Attenuation of the TE_{01} Wave Within the Curved Helix Waveguide*

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(Manuscript received April 1, 1958)

The change in the field pattern of the TE_{01} mode in the helix waveguide with constant curvature is calculated by means of a perturbation method. The helix waveguide has a coating of a lossy dielectric and is shielded by a metallic pipe. By bending this waveguide, the original field pattern of the TE_{01} wave is slightly perturbed. The perturbation produces additional field components which give rise to an electromagnetic field in the lossy dielectric. This field causes energy dissipation and gives rise to an additional attenuation. In this paper formulae for the curvature attenuation caused by the dielectric losses are given. It is shown that the attenuation due to curvature can be remarkably reduced by proper choice of the thickness of the dielectric layer between helix and metallic shield.

I. INTRODUCTION

If the system of cylindrical coordinates, in which the cylindrical waveguide ordinarily is described, is bent in the same way as the axis of the curved waveguide, Maxwell's equations change. Jouguet[1] has shown an approximate solution of Maxwell's equations written in a system of toroidal coordinates. This solution may be applied to calculate the field pattern of a TE_{01} wave in the curved helix waveguide.[2] The waveguide[3] consists of a helix made of very thin wire and a coating of a lossy dielectric which may be shielded by a metallic pipe. The lossy dielectric serves to suppress unwanted modes which may be excited in the helix waveguide. Although the lossy dielectric does not affect the TE_{01} wave in a perfectly straight guide, it does give rise to an additional attenuation if the waveguide is bent. It is the purpose of the present paper to show how a helix waveguide should be designed in order to keep this additional attenuation as low as possible. On the other hand, the attenuation of the TE_{01} wave in bends is not the only effect which is to be considered

* The author used the same method of analysis to calculate the attenuation and field structure of the TE_{01} wave in a curved spaced-ring type waveguide while he was working with the Siemens and Halske Company in Germany.

in designing the waveguide. In practice, a compromise should be made between the desire to keep the attenuation of the TE_{01} wave in a curved waveguide low and the desire to suppress spurious modes.[4] For a short part of the guide with a wanted high curvature, a helix waveguide which is designed for minimum curvature attenuation of the TE_{01} wave might be useful.

The mathematical model used for calculating the helix waveguide neglects the pitch angle of the helix wire.[5] It is assumed that the closely wound helix may be idealized as an infinitely thin sheet with anisotropic conductivity, conducting only in the direction perpendicular to the axis of the waveguide.

II. SOLUTION OF THE BOUNDARY VALUE PROBLEM

In the following calculation it is assumed that the helix and the pipe which serves as a shield around the helix waveguide are of perfect conductivity. The toroidal coordinates have the variables r, φ and z. These are the same variables as for an ordinary system of cylindrical coordinates. The only difference is that the z axis is curved in the same way as the axis of the helix waveguide. The line element is given as

$$ds^2 = \left(1 + \frac{r}{R}\sin\varphi\right)^2 dz^2 + dr^2 + r^2\, d\varphi^2,$$

where R is the radius of curvature which is assumed to be constant. Maxwell's equations are then given by:

$$\begin{aligned}
\frac{\partial}{\partial\varphi}\left[\left(1 + \frac{r}{R}\sin\varphi\right)H_z\right] - \gamma(rH_\varphi) &= j\omega\epsilon r\left(1 + \frac{r}{R}\sin\varphi\right)E_r\,,\\
\gamma H_r - \frac{\partial}{\partial r}\left[\left(1 + \frac{r}{R}\sin\varphi\right)H_z\right] &= j\omega\epsilon\left(1 + \frac{r}{R}\sin\varphi\right)E_\varphi\,,\\
\frac{\partial}{\partial r}(rH_\varphi) - \frac{\partial}{\partial\varphi}H_r &= j\omega\epsilon r E_z\,,\\
\frac{\partial}{\partial\varphi}\left[\left(1 + \frac{r}{R}\sin\varphi\right)E_z\right] - \gamma r E_\varphi &= -j\omega\mu r\left(1 + \frac{r}{R}\sin\varphi\right)H_r\,,\\
\gamma E_r - \frac{\partial}{\partial r}\left[\left(1 + \frac{r}{R}\sin\varphi\right)E_z\right] &= -j\omega\mu\left(1 + \frac{r}{R}\sin\varphi\right)H_\varphi\,,\\
\frac{\partial}{\partial r}(rE_\varphi) - \frac{\partial}{\partial\varphi}E_r &= -j\omega\mu r H_z\,,
\end{aligned}\tag{1}$$

if the z and t dependence of all field components are of the form

$$e^{j\omega t + \gamma z}. \tag{2}$$

If R tends to ∞ the equations (1) reduce to the ordinary Maxwell's equations in cylindrical coordinates. Since $r/R \ll 1$ in all practical applications, the solutions of (1) will be nearly the same as the solutions for the system with straight z axis; they will differ only by additional perturbation terms. Therefore, the following statement is made:

$$\begin{aligned} \mathbf{H} &= \mathbf{H}_0 + \mathbf{h}, \\ \mathbf{E} &= \mathbf{E}_0 + \mathbf{e}, \end{aligned} \tag{3}$$

where $\mathbf{H}_0$ and $\mathbf{E}_0$ are the solutions of (1) for $R \to \infty$. Since we are concerned with the special solutions which give the TE_{01} wave in the curved helix waveguide, the field of zero-order approximation will be the ordinary TE_{01} wave:

$$\begin{aligned} H_{z0} &= AJ_0(\chi_0 r)e^{j\omega t+\gamma z}, \\ H_{r0} &= -\frac{\gamma}{\chi_0} AJ_1(\chi_0 r)e^{j\omega t+\gamma z}, \\ E_{\varphi 0} &= -j\frac{\omega\mu}{\chi_0} AJ_1(\chi_0 r)e^{j\omega t+\gamma z}, \\ \chi_0^2 &= \beta_0^2 + \gamma^2, \\ \beta_0^2 &= \omega^2\epsilon_0\mu_0 . \end{aligned} \tag{4}$$

Equations (4) and (3) are now substituted into (1). Since $\mathbf{h}$ and $\mathbf{e}$ must be of the same order of magnitude as r/R, products of these terms with r/R are omitted. We therefore get only the first-order approximation.

If we consider that $\mathbf{H}_0$ and $\mathbf{E}_0$ are solutions of the unperturbed system, (1) becomes

$$\begin{aligned} \frac{\partial}{\partial r}(rh_\varphi) - \frac{\partial}{\partial \varphi} h_r - j\omega\epsilon r e_z &= 0, \\ \frac{\partial}{\partial \varphi} h_z - \gamma r h_\varphi - j\omega\epsilon r e_r &= -\frac{r}{R}\cos\varphi AJ_0(\chi_0 r), \\ \gamma h_r - \frac{\partial}{\partial r} h_z - j\omega\epsilon e_\varphi &= -\frac{r}{R}\frac{\gamma^2}{\chi_0}\sin\varphi AJ_1(\chi_0 r) \\ &\quad + \frac{1}{R}\sin\varphi AJ_0(\chi_0 r), \end{aligned} \tag{5}$$

$$\begin{aligned} \frac{\partial}{\partial r}(re_\varphi) - \frac{\partial}{\partial \varphi} e_r + j\omega\mu r h_z &= 0, \\ \frac{\partial}{\partial \varphi} e_z - \gamma r e_\varphi + j\omega\mu r h_r &= \frac{r}{R}\gamma r\frac{j\omega\mu}{\chi_0} AJ_1(\chi_0 r), \\ \gamma e_r - \frac{\partial}{\partial r} e_z + j\omega\mu h_\varphi &= 0. \end{aligned}$$

For $R \to \infty$, these equations are the ordinary unperturbed Maxwell equations for $\mathbf{e}$ and $\mathbf{h}$. The terms with $1/R$ make the system of equations (5) inhomogeneous. Its solution is therefore given as the general solution of the homogeneous system and a particular solution of the inhomogeneous system. The solution of the inhomogeneous equations is found by the statement:

$$\begin{aligned} e_z &= X \cos \varphi, & h_z &= U \sin \varphi, \\ e_r &= Y \cos \varphi, & h_r &= V \sin \varphi, \\ e_\varphi &= \frac{Z}{r} \sin \varphi, & h_\varphi &= \frac{W}{r} \cos \varphi. \end{aligned} \tag{6}$$

If we substitute this into (5) we get a system of ordinary differential equations with the only variable r. This system is solved by a statement of the form

$$X = PJ_0(\chi_0 r) + QJ_1(\chi_0 r),$$

where P and Q are polynomials of r only. After determining the coefficients of the polynomials we finally get the solution of (5), after adding the solution of the homogeneous equation:

$$\begin{aligned} h_z &= \left[BJ_1(\chi_0 r) - \frac{\chi_0^2 + 2\gamma^2}{2\chi_0^2} r \frac{A}{R} J_0(\chi_0 r) \right. \\ &\qquad \left. + \frac{\gamma^2}{2\chi_0} r^2 \frac{A}{R} J_1(\chi_0 r) \right] e^{\gamma z} \sin \varphi, \\ h_r &= \left[B \frac{\gamma}{\chi_0} J_1'(\chi_0 r) - jC \frac{\omega\epsilon_0}{\chi_0^2} \frac{1}{r} J_1(\chi_0 r) \right. \\ &\qquad + \frac{\gamma}{2\chi_0^2} (r^2\gamma^2 - 1) \frac{A}{R} J_0(\chi_0 r) \\ &\qquad \left. + \frac{\gamma}{2\chi_0^3} (\beta_0^2 + 2\chi_0^2) r \frac{A}{R} J_1(\chi_0 r) \right] e^{\gamma z} \sin \varphi, \\ h_\varphi &= \left[B \frac{\gamma}{\chi_0^2} \frac{1}{r} J_1(\chi_0 r) - jC \frac{\omega\epsilon_0}{\chi_0} J_1'(\chi_0 r) - \frac{1}{2} \frac{\gamma}{\chi_0^2} \frac{A}{R} J_0(\chi_0 r) \right. \\ &\qquad \left. + \frac{1}{2} \frac{\gamma}{\chi_0^3} (\beta_0^2 + \chi_0^2) r \frac{A}{R} J_1(\chi_0 r) \right] e^{\gamma z} \cos \varphi, \\ e_z &= \left[CJ_1(\chi_0 r) - j\omega\mu \frac{\gamma}{\chi_0^2} r \frac{A}{R} J_0(\chi_0 r) \right] e^{\gamma z} \cos \varphi, \end{aligned} \tag{7}$$

$$e_r = \left[-jB \frac{\omega\mu}{\chi_0^2} \frac{1}{r} J_1(\chi_0 r) + C \frac{\gamma}{\chi_0} J_1'(\chi_0 r) - \frac{j\omega\mu}{2\chi_0^2} \frac{A}{R} J_0(\chi_0 r) + \frac{j\omega\mu}{2\chi_0^3} r\gamma^2 \frac{A}{R} J_1(\chi_0 r) \right] e^{\gamma z} \cos\varphi,$$

$$e_\varphi = \left[jB \frac{\omega\mu}{\chi_0} J_1'(\chi_0 r) - C \frac{\gamma}{\chi_0^2} \frac{1}{r} J_1(\chi_0 r) + \frac{1}{2} \frac{j}{\omega\epsilon_0} \frac{\beta_0^2}{\chi_0^2} (1 + r^2\gamma^2) \frac{A}{R} J_0(\chi_0 r) + \frac{1}{2} j\omega\mu \frac{\beta_0^2}{\chi_0^3} r \frac{A}{R} J_1(\chi_0 r) \right] e^{\gamma z} \sin\varphi.$$

The validity of these solutions can be proven by substituting into (5). The solution of the homogeneous equations is chosen so as to have the same φ dependence and the same propagation constant as the solution of the inhomogeneous equation.

Up to this point the theory has followed the solution given in Jouguet's paper and is not restricted to the application to the helix waveguide.

Equations (7) and (4) give the field pattern of a normal mode in the curved helix waveguide, and $\epsilon = \epsilon_0$ is taken for a guide filled with air. The field in the dielectric outside of the helix should also be a solution of (1). But since the TE_{01} wave in a straight guide has no field outside of the helix, the field in the dielectric layer is excited only by the additional field components (7) and is therefore of the same order of magnitude. If we again neglect terms of the order $(r/R)^2$ we get Maxwell's equations for the straight waveguide for the small field within the dielectric layer. This field should have the same φ dependence as (7). We therefore state:

$$\begin{aligned}
h_z^e &= FW_1(\chi' r) \sin\varphi e^{\gamma z}, \\
h_r^e &= \left[F \frac{\gamma}{\chi'} W_1'(\chi' r) - jD \frac{\omega\epsilon}{\chi'^2} \frac{1}{r} V_1(\chi' r) \right] \sin\varphi e^{\gamma z}, \\
h_\varphi^e &= \left[F \frac{\gamma}{\chi'^2} \frac{1}{r} W_1(\chi' r) - jD \frac{\omega\epsilon}{\chi'} V_1'(\chi' r) \right] \cos\varphi e^{\gamma z}, \\
e_z^e &= DV_1(\chi' r) \cos\varphi, \\
e_r^e &= \left[-jF \frac{\omega\mu_0}{\chi'^2} \frac{1}{r} W_1(\chi' r) + D \frac{\gamma}{\chi'} V_1'(\chi' r) \right] \cos\varphi e^{\gamma z}, \qquad (8) \\
e_\varphi^e &= \left[jF \frac{\omega\mu_0}{\chi'} W_1'(\chi' r) - D \frac{\gamma}{\chi'^2} \frac{1}{r} V_1(\chi' r) \right] \sin\varphi e^{\gamma z}, \\
\chi'^2 &= \beta'^2 + \gamma^2, \\
\beta'^2 &= \omega^2 \epsilon\mu_0 , \\
\frac{\epsilon}{\epsilon_0} &= \epsilon' - j\epsilon'',
\end{aligned}$$

with ϵ' = dielectric constant, $\epsilon'' = \epsilon' \tan \delta$, where $\tan \delta$ is the loss factor of the dielectric material. The functions V and W are abbreviations for

$$V_1(\chi' r) = H_1^{(2)}(\chi' r) - \frac{H_1^{(2)}(\chi' b)}{H_1^{(1)}(\chi' b)} H_1^{(1)}(\chi' r),$$
$$W_1(\chi' r) = H_1^{(2)}(\chi' r) - \frac{H_1^{(2)\prime}(\chi' b)}{H_1^{(1)\prime}(\chi' b)} H_1^{(1)}(\chi' r), \tag{9}$$

where $H_1^{(1)}$ and $H_1^{(2)}$ are Hankel's functions of the first and second kind respectively. The prime on the V, W and Hankel function denotes differentiation with respect to the argument. The functions V and W are made such that e_z and e_φ vanish at the shield $r = b$. The boundary conditions on the shield are thus satisfied by the statements (8) and (9). The field of zero-order approximation has to be matched to the boundary independent of the additional field components because of their different dependence on the variable φ. It must be $E_{0\varphi} = 0$ at $r = a$. That means $J_1(\chi_0 a) = 0$, $\chi_0 a = 3.83$, which is exactly the same eigenvalue as that for the straight helix waveguide or even the ordinary circular solid-wall waveguide. To first order of approximation there is no change in eigenvalue. The field components (7) and (8) must be matched at the helix $r = a$ by means of four boundary conditions which determine the four constants B, C, D and F.

We have at $r = a$ the following boundary conditions:

(1) $e_\varphi = 0$;
(2) $e_\varphi^e = 0$;
(3) $e_z = e_z^e$;
(4) $h_\varphi = h_\varphi^e$.

These conditions regard the special features of the helix waveguide: $e_\varphi = 0$ at $r = a$ refers to the assumed perfect conductivity of the helix; h_φ continuous at $r = a$, means that the current in the helix runs strictly in the φ direction. But there is no condition for h_z because the change in h_z on both sides of the helix takes into consideration the connection between the magnetic field and the electric current. But this connection does not determine the relation between the field components on both sides of the helix but gives only the dependence of the excitation constant A of the TE_{01} wave on the wall current. However, this connection is of no interest to the problem we are concerned with. The solution of these four equations is:

$$B = -\frac{1}{2} \frac{1}{\chi_0 R} (1 + a^2 \gamma^2) A,$$

$$
\begin{aligned}
C &= -j \frac{\gamma}{\omega\epsilon_0\chi_0 R}\left[\frac{\gamma^2}{\chi'^3 a}\frac{W_1(\chi' a)}{W_1'(\chi' a)} + \frac{\epsilon}{\epsilon_0}\frac{\beta_0^2 a}{\chi'}\frac{V_1'(\chi' a)}{V_1(\chi' a)} - \frac{1}{2}\right]A, \\
D &= -j\omega\mu_0 \frac{\gamma}{\chi_0^2}\frac{a}{R}\frac{J_0(\chi_0 a)}{V_1(\chi' a)}A, \qquad (10) \\
F &= -\frac{\gamma^2}{\chi_0^2}\frac{1}{\chi' R}\frac{J_0(\chi_0 a)}{W_1'(\chi' a)}A.
\end{aligned}
$$

III. SOME CONCLUSIONS

The equations (4), (7), (8) and (10) give the complete solution to the first order of approximation of the field perturbation of the TE_{01} wave due to curvature of the helix waveguide.

The additional field components (7) and (8) are of the same order of magnitude as a/R and are therefore very small for gentle curvature. This means that the helix waveguide preserves the TE_{01} wave in its general shape even within its curved parts. But the magnitude of the additional field components depends obviously on the construction of the waveguide, the values of ϵ and the distance $(b - a)$ between helix and shield.

If b tends toward a; that is, if the helix waveguide is converted into an ordinary copper pipe with perfectly conducting walls, the TE_{01} wave becomes seriously perturbed. This is shown by the behavior of the constant C. If b tends toward a, $W_1'(\chi' a)$ tends to zero according to (9); this means that C becomes infinitely large. Of course, the perturbation theory has no meaning for infinitely large perturbation terms, but the tendency shows that something is wrong with the TE_{01} wave if the curved helix waveguide tends towards a perfectly conducting ordinary waveguide. The limit of the perturbation method is shown by using the asymptotic approximations for the Hankel functions. This is possible as long as $\epsilon' > 1$ and the TE_{01} wave is far from cutoff:

$$\frac{W_1(\chi' a)}{W_1'(\chi' a)} \approx \cot\,[\chi'(b - a)],$$

or if $|\,\chi'(b - a)\,| \ll 1$,

$$\frac{W_1(\chi' a)}{W_1'(\chi' a)} \approx \frac{1}{\chi'(b - a)}.$$

The perturbation method is therefore applicable as long as

$$\frac{a}{R\,|\,\chi'(b - a)\,|} \ll 1.$$

It is interesting to look at the meaning of C in the field pattern. According to (7), C is the excitation constant of the e_z component or, better, of that part of the e_z component which refers to the solution of the homogeneous Maxwell equations and therefore represents a wave of the TM_{11} type. The fact that C tends to infinity with b tending toward a means that the coupling between TE_{01} and TM_{11} waves becomes very strong, as is well known. The coefficient B is not affected by this limit process. That means that, since B is the excitation coefficient of h_z in (7) and therefore represents a TE_{1n} wave, the coupling between TE_{01} waves and TE_{1n} waves is not affected by the conversion of the helix waveguide into an ordinary solid-pipe waveguide. For a real helix waveguide, however, the perturbation terms h and e are small and our first order approximation is valid. It may be expected that there exists an optimum design for the helix waveguide with regard to mode conversion in bend parts, because the constant C is also affected by the choice of ϵ. For instance, for $\epsilon'' \rightarrow \infty$, C tends to ∞ too, because of the second term in parenthesis which tends to ∞ like $\sqrt{\epsilon''}$.

It should be mentioned that the additional field components (7) are not simply a superposition of TE_{11} and TM_{11} modes. Though they have the same angular distribution, their radial dependence is different from those of normal modes of the helix waveguide. In order to match these modes to normal modes of the straight guide an infinite series of TE_{1n} and TM_{1n} modes would be needed.

IV. THE ATTENUATION CAUSED BY CURVATURE

According to our assumption of perfectly conducting helix, the TE_{01} wave in the straight waveguide would have no attenuation. But the curvature gives rise to a field in the lossy dielectric between helix and shield. The power dissipation due to this perturbation field causes an attenuation of the whole wave. The power dissipation in the lossy dielectric may be found by calculating the radial power flow through the surface of the helix into that space.

The power dissipation P_d per unit length of the waveguide divided by twice the power P flowing through the cross section of the waveguide in the axial direction gives the attenuation $\alpha_\epsilon = P_d/2P$ (the index ϵ refers to attenuation due to losses in the dielectric). The average power flowing into the dielectric at $r = a$ is:

$$P_d = \frac{1}{2} \operatorname{Re} \int_0^{2\pi} (e_\varphi^e \bar{h}_z^e - e_z^e \bar{h}_\varphi^e) a \, d\varphi \qquad \text{at } r = a.$$

The dash over h means the transition to the conjugate complex value. In this way we get

$$\alpha_\epsilon = \frac{1}{2}\frac{|\gamma|}{\chi_0^2 a}\frac{1}{R^2}\operatorname{Re}\left\{\frac{j}{\bar{\chi}'}\left[\frac{|\gamma|^2}{\bar{\chi}'^2}\frac{\bar{W}_1(\chi' a)}{\bar{W}_1'(\chi' a)} - \frac{\bar{\epsilon}}{\epsilon_0}\beta_0^2 a^2 \frac{\bar{V}_1'(\chi' a)}{\bar{V}_1(\chi' a)}\right]\right\}. \quad (11)$$

For TE_{01} far from cutoff and for $\epsilon' > 1$, we have $k'a \gg 1$ and may therefore substitute for the Hankel functions their approximations for large arguments. If we do this we get the much simpler formula:

$$\alpha_\epsilon = \frac{1}{2}\frac{|\gamma|}{a\chi_0^2}\frac{1}{R^2}\operatorname{Re}\left[\frac{j}{\bar{\chi}'}\left(\frac{|\gamma|^2}{\bar{\chi}'^2} + \frac{\bar{\epsilon}}{\epsilon_0}\alpha^2\beta_0^2\right)\cot \bar{\chi}'(b-a)\right]. \quad (12)$$

The transition to the real part gives finally:

$$\alpha_\epsilon = \frac{1}{2}\frac{|\gamma|}{a\chi_0^2}\frac{1}{KR^2} \frac{\begin{aligned}&\cdot\sin u\cos u\left[\sin\delta\left(\frac{|\gamma|^2}{K^2}\cos 2\delta + \epsilon'\beta_0^2 a^2\right)\right.\\ &\qquad\left. + \cos\delta\left(\frac{|\gamma|^2}{K^2}\sin 2\delta - \epsilon'' a^2\beta_0^2\right)\right]\\ &\qquad + \sinh v\cosh v\left[\cos\delta\left(\frac{|\gamma|^2}{K^2}\cos 2\delta + \epsilon' a^2\beta_0^2\right)\right.\\ &\qquad\qquad\left. - \sin\delta\left(\frac{|\gamma|^2}{K^2}\sin 2\delta - \epsilon'' a^2\beta_0^2\right)\right]\end{aligned}}{[(\sin u\cosh v)^2 + (\cos u\sinh v)^2]}, \quad (13)$$

where

$$K^2 = \beta_0^2\sqrt{\left[(\epsilon' - 1) + \frac{\chi_0^2}{\beta_0^2}\right]^2 + (\epsilon'')^2}, \qquad 2\delta = \arctan\frac{\epsilon''}{\epsilon' - 1 + \dfrac{\chi_0^2}{\beta_0^2}},$$

$$u = K(b-a)\cos\delta, \qquad v = K(b-a)\sin\delta.$$

Equation (13) gives the attenuation of the TE_{01} wave in the curved helix waveguide due to losses in the dielectric between helix and shield. If ϵ'' tends to zero, δ and v tend to zero and therefore α_ϵ vanishes. In the shielded helix waveguide the additional losses of TE_{01} wave due to curvature may be very small if the losses in the dielectric could be kept small.

Equation (13) may be simplified for two special cases:

i. If b tends to ∞, that is, for the unshielded helix waveguide, the attenuation due to curvature becomes:

$$\alpha_\epsilon = \frac{1}{2}\frac{|\gamma|}{a\chi_0^2}\frac{1}{KR^2}\left[\cos\delta\left(\frac{|\gamma|^2}{K^2}\cos 2\delta + \epsilon' a^2\beta_0^2\right) + \sin\delta\left(\epsilon'' a^2\beta_0^2 - \frac{|\gamma|^2}{K^2}\sin 2\delta\right)\right]. \quad (14)$$

Now we get an additional attenuation, even for $\epsilon'' = 0$ due to radiation losses into the outer space.

ii. If $\epsilon'' \ll 1$ we get, as long as $0 < K(b - a) < \pi$:

$$\alpha_\epsilon = \frac{1}{4}\frac{a}{R^2}\frac{|\gamma|\beta_0^2}{K\chi_0^2} \cdot \frac{K(b-a)\epsilon' - \sin K(b-a)\cos K(b-a)(\epsilon' - 2)}{[\sin K(b-a)]^2}\frac{\epsilon''}{\epsilon' - 1}. \tag{15}$$

Finally, there is some additional curvature attenuation due to the wall currents on the shield and on the helix caused by the additional field components. This attenuation can be neglected when compared with the attenuation caused by losses in all realizable dielectrics.

It may be of interest to mention that the attenuation due to curvature of the helix waveguide is always proportional to $1/R^2$. That is, for the first order of approximation, which considers only terms proportional to $1/R$, there is no attenuation due to curvature. Therefore, the eigenvalue $\chi_0 a$ is unchanged. Instead of calculating the second-order approximation which would also give the attenuation of the TE_{01} wave due to curvature, we have calculated this attenuation by calculating the energy dissipation

V. SOME NUMERICAL EXAMPLES

The following figures show the behavior of the attenuation of TE_{01} mode caused by bends as a function of different parameters:

Fig. 1 gives the attenuation of the TE_{01} mode for fixed frequency and several values of ϵ' as a function of ϵ''. The shield is assumed to be at an

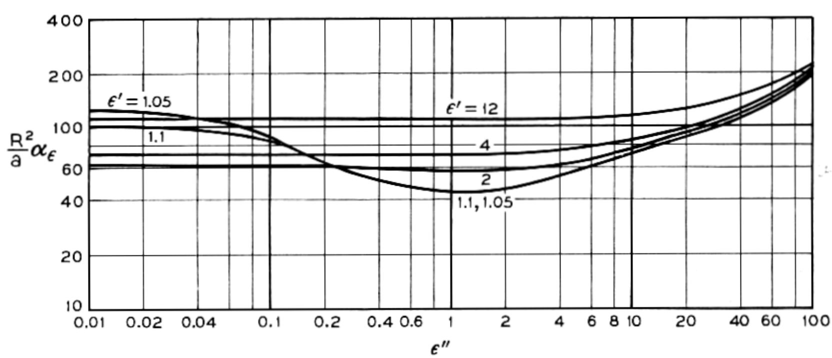

Fig. 1 — Attenuation of the TE_{01} mode due to the curvature of the helix waveguide, with $a\beta_0 = 29.5$.

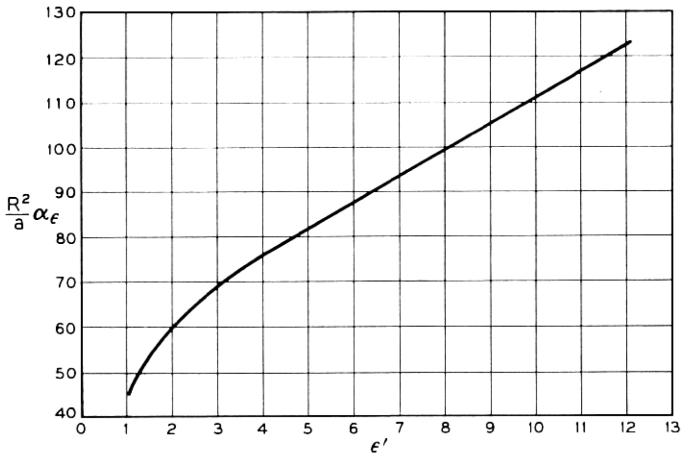

Fig. 2 — Curvature attenuation of the TE_{01} mode, with $\epsilon' = \epsilon''$.

infinite distance $b \rightarrow \infty$. The curves were calculated with the help of (14). The attenuation α is multiplied by R^2/a in order to make it dimensionless. The value of $a\beta_0 = 2\pi(a/\lambda_0) = 29.5$ corresponds to $a = 1$ in. and $f = 55.5$ kmc.

It is interesting that, for greater values of ϵ', the attenuation is nearly constant and depends on ϵ'' only for very large values. Diminishing the value of ϵ' gives a decrease in α. The attenuation has increasing values for decreasing ϵ'' only for values of ϵ' very close to one. The best choice is $\epsilon' = \epsilon''$. It is advisable to make $\epsilon' = \epsilon''$ as close to one as possible, if low values of attenuation are wanted.

Fig. 2 shows the dependence of attenuation on $\epsilon' = \epsilon''$. Let us consider a practical example. If $a = 1$ in., $f = 55.5$ kmc and $\epsilon' = \epsilon'' = 2$:

$$\alpha = \frac{43.3}{R^2}\ \text{db/ft}$$

if R is measured in feet. The attenuation of the straight waveguide with copper helix is about $\alpha = 2.92 \times 10^{-4}$ db/ft. The attenuation due to curvature has the same value for a radius of curvature of $R = 385$ ft. $=$ 117 meters.

Fig. 3 shows the frequency dependence of curvature attenuation for $\epsilon' = 2$, $\epsilon'' = 2$. If $a = 1$ in., $a\beta_0 = 2\pi(a/\lambda_0) = 18.6$ for $f = 35$ kmc and $a\beta_0 = 40$ for $f = 75$ kmc. In this range, the curvature attenuation at the upper frequency limit is five times the attenuation at the lower frequency limit.

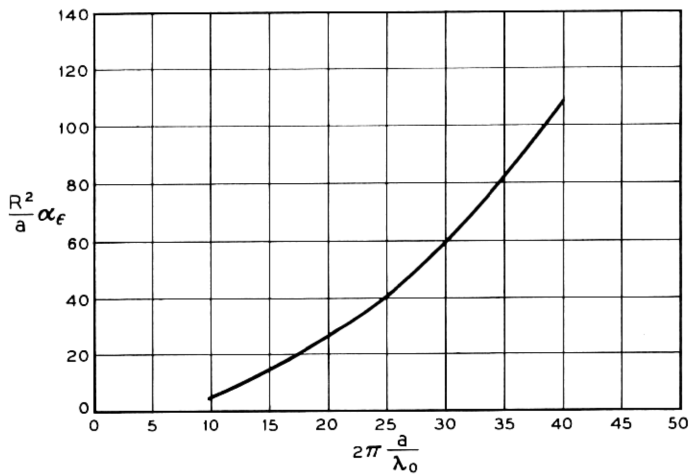

Fig. 3 — Curvature attenuation of the TE_{01} mode, with $\epsilon' = \epsilon'' = 2$.

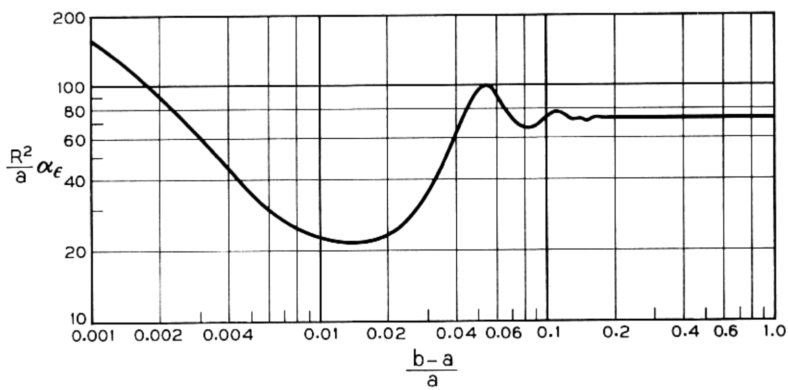

Fig. 4 — Curvature attenuation of the TE_{01} mode, with $\epsilon' = 4$, $\epsilon'' = 2$.

The attenuation due to curvature may very effectively be reduced by proper choice of the distance between helix and shield.

Fig. 4 shows the dependence of attenuation on the relative distance between shield and helix $(b - a)/a$; $\epsilon' = 4$, $\epsilon'' = 2$, $a\beta_0 = 29.5$ in this example. The attenuation first oscillates with increasing $(b - a)/a$ and reaches then a fixed value equal to the value for $b = \infty$ in Fig. 1. But the lowest value is only $\frac{1}{3}$ of the value for $b = \infty$. This favorable effect of the shield can still be improved with decreases in ϵ''. Fig. 5 shows the dependence of $(R^2/a)\alpha$ on $(b - a)/a$ for fixed $\epsilon''/\epsilon' = 0.01$ for different values of ϵ'.

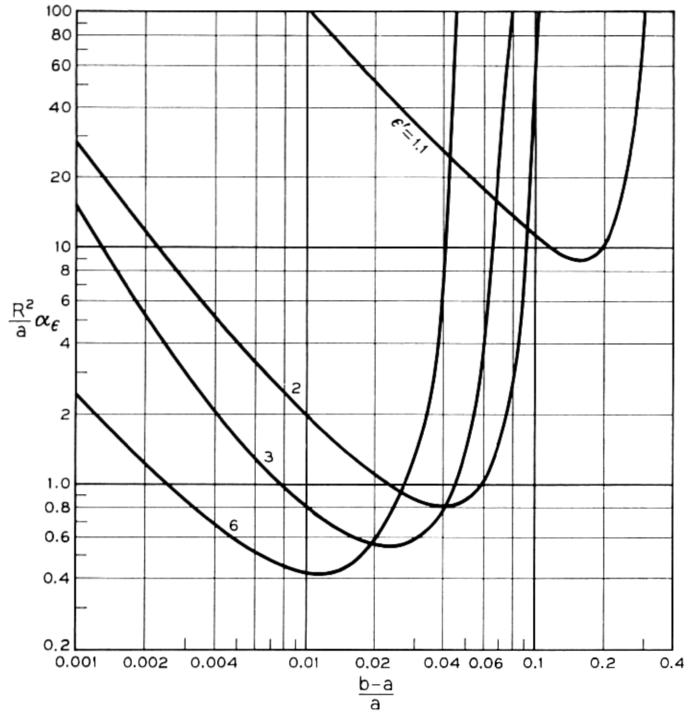

Fig. 5 — Curvature attenuation of the TE_{01} mode, with $\epsilon''/\epsilon' = 0.01$.

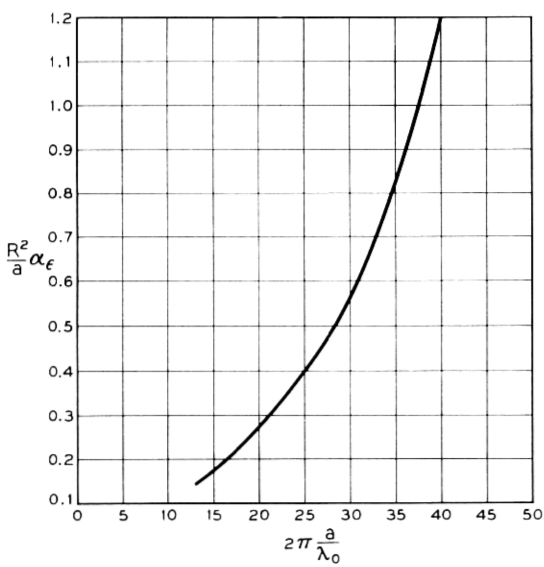

Fig. 6 — Curvature attenuation of the TE_{01} mode, with $\epsilon' = 3$, $\epsilon'' = 0.03$; $(b - a)/a = 0.025$.

It is surprising that the values of the minima are reduced by increase of ϵ'. This is just the opposite behavior as for the unshielded waveguide of Fig. 1. According to (15), α is proportional to ϵ'' for small values. The attenuation may therefore be simply calculated for other values of ϵ'' by means of Fig. 5.

If we again consider $a = 1$ in., $f = 55.5$ kmc and allow α due to curvature to equal the attenuation of TE_{01} mode in straight waveguide, the radius of curvature may be $R = 32.1$ ft. $= 9.8$ meters if $\epsilon' = 6$, $\epsilon'' = 0.06$ and $(b - a)/a = 0.01$.

Proper choice of $(b - a)/a$ allows, therefore, a 10 times larger curvature than in the previous example. That the frequency dependence is not worse for the shielded than for the unshielded guide is shown in Fig. 6. The ratio between the attenuation for $a\beta_0 = 40$ and $a\beta_0 = 18.5$ is again about 5.

If we are only concerned with the attenuation of the TE_{01} mode due to curvature, the shielded waveguide is very much better than the unshielded guide.

VI. SUMMARY

The additional attenuation of a TE_{01} mode due to curvature of the helix waveguide was calculated by means of a perturbation theory. The effects of dielectric constant and of a shield around the helix waveguide were discussed. It was shown that it is advisable to make $\epsilon' = \epsilon''$ as small as possible if low attenuation for a guide without shield and a very thick dielectric coating is wanted. But very much lower attenuation can be obtained if a shield is placed at a proper distance from the helix. To get low attenuation by aid of a shield, the real part of the dielectric constant ϵ' should be as large as possible but its imaginary part as low as possible.

The attenuation due to wall currents in the shield and additional currents due to curvature in the helix may be neglected when it is compared with the losses in the dielectric jacket for all real dielectric materials.

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