Experiments on the Timing of Regenerative Repeaters

By O. E. DE LANGE and M. PUSTELNYK

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This paper describes a number of experiments performed with self-timed binary regenerative repeaters to determine the behavior of the timing portion of a chain of such repeaters.

I. INTRODUCTION

Three other papers[1, 2, 3] appearing in this issue of the Bell System Technical Journal discuss and analyze the timing of regenerative repeaters. It is the purpose of this paper to describe some experiments performed on repeaters of the same general type to determine their performance in the presence of random noise.

Although our interest is largely in microwave systems, the repeaters for such systems are relatively complicated and expensive. Thus, it is impractical to build a long chain of them expressly for experimental purposes. The recovered timing information is at baseband frequency in a carrier system as well as in a baseband system. For these reasons, we built a chain of baseband repeaters to simulate, as nearly as possible, the microwave repeaters of interest. A pulse-repetition frequency of 10 mc was chosen. The experiments had two objectives: first, to determine the effects of noise in producing pulse errors through its action on the timing circuit and second, to determine the effects of noise in producing time or phase deviations of signal pulses, as discussed in Refs. 1, 2 and 3. From these results, we hope to determine what characteristics the timing circuits must have in order to meet the over-all system requirements. The experiments concern only self-timed repeaters.

It was found that, with a fixed pulse pattern and the simple timing circuits employed, the number of errors produced by the effects of noise upon timing was negligibly small in comparison to the number produced by other effects of noise. This was true even when the system was operating under very adverse conditions. It might be expected from these results that random noise in the timing circuit will not be a major cause of errors in an actual system. The results also indicate that, as far as random noise is concerned, the amount of time deviation of pulses at the output of a long chain of repeaters can be kept within tolerable bounds.

II. SCOPE OF THE EXPERIMENTS

Self-timed regenerative repeaters in general appear to have a common defect in that they convert changes of pulse pattern into changes in the time of occurrence of the pulses out of the regenerator. When a PCM system is transmitting information the pulse pattern changes constantly; therefore, if the system is to be useful its timing circuits must function satisfactorily in the presence of such changes. The degree of conversion depends upon the characteristics of the particular repeater employed and probably can be reduced to a tolerable value by proper design. However, before one decides to instrument a timing circuit which will tolerate a changing pulse pattern it is desirable to obtain an answer to the following question: Aside from the effects of pulse pattern changes, can a system be built which will function in the presence of random noise, and, if so, what are the requirements placed on the equipment by noise considerations alone? After this study has been made we can then face the practical problem of designing a timing circuit which will be sufficiently free of the defects of instrumentation to permit satisfactory operation of the over-all system. In order to avoid the effects of dynamic changes of pulse pattern, the experiments described here were performed with repetitive patterns.

III. TIMING ERRORS

By "timing errors" we mean changes in pulse pattern produced during transmission by imperfections in the process of pulse timing. For example, if one pulse is lost from a time slot which should contain a pulse this constitutes one error. In our experiments the number of errors was determined by comparing the pulse pattern at the input to the system with the pattern at the output of the system, pulse by pulse and space by space. Whenever there was a difference between the two patterns the comparator, which consisted of an "exclusive OR" circuit, emitted a pulse. These pulses were counted by a high-speed binary counter.

Timing errors result from misalignment between signal pulses and timing pulses. It is obvious that, if a timing pulse is displaced by a sufficient amount, it will occur at a time when the corresponding signal pulse is below standard amplitude and there will be no output from the regenerator. Even when the displacements alone are not great enough to produce errors they can increase the probability of errors due to amplitude effects. The number of errors produced in any given case will depend upon the widths and shapes of both signal and timing pulses as well as upon the amount of misalignment. Because of the number of factors influencing the production of timing errors, it is felt that experimental results obtained from a typical repeater might be more meaningful than those obtained from an analysis based on uncertain assumptions.

3.1 *Comparison of Timing and Amplitude Errors — Single Repeater*

The first experiment involved only a single repeater, a block diagram of which is shown in Fig. 1. The signal path is seen to consist of a filter, a baseband amplifier, the regenerator, another filter and an amplifier stage to couple the output to the succeeding repeater when the unit is part of a chain. The timing path consists of an amplifier, a tuned circuit, a limiter, a second amplifier and a pulse generator. Means are provided for adding broadband random noise from an external source to each repeater. The repeaters are set up in such a way that this noise can be inserted in the timing path only, in the slicer circuit only or in both simultaneously.

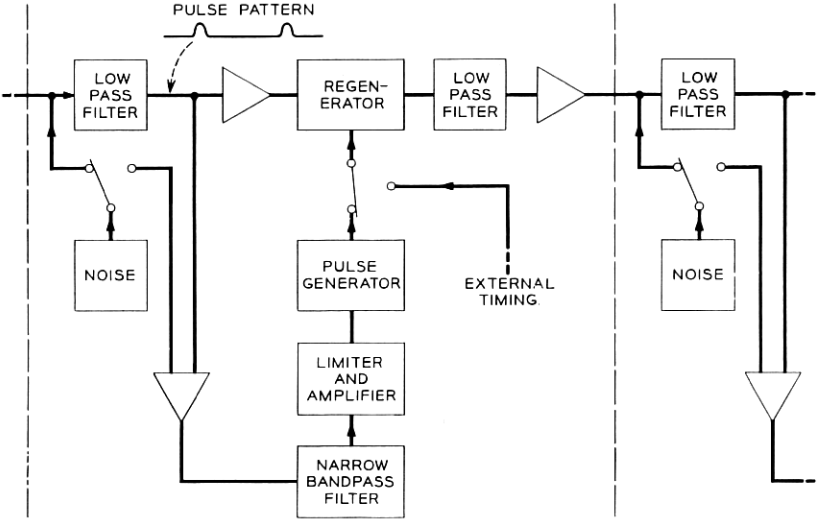

Fig. 1 — Block diagram of the baseband repeater.

The regenerator is essentially an AND gate; i.e., there is an output pulse only when there is a pulse on both of two inputs simultaneously. One input consists of a train of timing, or "clock" pulses. The other input is the train of signal pulses which are to be regenerated. The timing pulses are about one third as long as the signal pulses, so that the time of occurrence of each output pulse is determined almost entirely by the time of occurrence of a corresponding timing pulse. Ideally, there would be an additional requirement placed on the regenerator: that it respond completely to voltages in the signal path with amplitudes greater than one half of normal peak pulse amplitude and respond not at all to voltages less than this value; i.e., it would provide an ideal slicer characteristic. However, the microwave repeater which is being simulated (described in Ref. 4) has a slicer characteristic which departs considerably from the ideal. The baseband slicer characteristic was made to match that of the microwave repeater. The regeneration is performed by a 6AS6 pentode vacuum tube with the signal pulses applied to its suppressor and the timing pulses to its control grid. Since the type of baseband repeater employed was designed solely for these experiments and would never be used in an actual system it will not be described in greater detail.

For most of the experiments, the input pulse pattern was adjusted to have only one pulse present for every nine possible pulse positions in order to set up very unfavorable conditions for timing. With noise applied to the main input of the repeater so as to affect both the timing circuit and the slicing circuit, the number of errors corresponding to each of a number of signal-to-noise ratios was determined. The experiment was repeated with noise applied only to the timing path of the repeater. Finally, with noise applied to the main input, the repeater was timed from an external, noise-free source so that noise affected only the slicer path of the repeater. It is apparent from Fig. 2 that, for a given signal-to-noise ratio,* many more errors are produced when noise is applied to the slicer only than when it is applied to the timing circuit only. It can also be seen that the number of errors produced when noise acts on both circuits together is considerably greater than the sum of the errors produced by noise acting on each circuit alone. This results from the fact that less amplitude noise is required to produce an error when there is misalignment between signal pulses and timing pulses. Although the number of errors produced by timing effects alone is very small in comparison to the number produced by noise

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* We define signal-to-noise ratio as the ratio of peak pulse power to mean noise power.

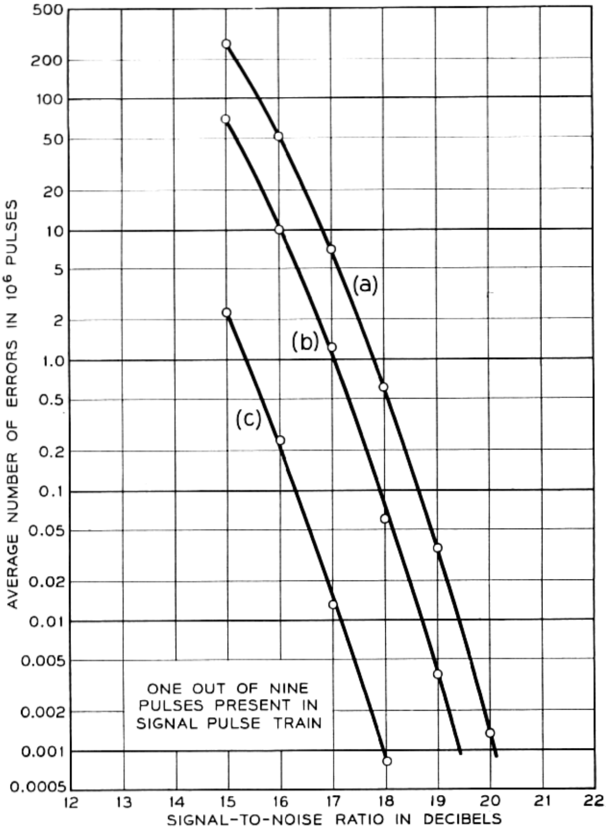

Fig. 2 — Average pulse error rate with linear timing amplifier: (a) self-timed, noise into both slicing and timing paths; (b) externally timed, noise into slicing path only; (c) self-timed, noise into timing path only.

acting on the slicer alone, timing noise is still exerting considerable influence on the total number of errors. It should be pointed out again that the conditions under which these counts were made are very adverse to timing; for example, an increase in the number of signal pulses to two out of a possible nine would be equivalent to improving the signal-to-noise ratio by 6 db, which would make timing errors entirely negligible.

For the above experiments, the timing circuit suffered a severe penalty from the fact that noise was applied to the timing filter continuously, whereas signal energy was applied to this circuit by only one out of each nine possible pulse positions. As a result, the component of the signal at pulse-repetition frequency had only one-ninth of the amplitude it would have had with all pulses present. In a later experiment, this penalty was reduced by inserting a nonlinear element between the main pulse path and the tuned circuit. This was accomplished by back-biasing one of the tubes in the timing amplifier so that it conducts only when the voltage on its grid exceeds some threshold value. In this way, most of the noise which occurs between signal pulses is eliminated, thus removing a considerable amount of noise power from the timing circuit when there are few signal pulses present. Comparing Fig. 3 with Fig. 2 makes it evident that the nonlinear amplifier has produced an apparent increase of about 3 db in the timing signal-to-noise ratio. Other experiments with the same equipment produced improvements of approximately 5 db. One might expect that such a peak amplifier would reduce the noise power applied to the tim-

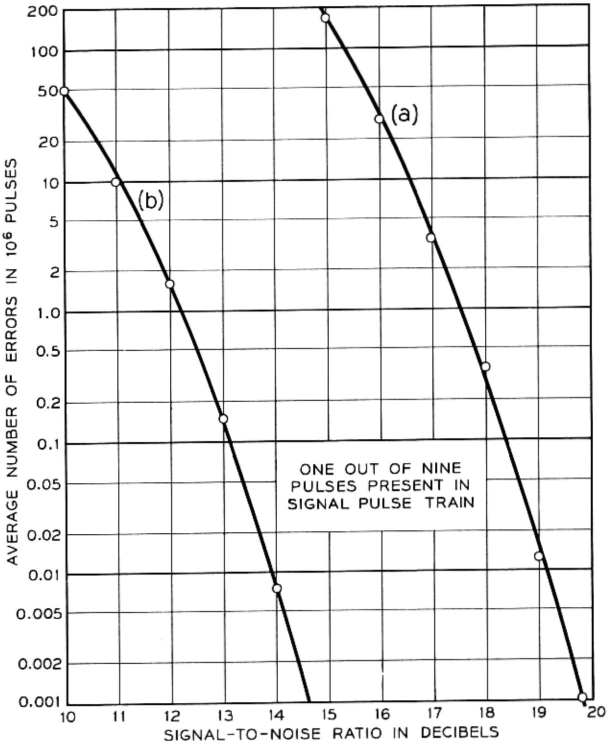

Fig. 3 — Average pulse error rate for a self-timed repeater with a nonlinear amplifier: (a) noise into both slicing and timing paths; (b) noise into timing path only.

ing circuit by a factor of nine to produce an improvement of 9 db. However, the reduction of noise power is accompanied by a redistribution of the noise frequency spectrum, which probably accounts for the fact that the improvement is less than might otherwise be expected. With the improvement obtained by use of the nonlinear circuit, it appears safe to assume that, for a single repeater employing a suitable timing circuit of simple design, the number of errors produced by the effects of random noise upon timing is completely negligible at all times in comparison with the number produced by other effects of noise.

3.2 *Comparison of Errors — Chain of Repeaters*

This experiment was performed to determine the manner in which the number of errors increases as the number of repeaters is increased. For this case, the signal-to-noise ratio at the input to each of five repeaters was held constant at 22 db. The pulse errors produced by employing first one, then two and so on up to five repeaters were counted. The timing filters for these experiments consisted of tuned circuits with an effective Q of about 80 in each repeater. It is obvious from Fig. 4 that, for five repeaters, the number of errors produced by the effects of noise on timing alone is very small in comparison with the number produced by other effects of noise. These results were obtained with the linear timing amplifier. With the nonlinear circuit, the number of errors produced by timing alone would have been much too small to measure with the equipment available.

Some discussion of the results indicated by Fig. 4 is needed at this point. Although these experiments were primarily directed toward the study of timing, we obtained as by-products some interesting results concerning the amplitude effects of noise. For ideal regeneration of pulses, the total number of errors produced in a system should vary almost linearly with the number of repeaters involved since, for such a system, no noise would be passed from one repeater to the next. For the system under discussion the regenerators do not have ideal input-amplitude vs. output-amplitude characteristics, and therefore some noise effects are passed from one repeater to the next.* If only the first repeater of the chain had noise at its input the noise power (except for peaks exceeding the slicing level) would be reduced at each successive repeater to such an extent that it would be negligible after traversing four or five of the partially regenerating repeaters. However, when each repeater has noise at its input there is a rapid build up of error rate for

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* For a discussion of this "partial regeneration" of pulses see page 17 of Ref. 4.

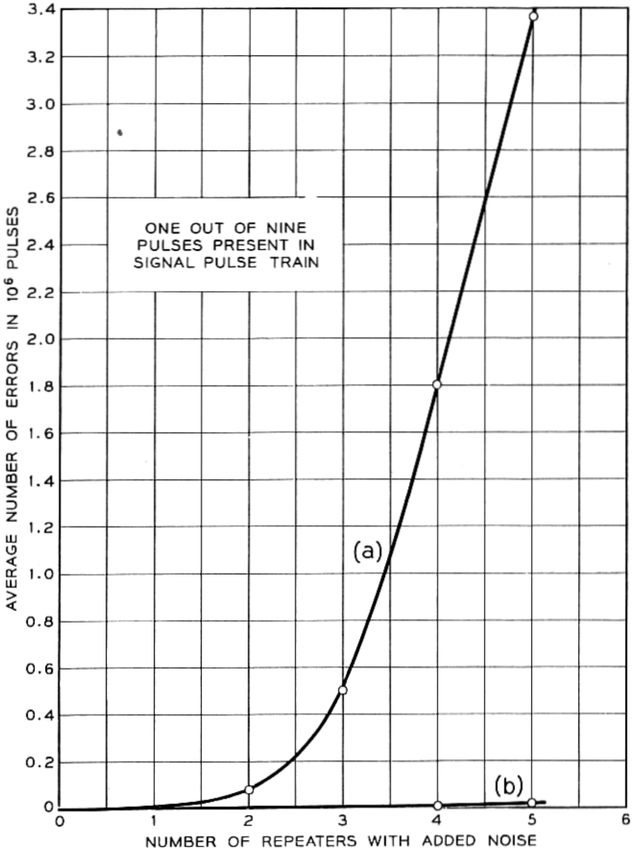

Fig. 4 — Error build up in a chain of self-timed repeaters: (a) noise into both timing and slicing paths; (b) noise into timing path only.

the first four or five repeaters, after which the rate of increase stabilizes, as shown on Fig. 4. From this point on the number of errors increases nearly linearly with the number of repeaters. The net result of partial amplitude regeneration appears to be a reduction in the effective signal-to-noise ratio in the main signal path. This should have little effect on timing. The noise which is passed from one repeater to the next is retimed in the regenerator in much the same way as signal pulses. We have found in our experimental systems that, for a long chain of repeaters, the signal-to-noise ratio required for a given error rate is three to four db greater than would be required for a system employing ideal regenerators.

IV. PHASE OR TIME DEVIATIONS

Having shown that noise has negligible effect in producing pulse errors we next proceed to investigate the effect of noise in producing phase deviations of the recovered timing wave. In the accompanying papers[1, 2, 3] it is shown that, for a chain of repeaters employing tuned circuits as timing filters, the effective bandwidth decreases as the number of repeaters is increased. Then, if there is noise only at the input to the first repeater its effect should decrease at successive repeaters along the chain. That this is actually the case is shown by the following experiment: A sufficient amount of noise was added to the signal at the input to the first repeater to provide a 20-db signal-to-noise ratio. The timing wave phase deviation at this repeater was measured to be 20°. Similarly, the phase deviations at the succeeding repeaters, which were free of added noise, were measured. With the measured deviation of 20° for the first repeater as a starting point, the expected deviations at succeeding repeaters were calculated on the basis that noise power should be directly proportional to effective bandwidth. The measured and calculated deviations, which are given in Table I, are seen to agree fairly well.

One of the early experiments was performed to determine, in a qualitative way, whether or not a long chain of repeaters can be made to perform satisfactorily in the presence of noise, especially with respect to timing. A chain of 22 repeaters of the type shown on Fig. 1 was set up for this purpose. Broadband noise was added to each repeater in such a way as to affect both the slicing and timing circuits and was adjusted in level to produce a 20-db signal-to-noise ratio at the input to each regenerator.

As far as could be determined by observing its output with an oscilloscope, the system performance was very satisfactory. Various repetitive pulse patterns were set up manually and found to be transmitted without discernible errors. (Our more sensitive error-detecting equipment would probably have detected some errors.) Timing-phase devia-

Table I

Number of Repeaters	Calculated Deviation	Measured Deviation
1	20.0°	20.0°
2	12.3°	12.8°
3	10.5°	10.0°
4	9.5°	7.6°
5	8.8°	7.1°

tions were determined by observing the timing wave recovered by the last repeater in the chain. Deviations of this wave were practically identical with deviations of the output pulses. Fig. 5 illustrates the degree of deviation produced by this amount of noise. The upper trace shows a jitter-free reference sine wave which can be compared with the timing wave recovered from the 22nd repeater. This timing wave is shown on the bottom trace. The apparent broadening of the lower trace makes it evident that there was some phase and amplitude modulation present on the recovered timing wave. Unfortunately the phase-measuring equipment was not available at the time of this experiment; however, it is apparent from the bottom trace that the phase deviations were small. By careful measurement of the horizontal displacement of the trace, it is estimated that the rms deviation was about 4.5°. If we adopt this figure of 4.5° for 22 repeaters we can calculate the deviation to be expected for a chain of 200 repeaters on the basis that deviation varies as the fourth root of the number of repeaters (see Refs. 2 and 3). Such calculations indicate a deviation of about 8° for the long chain operating under the conditions of this experiment. These results are consistent with those calculated in Ref. 2 in indicating that, from the standpoint of noise, even simple timing circuits can meet the very stringent system requirements.

V. CALCULATION OF PHASE DEVIATIONS

To make possible comparisons between measured deviations and expected deviations of phase, the expected values are calculated here for conditions corresponding to those under which the measurements were made. Measured and calculated deviations are found to agree within the range of experimental error, both for a single repeater operat-

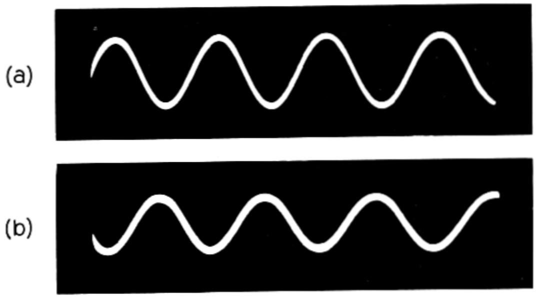

Fig. 5 — Comparison of (a) a reference 10-mc sine wave and (b) the recovered 10-mc timing wave at the output of 22 self-timed repeaters.

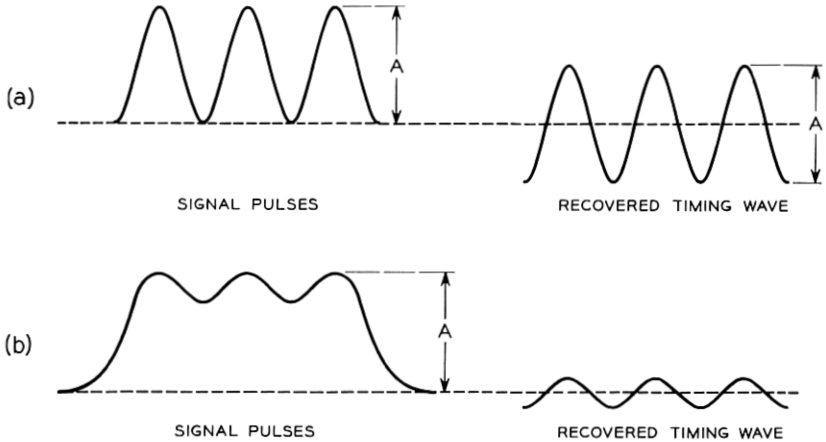

Fig. 6 — Signal pulses and recovered timing wave: (a) broadband input filter, (b) narrowband input filter.

ing under very adverse conditions and for a chain of 22 repeaters operating under more favorable conditions.

Although the method of calculation employed is basically the same as that employed in Ref. 2, it is somewhat different, because here we are dealing with systems which are completely baseband whereas the previous article considers only carrier systems. For this reason, calculations are given in some detail.

From the standpoint of filtering we choose the same conditions that were assumed in the previous article; i.e., the filtering following each regenerator is such that the signal pulses applied to the narrow filter are of raised cosine form, as shown in Fig. 6(a). If the filter at the input to each repeater has an amplitude characteristic as shown by the dashed line of Fig. 7 and a linear phase characteristic the signal pulses will not be altered in form by passage through it.* The relationship between signal pulses and recovered timing wave will be as shown on Fig. 6(a). If the noise power density is W_0 the total noise power, W_N, contained in the band B ahead of the timing filter is given by $W_N = W_0B$. In the same way, the total noise power at the output of the timing filter is given by $W_{na} = W_0b$, where b is the bandwidth of the timing filter. Peak pulse power W_p is A^2. The mean timing wave power is given by $W_s = A^2/8$ [see Fig. 6(a)]. From these relationships, we find

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* This is strictly true only when all pulses are present. In any case, the frequency component at the pulse-repetition frequency of 10 mc will not be affected.

$$\frac{W_s}{W_{na}} = \frac{W_p}{W_N}\frac{B}{8b}, \tag{1}$$

where W_s/W_{na} is the timing signal-to-noise ratio after filtering but before limiting and W_p/W_N is the ratio of peak pulse power to mean noise power at the input to the timing filter. After limiting,

$$\frac{W_s}{W_n} = \frac{W_p}{W_N}\frac{B}{4b} = \frac{W_p}{W_N}\frac{Q}{2\pi}, \tag{2}$$

since $B = f_r$ and $b = \pi f_r/2Q$.

By the same procedure employed in Section 3.5 of the previous article, we find the rms phase deviation to be given by

$$\Psi_{\text{rms}} = \sqrt{\frac{W_n}{W_s}} = 2.51\sqrt{\frac{W_N}{QW_p}}. \tag{3}$$

The filter used at the input to each repeater actually has a frequency characteristic as shown by the solid line of Fig. 7, which results in signal pulses as shown by Fig. 6(b). It is evident from Fig. 7 that the additional band limiting does not affect the timing signal-to-noise ratio to any extent, since both timing signal and timing noise are recovered from the narrow band b and are reduced by very nearly the same amount by the frequency discrimination. Equation (3), with W_s/W_n defined by (2) is still accurate for the limited band. Note that W_p/W_N is the signal-to-noise ratio which would exist for the filter whose characteristic is depicted by the dashed curve of Fig. 7.

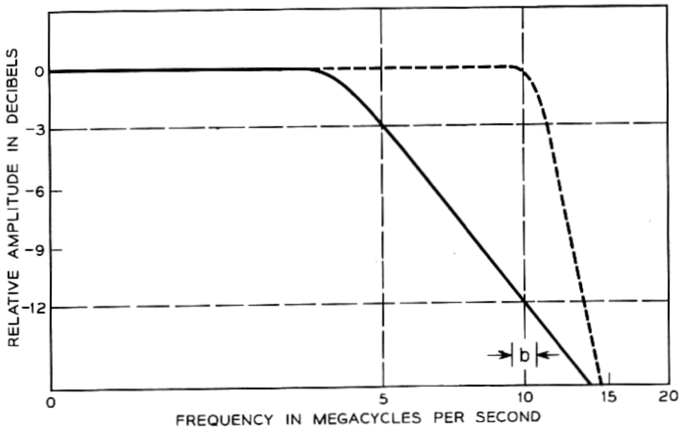

Fig. 7 — Frequency response of the lowpass input filter. The characteristic of the experimental filter is shown by the solid line. The timing filter bandwidth is shown as b.

In one experiment we operated a single repeater with a pulse signal-to-noise ratio of 20 db and with only one out of a possible nine pulses present. Input noise and signal were limited in bandwidth as shown by the solid curve of Fig. 7; i.e., the effective bandwidth was approximately 5 mc. The timing circuit had an effective Q of 80 at 10 mc.

Since (3) is based on the signal-to-noise ratio which would exist in a 10-mc band, whereas the ratio was measured in a 5-mc band, it is necessary to make a conversion to obtain the proper value to use in the equation. This value was calculated to be approximately 21 db. The ratio varies slightly with pulse pattern due to slight changes in pulse shape; however, these variations should be small.

Then, from (3),

$$\Psi_{\text{rms}} = 2.51 \sqrt{\frac{1}{80} \times \frac{1}{125}} = 2.51 \times 0.01 = 0.025 \text{ radian}$$

for the case where all pulses are present. With a linear timing input amplifier and only one out of nine pulses present, the timing wave amplitude is reduced by a factor of nine, whereas noise is unaffected by the number of pulses. The phase deviation then becomes $9 \times 0.025 = 0.225$ radian or 13°. This is in fair agreement with the value of 17° measured under the same conditions.

A chain of 22 repeaters was operated under the conditions described above but with seven out of nine pulses present. Then, for the first repeater in the chain, the deviation should be $\Psi_{\text{rms}} = 9/7 \times 0.025 = 0.032$ radian. From (30) of the previous article, it is evident that the effective bandwidth of a chain of N repeaters is

$$B_{\text{eff}} = \frac{1.77 f_r}{Q\sqrt{N}}.$$

The bandwidth of a single repeater is

$$\frac{\pi f_r}{2Q} = \frac{1.57 f_r}{Q}.$$

The ratio of noise power at the output of the Nth repeater to that at the output of the first repeater is

$$\frac{(W_n)_N}{(W_n)_1} = 1.13 \sqrt{N}.$$

Then

$$\Psi_N = 1.06\, N^{1/4} \Psi_1 .$$

For our example:

$$\Psi_{22} = 1.06 \times 2.17 \times 0.032 = 0.074 \text{ rad.} = 4.2^\circ.$$

This is consistent with the estimate made by means of the photograph of Fig. 6.

VI. PHASE CONVERSION

Changes of pulse pattern produced by manual switching were found to result in changes of delay, or phase, through a repeater of the type employed in these experiments. This change amounted to about 10° at the 10-mc pulse-repetition frequency when the pattern was changed from one pulse out of nine to seven out of nine. Any practical system will have to be designed in such a way that the total phase deviation resulting from expected changes of pulse pattern will not exceed system requirements. In an experimental timing circuit designed for the microwave system, phase conversion has been reduced to about one-tenth of that produced in the baseband repeaters.

VII. CONCLUSIONS

It was found that, with a fixed pulse pattern and the simple timing circuits employed, the number of errors produced by the effects of noise upon timing was negligibly small in comparison to the number produced by other effects of noise. This was true even when the system was operating under very adverse conditions. It might be expected from these results that random noise in the timing circuit will not be a major cause of errors in an actual system. The results also indicate that, as far as random noise is concerned, the amount of time deviation of pulses at the output of a long chain of repeaters can be kept within tolerable bounds.

The above experiments were performed with repetitive pulse patterns. We have not made an adequate study of the effects of rapid changes of pulse pattern upon timing. Indications are that these effects may be much more serious than the effects of random noise on timing and should, therefore, be studied more extensively.

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