High-Frequency Negative-Resistance Circuit Principles for Esaki Diode Applications

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Certain fundamental principles are presented for analyzing and designing high-frequency amplifiers and oscillators utilizing simple negative resistance elements such as the Esaki or tunnel diodes. The first part of the paper covers the conditions necessary for oscillation and amplification with a single negative-resistance diode, including stability criteria, gain and bandwidth. It is shown that the highest-frequency circuits require diodes with very small dimensions, so that a single-spot diode will have a very low power capacity. In order to obtain higher power at high frequencies, distributed circuits must be used, either with narrow-strip diodes or a multiplicity of small spot diodes. Such circuits present special stabilization problems in suppressing unwanted modes of oscillation. Methods of avoiding such difficulties are presented for one-port oscillator circuits and for traveling-wave amplifier circuits. In the latter case, nonreciprocal attenuation of the gyromagnetic type is recommended.

I. INTRODUCTION

The Esaki[1] diode (or tunnel diode) exhibits a negative-resistance characteristic in the forward-biased region as shown in Fig. 1. This is a "voltage-controlled" type of characteristic in that the current is a single-valued function of voltage. Fig. 1 is a static curve, but the negative resistance is believed to remain effective at extremely high frequencies. Oscillation in the microwave range has been observed by several workers.[2,3,4] It is suspected that useful negative-resistance effects will also become obtainable in the millimeter wave range as our technology improves.

There is also a capacitance across the junction. This is quite high by comparison with other junction diodes, when measured per unit area

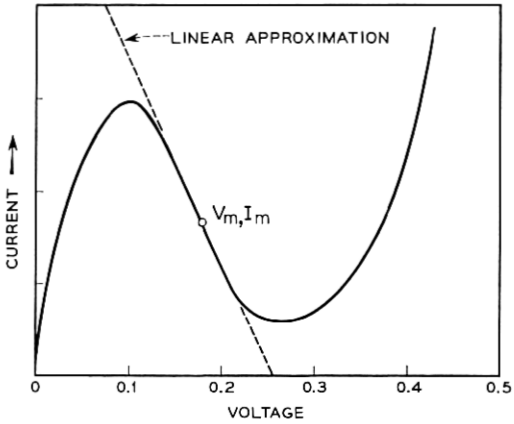


Fig. 1 — The current-voltage characteristic of an Esaki diode. The general curve shape is variable to a small degree, and the current density to a large degree, depending upon the semiconductor materials and processing. The total current is also proportional to diode junction area.

of the junction. The negative conductance, however, is also high, so that the negative time constant (negative R times C) is usually substantially less than 10^{-9} seconds. Negative time constants on the order of 10^{-11} to 10^{-12} seconds are believed to be obtainable in special diodes using intermetallic semiconductor compounds such as indium antimonide.[5]

The basic purposes of this paper are to evaluate the Esaki diode principle as a useful element in practical microwave devices, to show its limitations and capabilities and to present certain elementary device design methods for microwave oscillators and amplifiers. We will discuss only the circuit aspects of Esaki diodes as negative-resistance devices at high frequency. The solid state physics of the tunneling process applicable to these diodes has been described by others.[1,6,7] Microwave devices must include substantial parts of the high-frequency circuits, in a manner similar to that of microwave electron tubes. Suitable circuit geometries will be described and analyzed, taking into account the junction capacitance, negative resistance, parasitic resistance, load coupling, etc. This work is mostly theoretical and little experimental work is described.

The paper begins with a discussion of the stability criteria for simple negative-resistance circuits, which presents the appropriate relationships among the circuit parameters for amplification, oscillation and switching. The analysis includes the limiting effects of circuit and load resistance, diode capacitance and negative resistance. It is shown that

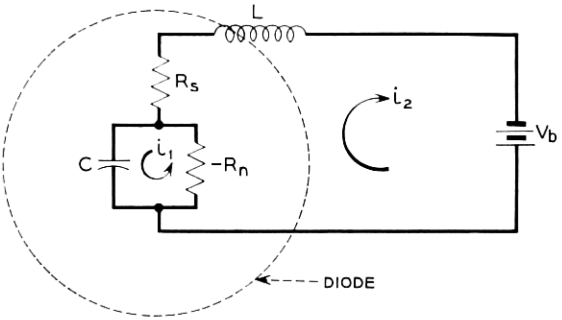


Fig. 2 — A simple equivalent circuit of an Esaki diode connected to a battery. Analysis for stability must include the lead inductance L, the parasitic resistance R_s and the capacitance C, as well as the negative resistance R_n.

very small diode dimensions will be required for high microwave frequencies.

If appreciable microwave power is to be obtained, distributed circuits will be required. These can take the form of extended narrow-strip diode junctions or a multiplicity of small-spot diodes in a filter-type structure. Such circuits pose difficult problems in device design and fabrication, and special precautions are necessary to avoid oscillation in spurious resonant modes. A substantial part of this paper is devoted to the latter problem, and several circuit possibilities are described.

II. SIMPLE NEGATIVE-RESISTANCE CIRCUITS

2.1 *Basic Stability Criteria*

We will begin with the simplest possible circuit, which is simply a diode connected to the terminals of a battery. This circuit, shown in Fig. 2, also may be interpreted to include a number of practical circuits in which additional inductance and resistance have been added. A meaningful analysis must include the finite lead inductance, the inductance of the battery loop, and at least the inherent passive resistance of the battery and diode.

R. L. Wallace has derived (4) below in unpublished work. We will repeat this and give a concise interpretation in the form of a stability diagram.

The V–I curve of Fig. 1 is nonlinear. We use a linearizing approximation valid in the immediate vicinity of the operating point V_m, I_m, shown in Fig. 1,

$$i = I_m - \frac{V - V_m}{R_n}, \tag{1}$$

where i and V are the instantaneous current and voltage, and the negative resistance R_n is the inverse of the V–I slope at the operating point. In a straightforward manner, one can write two loop equations and make appropriate elementary substitutions to obtain a differential equation for the current i_2 in the battery loop. This is

$$-L\frac{d^2 i_2}{dt^2} + \left(\frac{L}{R_n C} - R_s\right)\frac{di_2}{dt} + \frac{R_s - R_n}{R_n C}\, i_2 = \frac{1}{R_n C}\,(V_m - V_b + R_n I_n). \tag{2}$$

The general solution of the above equation is

$$i_2 = A_1 e^{p_1 t} + A_2 e^{p_2 t} + \frac{V_b - V_m - R_n I_m}{R_s - R_n}, \tag{3}$$

where the third term is the dc bias current and p_1 and p_2 are the two values (taking the + and − signs) given below.

$$p_{1,2} = \frac{1}{2}\left(\frac{1}{R_n C} - \frac{R_s}{L}\right) \pm j\sqrt{\frac{1}{LC}\left(1 - \frac{R_s}{R_n}\right) - \frac{1}{4}\left(\frac{R_s}{L} - \frac{1}{R_n C}\right)^2}. \tag{4}$$

In (3), A_1 and A_2 are arbitrary constants depending upon the initial current in the inductor and charge on the capacitor. The exponential factors p_1 and p_2 may be real, complex or imaginary, depending upon the choice of circuit parameters. If either value has a positive real part, the circuit will be unstable. If the p's are real, an initial disturbance will either grow or decay exponentially to the steady bias condition. If the p's are complex, the transient waves will be growing or decaying sinusoids.

Equation (4) has four parameters. We can reduce this to a two-parameter function by the substitutions

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{5}$$

and

$$Q_n = \omega_0 R_n C = \frac{R_n}{\sqrt{\frac{L}{C}}}. \tag{6}$$

These yield

$$\frac{p}{\omega_0} = \frac{1}{2Q_n}\left(1 - \frac{R_s}{R_n}\,Q_n^{\,2}\right) \pm j\sqrt{\left(1 - \frac{R_s}{R_n}\right) - \frac{1}{4Q_n^{\,2}}\left(1 - \frac{R_s}{R_n}\,Q_n^{\,2}\right)^2}. \tag{7}$$

Fig. 3 shows a set of curves for the above function, and Fig. 4 is a stability diagram suggested by W. W. Anderson. These show that the circuit will be unstable if the ratio of R_s to R_n is either too small or too large, or if the ratio of inductance to capacitance is too large. For many Esaki diodes, the negative conductance is so large that the inductance of the shortest pigtail leads is sufficient to cause oscillations, and a special low-inductance case is necessary to allow stable biasing at the operating point. For operation at VHF or lower frequencies, it is usually necessary to add capacitance in order to permit a lower inductance, and thereby get a value for Q_n that is large enough to obtain sinusoidal operation as opposed to exponential or "blocking" type oscillations. A typical value for the product R_nC might be 2×10^{-10} in a germanium Esaki diode. For such a diode the maximum allowable inductance for sinusoidal oscillations would resonate with the capacitance at about 400 mc. If a

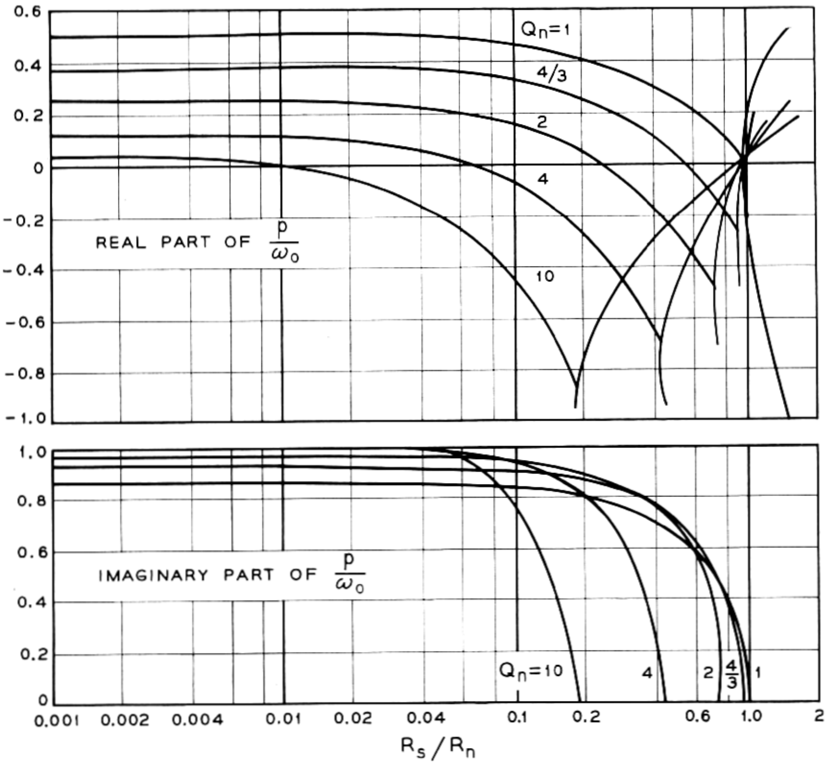


Fig. 3 — Exponential transient characteristics for the circuit of Fig. 2. Transient currents vary as e^{pt}. Here ω_0 is the resonant frequency of L and C, and Q_n is defined as $\omega_0 R_n C$.

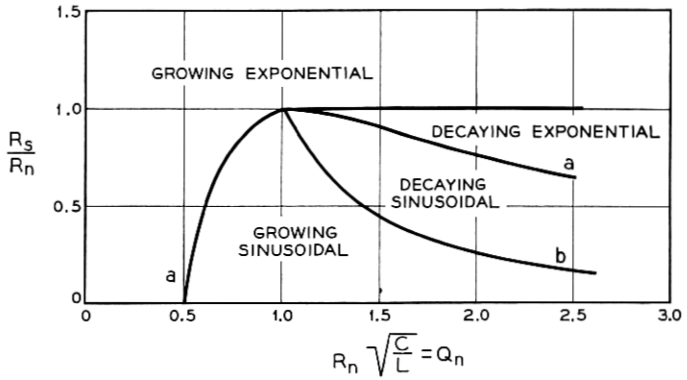


Fig. 4 — A regional stability chart for the circuit of Fig. 2, showing the allowed ranges of the parameters for particular types of transient waves. For curve a, $R_s/R_n = 2/Q_n - 1/Q_n^2$; for curve b, $R_s/R_n = 1/Q_n^2$.

stable condition is required, the resonant frequency must be at least twice this value, and preferably several times higher. If extra capacitance is to be added, it must be connected without adding appreciable inductance between the diode and the capacitor. One method would be to include the capacitance in the diode case or to mount the diode between the plates of a capacitor.

2.2 *An RF Circuit with External Battery*

It is seldom possible or desirable to include the battery or power supply in the high-frequency portion of an RF circuit. It is possible to isolate the RF region through the use of a bypass capacitor and an RF choke, but special precautions are necessary to avoid instabilities because of the inductance of the choke or of power leads alone. The use of capacitance to stabilize negative-resistance circuits has been described by Thomas.[8]

Fig. 5 shows a simple and useful RF circuit with such isolation; the parts drawn with heavier lines are the RF region, with C_2 being a large bypass capacitor and L_2 the inductance of the power leads and any added choke coil. The load resistance R_L should be inductively coupled, either as shown or by mutual inductance through a transformer. A stabilizing resistance, R_{s2}, may simply be the internal impedance of the battery or power supply. The ranges of values for L_2, C_2 and R_{s2} are strictly limited; otherwise we may expect instabilities at low frequencies.

To analyze the circuit of Fig. 5, we assume that C_2 is very large and

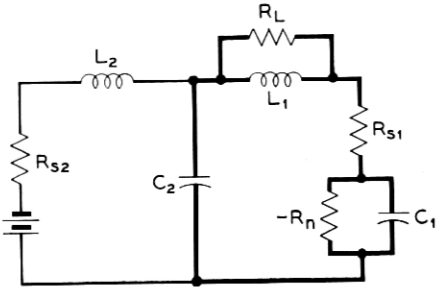
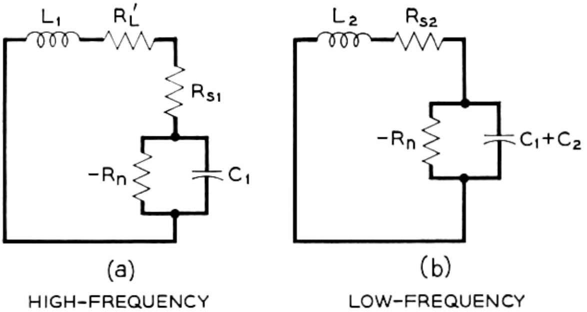


Fig. 5 — An RF Esaki diode circuit in which the RF and power supply are isolated at high frequencies. The bypass capacitor C_2 serves to confine the RF currents to the part of the circuit drawn with heavier lines. It is also helpful in stabilizing the circuit against instabilities involving the inductance of the power leads. Low-frequency stabilization can be further helped by shunting a low resistance across the capacitor C_2.

break the circuit into separate high-frequency and low-frequency equivalents as seen from the RF and dc terminals respectively. These are shown in Fig. 6. The circuits are in the same form as in Fig. 2, and we may use the criteria of Fig. 4 and (7) to determine the stability of each circuit and the nature of the transient waves involved. In the RF equivalent circuit we have replaced the load resistance by its series equivalent and assumed that C_2 is an RF short circuit. For the low-frequency equivalent circuit, we have assumed that L_1 is a short circuit. Four transient wave types will be obtained from these two circuits, and four would be obtained by an exact analysis of the complete circuit. We can presume that the results from the two equivalent circuits are close approxima-



(a)
HIGH-FREQUENCY EQUIVALENT

(b)
LOW-FREQUENCY EQUIVALENT

Fig. 6 — Two equivalent circuits applicable to Fig. 5. These may be separately analyzed by the criteria of Fig. 4 to determine the four transient wave types for the circuit of Fig. 5.

tions, provided that the values of p obtained are consistent with the assumptions about the low- and high-frequency impedance of L_1 and C_2 that allowed us to separate the circuits. For the low-frequency equivalent, the criteria for stability are

$$R_n \sqrt{\frac{C_1 + C_2}{L_2}} > 1, \tag{8}$$

$$R_n > R_{s2} > \frac{L_2}{C_1 + C_2} \frac{1}{R_n}. \tag{9}$$

It is possible to use a small stabilizing resistance R_{s2} provided that C_2 can be sufficiently large compared to L_2.

Fig. 7 shows a coaxial cavity version of the circuit we have been discussing in this section. An actual circuit of this type has been used by A. Yariv and E. Dickten at Bell Telephone Laboratories to obtain oscillation at frequencies above 8000 mc, using a germanium Esaki diode.

2.3 *Oscillation Conditions*

The conditions for obtaining oscillations can be deduced from the stability criteria; this requires a transient wave with a positive real part for p. If p is also complex, a growing sinusoidal transient is indicated.

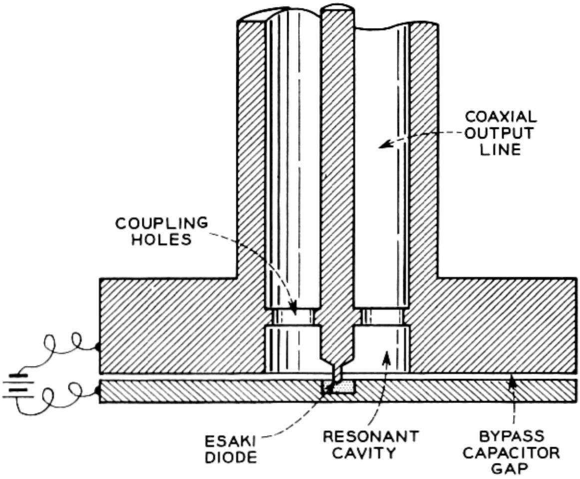


Fig. 7 — A schematic sketch of a coaxial cavity microwave circuit utilizing the isolation principle of Fig. 5. Oscillations at frequencies above 8000 mc have been obtained by A. Yariv and E. Dickten in a circuit of this type using a germanium Esaki diode.

Nonlinearities must limit the maximum growth of such a transient because large voltage swings will extend into the positive resistance region of the diode. Under these conditions, we may usually expect a steady oscillation. The region of growing sinusoidal transients is clearly shown in Fig. 4. Useful oscillations are also obtainable for low values of R_s/R_n and low Q in the lower left part of Fig. 4, where growing exponential transients are indicated. Because of the nonlinearity, we can expect the voltage to "switch" back and forth at high speed. Steady oscillations will not be obtained, however, for $R_s > R_n$. In that case, the circuit tends to stabilize at a voltage either above or below the region of maximum negative conductance. This condition is useful for logic circuitry.

2.4 *Amplification*

For amplification, a number of circuit configurations are possible. Fig. 8 shows the circulator method of obtaining useful gain with the negative resistance circuit of Fig. 5 or Fig. 7. The input wave passes through the circulator to the amplifier, and the reflected wave is diverted by the circulator to pass out by another port. When the impedances are properly adjusted, the circuit will be stable, but the reflected wave will be greater than the incident wave, resulting in a net power gain. Another method of obtaining amplification has been described and analyzed by Chang.[9] His scheme uses input and output lines separately coupled to the negative-resistance circuit.

A third method is to use a 3-db directional coupler or a hybrid junction (magic tee) with two negative-resistance circuits. This method is shown in Fig. 9. The waveguides connecting the tee to the amplifiers are unequal in length, with an additional one-fourth wavelength in one arm. This causes the reflections from the two amplifiers to be out of phase at the tee and pass out by the fourth port. Of the three methods, the circulator approach gives unilateral gain, and multiple stages can be used without increasing the danger of oscillation. The hybrid and two-port amplifiers are more sensitive to mismatches at input and output, and isolators are essential if a high total gain is obtained.

In order to analyze the reflection-type amplifier, we may conveniently use a shunt equivalent form, as shown in Fig. 8. Here we have a parallel-resonant combination and have replaced $-R_n$ by its inverse $-G_n$. We also include a shunt positive conductance G_p to account for parasitic losses in the diode and circuit.

The power gain of the circuit is given by the square of the magnitude of the voltage reflection coefficient Γ. This is the familiar expression

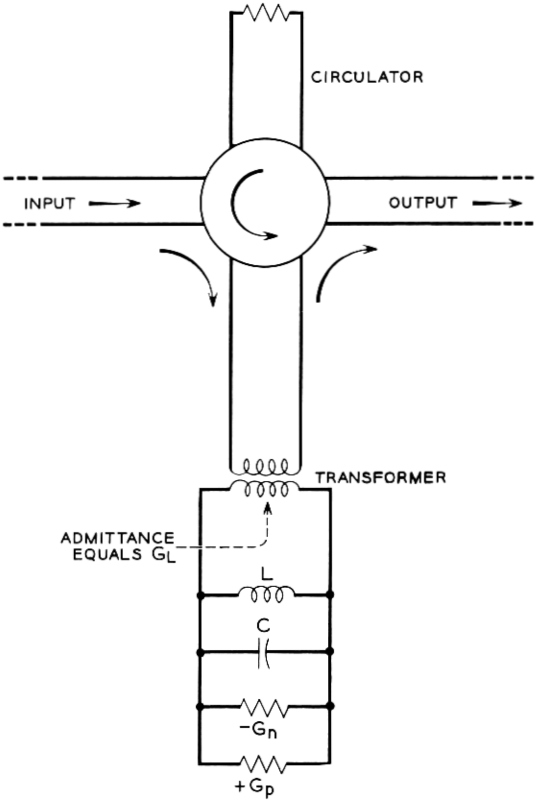


Fig. 8 — The circulator method of connecting a negative-resistance circuit to give useful amplification. The circuit of Fig. 7 is suitable for this kind of amplifier. An equivalent circuit in shunt form is shown for the amplifier in this figure.

$$\Gamma = \frac{G_L - Y}{G_L + Y}, \tag{10}$$

where G_L is the characteristic admittance of the connecting transmission line and Y is the admittance of the "amplifier" circuit that forms the terminating admittance of the line. For the circuit of Fig. 8,

$$Y = G_p - G_n + j\omega C - \frac{j}{\omega L}, \tag{11}$$

so that the power gain g_p will be

$$g_p = \left| \frac{G_L - G_p + G_n - j\omega C + \dfrac{j}{\omega L}}{G_L - G_n + G_p + j\omega C - \dfrac{j}{\omega L}} \right|^2 . \tag{12}$$

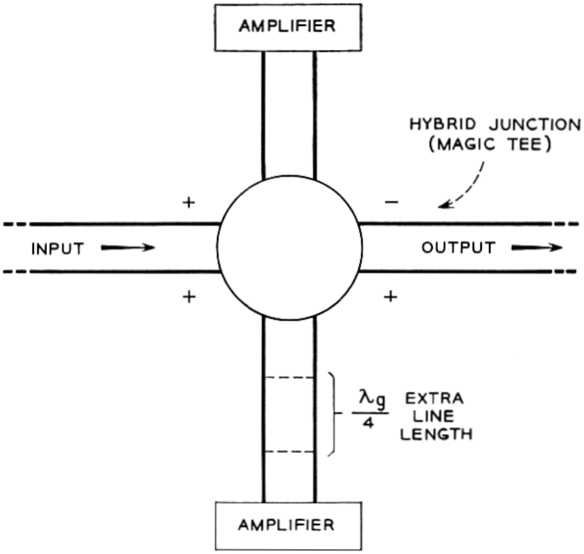


Fig. 9 — The hybrid-junction (magic-tee) method of obtaining amplification using two negative-resistance circuits. The extra length in the lower arm reverses the phase of one reflected signal so that the two reflected waves will combine in the output line of the hybrid junction. A 3-db directional coupler may be used as an alternative to the hybrid, but the proper phase conditions will depend upon the particular coupler used.

The gain will be greater than unity if G_n is greater than G_p and will approach infinity at resonance if G_n approaches G_p plus G_L. Oscillations will occur if G_n is greater than G_p plus G_L.

As in other negative-resistance amplifiers, the bandwidth decreases as the gain is increased. If a simple resonant circuit is used and the gain is high, the product of voltage gain and bandwidth is more or less invariant as the gain is changed over a wide range. At high gain, G_L is approximately equal to $G_n - G_p$, so that at resonance the voltage gain is approximately

$$g_v = \frac{2(G_n - G_P)}{G_L - (G_n - G_P)} \tag{13}$$

and the 3-db bandwidth is

$$B_{3\text{db}} = \frac{f_0}{Q_{\text{net}}} = f_0 \frac{G_2 - (G_n - G_P)}{\omega_0 C}. \tag{14}$$

The gain-bandwidth product, therefore, is

$$g_v B_{3\text{db}} = \frac{G_n - G_P}{\pi C}. \tag{15}$$

In the special case of negligible parasitic resistance,

$$g_v B_{3\text{db}} = \frac{G_n}{\pi C} = \frac{1}{\pi R_n C}. \tag{16}$$

As shown by Seidel and Herrmann,[10] however, this limitation does not apply for circuits of greater complexity.

2.5 *Maximum Frequency and Diode Geometry*

At present there are no indications of frequency limitations in the microwave range in the negative resistance of the diode junction itself. We assume here that the only significant frequency limitations are those resulting from diode capacitance and parasitic resistance. These are quite sufficient. In this section we shall derive an expression for the maximum frequency and show how it depends upon the characteristics of the diode junction and upon the diode geometry.

Let us assume that we are successful in obtaining a very small value for the series parasitic resistance R_s and that we are attempting to work at a high frequency where Q_n is also large. In this case, we can assume that

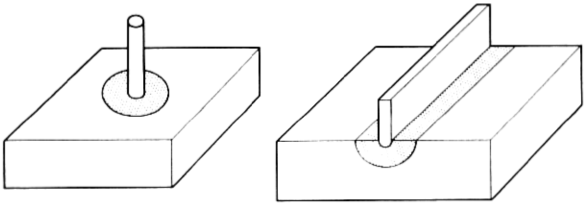
$$R_n \gg \frac{1}{\omega_0 C} \gg R_s. \tag{17}$$

For this condition, we can redraw the circuit as a shunt combination of a capacitor, a positive resistor and a negative resistor. To a close approximation, $-R_n$ and C will remain the same and we can take account of R_s by adding a shunt resistor of value $(\omega_0^2 C^2 R_s)^{-1}$. Thus, $\omega_0^2 C^2 R_s$ is the value of G_p in the circuit of Fig. 8. There is a value of ω_0 for which G_p is equal to G_n or R_n^{-1}, and at higher frequencies G_p will be greater. This is the maximum frequency for negative resistance effects, given by

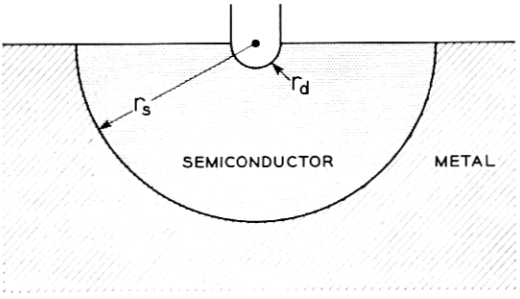
$$f_{\max} = \frac{\sqrt{\dfrac{R_n}{R_s}}}{2\pi R_n C}. \tag{18}$$

The denominator of this expression is dependent upon the properties of the junction and is independent of geometry. The numerator can be affected by the mechanical and electrical design of the diode mount and by the size and shape of the junction. In small alloy-junction diodes (most Esaki diodes have been of this type) most of the parasitic resistance will be found in the semiconductor material in the vicinity of the junction. This is the familiar "spreading-resistance" of point-contact diode theory.

In order to compare the useful types of diode geometry, we assume that this spreading-resistance is the only significant contributor. Actually, skin-effect is also important in many cases, but it can be treated by standard methods. Fig. 10 shows two widely divergent types of diode construction, idealized in shape in order to permit a simple analysis. For these cases we assume that the radius r_s is smaller than the skin depth in the semiconductor material, and that the metallic parts have negligible resistance. The junction has a capacitance of C_{d1} farads per square meter and a negative conductance of $-G_{d1}$ mhos per square meter. The semiconductor material has a resistivity of ρ ohm-meters. From these parameters, it is a straightforward matter to derive expressions for the maximum frequency for the two geometries of Fig. 10.



SPOT GEOMETRY LINE GEOMETRY



SECTION DETAIL

Fig. 10 — Two idealized types of Esaki diode geometry suitable for microwave applications. The circular cross section of the semiconductor is not likely to be a practical structure, but it has been assumed here to simplify the analysis and allow direct comparisons.

For the spot,

$$f_{\max} = \frac{G_{d1}^{\frac{1}{2}}}{2\pi C_{d1}\rho^{\frac{1}{2}}} \left[\frac{1}{r_d \left(1 - \frac{r_d}{r_s}\right)} \right]^{\frac{1}{2}}, \tag{19}$$

and for the strip,

$$f_{\max} = \frac{G_{d1}^{\frac{1}{2}}}{2\pi C_{d1}\rho^{\frac{1}{2}}} \left(\frac{1}{r_d \ln \frac{r_s}{r_d}} \right)^{\frac{1}{2}}. \tag{20}$$

The first term in each expression is the same, and is dependent upon the semiconductor and junction properties only. The second term depends upon geometry only. It is clear that r_d must be small for both cases if the semiconductor is of appreciable thickness. If $r_d/r_s \rightarrow 1$, the frequency limits are equal, but for small r_d/r_s the spot geometry has a higher limit. For example, if $r_d/r_s = 0.1$, the frequency limit for the spot case will be 1.6 times that for the strip case.

III. DISTRIBUTED CIRCUITS — GENERAL CONSIDERATIONS

In Section II the conditions for obtaining useful negative-resistance effects were presented. It was shown that single-spot diodes must be of small area if the highest frequencies are to be usable. It was also shown that narrow-strip diodes can be utilized at high frequencies and that these can have substantially greater power capacity. If significant amounts of power are to be obtained, it will be necessary to use a large number of single-spot diodes or to use narrow-strip diodes of appreciable total length. The remainder of this paper will be devoted to such distributed circuits.

A basic characteristic of distributed circuits in this sense is that more than one resonant mode is possible in the general frequency range of interest. With negative-resistance elements distributed along such a structure, spurious oscillations are a major hazard. This is the familiar *moding* problem which has plagued the development of multicavity magnetrons since the early days of World War II. If we attempt to design ordinary Esaki-diode distributed circuits of equivalent complexity, the problems are likely to be even more serious, because of the broadband nature of the Esaki-type negative resistance. There appear to be two methods of avoiding such difficulties. One is to use circuits of essential simplicity with few diodes, so that the modes are clearly distinct in character and/or frequency and can be separately damped by resistance. The other is to use traveling-wave circuits combined with a non-reciprocal (gyrator) type of attenuation. These are the methods to be discussed here.

IV. TWO-DIODE CIRCUITS

Fig. 11 shows a two-diode circuit for push-pull operation. This circuit has two resonant modes, which we will call the push-pull mode and the unison mode. In the former, the ac voltages are out of phase at the diodes and an ac voltage null is found at the center between the two inductors. The resistance R_s does not affect the Q of this resonance, but the load resistance R_L is energized instead. In the unison mode, the load resistance is not effective, but the alternating currents combine and pass through the resistor R_s. Thus, we have a kind of orthogonality distinguishing the two modes so that they are loaded separately. Fig. 11 shows two

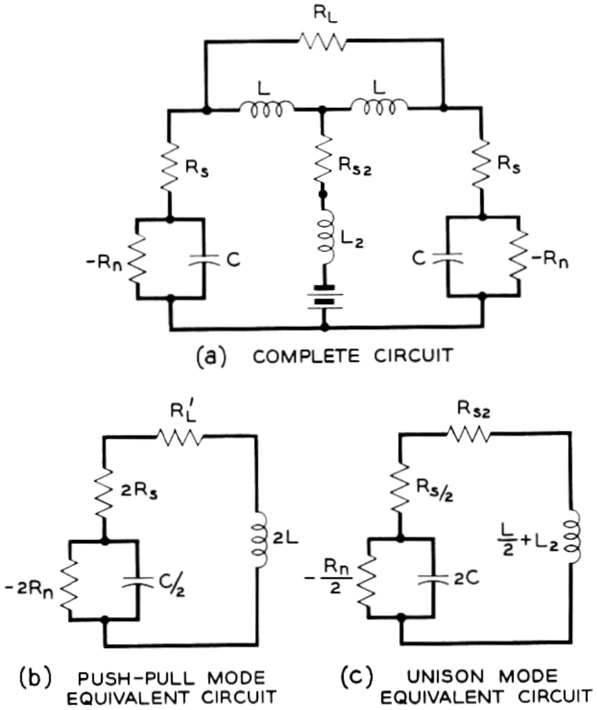


Fig. 11 — A simple push-pull circuit using two Esaki diodes. Two resonant modes are possible, and it is necessary to stabilize the unison mode that involves the power connections. The stability conditions may be determined from the two equivalent circuits shown.

equivalent circuits for the two modes. In Fig. 11(c) the load resistance R_L has been replaced by its series equivalent R_L'. As before, these equivalents can be analyzed for stability by the diagram of Fig. 4. The unison mode must be stable, and the push-pull mode should give a growing sinusoidal transient if oscillations are desired.

Figs. 12 and 13 show two possible microwave applications of the orthogonal mode-separation principle. In Fig. 12 we have two diodes in a double-ended coaxial cavity operating in a "half-wave-length" mode. This circuit is similar to one suggested by R. L. Wallace. The desired push-pull resonance is similar to that of a half-wavelength coaxial line with the ends open-circuited. The actual cavity would be much shorter than a half wavelength because of the excess capacitance at the ends that would be provided by the diodes. Power is fed through a resistor to the middle of the center conductor. This point is a voltage null for the desired push-pull mode. The inductance of this resistor must be included in the equivalent circuit [Fig. 11(c)], and we can analyze the unison mode including the bypass capacitor and power leads in the manner of Fig. 6. Coupling to a waveguide can be accomplished through a window as shown.

Fig. 13 shows a strip-line type of circuit using two diodes, which are mounted on short posts between the plates of the line. The strip line should be sufficiently narrow that it will propagate only the TEM mode at the frequency of interest. For the desired push-pull mode of operation we will have a local resonant circuit involving the inductance of the two posts and the capacitance of the diodes. This mode, if symmetrical, cannot excite a wave in the strip line. The unison mode, however, is

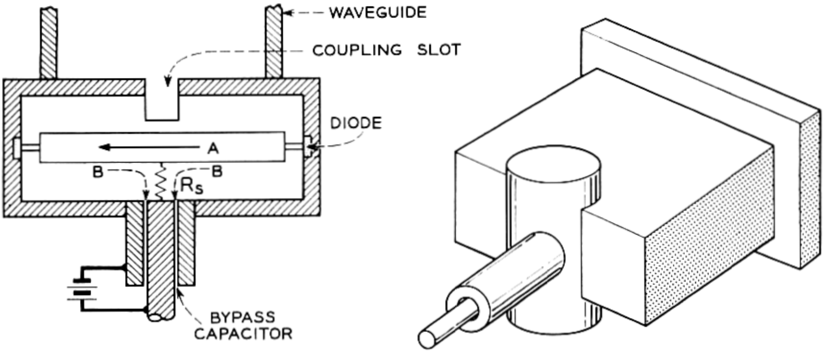
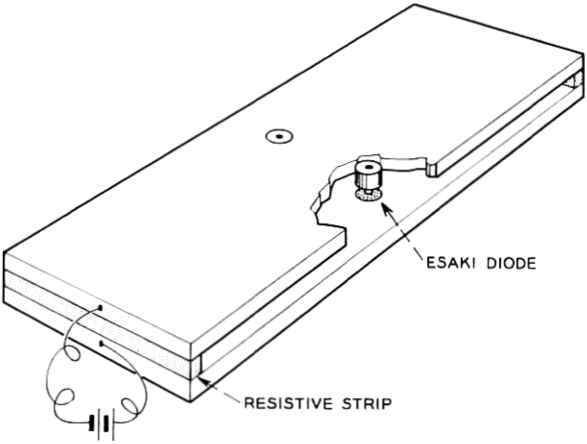
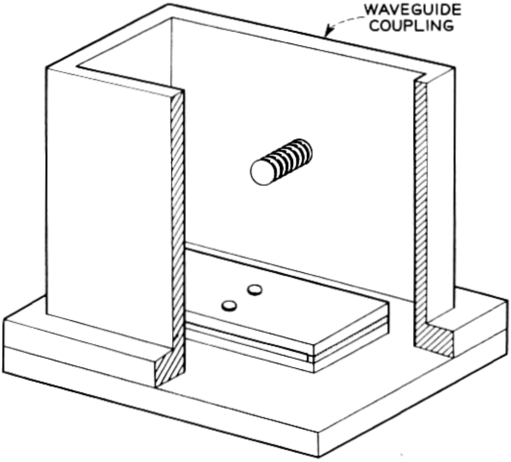


Fig. 12 — A possible coaxial cavity arrangement for a push-pull microwave circuit. The window between the cavity and waveguide provides the loading resistance. The resistance in the cavity stabilizes the unison mode without loading the push-pull mode.

directly coupled to waves on the line, and oscillations can be suppressed by choosing an appropriate matching resistance at a suitable distance from the diode. Also shown, in Fig. 13(b), is a method of mounting the strip line across the end of a waveguide to provide a method of output coupling. If the strip lines are terminated in resistive films at the corners



(a) STRIP–LINE CIRCUIT



(b) METHOD OF MOUNTING

Fig. 13 — A possible push-pull circuit in strip-line form. The strip resistances stabilize the unison mode, but the balanced push-pull mode cannot induce waves along the strip line. The push-pull mode can be coupled to a waveguide if the strip line is placed across the end as shown.

of the waveguide, these resistive films will not be coupled to the waveguide fields. The degree of coupling of the desired mode to the waveguide can be decreased or increased by using a wide or narrow strip line respectively. An alternative method would be to place an iris or post in the waveguide at some distance from the end, as shown.

V. TRAVELING-WAVE DISTRIBUTED CIRCUITS

Fig. 14 shows the simplest possible form of "smooth" distributed circuit. As drawn here, it would be unsuitable for practical use, since it does not include a method of applying dc power and avoiding instability in a uniform-phase mode. This is simply a block of semiconductor with an Esaki-type p-n junction separating p and n regions in the block. The barrier layer between the p and n sides can act as a gap, forming a kind of strip-line.

R. L. Wallace has analyzed this case in an unpublished work, and we will follow his line of attack, using MKS units. We will assume that the junction will act as a parallel-plate transmission line with a very narrow gap. This gap will be the barrier region of the diode, which may be as narrow as 50–100 angstroms. We will simply assume that the gap has a capacitance of C_{d1} farads per square meter and a negative conductance across the gap of $-G_{d1}$ mhos per square meter. Outside the gap on either side we will have a region of poor positive conductance. Skin effect in these regions will have a dominant effect on the transmission characteristics of the line.

As shown in Fig. 14, we assume a strip of width w, and a resistivity of ρ_1 and ρ_2 for the two outer regions. We will be concerned with the propagation of the TEM mode along the strip.

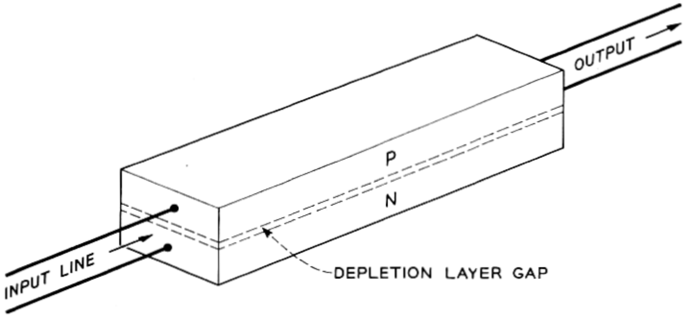


Fig. 14 — An Esaki p-n junction "strip line" that has been analyzed for traveling-wave type gain. This does not appear to be a practical geometry, but the results of the analysis are interesting in considering the stability of large-area Esaki diode junctions.

The familiar formula for skin-effect resistance is

$$R_{\text{sk}} = \sqrt{\rho\pi f\mu} \qquad \text{ohms per square,} \tag{21}$$

where ρ is in ohm-meters and μ is the free-space permeability (value: $4\pi \times 10^{-7}$ henrys per meter). There is also an inductive reactance associated with skin effect, which, in planar geometry, has the same magnitude as the resistance:

$$jX_s = j\sqrt{\rho\pi f\mu} \qquad \text{ohms per square.} \tag{22}$$

The inductance associated with the barrier-region gap itself can be neglected, because this gap is very narrow compared to the skin depth. An expression for the series impedance of the line is, therefore,

$$Z = \frac{1}{w}(1 + j)\sqrt{\pi f\mu}(\sqrt{\rho_1} + \sqrt{\rho_2}) \qquad \text{ohms per meter}, \tag{23}$$

where ρ_1 and ρ_2 are the semiconductor resistivities on either side of the junction.

The shunt admittance of the gap as a transmission line is the sum of the capacitive susceptance of the gap plus the negative conductance of the region as an Esaki diode,

$$Y = (jw\omega C_{d1} - G_{d1}w) \qquad \text{mhos per meter,} \tag{24}$$

where C_{d1} and G_{d1} are the capacitance and negative conductance per square meter respectively.

The propagation of waves along a transmission line is given by the expression

$$V(z,t) = V_0 e^{j\omega t \pm \sqrt{ZY}z}, \tag{25}$$

and we are particularly interested in the propagation constant, $\sqrt{ZY}$. If its real and imaginary parts have opposite signs, we can expect gain; if they are of the same sign, we will have loss. From (23) and (24) we obtain

$$\begin{aligned}\sqrt{ZY} &= [(1 + j)\sqrt{\pi f\mu}(\sqrt{\rho_1} + \sqrt{\rho_2})(j\omega C_{d1} - G_{d1})]^{\frac{1}{2}} \\ &= j[\sqrt{\pi f\mu}(\sqrt{\rho_1} + \sqrt{\rho_2})G_{d1}]^{\frac{1}{2}} \\ &\qquad \cdot \left[1 + \frac{\omega C_{d1}}{G_{d1}} + j\left(1 - \frac{\omega C_{d1}}{G_{d1}}\right)\right]^{\frac{1}{2}}.\end{aligned} \tag{26}$$

We will obtain gain if the real and imaginary parts of $\sqrt{ZY}$ are of opposite sign, which requires the imaginary part inside the second brackets to be positive. Thus, the highest frequency for gain is

$$f_{\max} = \frac{G_{d1}}{2\pi C_{d1}}. \tag{27}$$

This frequency is independent of the strip width (for our assumed wide strip) and depends only upon the properties of the junction and the semiconductor materials involved. In comparison with the results of the spot and narrow-line geometry cases, we are much more severely limited here.

It is a straightforward matter to compute the wavelength, gain per wavelength and "characteristic impedance" of this kind of structure as a transmission line. These are given by the expressions

$$\text{wavelength} = \frac{2\pi}{\text{imaginary part of } \sqrt{ZY}} \quad \text{meters}, \qquad (28)$$

$$\text{gain per wavelength} = 54.7 \frac{\text{real part of } \sqrt{ZY}}{\text{imaginary part of } \sqrt{ZY}} \quad \text{db}, \qquad (29)$$

$$\text{characteristic impedance} = \sqrt{\frac{Z}{Y}} \quad \text{ohms} \quad \text{(will be complex)}. \qquad (30)$$

A little computation will show that the wavelength will be much shorter than that in free space, that the gain will be very high (for frequencies below the maximum frequency) and that the characteristic impedance will be very low if the strip has appreciable width.

This wide-strip diode geometry does not appear to be particularly interesting at this time for a useful traveling-wave device. The frequency limitation is relatively low, the impedance will be *very* low and there will be serious stability problems in providing a suitable power feed.

A very-narrow-strip diode geometry appears to be more promising. One method of using such a strip diode would be as shown in Fig. 15. Here we have a long, narrow negative-resistance diode placed along the center line of a metallic strip line of appreciable width. At high frequencies, this will propagate a growing wave whose phase velocity will be slower than the velocity of light because of the capacitance loading of the diode. For such slow wave propagation, the magnetic and electric fields in the gap will decay exponentially in the transverse direction away from the diode strip. The field patterns are shown in Fig. 16. At high frequencies, the fields will decay more rapidly with transverse distance than at low frequencies. Thus, a resistive shunt can be placed continuously along both outer edges of the strip line where the high-frequency fields will be weak, and this will not cause a serious amount of high-frequency attenuation. At low frequencies, however, this shunt can cause very substantial attenuation, and if the net positive conductance per unit length exceeds the net negative conductance there will be no problem with low-frequency stability or in connecting the power leads.

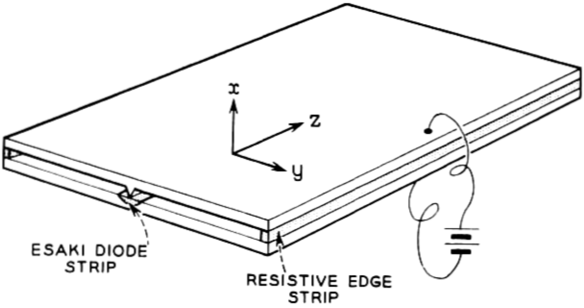


Fig. 15 — A possible traveling-wave Esaki diode amplifier using a narrow-line junction diode. The edge resistances can stabilize the circuit at low frequencies and allow the connection of power leads. At high frequencies, the ac fields are confined to the region near the diode strip, so that transmission is little affected by the resistance strips.

This mode of propagation will be substantially different from the TEM mode of a simple strip line with a central *metallic* ridge with a narrow gap, for which we would expect no velocity reduction. This diode strip has a capacitance corresponding to the barrier gap that is in the order of 10^{-6} cm, while the inductance for currents along the diode

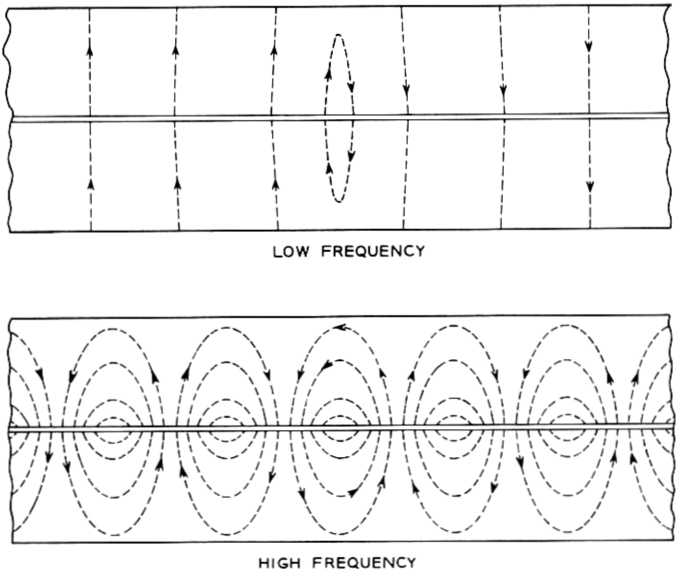


Fig. 16 — The ac magnetic field configuration in the gap region of the circuit of Fig. 15. At high frequencies the ac fields decay exponentially with distance from the center line.

strip corresponds to a gap equal to the skin depth in the semiconductor which is in the order of 10^{-2} to 10^{-3} cm. Thus, propagation would more closely resemble that for a strip line loaded capacitively with a central web of dielectric with a high dielectric constant.

The field pattern in the gap region outside the diode will be a TE mode. If we assume perfectly conducting walls, the field equations in MKS units are

$$H_z = Ae^{j\omega t-\gamma z+qy} + Be^{j\omega t-\gamma z-qy}, \tag{31}$$

$$H_y = \frac{\gamma q}{\gamma^2 + k^2} (Ae_{as}^{j\omega t-\gamma z+qy} - Be^{j\omega t-\gamma z-qy}), \tag{32}$$

$$E_x = \frac{j\omega\mu q}{\gamma^2 + k^2} (Ae^{j\omega t-\gamma z+qy} - Be^{j\omega t-\gamma z-qy}), \tag{33}$$

where z is the direction of propagation, x is perpendicular to the plane of the strips and y is transverse in the plane of the strips. In the above, γ is the complex propagation constant we wish to determine, and

$$k^2 = \omega^2\mu\epsilon, \tag{34}$$

$$q = \pm j\sqrt{\gamma^2 + k^2}. \tag{35}$$

If there were little gain or attenuation in propagation, q would be very nearly purely real, and would be substantially greater than k at high frequencies. The propagation constant γ would be nearly purely imaginary, and would have a magnitude nearly equal to q at high frequency.

We wish to determine γ and q when we include the negative conductance along the center, the spreading resistance, the skin resistance of the strip line and the positive conductance along the edges. These resistances will modify γ so that it will have a finite real part, indicating gain or loss for traveling waves. The method of attack is to find the admittance per unit length looking outward from the diode strip as a function of q and ω. This admittance must be the negative of the admittance per unit length of the diode strip, giving an equation for q as a function of frequency. This is based upon an argument that the current leaving the diode must enter the strip line, and, if a voltage is to exist, the admittances must be equal but of opposite signs. There is a hidden assumption here that the longitudinal current under the diode junction is negligible. This will be valid provided that the diode strip is *very* narrow compared with a wavelength on the structure and the strip-line gap is quite thin and of the same order of magnitude as the skin depth in the semiconductor under the junction. In this case, there will be a low-impedance current path from one part of the diode to another through the strip-line gap and only a small fraction of the total z-directed current will be in the semiconductor material under the junction.

We can find the transverse admittance of the strip line by solving the boundary-value problem for the outer edge, temporarily neglecting skin-effect losses. At the outer edge we assume that the resistive film forms the total terminating admittance and that there is no y-directed current leaving the gap region. The wall resistivity at the boundary is taken as

$$\rho_s = \left(r\sqrt{\frac{\epsilon}{\mu}}\right)^{-1} \qquad \text{ohms per square,} \tag{36}$$

where $1/r$ is the dimensionless ratio of the free-space impedance $\sqrt{\mu/\epsilon}$ to the actual resistivity ρ_s. At the boundary where $y = y_0$, therefore,

$$\left(\frac{H_z}{E_x}\right)_{y=y_0} = r\sqrt{\frac{\epsilon}{\mu}}. \tag{37}$$

Using (31) and (33), we can write the boundary equation as

$$\left(\frac{\gamma^2 + k^2}{(j\omega\mu q)}\right)\frac{Ae^{qy_0} + Be^{-qy_0}}{Ae^{qy_0} - Be^{-qy_0}} = r\sqrt{\frac{\epsilon}{\mu}}. \tag{38}$$

We may substitute for $(\gamma^2 + k^2)$ from (35) to obtain

$$\frac{\dfrac{A}{B}\,e^{2qy_0} + 1}{\dfrac{A}{B}\,e^{2qy_0} - 1} = -jr\frac{k}{q}, \tag{39}$$

from which we may solve for A/B, obtaining

$$\frac{A}{B} = e^{-2qy_0}\,\frac{jr\dfrac{k}{q} - 1}{jr\dfrac{k}{q} + 1}. \tag{40}$$

The admittance at $y = 0$ is obtainable by substituting the above and performing some algebra, giving

$$(Y_s)_{y=0} = \frac{1}{h}\left(\frac{H_z}{E_x}\right)_{y=0} = \frac{r}{h}\sqrt{\frac{\epsilon}{\mu}}\,\frac{1 - j\dfrac{q}{rk}\tanh qy_0}{1 + j\dfrac{rk}{q}\tanh qy_0} \qquad \text{mhos per meter.} \tag{41}$$

We must also include the diode spreading resistance and the effect of skin resistance in the strip-line region. The spreading resistance under the diode can be taken as a long strip resistance with a conductivity

G_{s1} mhos per meter of length, and we may postulate another strip resistance with a conductance of G_{s2} mhos per meter to account for the skin resistance of the strip line. We include these terms to obtain the total admittance for the circuit looking outward from the diode junction:

$$Y_c = \frac{1}{\dfrac{1}{G_{s1}} + \dfrac{1}{2G_{s2}} + \dfrac{1}{2Y_s}} \qquad \text{mhos per meter.} \tag{42}$$

The factors of 2 in the above account for the two sides of the strip line.

We can now write the admittance equation, which is solvable for q, the transverse propagation constant. This is

$$Y_d = -Y_c \tag{43}$$

or

$$\frac{1}{\dfrac{1}{G_{s1}} + \dfrac{1}{2G_{s2}} + \dfrac{1}{2Y_s}} = G_d - j\omega C_d\,, \tag{44}$$

where C_d and G_d are the capacitance and negative conductance per meter of length.

We need expressions for the terms G_{s1} and G_{s2}. The term G_{s1} is the inverse of the spreading resistance for one meter of diode length,

$$R_{s1} = \frac{1}{G_{s1}} = \frac{\rho}{\pi} \ln \frac{r_s}{r_d} \qquad \text{ohm-meters,} \tag{45}$$

where ρ is the resistivity of the semiconductor material. Both G_{s2} and Y_s are transcendental functions of wavelength and frequency and involve the unknown q. We will have to make further simplifying assumptions to obtain an equation which we can use readily. For this, we assume that the strip line is sufficiently wide and the frequency is sufficiently high that $\tanh qy_0$ can be taken as equal to one. This is the same as taking A/B equal to zero in (31). This assumption is valid for the high-frequency region, where we can expect substantial gain and where the ac fields are weak at the edges of the strip line. This approximation gives

$$(Y_s)_{\text{high freq.}} = -j\,\frac{q}{h\omega\mu}\,. \tag{46}$$

To determine $1/G_{s2}$ we use the same high-frequency approximation. We also assume that skin effect perturbs the field equations but little, and compute the skin-effect power loss per unit length by an integral over the two faces of the strip line,

$$P = 2 \int_0^{y_0(\to\infty)} R_{\mathrm{sk}}(H_y^2 + H_z^2)\, dy \qquad \text{watts per meter,} \tag{47}$$

where R_{sk} is the metallic skin resistance in ohms per square. We may note that H_z and H_y have nearly equal magnitudes at high frequency provided that $q^2 \gg k^2$. This allows us to integrate into a simple form:

$$P = \frac{2H_z^2(0)R_{\mathrm{sk}}}{\mathrm{Re}\,(q)} \qquad \text{watts per meter,} \tag{48}$$

where Re (q) is the real part of q. We postulate an equivalent strip resistance R_{s2} (or G_{s2}^{-1}) at the inside edge adjacent to the diode. This would give a power loss of

$$P = R_{s2}H_z^2(0) \qquad \text{watts per meter.} \tag{49}$$

Equating these powers gives an approximate value for R_{s2}:

$$\frac{1}{G_{s2}} = R_{s2} = \frac{2R_{\mathrm{sk}}}{\mathrm{Re}\,(q)} \qquad \text{ohm-meters.} \tag{50}$$

Let us now write (44) in a complete form valid at high frequencies only:

$$\frac{1}{\dfrac{\rho}{\pi} \ln \dfrac{r_s}{r_d} + \dfrac{R_{\mathrm{sk}}}{\mathrm{Re}\,(q)} + j\,\dfrac{h\omega\mu}{2q}} = G_d - j\omega C_d\,. \tag{51}$$

We may solve this for q by using still another approximation — that q is substantially a real quantity — and substitute q for Re (q). This is a valid approximation because the term involving Re (q) is small compared with the one involving q:

$$q = \frac{\omega^2 C_d\mu h}{2}\left[\frac{\left(1 - j\,\dfrac{2R_{\mathrm{sk}}}{\omega\mu h}\right)\left(1 + \dfrac{jG_d}{\omega C_d}\right)}{1 - \left(\dfrac{\rho}{\pi}\ln\dfrac{r_s}{r_d}\right)G_d + j\left(\dfrac{\rho}{\pi}\ln\dfrac{r_s}{r_d}\right)\omega C}\right]. \tag{52}$$

At high frequencies, the imaginary quantities in the numerator of (52) are small compared with one, and with low spreading resistance the second and third terms in the denominator are also small compared with one. If we bring up the denominator with a binomial expansion, multiply the result and drop terms involving the products of two or more small quantities, we obtain an approximate first-order result:

$$q \approx \frac{\omega^2 C_d\mu h}{2}\left(1 + \frac{jG_d}{\omega C} - j\,\frac{2R_{\mathrm{sk}}}{\omega\mu h} - j\,\frac{\rho}{\pi}\left(\ln\frac{r_s}{r_d}\right)\omega C_d + \frac{\rho}{\pi}\,G_d \ln\frac{r_s}{r_d}\right). \tag{53}$$

From q we can obtain γ by (35). This is

$$\gamma = \pm j\sqrt{q^2 + k^2}. \tag{54}$$

In the high-frequency region where (46) is valid, γ^2 will be much larger than k^2, so that

$$\gamma \approx \pm jq. \tag{55}$$

We will obtain gain if the real and imaginary parts of γ have opposite signs, and this requires a positive imaginary part in the expression for q given in (53). The maximum frequency for this is

$$f_{\max} = \frac{G_d}{2\pi C_d}\sqrt{\frac{1 - \dfrac{2R_{\text{sk}}C_d}{\mu h G_d}}{G_d \dfrac{\rho}{\pi} \ln \dfrac{r_s}{r_d}}}. \tag{56}$$

If we ignore skin-effect losses, this is the same limit we obtained in (20). This can easily be shown by substituting $G_d = \pi r_d G_{d1}$, $C_1 = \pi r_d C_{d1}$ and $R_{\text{sk}} = 0$.

There are several drawbacks to this structure as a practical amplifier. The major one is that the circuit will show gain in both directions and there will be considerable difficulty in stabilization. Slight mismatches will establish standing waves, and oscillations can occur if the total gain is substantial. Gain will be found over a wide frequency range, and it will be difficult to obtain a good match to input and output transmission lines throughout this range. The impedance of the structure is approximately proportional to frequency over a wide range, and this increases the matching difficulties.

A method of avoiding oscillations is to combine this circuit with a nonreciprocal attenuator to give a net loss in one direction and permit gain in the other. One method of accomplishing this is illustrated in Fig. 17. (This proposal was made in collaboration with W. W. Anderson of Bell Telephone Laboratories.) It involves the placement of thin ferrite slabs in both sides of the strip-line gap. The ferrite material is magnetized approximately in the $\pm x$ direction in a nonuniform manner by the shaping and placement of the magnetic pole pieces. This field is strong near the center and weaker near the edges of the strip line. This provides for a region of high-frequency magnetic resonance near the diode strip, continuously graded to a lower frequency resonance at greater distances from the center. It will be noted in (31) and (33) that the ratio of H_y to H_z at high frequency is

$$\frac{H_y}{H_z} = \pm j\frac{\gamma}{\sqrt{\gamma^2 + k^2}} \tag{57}$$

provided that A/B can be taken as approximately zero. This indicates that the magnetic field will be elliptically polarized. If the wave velocity is so slow so that $\gamma^2 \gg k^2$, the polarization will be quite nearly circular. The ellipticity will be pronounced only at low frequencies, where the wave velocity is more nearly the velocity of light, and near the outer boundary, where Ae^{qy} may be roughly equal to Be^{-qy}. As in other devices of this type, the sense of rotation of the magnetic vector is reversed if the direction of propagation is reversed. This is the condition for nonreciprocal resonance absorbtion in ferrite materials that are magnetized perpendicular to the plane of polarization.

A unique property of this circuit is that the circularity condition is met over a wide frequency range and also over a large fraction of the available space. This allows us to obtain broadband isolation by using a nonuniform magnetization for the ferrite isolator, so that resonance for the highest frequencies occurs near the center and, at lower frequencies, at greater distances from the axis. The steady H_x field is to be nonuniform and vary roughly as y^{-1}. This requires that H_y be nonzero if Laplaces's equation is to be satisfied. However, if the slab is thin, the H_y component will be small and give little trouble.

A complete description and theory for this type of isolator is beyond the scope of this paper. The author is not aware of any theoretical solu-

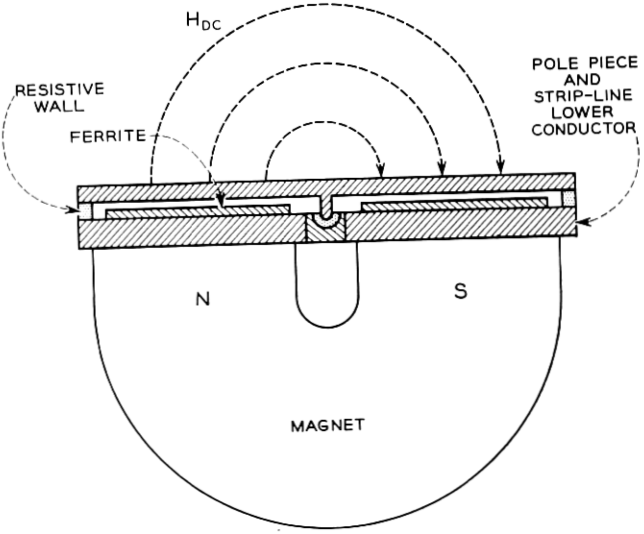


Fig. 17 — A proposed method of obtaining unidirectional gain for the circuit of Fig. 15. The ferrite acts as a broadband resonance isolator distributed along the circuit, introducing substantial attenuation for only one direction of wave propagation.

tion to the wave propagation problem in a gyromagnetic medium that is nonuniformly magnetized. However, initial experiments by W. W. Anderson and the author are most encouraging. An isolation tester was built using a closely-spaced array of passive capacitors of 7 $\mu\mu$f each, spaced 0.065 inch apart along the axis of a thin waveguide 2 inches wide and 0.025 inch high. This gave an upper cutoff frequency as a filter at about 8 kmc and a lower cutoff due to the metallic outer boundary at about 1 kmc. Through most of the intervening band, the wave propagation characteristics were similar to the continuously loaded strip line we have been discussing. The insertion of a single type of ferrite material into the 0.025-inch space on both side of the gap gave a maximum forward additional attenuation of 0.5 db per inch and a minimum additional reverse attenuation of 15 db per inch over the band from 1.5 kmc to 6.0 kmc. These measurements were made with a single setting of the magnetic field.

It is expected that the first practical traveling-wave amplifier of this type may involve the use of a closely spaced array of spot diodes rather than a continuous strip diode. If the gain per unit length is to be sufficiently low that the ferrite material can suppress reverse gain, it may be necessary to use relatively few active diodes, interspersed with passive capacitors to keep the wave velocity low.

VI. STRIP-DIODE OSCILLATOR CIRCUITS

The simplest method of obtaining oscillations with a narrow-strip diode might be to use a short section of a circuit like that of Fig. 15. The length ΔZ should be equal to one-half of a wavelength on the circuit at the desired frequency. In this case, the circuit should be shorter than one free-space wavelength at the *maximum frequency of amplification* as given by (56) if we desire absolute stability in the next-order mode of resonance. Probably, however, this limit can be exceeded to some extent in a practical oscillator, as nonlinearities are sometimes effective in allowing only one mode of oscillation at full power. Fig. 18 shows such a circuit, which can be mounted in a waveguide in a manner similar to the circuit of Fig. 13.

Fig. 19 shows a possible arrangement utilizing two strip diodes, intended to operate in a push-pull mode. This is still closer in concept to the circuit of Fig. 13 and could be mounted in the same way. It should operate at substantially higher frequencies than that of Fig. 18, and allow a greater total diode area. In this case, the current paths in the desired mode are shorter, reducing the effects of skin-effect resistance in the circuit.

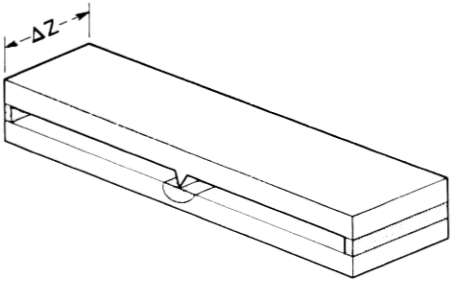


Fig. 18 — A possible high-frequency oscillator circuit using a narrow-line Esaki diode. This is simply a short section of the circuit of Fig. 15. The length ΔZ is one-half wavelength at the desired frequency. This could be mounted in the same way as the circuit of Fig. 13.

In the desired mode of operation, each of the two diode strips should have uniform phase along its length, but the two should be 180° out of phase with respect to each other. The slot between the diodes acts as an inductive chamber to resonate the capacitance of the diodes in series. We must be concerned also with three spurious modes if we desire to use the maximum possible diode strip length. The four lowest-order modes are illustrated in Fig. 20. Here the surface current flow is shown on one internal face of the strip line for each mode. In the desired push-pull mode, substantially all of the current flows directly across the inductive slot between the strips, but some small amount flows outward to charge the capacitance between the plates on the "land" regions on either side of the strips. Transverse slots are placed at each end of the strips to provide an inductive impedance for currents attempting end-runs around

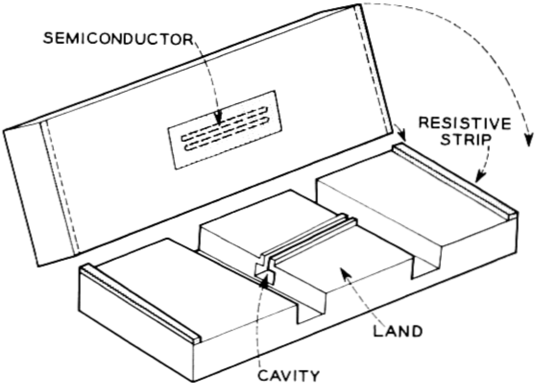
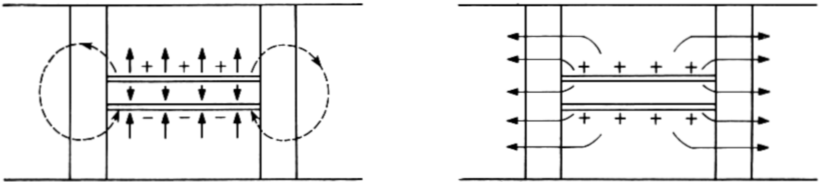
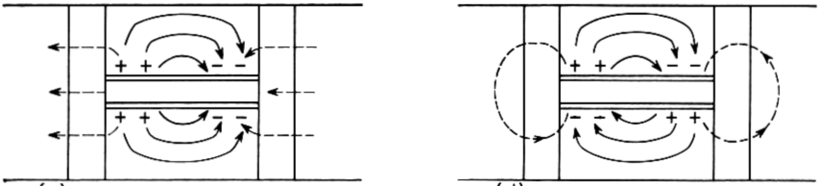


Fig. 19 — A possible push-pull circuit using two narrow-line Esaki diodes.



(a) DESIRED PUSH-PULL MODE

(b) UNDESIRED UNISON MODE



(c) UNISON STANDING-WAVE MODE, UNDESIRED

(d) PUSH-PULL STANDING-WAVE MODE, UNDESIRED

Fig. 20 — Four possible modes of oscillation for the circuit of Fig. 19. The surface current flow is illustrated for the semiconductor side of the strip line. The resistance strips can suppress the unison mode of (b). The standing-wave modes of (c) and (d) can be suppressed by sufficiently short strips and using close gap spacing in the "land" region of Fig. 19.

the ends of the strips from one side to the other. In the undesired unison mode the two diodes are in equal phase and this mode would launch waves along the strip lines, which must have very narrow gaps and low impedances if this mode is to be suppressed. The two other undesired modes involve standing waves along the diode junctions. In one, the opposing diode strips are in phase, and in the other they are 180° out of phase.

The problems of analysis will be to determine the frequency for the desired mode, to determine whether or not the desired oscillation will occur, and to determine whether or not oscillations in the three listed spurious modes will be troublesome. Of course, still higher-order standing-wave modes are possible, but these will be at still higher frequencies and therefore less troublesome.

For the desired mode, we can determine the inductance of the central cavity for a given length. At the desired frequency this inductive reactance should equal that of the capacitance of the two diodes in series. If the "lands" have appreciable capacitances, these must be added to the diode capacitances. The methods of accounting for spreading resistance and circuit resistance are similar to that used for the traveling-wave amplifier described in Section V.

For the undesired standing-wave modes we can determine the resonant frequencies by first considering that the circuit extends indefinitely in the direction of the diode strips. We can find two propagation characteristics of such a structure, one for the diodes operating in unison and one for push-pull phase opposition. This can be done by an extension of the methods of the last section, taking proper account of the central cavity. The resonant frequencies for a limited diode length will be those frequencies for which the diode strips are a half-wavelength in extent. These frequencies must be above the maximum frequency for gain as a traveling-wave device if there is to be no danger of oscillation in spurious modes. This condition will establish a maximum length for the diode strips. If the land regions have a very narrow gap, this will allow longer strips but will also increase the circuit losses. A compromise can be reached, however, which will allow a substantial diode length with only slight degradation of the desired operation.

This type of circuit has been analyzed rather completely in unpublished work by the author, but the theory is too lengthy to include in this paper. A particular fictitious example to operate at about 9000 mc would require an R_nC product of 10^{-10}, diode strips only 0.00016 inch wide and strip-line and land gaps of 0.00016 inch. The resonant cavity required would be approximately 0.003 × 0.003 inch in cross section! However, diode strips on the order of 3 mm long would be allowable without danger of standing-wave oscillations of the types shown in Fig. 20. For this example, the total diode direct current would be about 300 ma, and the power output might be a few milliwatts, compared to the few microwatts that could be obtained from a single-spot diode small enough to oscillate at this frequency. It is obvious that a practical utilization of this principle will involve some difficult fabrication problems.

A third method of using a narrow strip diode as an oscillator is to use an axially symmetric geometry with a ring-shaped diode. The structure is illustrated in Fig. 21. The desired mode of operation involves no angular variations of voltage around the ring, and the cavity is an annular space adjacent to the ring. Outside of the cavity, there is a large bypass capacitance.

In this structure, the ring diameter would be limited by the possibility of oscillations in modes involving angular variations of voltage around the ring. To assist in obtaining a larger ring diameter we can use a very narrow gap spacing in the region inside the ring. In a manner analogous to the action of the lands in the two-strip structure, this space acts as a low inductance transverse current path for standing-wave patterns around the ring, raising their resonant frequencies and inducing extra loss. In the desired mode, the central cavity region acts as an additional capacitance to be added to that of the diode. Current paths for the lowest-order mode of this type are shown in Fig. 21.

Methods of analysis for this structure in the desired mode are quite straightforward. The methods outlined in previous sections of this paper should be adequate. For the undesired θ-varying modes, we can use a method analogous to that used for the traveling-wave amplifier. A complete analysis will not be given here, but a method of attack will be described.

As before, we can determine the admittance per unit length along the diode ring looking inward and outward. Looking outward, we simply have the inductive reactance of the cavity plus the circuit losses. Looking inward, we have a simple capacitance for the desired unison mode. For the lowest-order standing-wave mode we can write the appropriate field equations for the space, presuming that E_z varies as

$$E_z = AJ_1(k_c r) \cos \theta, \tag{58}$$

from which we can determine H_r and H_θ.

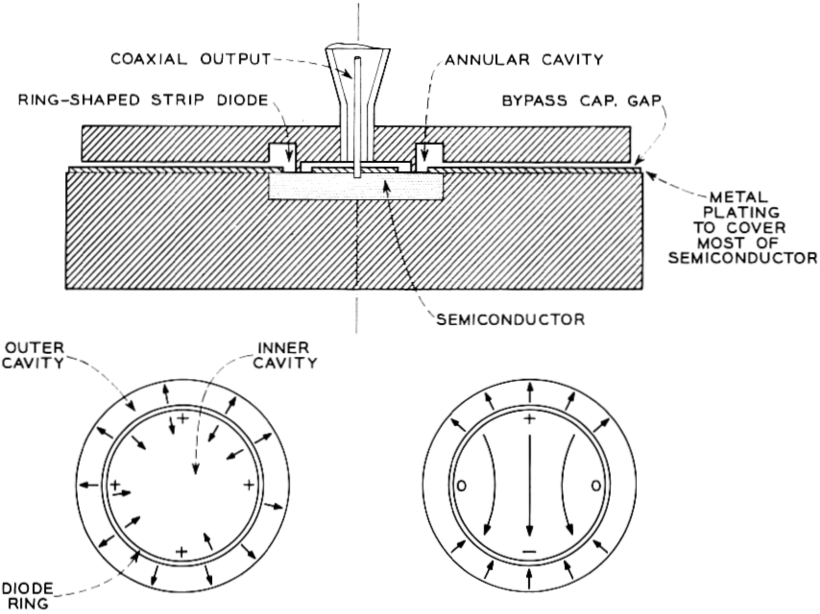


Fig. 21 — A possible oscillator circuit using a narrow-line Esaki diode in ring form. The desired mode involves uniform phase around the ring, as shown at the lower left. The undesired modes will involve θ-variations, as shown at the lower right. The latter can be suppressed by keeping the ring diameter sufficiently small and using a narrow gap in the inner cavity.

This will give the reactance for a unit length along the circumference,

$$jX = \frac{hE_2}{H_\theta} \qquad \text{ohm-meters,} \tag{59}$$

which will be inductive if the inner cavity is small compared to a wavelength.

The ring structure and the two-strip structure are quite similar circuits in many ways. The ring structure has a disadvantage in that it will be more difficult to obtain low circuit losses at the highest frequencies in the desired mode. The extra circuit losses will be found in the resonant cavity because it must have a larger ratio of perimeter to cross-section area, in the semiconductor material because the gap is not shared by two diodes in series, and in the bypass condenser that is avoided in the two-strip structure. It may also prove to be more difficult to arrange suitable coupling to a waveguide or coaxial line output for the ring structure. The coaxial output shown does not appear to be an entirely satisfactory answer.

VII. NOISE FIGURE FOR A SMOOTH DISTRIBUTED AMPLIFIER

In this section we will derive an expression for the noise figure of a rather generalized type of distributed amplifier in which isolation is not used. Chang[9] and Anderson and Hines[11] have derived expressions for the single-stage amplifier. We assume that we have a transmission line with a series reactance jX_1 and a series passive resistance of R_1 ohms per meter. The gain is provided by a distributed negative conductance of $-G_d$ mhos per meter, and we have an additional passive shunt conductance G_1 and a susceptance of jB_1 mhos per meter. We may use the well-known equation for such a line, which states that waves propagate as

$$\begin{aligned} V(z,t) &= V(0,0)e^{j\omega t-\sqrt{ZY}\,z} \\ &= V(0,0)e^{j\omega t-j\beta z+\alpha z}, \end{aligned} \tag{60}$$

$$Z = R_1 + jX_1\,, \tag{61}$$

$$Y = G_d + G_1 + jB_1\,. \tag{62}$$

The characteristic impedance is given by

$$Z_0 = R_0 + jX_0 = \sqrt{\frac{Z}{Y}} = \frac{V(z,t)}{i(z,t)}\,. \tag{63}$$

The noise figure of an amplifier system depends upon the termination

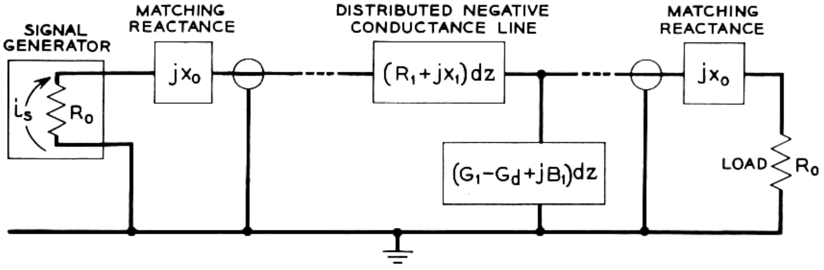


Fig. 22 — A generalized type of negative-conductance traveling-wave transmission line. Expressions have been derived for the noise figure of such a line.

presented at its source and in some cases at the load also. Ordinarily, matched impedances are considered desirable. In this case, the impedance of the amplifier line is complex, which presents a special problem, since we cannot match the amplifier impedance to the input and output lines for both directions of propagation. To prevent internal standing waves, a line with characteristic impedance $R_0 + jX_0$ should see an impedance $R_0 + jX_0$ at either end. In a conventional network, we would "match" by terminating with $R_0 - jX_0$ to obtain maximum power transfer; here, however, we wish to terminate with the characteristic impedance rather than its complex conjugate. Let us terminate our lines in this way, as shown in Fig. 22. This will cause some power reflection at the input but will avoid internal standing waves and undesired regeneration effects.

A current i which enters the amplifier grows with distance as

$$i(z) = i_1 e^{\sqrt{ZY}\, z} = i_1 e^{-j\beta z + \alpha z}. \tag{64}$$

At the output

$$i_2 = i_1 e^{\alpha l}, \tag{65}$$

where l is the total length. We define the "internal gain" g_i as

$$g_i = e^{2\alpha l}. \tag{66}$$

For the circuit of Fig. 22, the rms current source i_s for a sine-wave input of available power p_i is

$$i_s = 2\sqrt{\frac{p_i}{R_0}}, \tag{67}$$

which divides between R_0 and the input in proportion to the relative admittances. The current i_1 is easily determined to be

$$i_1 = \sqrt{\frac{p_i}{R_0}}\,\frac{R_0}{\sqrt{R_0^2 + X_0^2}}\,. \tag{68}$$

The output power is $i_2^2 R_0$, which determines the net gain g_p,

$$g_p = g_i\,\frac{R_0^2}{R_0^2 + X_0^2}\,. \tag{69}$$

We will use the following expression as our definition of noise figure F:

$$F = 1 + \frac{\text{output noise power from internal sources}}{(\text{net gain})\ (kT_0B)}\,, \tag{70}$$

where k is Boltzmann's constant, T_0 is the noise reference temperature (290°K) and B is the bandwidth of interest. Internal sources include the shot noise of the diode direct current plus thermal noise in the internal passive resistance. We assume a direct current in the diode of I_0 amperes per meter. For the diode noise we might assume shot noise or something proportional to shot noise. In an infinitesimal length dz, we can assume a differential amount of mean-square noise current,

$$d(\bar{\imath}^2) = \gamma^2 2eI_0B\,dz \qquad \text{amperes}^2, \tag{71}$$

where γ^2 is our unknown factor of proportionality and e is the electronic charge. This current from an infinitesimal section of line must divide equally into a forward and a backward wave, with the backward wave presumed to be lost. The forward waves from each length dz add at the output with an appropriate gain factor depending upon the distance from the point z and the output. We can find the total mean-square current at $z = l$ by a simple integration:

$$\begin{aligned}(\bar{\imath}_2^2)_{\text{shot noise}} &= \frac{\gamma^2 eI_0B}{2}\int_0^l e^{2\alpha z}\,dz \\ &= \left(\frac{\gamma^2 eI_0B}{2}\right)\left(\frac{g_i - 1}{2\alpha}\right) \qquad \text{amperes}^2.\end{aligned} \tag{72}$$

We can find the total mean-square current at $z = l$ for the thermal noise in the shunt conductance G_1 in the same manner. For an infinitesimal conductance, G_1dz, the noise can be considered to arise in a current generator,

$$d(\bar{\imath}^2)_{G_1} = 4kTBG_1\,dz, \tag{73}$$

and we integrate as before to obtain

$$(\bar{\imath}_2^{\,2})_{G_1} = kTBG_1 \frac{g_i - 1}{2\alpha} \qquad \text{amperes}^2. \tag{74}$$

For the series resistance R_1 we assume the alternative Thevenin form of a voltage source of noise for a length dz:

$$d(\bar{v}^2)_{R_1} = 4kTBR_1\, dz. \tag{75}$$

There is an impedance $2(R_0 + jX_0)$ in series with this generator, giving

$$d(\bar{\imath}^2)_{R_1} = \frac{kTBR_1}{R_0^2 + X_0^2}\, dz, \tag{76}$$

which we integrate as before to obtain

$$(\bar{\imath}_2^{\,2})_{R_1} = \left(\frac{kTBR_1}{R_0^2 + X_0^2}\right)\left(\frac{g_i - 1}{2\alpha}\right). \tag{77}$$

To obtain the total output noise power from internal sources we add the three mean-square currents and multiply by R_0. From (70) we obtain

$$F = 1 + \left(\frac{g_i - 1}{g_i}\right)\left(\frac{R_0^2 + X_0^2}{R_0^2}\right)\left(\frac{T_d}{T_0}\right)\left(\frac{R_0G_d}{2\alpha}\right) \cdot\left(\frac{\gamma^2 eI_0}{2kT_dG_d} + \frac{R_1}{G_d(R_0^2 + X_0^2)} + \frac{G_1}{G_d}\right). \tag{78}$$

It is evident that we want R_1 and G_1 to be small. These are parasitic elements which are generally undesirable. We would also like to reduce the magnitude of the term $eI_0/2kT_0G_d$. This term depends upon the properties of the p-n junction itself and is independent of junction area or circuitry. To obtain the lowest noise figure we should seek a diode which has a high negative conductivity per unit of direct current.

Equation (78) can be simplified for the limiting case of no internal passive resistance, that is for $R_1 = 0$ and $G_1 = 0$. This allows us to drop two of the three terms in the last factor. Also, the terms $(R_0^2 + X_0^2)/R_0^2$ and $R_0G_d/2\alpha$ are reciprocals in this special case and cancel.

This leaves

$$(F)_{\substack{R_1=0\\G_1=0}} = 1 + \left(\frac{g_i - 1}{g_i}\right)\left(\frac{\gamma^2 eI_0}{2kT_0G_d}\right). \tag{79}$$

This is essentially the same expression obtained by Hines and Anderson[11] for the case of a single diode amplifier without parasitic resistance.

VIII. GENERAL DISCUSSION AND CONCLUSIONS

In this paper at least some of the methods have been described by which we may hope to obtain appreciable amounts of power in the microwave range with circuits using Esaki diodes. The circuits proposed would utilize diodes in pairs, in narrow strip form, and in traveling-wave distributed circuits with ferrite nonreciprocal attenuation.

The situation appears very hopeful to the author. As our semiconductor technology improves, we should be able to develop useful solid state amplifiers and oscillators for the high microwave and probably for the millimeter-wave range as well.

The author suspects that solid state device research for the microwave and millimeter-wave range will probably continue to advance through the discovery of better methods of obtaining negative-resistance effects. Esaki diodes are usable devices at high frequencies, and we must exploit their possibilities. What we learn in the process about the useful application of negative resistance will probably be helpful in designing better devices to come.

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