# A Transversal Equalizer for Television Circuits

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(Manuscript received November 23, 1959)

*Transmission of television signals over long systems requires a fine degree of equalization. An adjustable type of equalizer to supplement fixed equalizers is discussed. It provides both gain and delay characteristics in the form of harmonically related cosine shapes which are independently adjustable. The design and operation of the equalizer is explained on the basis of paired echo theory. The 336A equalizer used in the L3 coaxial system is discussed as an example of this equalization technique.*

## I. INTRODUCTION

In an ideal transmission system the received signal is an exact replica of the transmitted signal. In all practical systems the received signal differs from the transmitted signal, due to imperfections of the transmission media. The lack of uniform gain and delay over the frequency band of the system constitutes a common form of imperfection. These gain and delay distortions are usually so severe as to require some form of equalization.* The amount of equalization required depends upon the nature of the signal being transmitted. Fortunately, most communication signals have qualities that make perfect equalization unnecessary.

For television transmission, the requirements are particularly stringent for both gain and delay equalization. Systems transmitting TV signals either as video circuits or carrier circuits usually use fixed gain and delay equalizers to correct the bulk of the system distortion and adjustable equalizers to correct the residual.

The residual gain and delay distortions will vary with system aging and manufacturing variations; thus, an unpredictable amount of equalization is necessary. These distortion characteristics are arbitrary functions of frequency, and will change from time to time. This requires

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* A theory of equalization of complex systems is described in Ref. 1.

that the adjustable equalizer be flexible enough to equalize arbitrary distortion characteristics, but still be easy to adjust.

It is well known that any reasonable function over a finite interval can be mathematically approximated by means of a finite number of terms of an infinite series. The degree of approximation determines the number of terms of the series. The Fourier cosine series is particularly useful because its orthogonal properties make the coefficients of each term independent. Furthermore, methods of producing cosine shapes of gain and delay are known. So a device that produces adjustable cosine shapes of gain and delay should be ideal for equalizing residual distortions. This paper describes an adjustable equalizer that produces harmonically related cosine shapes of both gain and delay. This device is called the *transversal equalizer*.*

## II. THEORY OF THE TRANSVERSAL EQUALIZER

Wheeler's theory of paired echoes[4] is useful in describing the operation of the transversal equalizer. Let the impulse response of a network be the two impulses as shown by Fig. 1. The larger response impulse will be called the *main signal* and the smaller one the *echo*. As shown in Fig. 1, the response is a main signal and a lagging echo. The transfer function of the network can be obtained from the Laplace transform of the response, and is given by

$$\begin{aligned} F(j\omega) &= L[\delta(t_1) + K\delta(t_1 + \tau)] \qquad p = j\omega \\ &= e^{-jt_1\omega}(1 + Ke^{-j\tau\omega}) \\ &= e^{-(\alpha + j\beta)}. \end{aligned}$$

The loss of the network is given by

$$\alpha = -20 \log | F(j\omega) | \qquad \text{in decibels,}$$

and the phase is given by

$$\beta = -\tan^{-1} \frac{\text{Im}\,[F(j\omega)]}{\text{Re}\,[F(j\omega)]} \qquad \text{in radians,}$$

which, for $K \ll 1$, results in

$$\begin{aligned} \alpha &\doteq -8.686K \cos \tau\omega, \\ \beta &\doteq t_1\omega + K \sin \tau\omega \end{aligned} \tag{1}$$

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* This name is a logical one in view of the use of "transversal filter" by Kallmann[2] and Linke.[3]

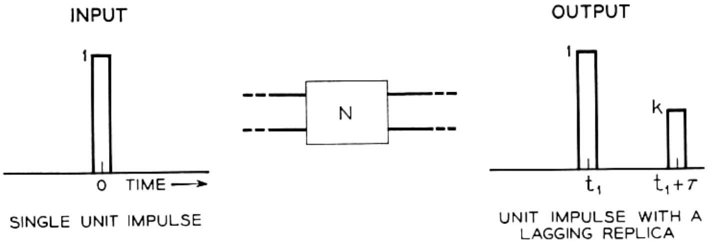

Fig. 1 — Single lagging echo response.

and the envelope delay

$$T \equiv \frac{d\beta}{d\omega} \doteq t_1 + \tau K \cos \tau\omega.$$

Thus, any network producing an echo pattern as shown by Fig. 1 has loss and delay responses in the frequency domain that are approximate cosines. Should the response be a main signal and a leading echo, the sign of $\tau$ would be negative and the resulting loss, phase and delay are given by

$$\begin{aligned} \alpha &\doteq -8.686K \cos \tau\omega, \\ \beta &\doteq t_1\omega - K \sin \tau\omega, \\ T &\doteq t_1 - K\tau \cos \tau\omega. \end{aligned} \tag{2}$$

Note that only the phase and delay are affected. Therefore, if the response of the network were to produce both leading and lagging echoes as well as a main signal the frequency characteristics would be the addition of (1) and (2). A number of echo patterns and their corresponding frequency responses are given by Fig. 2.

This shows that a pair of echoes with even symmetry about the main signal is associated with cosine loss deviations and constant delay, while echoes with odd symmetry are associated with cosine shapes of delay and approximately constant loss. Note also that the cosine function completes one-half cycle in a frequency band $B = 1/2\tau$.

Consider now a network with the multiple echo response as shown by Fig. 3. The frequency response is given (see the Appendix) by

$$\alpha \doteq -8.686\,[2K_1 \cos \tau\omega - 2K_2 \cos 2\tau\omega + 2\,K_3 \cos 3\tau\omega + \cdots],$$

$$\beta = t_1\omega$$

and

$$T = t_1 .$$

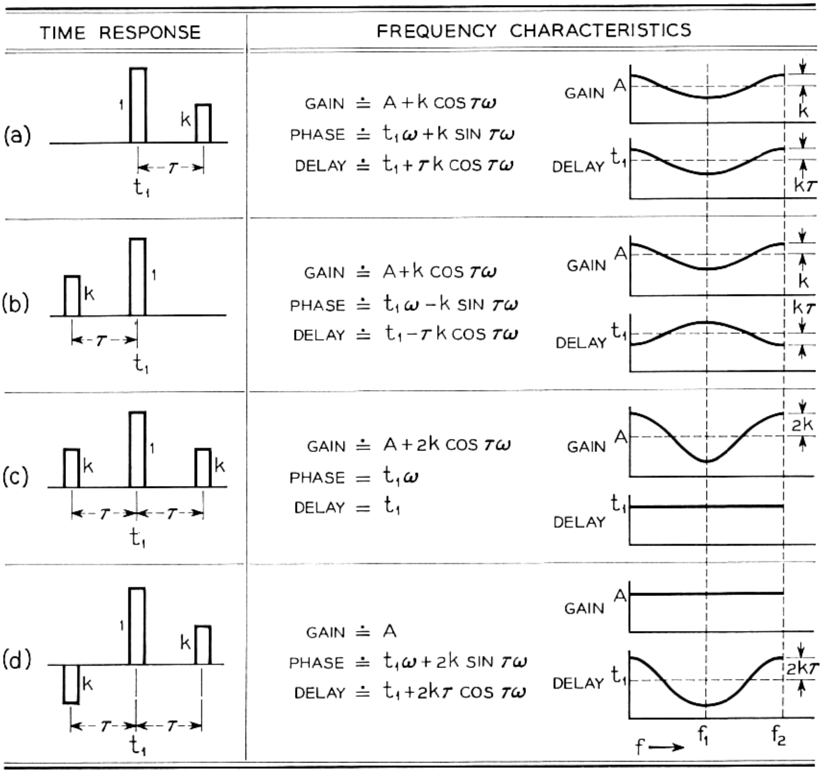

Fig. 2 — Corresponding time and frequency responses.

Thus, an echo pattern with even symmetry in which the spacing between individual echoes is constant produces harmonically related cosines of loss.

Similarly, it can be shown that such echoes with odd symmetry give harmonically related cosines of delay, as given by the following equations:

$$\alpha \doteq K,$$

$$\beta \doteq t_1\omega + 2K_1 \sin \tau\omega + 2K_2 \sin \tau 2\omega + 2K_3 \sin \tau 3\omega + \cdots$$

and

$$T \doteq t_1 + 2K_1\tau \cos \tau\omega + 2K_2\,(2\tau) \cos \tau 2\omega + 2K_3\,(3\tau) \cos \tau 3\omega + \cdots.$$

Observe that the amplitudes of the harmonic terms of loss are propor-

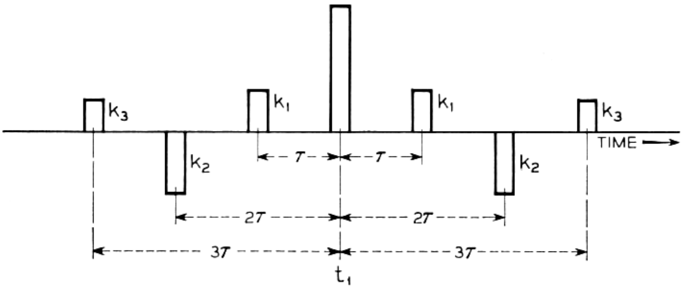

Fig. 3 — Multiple echo pairs with even symmetry.

tional to the amplitude of the echoes, while the amplitudes of the harmonic terms of delay are proportional to the product of the amplitudes of the echoes and the time interval between the corresponding echo and the main signal.

Networks capable of producing such multiple echo patterns have been used for Fourier series types of filters and equalizers by Wiener and Lee[5] and by others.[2,3,6,7]

A network capable of producing a multiple echo pattern can be constructed from a delay line with a number of equally spaced taps along the line, as shown by Fig. 4. The tap at the center of the line is for the so-called main signal. The other taps symmetrically spaced from the main signal tap are to provide leading and lagging echoes. In a restricted band $B$ the Nyquist sampling interval is $1/2B$; therefore the impulse response measured at $1/2B$ intervals is sufficient to describe the system completely. It follows that placing the taps $\tau$ seconds apart, where $\tau = 1/2B$ (which corresponds to a half cycle of the first cosine term), will

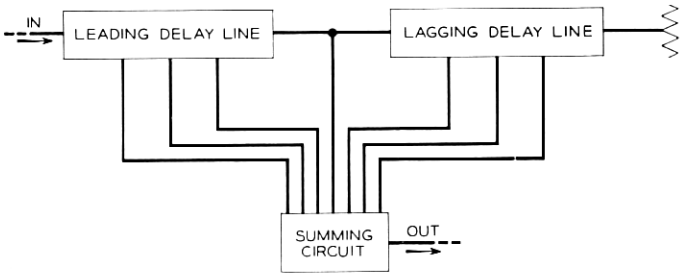

Fig. 4 — Block diagram of an echo-type equalizer.

completely equalize the system in the frequency band $B$, provided a sufficient number of echoes are available with appropriate amplitudes.

To have an adjustable equalizer it is necessary to have echoes available with adjustable amplitude and either polarity. In previous equalizers[3] this was accomplished by using either balanced delay lines or balanced summing amplifiers. The transversal equalizer uses an unbalanced delay line and a balanced summing network without active elements.[8]

Proper combinations of leading and lagging echoes can provide the desired harmonically related cosine shapes of loss only or delay only as shown by Fig. 2. The echoes can be combined electrically by introducing both a leading and a lagging echo to the same potentiometer, as shown in Fig. 5, if $r_1$ and $r_2$ are large compared to the potentiometer resistance. If it is necessary to have a single control adjust a pure loss term, both a leading and a lagging echo must be combined at the potentiometer corresponding to the desired term. These echoes should have the same amplitude and polarity to give a pure loss term. Now, if the same harmonic of delay is to be available and independently adjustable, the same two echoes must be introduced at the delay potentiometer. However, at this potentiometer one of the echoes must be of the opposite polarity to that introduced at the loss potentiometer. It follows then that, to adjust the equalizer in terms of loss and delay independently, both polarities of either the leading or lagging echoes will be simultaneously needed. This requires an additional delay line or a phase-inverting device. If independent adjustment is not necessary, a single echo can be used to adjust either loss or delay, and a pair of echoes can be used for the other. This would require that the single echo adjustment be made first and the paired echo adjustment be made later.

Some advantage accrues from using single echoes for the loss terms rather than the delay terms. Delay terms obtained from the combination of leading and lagging echoes give relatively pure delay and twice the range of those that could be obtained from a single echo. This is important for the lower harmonics because, as shown above, the amplitude of a delay term is proportional to the time interval between the

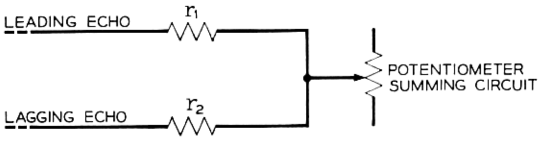

Fig. 5 — Method of combining leading or lagging echoes to produce pure loss or pure delay.

echo and the main signal, and hence proportional to the harmonic number. The disadvantage of this pattern is that the loss shapes also introduce a corresponding delay shape that may increase the over-all delay distortion. If the system delay distortion were of the minimum-phase type it would be advantageous to obtain the loss shapes from lagging echoes alone, since minimum-phase networks are lagging echo devices. However, if the system contains fixed-delay equalizers, the over-all distortion is not necessarily minimum phase. In order to secure optimum equalization, a switch may be provided to select either a leading or a lagging echo for each of the loss terms. This arrangement allows maximum utilization of the delay terms.

## III. ADJUSTING TECHNIQUE

The operation of the transversal equalizer can be described equally well in terms of paired-echo theory or in terms of its steady-state frequency response. These two descriptions, although equally correct, lead to two distinct methods of adjustment and evaluation of performance. The echo analysis leads to adjustment in terms of individual echoes and is described by Linke.[3] However, existing equipment suitable for adjusting the transversal equalizer utilizes a sweeping-frequency technique. This necessitates the evaluation of loss and delay characteristics separately, hence combinations of leading and lagging echoes are required.

The adjusting set is a modified version of that used for the "cosine equalizers."[1] Its operation is briefly described here in order to clarify the factors affecting the design of the transversal equalizer. For the loss adjustment, the adjusting set transmitter sends a swept-frequency signal through the system, as shown in Fig. 6. This signal is amplitude-modulated by the system loss distortion. The signal then travels through the equalizer and into the receiver of the adjusting set, where it is

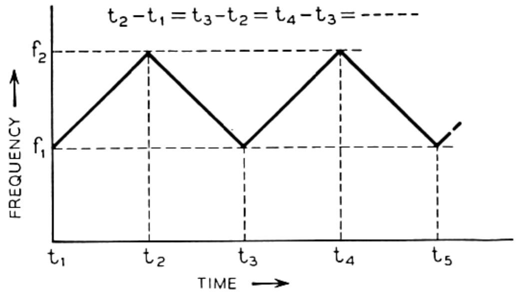

Fig. 6 — Sweep frequency of the cosine-adjusting set.

demodulated, and the resulting voltage is fed into a power meter. The waveform of this voltage represents the remaining distortion of the line plus the equalizer. Considering this distortion as a summation of its Fourier harmonics, it is apparent that removal of any harmonic by the equalizer reduces the total energy of the distortion voltage, and the power meter reading is lowered.

Delay adjustment is made similarly, except that two frequencies separated in frequency by a constant amount are swept across the band simultaneously. The difference in phase between these two frequencies at any instant is an indication of the delay characteristic of the line, and a discriminator converts this phase difference into a voltage which energizes the power meter.

Note that, in sweeping up and down the band, distortion versus frequency is converted into a periodic function of voltage versus time. It is also an even function about the time corresponding to either extreme of the sweep; therefore, to synthesize such a waveform, only Fourier cosine series terms are required. This restricts the equalizer to integral multiples of half cosines of frequency in the required band, which is consistent with the Nyquist rate. The up-and-down nature of the sweep converts these half-cycles into full cosine shapes in the repetition period.

## IV. PHYSICAL REALIZATION OF A TRANSVERSAL EQUALIZER

Several variations of the transversal equalizer have been constructed at Bell Telephone Laboratories. One variation is the 336A equalizer designed for use in the TV branch of the L3 coaxial system, as shown in Fig. 7. It provides 23 continuously adjustable terms of loss and 15 terms

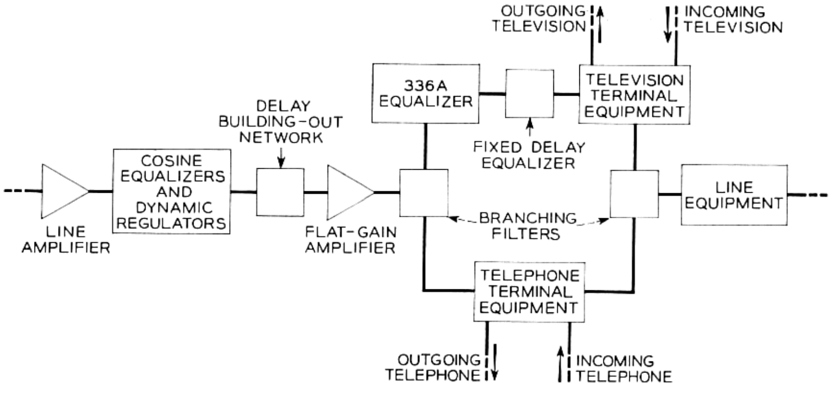

Fig. 7 — 336A equalizer location in L3 system.

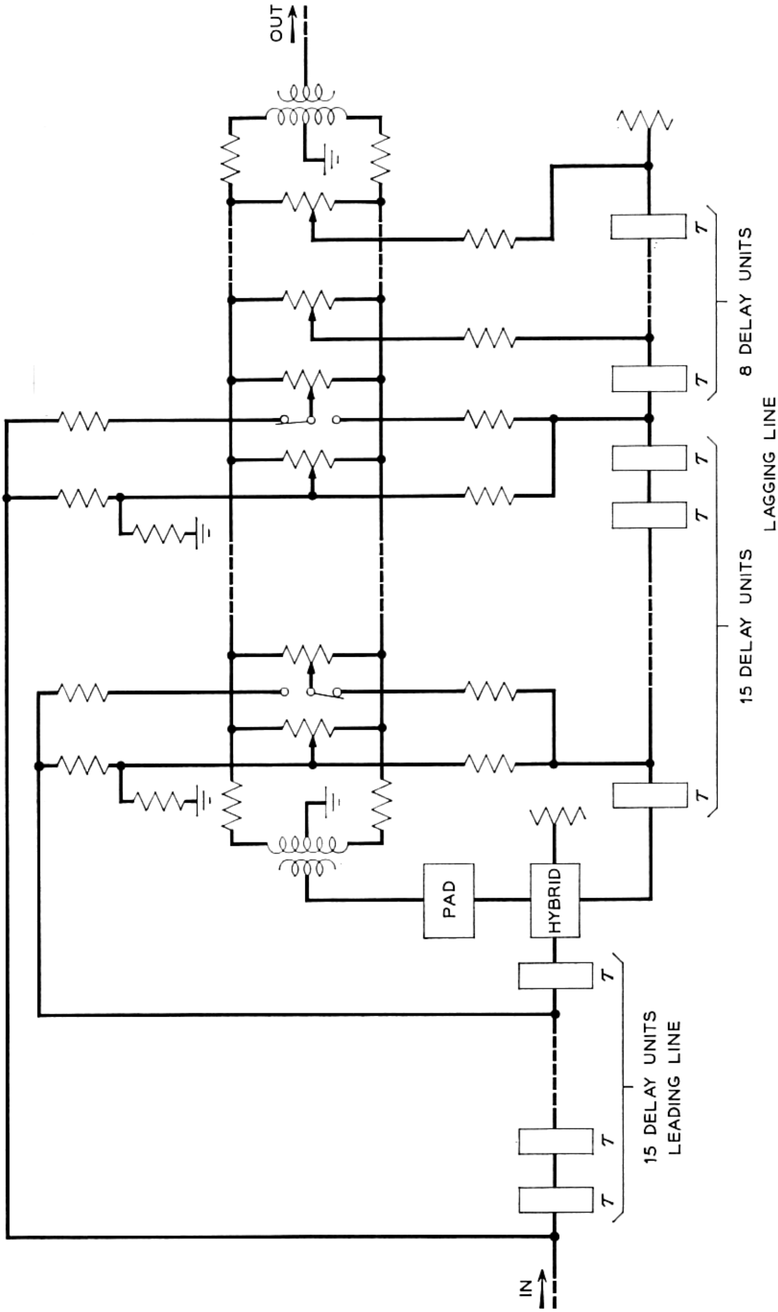

Fig. 8 — Block schematic of transversal equalizer.

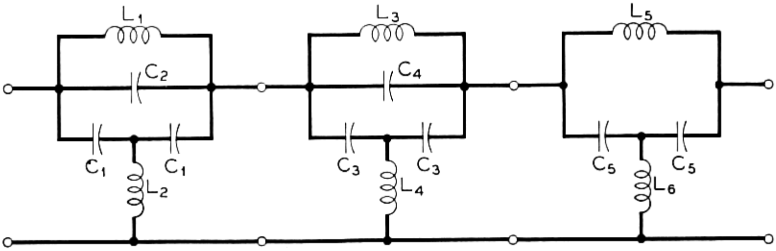

Fig. 9 — Schematic for the delay units.

of delay. These terms correspond to the desired Fourier cosine series over the frequency range of 3.75 to 8.25 mc. The loss terms are obtained from single echoes only, and the delay terms are obtained by using pairs of echoes with odd symmetry.

The equalizer is composed of a delay line, a hybrid coil and a summing network, as shown in Fig. 8. The delay line is composed of 38 identical sections, each of which is required to have a linear phase change of 180° over the frequency band. In order that the terms be cosines the phase shift at each band edge must be a multiple of 180°.

The present design of the delay units uses three all-pass sections, as shown in Fig. 9. The internal construction of these sections is shown in Fig. 10. Fig. 11 shows the loss and phase shift of a typical delay unit. It should be noted that any variation of the loss through the delay units modifies the amplitude of the cosine shapes produced by the equalizer. A Fourier analysis of the resulting "cosines" showed that the loss characteristic of these sections introduced negligible distortion.

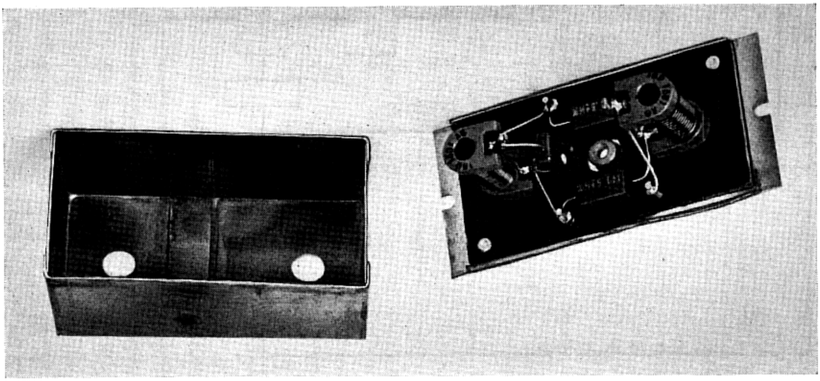

Fig. 10 — Internal construction of the delay units.

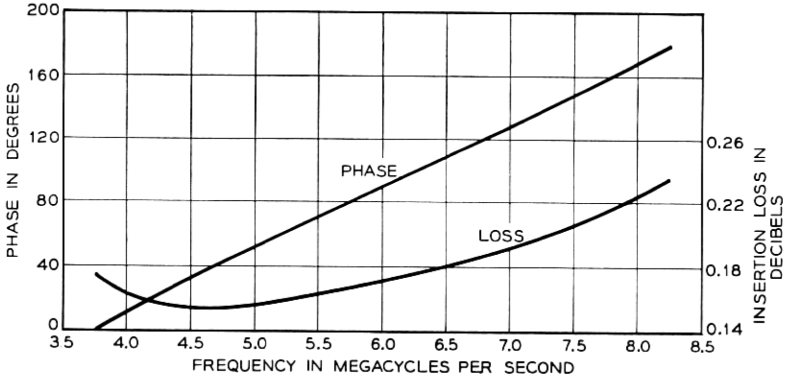

Fig. 11 — Loss and phase characteristics of a typical delay unit.

The hybrid coil serves several purposes. It splits the signal from the leading line into the component called the main signal and the component that energizes the lagging line. It inverts the phase of the signal to the lagging line thus providing the odd symmetry required for the delay terms. And it also provides isolation between the lagging line and the main signal; this is desirable because mismatches in the lagging line produce reflections that appear as lagging echoes if they are transmitted without attenuation to the output of the equalizer.

Combining the main signal with the tapped echoes requires a scheme that allows the magnitude and the sign of the echoes to be controlled. Using a balanced summing circuit obtained by means of transformers as shown by Fig. 8, the echo voltages are introduced through tapping resistors to the variable arms of the potentiometers that are connected across the balanced transformers. The position of the variable arm of the potentiometer controls the magnitude as well as the sign of the echo with respect to the main signal. The delay terms are obtained by combining corresponding leading and lagging echoes at the variable arm of a single potentiometer. In order that these echoes have equal intensities regardless of the setting of the potentiometer, a dissymmetrical pad is used at each delay potentiometer to compensate for the loss of the delay line and the hybrid coil.

To reduce the loading of the delay line at the tapped points, high values of resistors in the summing networks are required. This inherently restricts the echo magnitudes — and thus the ranges — unless the magnitude of the main signal is reduced. The pad between the hybrid and summing transformers serves this purpose; it is in the main transmission path and, if made arbitrarily large, will degrade the signal to noise ratio. Thus, the range of the loss and delay shapes is limited by the over-all loss of the equalizer and the loading of the line at the taps. The ranges provided by the 336A equalizer are listed in Table I.

It should be noted that the harmonic terms are not completely independent in the 336A. To make the terms completely orthogonal, very precise adjustment of the phase shift from the input through the delay line to the summing potentiometers for each path is required. A lack of precision here introduces term interactions, and consequently several iterations may be required to adjust the equalizer. This embodiment of the equalizer represents a compromise among cost factors, design schedules and ease of adjustment. Fig. 12 shows the front view of the 336A equalizer. It is equipped with a sliding mask to allow access to either the delay controls only or loss controls only. An additional knob is provided near each loss control to select leading or lagging echoes for loss equalization. The controls near the base are for simple all-pass networks that are switched in and out to supplement the first and second delay terms. Fig. 13 shows the equalizer with the sliding mask removed, and the controls for the selection of leading or lagging echoes can be seen.

## V. FIELD TRIAL

Models of the 336A equalizer were built at the Laboratories and tested in a 400-mile link of the L3 cable system. At the time of the tests, the

TABLE I

| $n$ | Loss Range, in db | Delay Range, in μsec |
|---|---|---|
| 1 | 1.1 | 0.03 |
| 2 | 1.1 | 0.06 |
| 3 | 1.1 | 0.09 |
| 4 | 1.1 | 0.12 |
| 5 | 1.1 | 0.14 |
| 6 | 0.8 | 0.09 |
| 7 | 0.7 | 0.10 |
| 8 | 0.7 | 0.11 |
| 9 | 0.7 | 0.12 |
| 10 | 0.7 | 0.13 |
| 11 | 0.5 | 0.11 |
| 12 | 0.5 | 0.12 |
| 13 | 0.5 | 0.12 |
| 14 | 0.5 | 0.13 |
| 15 | 0.5 | 0.13 |
| 16 | 0.4 | — |
| 17 | 0.4 | — |
| 18 | 0.4 | — |
| 19 | 0.4 | — |
| 20 | 0.4 | — |
| 21 | 0.4 | — |
| 22 | 0.4 | — |
| 23 | 0.4 | — |

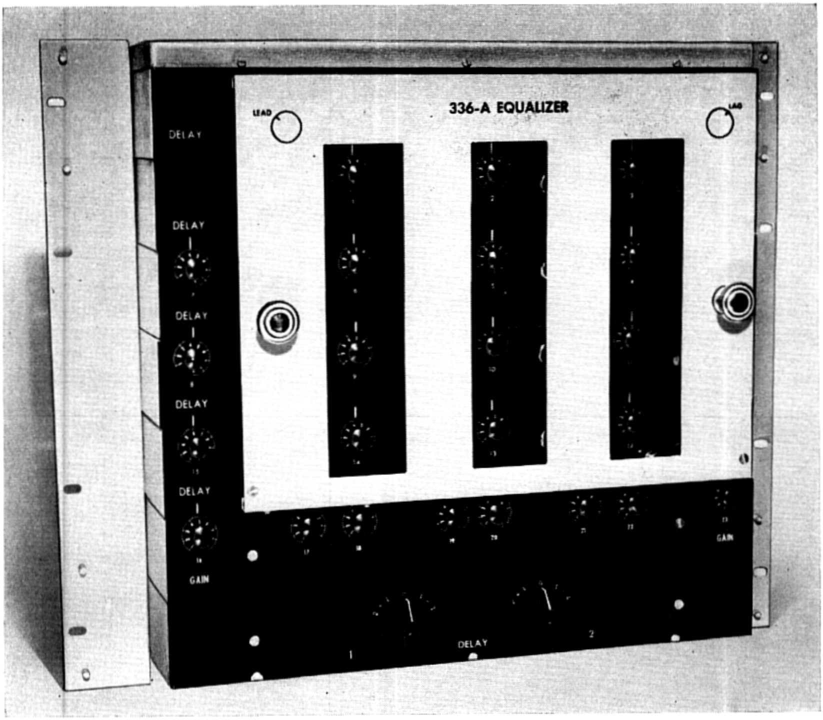

Fig. 12 — Front view of the 336A equalizer.

lines were equipped with only three of the full complement of six automatic regulators and therefore the system distortion cannot be considered that of a typical L3 system. The lines were otherwise in normal operating condition, including recent equalization of the entire L3 band with "cosine equalizers." Fig. 14(a) shows the gain and delay characteristics of a 400-mile line without a 336A equalizer. These transmission distortions do not represent any fixed type of deviations. In fact, the deviations from line to line are unpredictable, and therefore a very flexible equalizer such as the 336A is required. For this particular line the distortions of ±0.7 db and ±0.15 microseconds are excessive. However, Fig 14(b) shows the corresponding characteristic after adding a 336A equalizer and adjusting the 23 gain and the 15 delay terms. Note that the distortion in gain is now ±0.3 db over the entire band and the delay distortion is ±0.05 microsecond. The improvement in gain up to about 8 mc is quite marked (±0.1 db) and only at the extreme edge of the band does the gain deviate appreciably. It should be mentioned that the three regulators not in the system at the time these measure-

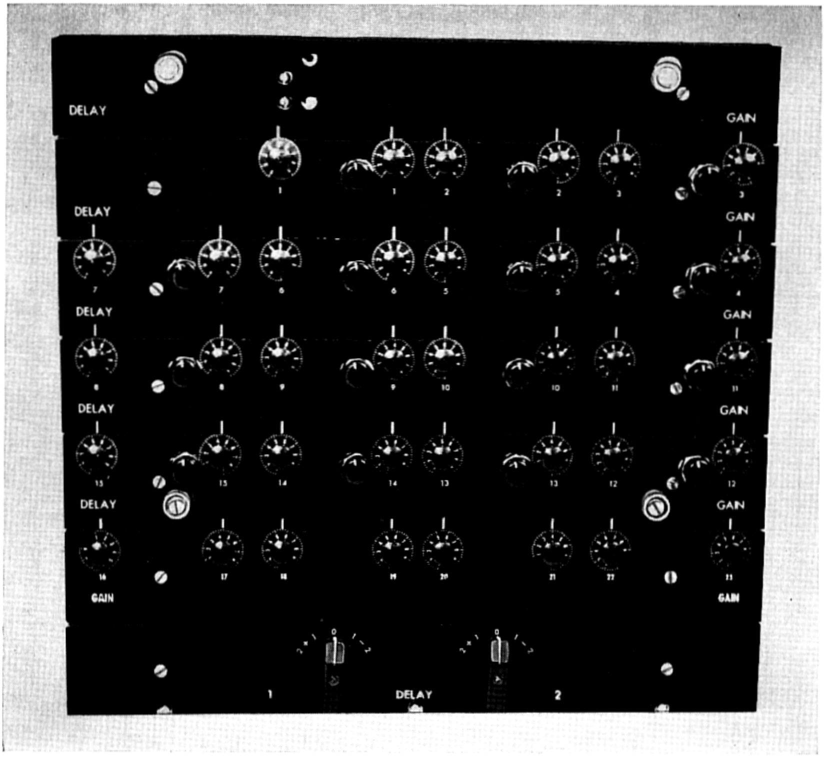

Fig. 13 — 336A equalizer with sliding mask removed.

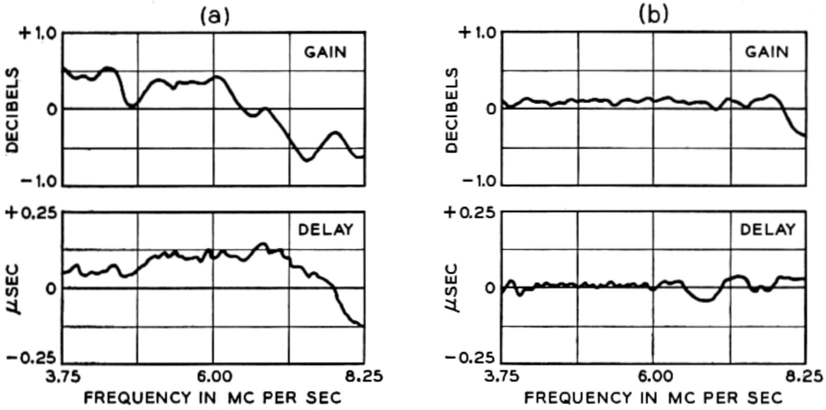

Fig. 14 — Delay and gain characteristics of a 400-mile circuit before and after equalization with a 336A equalizer.

ments were taken would be effective in reducing the gain distortion at the top edge of the band. Fig. 15 shows the gain and delay characteristic of a 800-mile loop after equalization with two 336*A* equalizers. Here the gain distortion is ±0.15 db and the delay distortion is ±0.07 microsecond over the desired band. The distortion over the 800-mile loop looks slightly better than the 400-mile line shown because the band edge characteristic happened to be less pronounced. There is no reason to expect that, in general, an 800-mile section with two equalizers would be better than a 400-mile section with one 336A.

## VI. CONCLUSIONS

The design of the transversal equalizer is a step forward in adjustable delay equalization. It is flexible because it uses the Fourier series approach; it can be made simple to adjust and is compatible with the existing adjusting techniques. It makes maximum use of only a few delay blocks to provide both loss and delay equalization. The flat loss of the equalizer is essentially independent of the number of terms; thus, it is particularly economical where a large number of terms is desired.

## VII. ACKNOWLEDGMENTS

The authors are indebted to many persons who have contributed to the realization of the 336A equalizer. Specifically, we wish to thank M. R. Aaron, F. J. Braga, R. Dempster, J. L. Garrison, R. S. Graham, E. S. Kuh, W. R. Lundry, O. L. Williams and G. F. Wyzga for their valuable suggestions and aid during the design and construction of the equalizer.

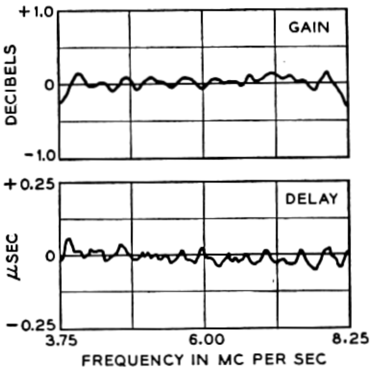

Fig. 15 — Delay and gain characteristics of a 800-mile circuit after equalization with two 336A equalizers.

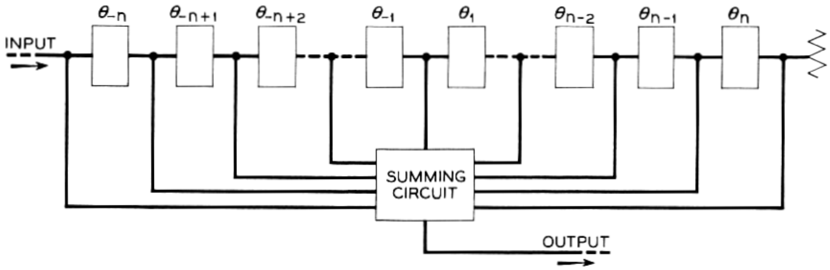

Fig. 16 — Block diagram of a transversal equalizer.

## APPENDIX

### *Multiple Echo Pattern Analysis*

Consider the performance of the circuit shown in Fig. 16, in which each box is a constant resistance network with

$$\frac{e_{\text{out}}}{e_{\text{in}}} = \epsilon^{\theta_k}$$

where

$$\theta_k = \alpha + j\beta \qquad k = 1, \cdots, n,$$

and both $\alpha$ and $\beta$ may be functions of frequency.

The insertion ratio of the circuit is given by

$$\frac{V_{\text{out}}}{V_{\text{in}}} = a_0 e^{n(\alpha+j\beta)} \left[ 1 + \sum_{k=1}^{n} \frac{a_{-k}}{a_0} e^{-k(\alpha+j\beta)} + \sum_{k=1}^{n} \frac{a_k}{a_0} e^{k(\alpha+j\beta)} \right],$$

where $a_i$ is the summing coefficient for the voltage from the $i$th box. The factor $a_0 e^{n(\alpha+j\beta)}$ is independent of the summing of the tapped voltages, and can be equalized by means of a fixed equalizer.

The factor

$$F = 1 + \sum_{k=1}^{n} \frac{a_{-k}}{a_0} e^{-k\alpha} e^{-j\beta k} + \sum_{k=1}^{n} \frac{a_k}{a_0} e^{+k\alpha} e^{j\beta k}$$

is the variable term that has the desired properties for equalization.

If we let

$$A_{-k} = \frac{a_{-k}}{a_0} e^{-k\alpha}$$

and

$$A_k = \frac{a_k}{a_0} e^{k\alpha},$$

the insertion gain variation of the circuit is given by

$$\begin{aligned} 20 \log F &= \\ & 20 \log_{10}\left[1 + \sum_{k=1}^{n} (A_k + A_{-k}) \cos k\beta + j \sum_{k=1}^{n} (A_k - A_{-k}) \sin k\beta\right] \\ &= G + j\Phi, \end{aligned}$$

where

$$\begin{aligned} G &= 10 \log_{10}\left\{1 + 2 \sum_{k=1}^{n} (A_k + A_{-k}) \cos k\beta \right. \\ & \left. + \left[\sum_{k=1}^{n} (A_k + A_{-k}) \cos k\beta\right]^2 + \left[\sum_{k=1}^{n} (A_k - A_{-k}) \sin k\beta\right]^2\right\} \qquad (3) \\ &= 10 \log_{10}\left[1 + 2 \sum_{k=1}^{n} (A_k + A_{-k}) \cos k\beta + D_1\right] \qquad \text{decibels,} \end{aligned}$$

and

$$\begin{aligned} \Phi &= \arctan\left[\frac{\sum_{k=1}^{n} (A_k - A_{-k}) \sin k\beta}{1 + \sum_{k=1}^{n} (A_k + A_{-k}) \cos k\beta}\right] \qquad (4) \\ &= \arctan\left[\sum_{k=1}^{n} A_k - A_{-k}) \sin k\beta - D_2\right], \end{aligned}$$

where $D_1$ and $D_2$ are distortion terms containing cosines with magnitudes proportional to $A_m A_n$ and arguments of

$$\left[(m \pm n)\beta + \frac{\delta\pi}{2}\right],$$

where $\delta = 0, 1$.

Now, if we can assume $A_k < 1$, which means $a_k/a_o < 1$ (i.e., small percentage echo), then (3) becomes

$$\begin{aligned} G = 8.686 &\left\{\sum_{1}^{n} (A_k + A_{-k}) \cos k\beta - \left[\sum_{1}^{n} (A_k + A_{-k}) \cos k\beta\right]^2 \right. \\ & \left. + \cdots + \frac{D_1}{2} - \frac{D_1^2}{4} + \cdots - D_1 \sum_{1}^{n} (A_k + A_{-k}) \cos k\beta + \cdots\right\}, \qquad (5) \end{aligned}$$

and (4) becomes

$$\begin{aligned} \Phi = & \sum_{1}^{n} (A_k - A_{-k}) \sin k\beta - D_2 \\ & - \frac{1}{3}\left[\sum_{1}^{n} (A_k - A_{-k}) \sin k\beta - D_2\right]^3. \qquad (6) \end{aligned}$$

If the $A_k$ are much smaller than unity, only the first term of either (5) or (6) need be considered. Thus the loss and phase are cosine and sine functions respectively only if these conditions are met.

Hence

$$G \doteq 8.686 \sum_1^n (A_k + A_{-k}) \cos k\beta,$$

$$\Phi \doteq \sum_1^n (A_k - A_{-k}) \sin k\beta.$$

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