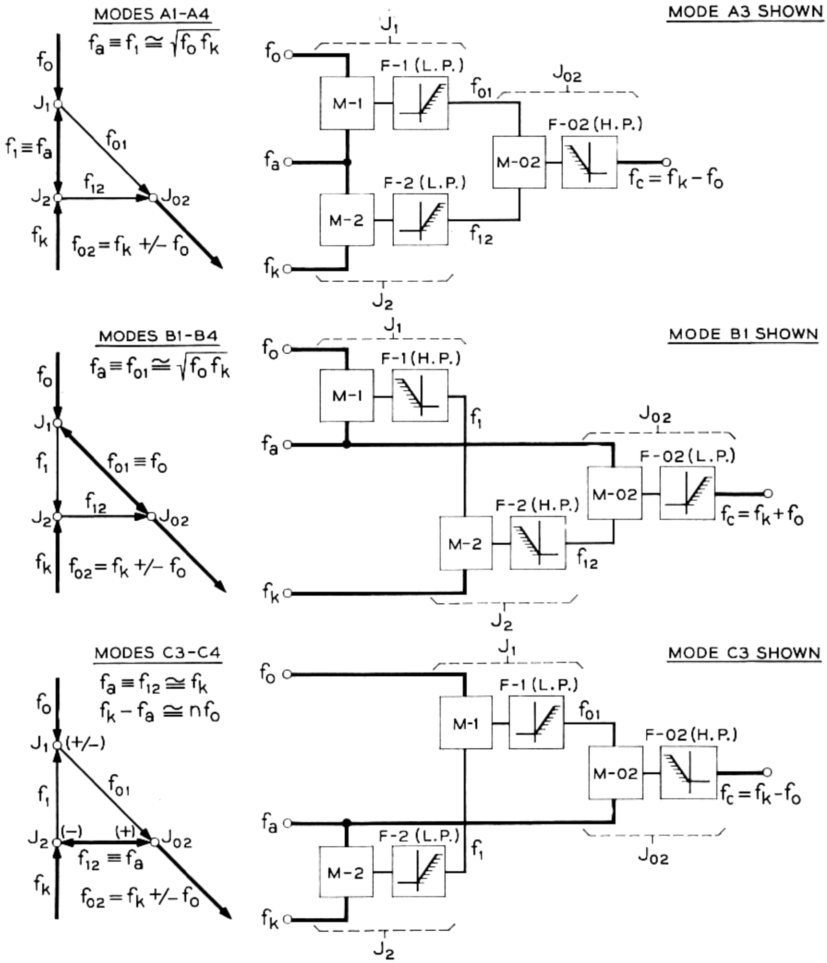
1.3.3 *Possible Network Configurations*

As mentioned above, any branch of a mesh can represent an auxiliary frequency. Thus, by changing the position of the auxiliary frequency within a mesh a certain number of possible network configurations, each of which represents a variant of the general solution, may be obtained. For a given symbolic network, the number of these variants depends upon the number of meshes as given by (4) and thus, eventually, on the parameter k.

It should be noted that the elemental frequency-combining scheme shown in Fig. 1 can be considered as a special case when $k = 1$; as indicated by (4), there is no auxiliary frequency present for this value of k.

For $k = 2$, (4) indicates one mesh, while from (5) the number of junction points can be found as three. In this case, therefore, the symbolic network consists of one three-branch mesh to which the original input frequencies, f_0 and f_k, are applied to two corners and the output frequency is taken from the third corner. Fig. 4 shows three basic network configurations (or *modes*) arrived at when the auxiliary frequency is

* It should be noted that a method of combining multiples or submultiples of two frequencies in order to obtain the sum or difference of these frequencies is excluded from the present considerations. It can be shown, moreover, that frequency "jitters" and phase variations associated with frequency multiplication and division cannot, for inherent reasons, be eliminated from the output frequency.



USEABLE SIDEBAND COMBINATIONS

MODE	J_1	J_2	J_{02}	f_k & f_o COMBINATION
A1	+	+	−	DIFFERENCE
A2	−	+	−	SUM
A3	−	−	+	DIFFERENCE
A4	+	−	+	SUM
B1	+	+	−	SUM
B2	−	+	−	DIFFERENCE
B3	−	−	+	SUM
B4	+	−	+	DIFFERENCE
C3	−	−	+	DIFFERENCE
C4	+	−	+	SUM

Fig. 4 — Possible network variations.

placed in branches f_1, f_{01} and f_{12} in succession. Additional variations for each of these modes are obtained by choosing different combinations of the sidebands for each of three junction points, as shown in the table in Fig. 4. It could be observed that out of eight (2^3) possible sideband combinations for each mode A, B and C, four combinations must be rejected as not providing cancellation of the auxiliary frequency component at the J_{02} junction point. Thus, the acceptable combinations are only those in which the filter sideband of the J_2 junction point is opposite to the sideband of the J_{02} junction. For configuration C, which uses an auxiliary frequency of the same order as the f_k frequency, two additional combinations must be rejected, since only the lower filter sideband at the J_2 junction can be used to produce an acceptable value for the f_1 frequency. It can be found, therefore, that either the sum or the difference, as desired, of the two original frequencies f_k and f_0, having the ratio index $k = 2$, can be provided in three modes (A, B and C, Fig. 4), two of which (A and B) have two sideband variants each, while the third one (C) has one variant. Thus, the total number of possible circuit arrangements in either case (sum or difference) is five.

Increasing the value of the frequency-ratio index from k to $k + 1$ extends the symbolic network by an additional four-branch mesh. It could be noted that one of the branches of this new mesh is common with the $f_{(k-1)k}$ (see Fig. 3) branch, while the other three branches, f_k, f_{0k} and $f_{k(k-1)}$, are independent. On this basis, the following formula, which expresses the number of modes, M, for any successive $(k + 1)$ order, can be derived:

$$M_k = 4M_{k-1} - M_{k-2}, \tag{9}$$

where M represents the number of basic circuit configuration for a given order.

Thus, for $k = 3$,

$$M_3 = 4M_2 - M_1 .$$

But, as stated above, $M_1 = 1$ and $M_2 = 3$. Therefore:

$$M_3 = 4 \cdot 3 - 1 = 11.$$

Similarly, for $k = 4$:

$$M_4 = 4M_3 - M_2 = 4 \cdot 11 - 3 = 41.$$

Thus, by using (9), the total number of possible circuit configurations can be found for any symbolic network of the $(k + 1)$ order.

II. APPLICATIONS

2.1 *Design Considerations*

2.1.1 *Practical Circuit Configurations*

A relatively large number of possible circuit arrangements, all of which are derivable from the symbolic frequency-combining diagram, may present considerable practical advantages. Since these arrangements can be systematically scrutinized, in any given case the most practical solution can be thus attained. Such a solution may take into account certain particular requirements — for instance, low content of modulation products, avoidance of phase and amplitude distortions and, especially, availability of certain components such as filters and oscillators.

In the design of systems concerned with frequency selection and discrimination, the problem of filter characteristics is usually one of the important considerations. In general, electric wave filters of a special design are relatively expensive, particularly when stringent requirements must be met. The method presented here allows the use of filters of any limited frequency discrimination characteristics in building systems that realize much higher frequency discrimination capabilities. Thus, the necessity for using filter networks of a difficult or impractical design is eliminated. Eventually, an optimal solution compromising the quality of the filters to be used in the given system with their over-all number can be established. Moreover, in many cases filters of an available design may be utilized.

Adaptability of this procedure to certain practical cases is illustrated in the examples of Section 2.2 below.

2.1.2 *Effects of Excessive Auxiliary Frequency Drifts*

As explained above, drifts of the auxiliary frequencies do not essentially affect stability of the output signal of the desired frequency combination. However, since variations of the auxiliary frequencies do affect the positions of the operational points within the filter passbands, excessive drifts may, under certain practical conditions, produce objectionable secondary effects in the form of phase and amplitude distortions. This may take place when the incremental characteristics of attenuation and phase versus frequency of certain pairs of filters carrying the same auxiliary frequencies differ appreciably within the bandwidth covered by the shifts of the operational points.

These amplitude and phase distortions, which rarely exceed acceptable limits, can be reduced, if necessary, in several ways such as:

(a) stabilization to a desirable degree of the auxiliary frequency sources;

(b) shifting, by a discrete amount, of any particular auxiliary frequency value so that an optimal operational section of the filter characteristics can be used;

(c) corrective shaping of the filter passband characteristics so that compensating effects can be achieved;

(d) introducing amplitude and phase equalizers at the auxiliary oscillator outputs.

By applying the above methods, separately or in combination, phase and amplitude distortions usually can be reduced to acceptable levels.

2.1.3 *Noise Levels*

It has been found experimentally that, by observing precautions usually applied in the design of high-quality transmission circuits, noise levels at the output from the systems described here are particularly low. This may be attributed to the fact that the virtual bandwidth of these systems actually is very narrow as a result of certain sequential combinations of filter networks. In many practical cases this property can be enhanced still further by using bandpass filters rather than low- or high-pass filters, as called for by the essential design requirements. Examples of such modifications, which are also desirable when certain filters of a standard design are available, are shown below.

2.2 *Practical Examples*

The following examples fall into three categories, which embrace some typical problems encountered in practice:

(a) combining two essentially fixed high-ratio signal frequencies;

(b) combining two high-ratio signal frequencies, one of which is adjustable over a wide frequency range;

(c) providing high-accuracy signal frequency sources that can be varied over wide frequency ranges either in digital or arbitrarily chosen frequency steps.

2.2.1 *Combining Two High-Ratio Essentially Fixed Frequencies*

In an application related to the phase-delay measurements, a solution of the following problem is required (see Fig. 5):

A frequency of 90 mc with drifts and random frequency variations of the order of ±1000 cps represents a "master" frequency of a measuring system. It is expected that some of these variations may occur at a rate too rapid for any mechanoelectronic servo system to follow without excessive tracking errors. It is required that another ac signal source be provided, the frequency of which should be higher than the master oscillator frequency by an increment of 55,560 ±10 cps. Thus, using denota-

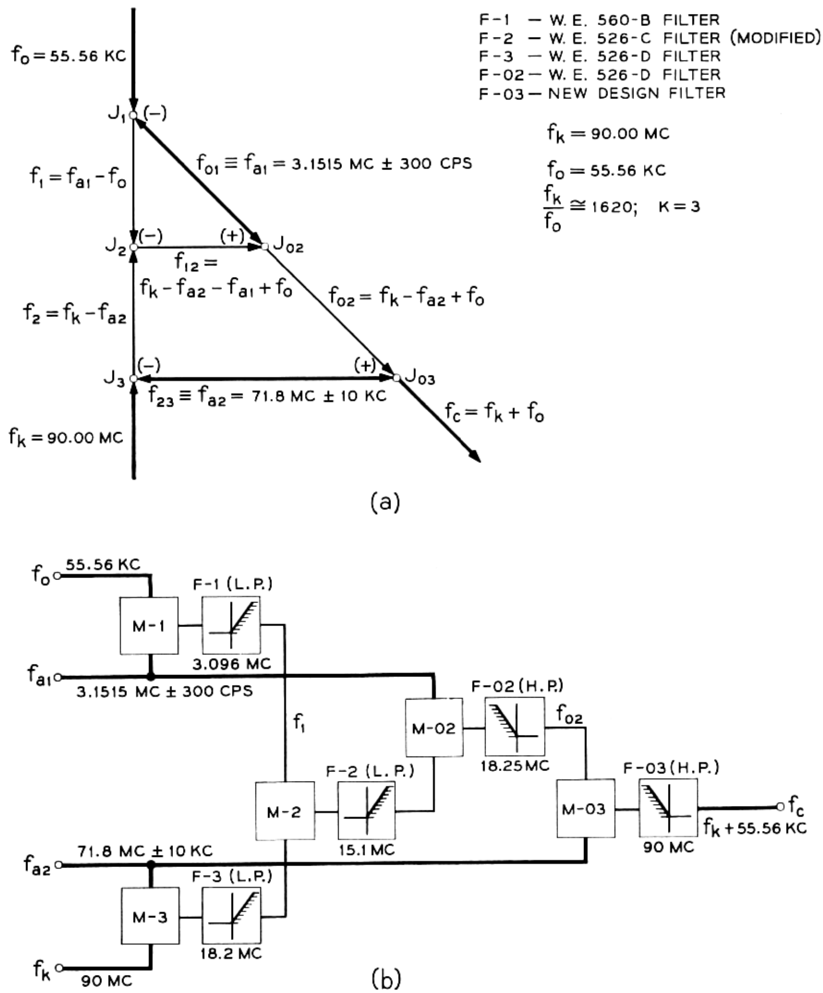


Fig. 5 — Fixed-range "slave" oscillator system: (a) symbolic diagram; (b) block diagram.

tions previously introduced:

$$f_k = 90{,}000 \text{ kc},$$

$$f_0 = f_c - f_k = 55.56 \text{ kc},$$

$$(n')^k = \frac{f_k}{f_0} \cong 1620.$$

For the solution of this problem, the following filters of an available design (manufactured by the Western Electric Company) were taken under consideration:

(a) Type 560-B filter with a passband of 3.096 mc ±1000 cps; n value over 100 (for 60-db minimum discrimination).

(b) Type 526-C filter (modified) with a passband at 15.1 mc ±100 kc; n value of the order of 10.

(c) Type 526-D filter with a passband at 18.2 mc ±100 kc; n value of the order of 10.

Using these filters, the k value has been found as follows:
Assuming

$$n_1 \cong 100,$$

$$n_2 \cong 10,$$

$$n_3 \cong 10,$$

$$n_1 n_2 n_3 = 100 \cdot 10 \cdot 10 > 1620.$$

Then,

$$k = 3,$$

and thus the number of auxiliary frequencies, from (4), is

$$m_a = k - 1 = 2$$

and the number of junction-points, from (5), is

$$m_j = 2k - 1 = 5.$$

Subsequently, the symbolic network shown in Fig. 5 was designed. From a study of this network it was found that an optimal use of the available filters may be achieved by placing one 3.096-mc filter (560-B) as a low-pass filter at the J_1 junction point, an 18.2-mc filter (526-D) as a low-pass filter at the J_3 point, and another filter of the same type as an essentially high-pass filter at the J_{02} point. Finally, one 15.1-mc filter (modified 526-C) was placed at the J_2 point as a low-pass filter.

By locating one of the auxiliary frequencies (3.1515 mc ±300 cps) in the f_{01} branch and the other (71.8 mc ±10 kc) in the f_{23} branch, all the other frequencies were found according to the rules described above. The only filter for which no existing design had been found is at the J_{03} junction-point. This filter, however, must pass the signal of 90.00 mc as the sum of f_{02} frequency (approximately 18.2 mc) and f_{23} frequency (minimum 71.8 mc), while rejecting the other modulation products. As these requirements indicate, the design of such a filter network is well within practical limits of art.

Fig. 5(b) shows a conventional block diagram, representing the practical solution derived as described, from the symbolic network of Fig. 5(a).

2.2.2 *Combining Two High-Ratio Signal Frequencies, One of Which is Adjustable Over a Discrete Range*

An example of a solution applicable to this case is shown in Fig. 6. The frequency of a "master" ac signal source, covering the range from 20 to 100 mc, has to be followed by a "slave" ac signal source having a frequency higher than the "master" frequency by a constant amount of 55.556 kcs.

Let

$$f_{v\,\max} = 100 \text{ mc},$$

$$f_{v\,\min} = 20 \text{ mc},$$

$$f_0 = 55.556 \text{ kc}.$$

The solution in this case can be arrived at in two steps. In the first step, a fixed frequency, f_k, is established so that a single-stage combining of this frequency with the frequency difference, $f_k - f_v$, is realizable in accordance with (2). Thus,

$$\frac{f_k}{f_k - f_{v\,\max}} = n_k',$$

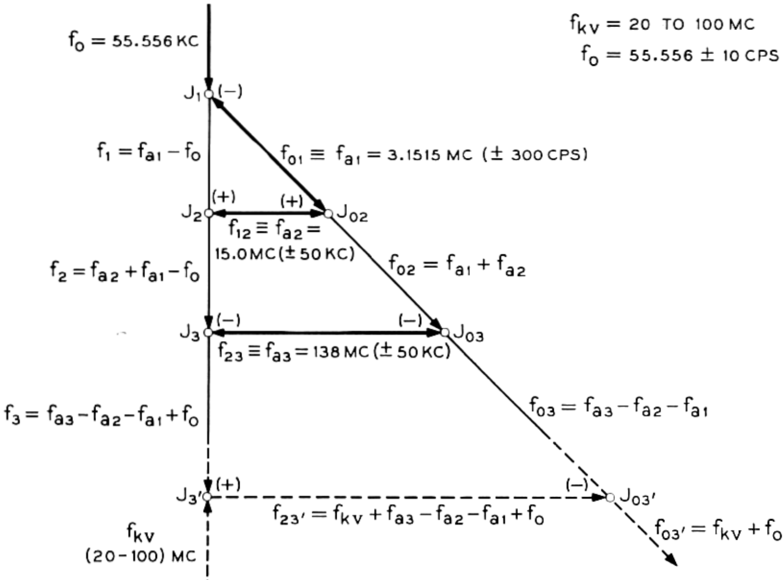
which gives

$$f_k = \frac{n_k'}{n_k' - 1} f_{v\,\max}. \tag{10}$$

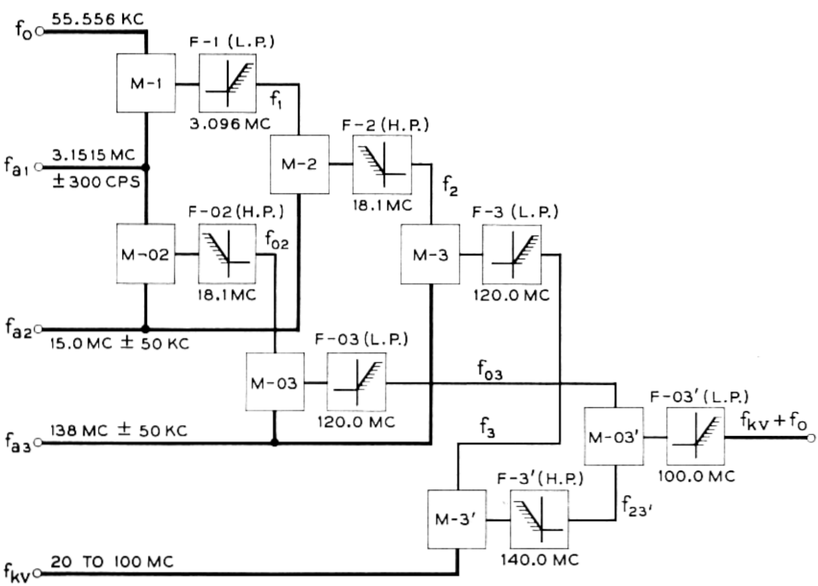
On the other hand, since the whole range of the variable frequency f_v, down to its minimum value $f_{v\,\min}$ must be combined with the f_k frequency, then:

$$\frac{f_k}{f_{v\,\min}} \leqq n_k',$$

or



(a)



(b)

Fig. 6 — Variable-range "slave" oscillator system: (a) symbolic diagram; (b) block diagram.

$$f_k \leqq n_k' f_{v\ \min} . \tag{11}$$

Assuming $n_k' \cong 6$, f_k can be found to be 120 mc.

In the second step the problem is thus reduced to adding two fixed signal frequencies of 55.556 kc (f_0) and of 120 mc (f_k) to each other. But this can be done in a manner similar to that described in the previous example. Thus,

$$\frac{f_k}{f_0} = \frac{120{,}000}{55.556} \cong 2200 .$$

Assuming, as previously, that $n_1 \cong 100$ and $n_2 \cong n_3 \cong 10$, we obtain

$$n_1 n_2 n_3 = 100 \cdot 10 \cdot 10 \geqq 2200.$$

Then

$$k = 3 .$$

Subsequently, the symbolic diagram, as shown by the solid lines in Fig. 6(a), can be constructed. To this diagram, a mesh comprising the J_3' and J_{03}' junction points is added, as shown by dotted lines. The variable "master" signal frequency is connected to the J_3' junction point (which comprises a filter with discrimination ratio of n_k'); and the variable "slave" signal frequency is available from the J_{03}' junction point (which comprises a similar filter).

One of the possible practical versions of this solution that could be derived from the symbolic network of Fig. 6(a), is shown in Fig. 6(b).

2.2.3 *Continuously Adjustable Signal Frequency Sources*

This special case of circuitry, which can be derived simply from a single-mesh frequency-combining network (see Fig. 4, mode C), is shown in Fig. 7. It provides a continuously adjustable range of signal frequencies, these being the sum or the difference of a desired harmonic of a standard frequency signal and an interpolating frequency source. In place of the signal frequency f_0, a low-frequency signal source that is continuously variable over a frequency interval equal to a fundamental frequency value is connected. In place of the signal frequency f_k, a multifrequency signal source in the form of a harmonic generator that supplies harmonics of the same fundamental frequency value is connected. Subsequently, in place of the f_{12} mesh frequency, a special auxiliary frequency source is applied. The frequency of this source can be varied in steps, each being essentially equal to the same fundamental frequency value. At the J_2 junction point, a narrow passband filter is placed. The center point of its passband is equal to the sum of one of the harmonics present at f_k and an auxiliary frequency available at f_{12}. In this manner, the following results are achieved:

i. The f_1 frequency derived from the combination of one of the harmonics and an auxiliary oscillator frequency remains essentially constant while successive harmonics enter the combination.

ii. In order to obtain f_{01} and f_{02} as single sideband frequencies, only fixed low-pass or high-pass filters must be used.

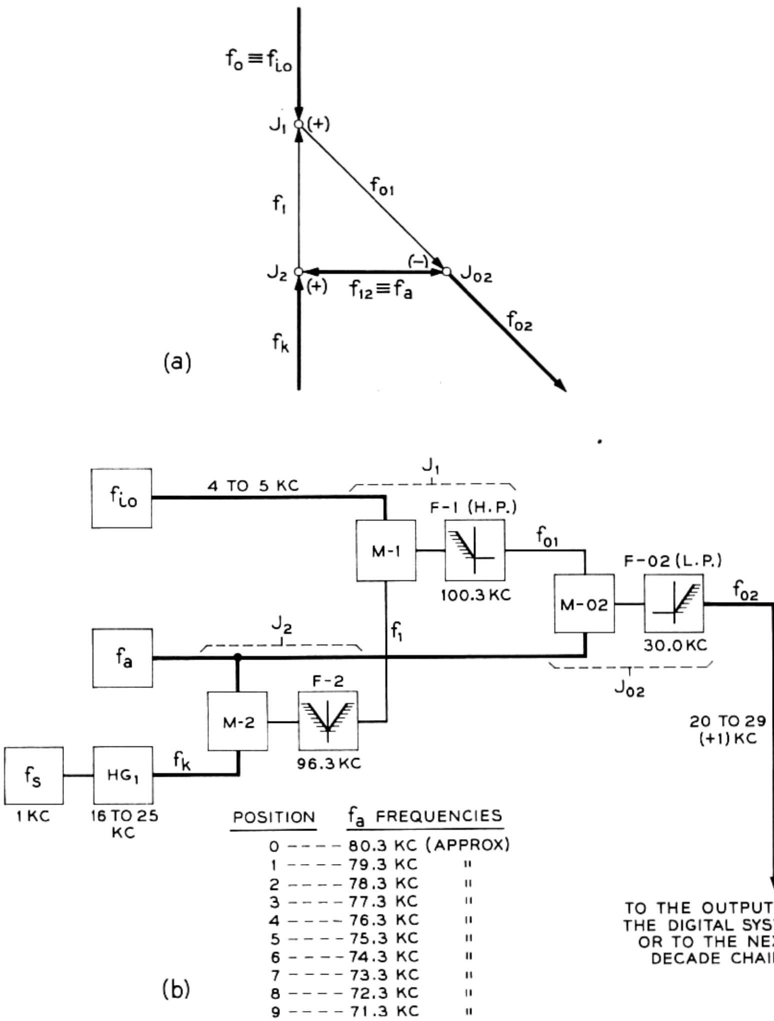


Fig. 7 — Decade "chain" (digital frequency-combining and frequency-selecting system: (a) symbolic diagram; (b) block diagram.

iii. At the output from the J_{02} junction point, a signal of a single sideband frequency derived as the sum or difference of a desired harmonic frequency and the interpolating oscillator frequency is available.

The above properties of this circuitry allow the design of precise signal frequency systems which cover extensive frequency ranges with any desired degree of resolution. In contrast to other systems,[10] the frequency ranges are adjustable here in digital steps without using tunable elements.

An example of an elemental practical circuit[11] is shown in Fig. 7(b). It represents a "step-up digital chain" covering the frequency range from 20 to 30 kc by 10-decade steps and an interpolating oscillator.

The interpolating oscillator covers the range from 4.000 to 5.000 kc and is calibrated in such a way that the "0" cps mark corresponds to 4.000 kc and the "1000" cps mark corresponds to 5.000 kc. A harmonic generator, connected to a 1-kc high-accuracy frequency standard, provides harmonics utilized in the range from the 16th through the 25th. The auxiliary frequency is controlled by the 0–9 positions of a decade switch and provides 10 frequencies, from 80.3 to 71.3 kc respectively, in approximately 1-kc intervals.

The proper operation of this step-up chain depends primarily on the filter at J_2 junction point (F-2 filter). It is a narrow passband filter that rejects the fundamental and all harmonics of the frequency standard available from the harmonic generator, but which passes a narrow frequency band between the numerical values of two successive integral multiples of the standard frequency. It is desirable that this passband be located well beyond the useful range of the harmonic generator, possibly in the vicinity of the point where the envelope of the harmonic spectrum is near, or crosses, the zero amplitude value. The types of filters particularly suited to this application are some of the quartz crystal filters developed for the carrier telephony[12] or mechanical filters used for certain communication radio receivers of an advanced type.[13] The filter used in a practical application of this case is the Western Electric Company's standard crystal filter (97A), which has the passband frequency at 96.3 kc and a flat characteristic within ±50 cps.

As an auxiliary frequency source, an oscillator adjustable in 10 steps of approximately 1-kc each is used. Its frequency, corresponding to the "0" decade step, is thus 80.3 kc, while the frequency corresponding to the ninth decade step is equal to 71.3 kc.

The F-1 and the F-02 filters are somewhat modified versions of standard carrier telephony filters.

The operation of the circuit, as shown in Fig. 7(b), may be illustrated by a numerical example. Let us assume that the setting of the frequency of 26.785 kc is desired. Thus, the 1-kc decade dial is set to position 6, in which setting a signal of 74.3 kc $\pm d_1$ (where d_1 represents an instantaneous value of the drift of the auxiliary frequency) is provided. The interpolating oscillator, set to "785" cps, provides a 4.785-kc signal. Subsequently, the F-2 filter passes the 96.3-kc $\pm d_1$ signal, which in this case will be a combination of the 74.3-kc $\pm d_1$ signal from the auxiliary oscillator and the 22nd harmonic of the 1-kc standard frequency signal. The adjacent signals of 95.3 kc $\pm d$ and 97.3 kc ± 1, as well as any other signals formed by the auxiliary frequency and the 1-kc harmonic spectrum available from the harmonic generator, are suppressed by the filter F-2 with substantially higher than 60-db attenuation.

In the modulator M-1 the signal of 96.3 kc $\pm d_1$ is combined with the 4.785-kc signal from the interpolating oscillator. The difference of these two signals, equal to 91.515 kc $\pm d_1$, is suppressed by the high-pass filter F_1 having cutoff frequency of 100.3 kc, while their sum, equal to 101.085 kc, passes this filter and is applied to the modulator M-02. In this modulator the same auxiliary frequency of 74.3 kc $\pm d_1$ is combined with the 101.035-kc $\pm d_1$ signal from the filter F-1. The sum of these two signals, equal to 175.385 kc $\pm 2d_1$, is rejected by the low-pass filter F-02 having a cutoff frequency of 30.0 kc. The difference of these two signals,

$$101.085 \pm d_1 - (74.3 \pm d_1) = 26.785 \pm 0,$$

is thus available at the filter output. It may be noted that the cancellation of the auxiliary frequency drift took place as the result of adding and then subtracting the auxiliary frequency, and that the desired output signal frequency is eventually the sum of the 22nd harmonic of the 1-kc standard frequency and the interpolating oscillator output only.

A number of step-up frequency digital chains, like the one just described, can be designed for various frequency levels that are related to each other by a desired order of any digital system. Each chain of the lower order can be connected as a source of the interpolating frequency for the higher chain. By combining a necessary number of such chains, wide-range systems having digital frequency readability and resetability and any desired degree of resolution can be thus realized. An example of a 3-decade system is shown in Fig. 8.

The decade chains of multidecade systems are similar to the one described above, with the exception that some of the auxiliary oscillator decade steps of the higher decades may be suppressed. Also, the auxiliary oscillators of the intermediate decades provide two alternative signal frequencies, depending upon the decade control settings of the higher decades. Detailed explanation and theory of these decade arrangements can be found in Ref. 11.

Output switching from the individual decade chains may also be coupled to the decade controls so that eventually a continuous coverage of the predetermined frequency range at the output terminals of the system may be achieved.

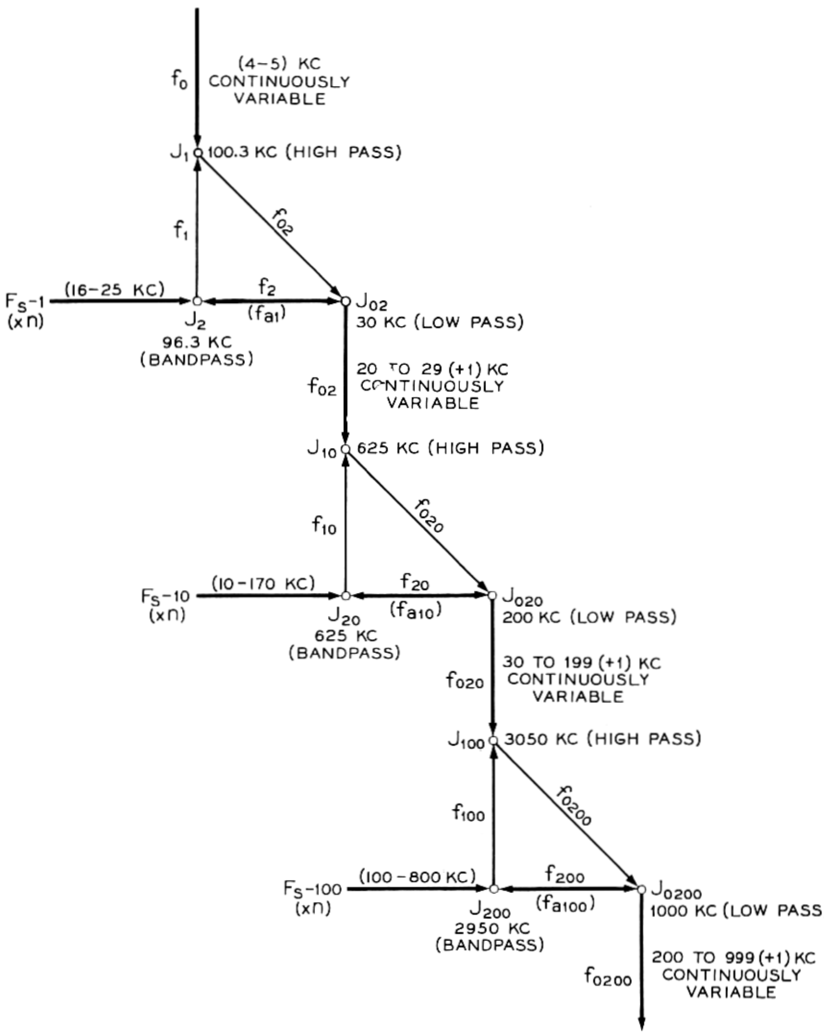


Fig. 8 — Symbolic diagram of a multidecade frequency-combining system.

III. CONCLUSIONS

The theory of a frequency-combining network presented here leads to a general solution of the problem of adding or subtracting two signal frequencies, independently of their ratio, while using electrical wave filters of a practical design with arbitrarily limited frequency discrimination characteristics.

A special symbolic network, established by means of a "sideband algebra", is applicable to signal frequency-combining problems. From this network not only can the minimum number of basic circuit elements be found, but also the fundamental circuitry and all its possible practical variations can be derived and scrutinized.

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